Chapter 14

A HETEROSKEDASTIC BLACK-LITTERMAN PORTFOLIO OPTIMIZATION MODEL WITH VIEWS DERIVED FROM A PREDICTIVE REGRESSION

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CHAPTER OUTLINE:

Abstract

The modern portfolio theory in Markowitz (1952) is a cornerstone for investment management, but its implementations are challenging in that the optimal portfolio weight is extremely sensitive to the estimation for the mean and covariance of the asset returns. As a sophisticate modification, the Black-Litterman portfolio model allows the optimal portfolio's weight to rely on a combination of the implied market equilibrium returns and investors' views (Black and Litterman, 1991). However, the performance of a Black-Litterman model is closely related to investors' views and the estimated covariance matrix. To overcome these problems, we first propose a predictive regression to form investors' views, where asset returns are regressed against their lagged values and the market return. Second, motivated by stylized features of volatility clustering, heavy-tailed distribution, and leverage effects, we estimate the covariance of asset returns via heteroscedastic models. Empirical analysis using five industry indexes in the Taiwan stock market shows that the proposed portfolio outperforms existing ones in terms of cumulative returns.

Keywords
Markowitz modern portfolio theory, Black-Litterman model, GARCH model, EGARCH model, the investor’s views

14.1 INTRODUCTION

Markowitz (1952) provided a framework for modern portfolio theory, in which the asset returns can be assumed as identical and independently distributed multivariate normal vector with mean μ and covariance Σ. In practice, investment theory seeks portfolios to diversify risk. However, a direct implementation of the Markowitz model is challenging as the optimal weight is extremely sensitive to the estimation of the assets means and covariances and may be a corner solution, which is however against the intuition about portfolio diversification. For example, Chopra et al. (1993) and Brianton (1998) have revealed that estimation errors in the parameters severely affect the optimized portfolio and efficiency frontier. In addition, Michaud (1989) points out that Markowitz model may lead to maximize the effect of errors in the input parameters assumptions.

Black and Litterman (1991) improve the Markowitz model by combining the implied market equilibrium return and investors' views, so that the estimated means and covariances are adjusted to reflect the market behaviour more realistically. Using a Bayes' formula, the return given investors' views is a weighted average between the implied market equilibrium returns and investors' views. When the investors have a higher confidence level in their views, the weighted average return given investors' views tend to be closer to the investors' views, and vice versa.

Detailed expositions to the Black-Litterman (BL) model have been extensively studied in the literature. For example, Christodoulakis (2002) and Walters (2014) provide detailed instructions on how to use Bayes formulas to derive BL model; Idzorek (2004) summarize the step-by-step guide to derive the Black-Litterman model's parameters; the standard BL model is further extended by Black and Litterman (1992) and He and Litterman (1999). On the other hand, O'toole (2013) revisits the BL model from a risk budgeting perspective to demystify the BL model in an insightful and intuitive manner. Similar to the BL model, Jurczenko and Teiletche (2018) introduce an analytic framework that allows investors to add active views on top of a risk-based investing solution.

The employment of the BL model requires an assignment of investors' views, which are subjective to investors and hence problem dependent. Examples include a simple statistical method in Meucci (2010) and a generalized autoregressive conditional heteroscedastic model in Beach and Orlov (2007). Therefore, in order to provide a simple and useful scheme for the BL model application, we first provide a predictive regression to form investors' views, where asset returns are regressed against their on lagged values and the market return.

Because the estimation of the covariance matrix for the asset returns is critical, various methods to estimate the covariance matrix have been studied Guo et al. (2017). Simple estimators, such as diagonal estimators or constant correlation estimators, are usually used as benchmarks for comparison. The former assumes that asset returns pairwise uncorrelated, whereas the former assumes that every pair of asset returns has the same correlation coefficients (Elton and Gruber, 1973). Covariance matrix can be estimated via linear factor models, which help to reduce the number of model parameters. Examples include Avellaneda and Lee (2010), Chen et al. (1986), Sharpe (1963), and Torun et al. (2011).

In addition to the above methods, Litterman and Winkelmann (1998) use Goldman Sachs decay rate covariance matrix model for portfolio optimization and risk management. Ledoit and Wolf (2003b) estimate the covariance matrix of stock returns by an optimally weighted average of the sample covariance matrix and the single-index covariance matrix. Ledoit and Wolf (2003a) and Ledoit and Wolf (2003b) consider a shrinkage estimator for the covariance matrix which combines the sample covariance and any covariance estimator, where the mixing weight can be obtained through cross-validation. But, Disatnik and Benninga (2007) show that there is no statistically significant gain from using more sophisticated shrinkage estimators, in terms of ex-post standard deviation of the global minimum variance portfolio.

To estimate the covariance, we implement a careful exploratory data analysis of five industry indexes in the Taiwan stock market. It is shown that returns exhibit stylized features, such as volatility clustering, heavy-tail distributions, and leverage effects. For these reasons, we estimate a time-varying covariance matrices using heteroscedastic models.

The rest of this paper is organized as follows. Section 14.2 reviews the BL model and presents a predictive regression to form investors views. In addition, both the absolute and relative views are considered for investors. Section 14.3 presents a methodology to dynamically estimate the covariance matrix to incorporate heteroscedasticity in the return. Section 14.4 summarizes empirical analysis. Section 14.5 concludes.

14.2 PRELIMINARIES

We first review the BL model. Then, we present procedures for deciding the implied market equilibrium returns and the investors' views.

14.2.1 The Black-Litterman model

The BL model uses a Bayes' formula to combine the investors' subjective views regarding the expected returns of one or more assets with the implied market equilibrium returns to form a new mixed estimate of expected returns. For detailed expositions, please see Meucci (2010), Satchell and Scowcroft (2000), Idzorek (2004), Walters (2014), and references therein.

Consider a market of m securities or asset classes. It is assumed that they have normally distributed returns $r_t = (r_{1t}, \cdots, r_{mt})'$.

$$r = N(\mu, \Sigma). \tag{14.2.a}$$

The expected return μ follows

$$\mu = N(\pi,\ \Sigma_\pi = \tau\Sigma), \tag{14.2.b}$$

Furthermore, the covariance matrix of μ is proportional to the covariance of the returns Σ, i.e., $\Sigma_\pi = \tau\Sigma$. For the selection of τ, please refer to Black and Litterman (1992), He and Litterman (1999), Satchell and Scowcroft (2000) and Allaj (2013). In this paper, τ is set as 0.025 (Idzorek, 2004).

The BL model inversely employs assets market weight to obtain the implied market equilibrium return

$$\pi = \lambda\, \Sigma \omega^M, \tag{14.2.c}$$

where λ is a constant denoting the risk aversion coefficient, Σ is an $m \times m$ covariance matrix of the assets, and the market weight $\omega^M = (\omega_1^M, \cdots, \omega_m^M)'$ in a portfolio is a $m \times 1$ vector calculated by the following equation

$$\omega_i^M = \frac{P_i Q_i}{\sum_{i=1}^{m} P_i Q_i}, \tag{14.2.d}$$

for $i = 1, \cdots, m$. Here, P_i is the market price of the *i*th asset and Q_i is the outstanding shares of the *i*th asset.

To construct the views for investors' expected returns, the BL model considers that the expectation of the views are related to the parameter μ. Thus, the uncertainty with the views, v, is a random vector that follows

$$v|\mu \sim N(P\mu, \Omega), \tag{14.2.e}$$

where P is a $K \times m$ matrix to represent K views, and the covariance matrix Ω quantifies investors' confidence level toward their views. For mathematical simplicity, it is assumed that

$$\Omega = \tau P \Sigma P', \tag{14.2.f}$$

where the constant τ characterises the uncertainty about the confidence of the views. We remark that there is no rule to set up the value of τ . For example, Almadi et al. (2014) set τ to be unity and Michaud et al. (2013) set τ to be $1/T$ where T is the number of return observations. In this paper, we set $\tau = 0.025$ to indicate a relatively high level about the confidence toward investors' views (Idzorek, 2004).

Given the expected returns μ and investors' views v, Bayes' formula yields the following critical formulas for the return,

$$r|v, \Omega \sim N(\mu_{BL}, \Sigma_{BL}), \tag{14.2.g}$$

where

$$\mu_{BL} = \pi + \tau \Sigma P' (\tau P \Sigma P' + \Omega)^{-1} (v - P\pi), \tag{14.2.h}$$

$$\Sigma_{BL} = (1 + \tau)\Sigma - \tau^2 \Sigma P' (\tau P \Sigma P' + \Omega)^{-1} P \Sigma. \tag{14.2.i}$$

The return given investors' views v and the confidence towards the views Ω remains a normal distribution with mean μ_{BL} and covariance Σ_{BL}. Here, μ_{BL} and Σ_{BL} modify coefficients for the return in the reference model $r \sim N(\mu, \Sigma)$ by incorporating the views $P\mu \sim N(v, \Omega)$. Finally, the optimal portfolio weight in the BL model solves the following constrained optimization problem,

$$\max_{\omega} \ \omega' \mu_{BL} - \frac{\lambda}{2} \omega' \Sigma_{BL} \omega$$

$$s.t. \ \sum_{i=1}^{m} \omega_i = 1, \omega_i \geq 0. \tag{14.2.j}$$

14.2.2 Views selection

P is a $K \times m$ matrix of the asset weights within each view that shows investors' K views on m assets (Walters, 2014). In this paper, we consider two types of views, the absolute view and the relative view. In added, different authors compute the various matrix to represent weights; He and Litterman (1999) and Idzorek (2004) have used market capitalization weighed scheme, whereas Satchell and Scowcroft (2000) used an equally weighted scheme in their examples.

Now, a predictive regression model is proposed as a part to establish investors' views on the return for each asset. In the predictive regression model, the asset return at time t is the dependent variable, and explanatory variables include the asset return at time $t-1$ and the market return at time *t*.

Let r_{it} denote the *i*th asset return at time t, and r_t^M denote the market return at time *t*. Then, r_{it} is modeled as the following prediction regression,

$$r_{it} = \alpha_i + \beta_{i1} r_{i(t-1)} + \beta_{i2} r_t^M + \varepsilon_{it},$$

$$\varepsilon_{it} \sim i.i.d.\, N(0, \sigma^2),\ i = 1, \cdots, m,\ t = 1, \cdots, T. \tag{14.2.k}$$

Recall that the capital asset pricing model (CAPM) assumes

$$r_{it} = \alpha_i + \beta_i r_t^M + \varepsilon_{it}, \tag{14.2.l}$$

where $\varepsilon_{it} \sim i.i.d.\, N(0, \sigma^2)$. Via empirical analysis, we generalize CAPM by including a lagged dependent variable as in Eq. (14.2.k) in the paper. Indeed, the prediction regression in Eq. (14.2.k) coincides with the standard market model with a lagged dependent variable in Cartwright and Lee (1987), where forty-nine US stocks are considered.

The expected returns for the *i*th asset at time t, denoted by $\hat{r}_{i,t}$ is calculated via the model in Eq. (14.2.k)

$$\hat{r}_{it} = \hat{\alpha}_i + \hat{\beta}_{i1} r_{i(t-1)} + \hat{\beta}_{i2} r_t^M,\ i = 1, \cdots, m,\ t = 1, \cdots, T. \tag{14.2.m}$$

Investors' views *v* are obtained via

$$v = P\hat{r}_t, \tag{14.2.n}$$

where $\hat{r}_t = (\hat{r}_{1t}, \cdots, \hat{r}_{mt})'$ with $\hat{r}_{i,t}$ is given in Eq. (14.2.l).

14.2.2.1 The absolute view

For the absolute view, *P* is defined as the *k*-dimensional identity matrix, which displays the weights of each asset in each of the m views. Specifically, *P* is

$$P_{m \times m} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}. \tag{14.2.o}$$

With the absolute view, investors have views towards the expected return of each single asset.

14.2.2.2 The relative view

Another approach for defining P, called the relative view, is a $1 \times m$ vector and is given below. We first use the prediction regression model in Eq. (14.2.k) to establish the expected return of each asset. Then, these assets are sorted based on their expected returns from the lowest to the highest. Suppose the ith asset has the highest expected return, and the jth variate has the lowest expected return. P is everywhere zeros, but with value 1 at the ith variate and with value -1 at the jth variate.

For example, five assets are included in a portfolio; they are sorted from the lowest to the largest based on their expected returns. Suppose that the first asset has the highest expected return, whereas the fifth asset has the lowest expected return. Then P is set to

$$P_{m\times m} = [1 \quad 0 \quad 0 \quad 0 \quad -1]. \tag{14.2.p}$$

Therefore, investors have relative view simply to the first and fifth assets in the portfolio. The relative view indicate that the investors only focus on assets with the highest or lowest returns.

14.3 OUR METHODOLOGY

Section 14.3.1 explains that statistical features of the data, which motivate the incorporation of a GARCH-typed model to adjust the standard BL model. Section 14.3.2 implements portfolio allocation using heteroskedastic information on the volatility.

14.3.1 Data and motivations

The return at time t is defined as $r_t = ln(p_t) - ln(p_{t-1})$, where pt is the closing price at time t. In this paper, five industry indexes in Taiwan are examined as research samples, retrieved from the Taiwan Economic Journal. The study period is from 2010/01/06 to 2014/12/31, and a dataset of 1240 daily returns is used.

Figure 14.1 shows that these five returns have no significant trend, and gives empirical evidence of volatility clustering and leverage effects.

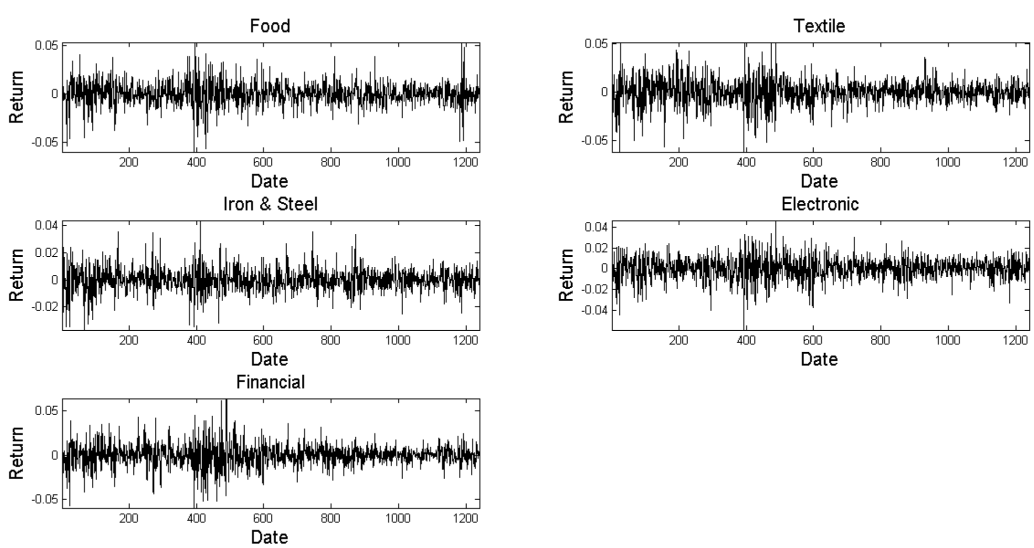

Figure 14.1: Time series plots for returns of five industry indexes.

Table 14.1 provides summary statistics, including mean, standard deviation, maximum, minimum, skewness, kurtosis, R^2 of the predictive regression in Eq. (14.2.k), Ljung-Box Q-test for heteroskedastisity and Engle's ARCH test for the log-returns of these five industry indexes.

The expected return of food industry index in the observed period is the highest and that of the steel industry index is the lowest. The textile industry index has the highest standard deviation, food industry index has the third highest, and the lowest is in the steel industry index. The finance industry index has the largest difference between the highest and the lowest return, whereas the steel industry index has the lowest difference.

From the skewness and kurtosis, it is shown that these index returns appear to be skewed and heavy tailed. The Ljung-Box Q-test and Engle's ARCH test show that these index returns exhibit heteroscedasticity.

Table 14.1: This table lists basic summary statistics of return for five industry indexces, the R^2 of fitting the predictive regression in Eq. (14.2.k), and test statistics of the Ljung-Box Q-test for heteroskedasticity at lag 10 and Engle's ARCH test at lag 10. * represents significance at the 0.05 level.

Industry	Market	Food	Textile	Iron & Steel	Electronic	Financial
Mean	0.0002	0.0005	0.0003	-0.0001	0.0001	0.0002
Std	0.0099	0.0121	0.131	0.0089	0.0107	0.0129
Max	0.0456	0.0551	0.0526	0.045	0.0477	0.0666
Min	-0.0558	-0.0579	-0.0595	-0.0366	-0.0574	-0.0578
Skewness	-0.4085	-0.0135	-0.177	0.1212	-0.3229	-0.0455
Kurtosis	5.5593	5.9805	5.3989	5.3694	4.8625	6.0743
R^2		0.5003	0.5954	0.4966	0.9172	0.7575
Ljung-Box Q(10)		175.1303*	176.3671*	123.0557*	202.9977*	219.5338*
ARCH test(10)		89.5639*	87.287*	72.6929*	102.7902*	104.7472*

Various empirical analysis has shown that asset returns feature conditional variance that changes with time, volatility clustering, and heavy-tailed distributions (Mandelbrot, 1963; Morgan, 1976). A better understanding of the volatility facilitates investment decision making and market stabilization. Compared with constant volatility models, GARCH models allows to model with features such as time-varying volatility and volatility clustering.

In addition to these informal plots, the mean-adjusted residuals are used to test for ARCH effects through the Ljung-Box test and Engle's test at lags 10. The Ljung-Box test considers the null hypothesis that the squared residuals of the returns are not autocorrelated. The Engle's ARCH test is a Lagrange multiplier test with the null hypothesis that no conditional heteroscedasticity exists. The testing results in Table 1 show that all the returns for these five indexes exhibit GARCH effects based on the Ljung-Box test and Engle's ARCH test.

Now, we consider three GARCH models. For a return series r_t we say that it follows a GARCH(1,1) model if

$$r_t = \mu_t + a_t, \tag{14.3.a}$$

$$a_t = \sigma_t \varepsilon_t, \tag{14.3.b}$$

$$\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + \alpha_1 a_{t-1}^2, \tag{14.b.c}$$

where $\{\varepsilon_t\}$ is a sequence of independently and identically distributed random variables with zero mean and unit variance. If ε_t is normal distribution, we call the model a GARCH(1,1) model with normal innovations, and abbreviate it as Gn model. If ε_t is a studentized-t distribution, we call the model a GARCH(1,1) model with t innovations, and abbreviate this model as Gt.

To incorporate the phenomenon of asymmetric shocks in the return (leverage effect), the EGARCH model is also considered (Nelson, 1991). The EGARCH model identifies markets with more downward movements and higher volatilities than upward movements under the same conditions. A sequence of data follows the EGARCH(1,1) model if

$$r_t = \mu_t + a_t, \tag{14.3.d}$$

$$a_t = \sigma_t \varepsilon_t, \tag{14.3.e}$$

$$ln(\sigma_t^2) = \alpha_0 + \alpha_1 \frac{|a_{t-1}| + \gamma_1 a_{t-1}}{\sigma_{t-1}} \beta_1 \ln(\sigma_{t-1}^2), \tag{14.3.f}$$

where α_0 is a constant, $\{\varepsilon_t\}$ is a sequence of independently and identically distributed random variables with zero mean and unit variance. Here, we assume ε_t to follow the Student's t distribution. We call this model an EGARCH(1,1) model with *t* innovations, and abbreviate the model as EG-t.

A positive at a_{t-1} contributes $\alpha_1(1+\gamma_1)|\varepsilon_{t-1}|$ to the log volatility, whereas a negative at a_{t-1} provides $\alpha_1(1-\gamma_1)|\varepsilon_{t-1}|$, where $\varepsilon_{t-1} = a_{t-1}/\sigma_{t-1}$. Thus, the γ_1 parameter signifies a leverage effect of a_{t-1}. Using the alternative form and noting that $\varepsilon_t = a_t/\sigma_t$, EGARCH(1,1) is written as

$$ln(\sigma_t^2) = \alpha_0 + \alpha_1(|\varepsilon_{t-1}| + \gamma_1 \varepsilon_{t-1}) + \beta_1 \ln(\sigma_{t-1}^2). \tag{14.3.g}$$

Since it is difficult to select the orders with the GARCH models, we choose GARCH(1,1) and EGARCH(1,1) because they are parsimonious yet provide a reasonable fit to the data.

14.3.2 Dynamic covariance matrices estimation using GARCH models

We follow Tsay (2010) to estimate the covariance matrix using the conditional covariance matrix with GARCH models, where applications are shown in time-varying correlation between returns of two stocks and time-varying betas under the capital asset pricing model (CAPM). This paper adopts this method for time-varying covariance estimation and applies it for portfolio allocation using the Black-Litterman model. This estimation method is simpler than the commonly used exponentially-weighted moving average (EWMA) method because it does not need to estimate the decay factor, yet the estimated time-varying volatility and correlation is of a similar pattern as those using the EWMA method.

Let $r_t = (r_{1t}, \cdots r_{mt})'$ denote the return vector of m assets at time *t*. It is shown in Tsay (2010) that a multivariate GARCH model can be used to estimate the conditional covariance matrix, denoted by

$$\Sigma_t = [\,\Sigma_{ijt}] = Cov(r_t | F_{t-1}), \tag{14.3.h}$$

where Σ_{ijt} is the *i*th row and *j*th column of Σ_t, F_{t-1} is the information available up to time *t*-1, and *Cov* is the covariance operator. For simplicity, we follow Tsay (2013) to estimate the conditional covariance matrix at time t with univariate GARCH models.

The diagonal element of the conditional covariance matrix $\Sigma_t$is the conditional variance of the *i*th return at time t. To estimate Σ_{iit}, we first estimate the conditional volatility of the *i*th return at time *t* with a specific GARCH model, denoted by σ_{it}. Here, we assume that

$$r_{it} = a_{it}, \tag{14.3.i}$$

$$a_{it} = \sigma_{it}\varepsilon_{it}, \tag{14.3.j}$$

where ε_{it} are innovations for $i = 1, \cdots m$ and $t = 1, \cdots T$. We estimate σ_{it} by maximum likelihood estimate using the series $\{r_{it}\}$ under the GARCH model, denoted by $\hat{\sigma}_{it}$. Then, we estimate Σ_{iit} by

$$\hat{\Sigma}_{iit} = \hat{\sigma}_{it}^2. \tag{14.3.k}$$

To estimate Σ_{ijt}, note that

$$V\left(r_{it} + r_{jt}\right) = V(r_{it}) + 2Cov\left(r_{it}, r_{jt}\right) + V\left(r_{jt}\right) = \Sigma_{iit} + 2\,\Sigma_{ijt} + \Sigma_{jjt}, \tag{14.3.l}$$

$$V\left(r_{it} - r_{jt}\right) = V(r_{it}) - 2Cov\left(r_{it}, r_{jt}\right) + V\left(r_{jt}\right) = \Sigma_{iit} - 2\,\Sigma_{ijt} + \Sigma_{jjt}, \tag{14.3.m}$$

where V(·) is the variance operator. Therefore, we obtain

$$\Sigma_{ijt} = \frac{V\left(r_{it}+r_{jt}\right)-V\left(r_{it}-r_{jt}\right)}{4}. \tag{14.3.n}$$

Now, $V(r_{it} + r_{jt})$ and $V(r_{it} - r_{jt})$are estimated using two artificial time series $r_{ijt}^{+} = r_{it} + r_{jt}$ and $r_{ijt}^{-} = r_{it} - r_{jt}$. Similarly, conditional volatilities for r_{ijt}^{+} and r_{ijt}^{-} are estimated, denoted by $\hat{\sigma}_{ijt}^{+}$ and $\hat{\sigma}_{ijt}^{-}$, respectively. Then, we estimate $V(r_{it} + r_{jt})$ and $V(r_{it} - r_{jt})$ by

$$V\left(r_{it} + r_{jt}\right) = \hat{\sigma}_{ijt}^{+}, \tag{14.3.o}$$

$$V\left(r_{it} - r_{jt}\right) = \hat{\sigma}_{ijt}^{-}. \tag{14.3.p}$$

Finally, we estimate $\hat{\Sigma}_{iit}$ by

$$\hat{\Sigma}_{ijt} = \frac{\hat{V}\left(r_{it}+r_{jt}\right)-\hat{V}\left(r_{it}-r_{jt}\right)}{4}. \tag{14.3.q}$$

14.4 EMPIRICAL ANALYSIS

In this section, the daily closing values of five industry indexes in Taiwan are used to demonstrate the superiority of our proposed heteroscedastic BL model. Recall that in Section 3.1 we collect daily log-returns using closing indexes for five major industry indexes, including food, textile, iron and steel, electronic, and financial sectors in Taiwan stock markets from 2010/01/06 to 2014/12/31, a total of 1024 observations. According to Taiwan Stock Exchange (TSE) on 2018/1/23, 74% of market capitalization is composed of these five industries: electronic sector (56%), financial sector (13%), food sector (2%), textile sector (2%), and iron and steel sector (2%), as summarized in Figure 14.2. Consequently, Taiwan stock markets are mainly composed of these major five industries, and they approximately span the whole stock market and make the reverse optimization possible.

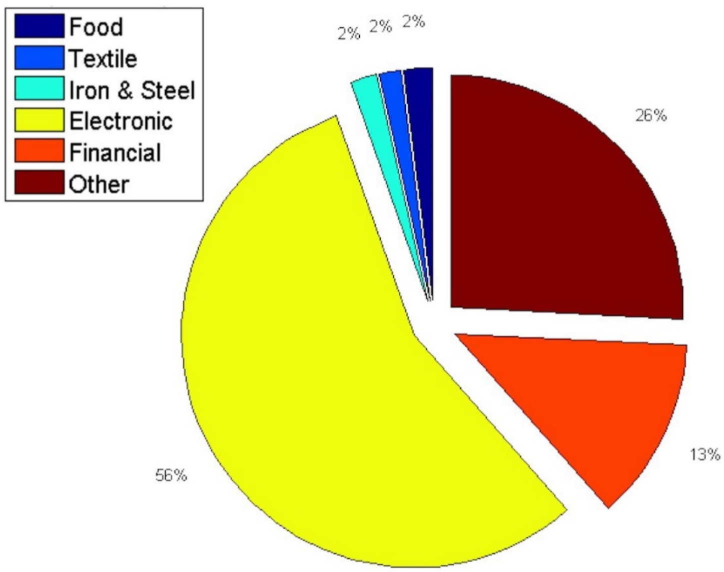

Figure 14.2: Percentage of market capitalization per industry. (Source: Taiwan Stock Exchange)

14.4.1 Study plan

To begin with, we consider three categories of risk preferences for investors: mildly risk averse (λ = 1), moderately risk averse (λ = 5), and severely risk averse (λ = 10).

Now, we propose a dynamic rolling window scheme as follows. A rolling window consists of two parts: the training period of size N_{train} days and the holding period of size N_{hold} days. Hence, the size of the rolling window is of $(N_{train} + N_{hold})$ days. Given a rolling window, we use the data in the training period to estimate parameters and obtain optimal portfolio weight, then we form the portfolio and hold it during the holding period to measure its performance in terms of various strategy summary statistics. Then, we move the rolling window N_{hold} days forward, and repeat the above procedures till the end of the study period.

In our study, we set N_{train} as 124 days and N_{hold} as 60 days to measure out of sample performance. In details, when N_{train} is 124 and N_{hold} is 60, the parameter is estimated using samples from days 1 to 124 to obtain the portfolio weight, and the portfolio is reallocated with the optimal portfolio weight and held for the next 60 days, i.e., days 125 to 184. Then we move the rolling window 60 days further. The next iteration uses samples from days 61 to 184 to estimate parameter and obtain the optimal portfolio weight, and the portfolio is held with the updated portfolio weight for the next 60 days, i.e., days 185 to 234.

By repeating this procedure, a dynamically adjusted portfolio allocation for a total of 1240 days is constructed. We remark that the selection of N_{train} to be 60 is subjective. However, our empirical analysis shows that it can reasonably avoid frequent trading and control transaction cost. How to select N_{train} and N_{hold} is beyond the scope of this paper.

For comparisons, we first consider the 1/*N* portfolio as a benchmark. The 1/*N* portfolio uses equal weight for each assets. Furthermore, we consider the Markowitz portfolio. The Markowitz model is also called Mean-Variance Model due to the fact that its portfolio weight is selected based on expected returns (mean) and the risk (standard deviation) of the various assets. The weight of the Markowitz portfolio solves the following constrained optimization problem,

$$\max_{\omega} \ \ \omega'\mu_{BL} - \frac{\lambda}{2}\omega'\Sigma\omega$$

$$s.t. \ \ \sum_{i=1}^{m}\omega_i = 1, \omega_i \geq 0. \tag{14.4.a}$$

To avoid short selling, a positive constraint is placed on portfolio weight. Here, μ and Σ are estimated as sample mean and sample covariance matrix using data in the training period.

For the Black-Litterman portfolio, we consider four models to estimate the covarinace matrix parameter: i.i.d. normal, Gn, Gt, and EGt. The later three models present volatility clustering, volatility clustering with a heavy-tailed distribution, and volatility clustering with a heavy-tailed distribution and asymmetric leverage effects, respectively. For notations, we abbreviate these BL portfolios by BL, BL-Gn, BL-Gt, and BL-EGt, respectively. In addition, the absolute and relative views are employed separately as investors' views. Consequently, there are eight versions of Black-Litterman portfolios. For BL portfolio, the covariance matrix Σ is estimated using sample covariance. For BL-G, BL-EG, and BL-EGt portfolios, the covariance matrix is estimated with univariate GARCH models as described in Section 14.3.2. We recall that the weight of the Black-Litterman portfolio solves the constrained optimization problem in Eq. (14.2.j).

Portfolio performance is compared in terms cumulative returns, standard deviations, averaged returns, Sharpe ratios, and beta values. Note the beta value is the covariance between portfolio returns and market returns divided by the variance of market returns.

14.4.2 Empirical comparisons

Figure 14.3 summarizes average weights in different portfolios: Markowitz portfolio, BL portfolio, BL-Gn portfolio, BL-Gt portfolio, and BL-EGt portfolio, at three levels of risk aversion λ = 1, 5, and 10. For BL portfolios, the absolute and relative views are separately considered. Compared with the Markowitz portfolio, it appears BL portfolios in general prefer the iron and steel sector to the textile sector.

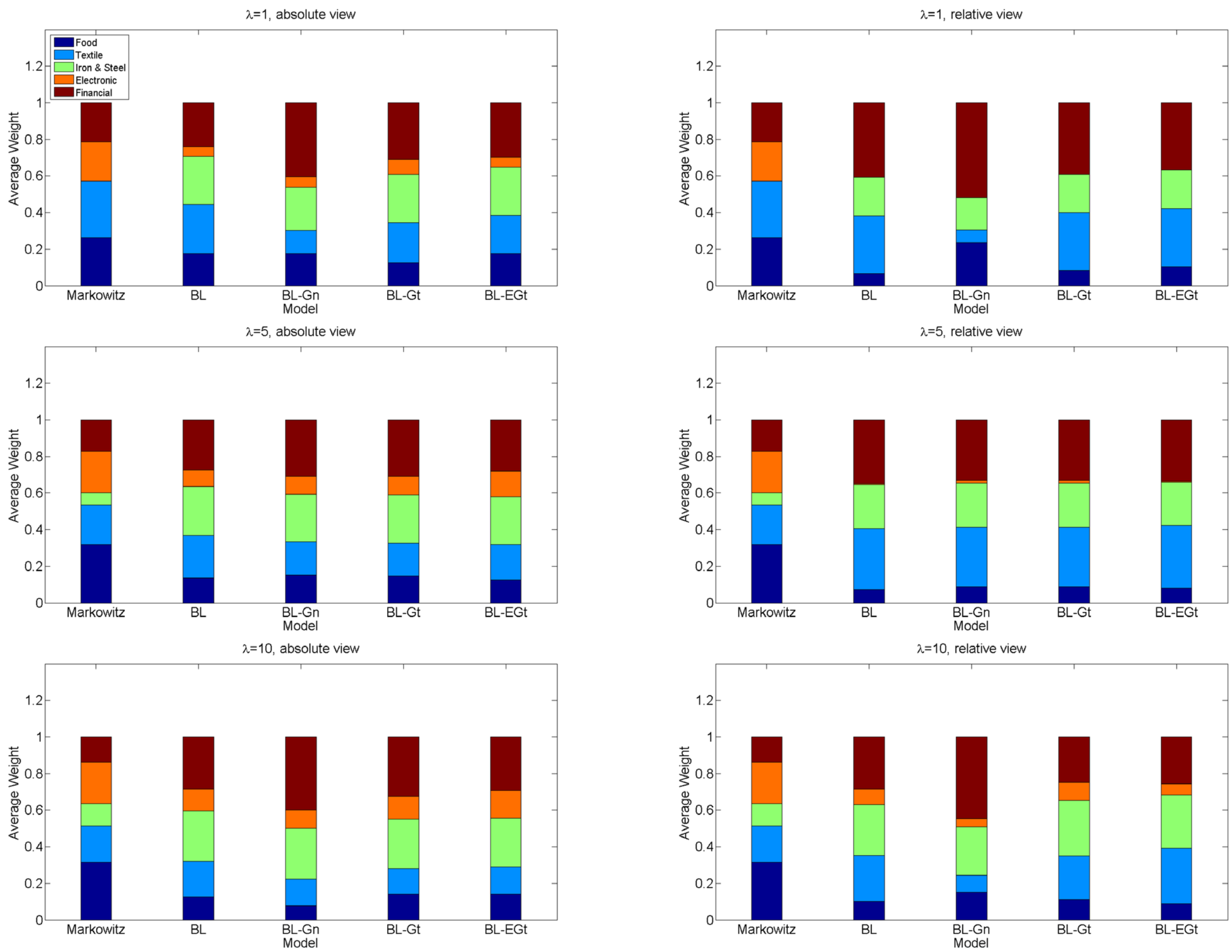

Figure 14.3: Average weight of the Markowitz portfolio, BL portfolio, BL-Gn portfolio, BL-Gt portfolio, and BL- EGt portfolio, during a total of 1240 days at various levels of risk aversion (λ= 1, 5, 10) with $N_{\text{train}} = 124$. For Black-Litterman portfolios, the absolute and relative views are separately considered.

Table 14.2 summarizes the empirical results in terms of cumulative returns, standard deviations, average returns, beta values, and ratios of the averaged return to the standard deviation.

Table 14.2: Strategy summary statistics of the 1/N portfolio, Markowitz portfolio, BL portfolio, BL-Gn portfolio, BL-Gt portfolio, and BL-EGt portfolio, at three levels of risk-aversion (λ=1, 5, 10) with $N_{\text{train}} = 124$. For Black-Litterman portfolios, the absolute and relative views are separately considered.

Portfolio	Averaged return		Std		Beta		Sharpe ratio	
λ =1								
1/N	0.1115		0.1422		0.8708		0.7225	
Markowitz	0.0656		0.1859		0.9544		0.3057	
	Absolute	Relative	Absolute	Relative	Absolute	Relative	Absolute	Relative
BL	0.1289	0.1436	0.163	0.175	0.8077	0.9031	0.7363	0.7705
BL-Gn	-0.0075	0.0353	0.1969	0.1978	0.9517	0.9372	-0.0827	0.1341
BL-Gt	0.1606	0.1655	0.1613	0.1735	0.8341	0.8779	0.9412	0.9032

BL-Egt	0.1336	0.1919	0.1638	0.1743	0.8269	0.8469	0.762	1.0507
λ =5								
1/N	0.1115		0.1422		0.8708		0.7225	
Markowitz	0.1062		0.1813		0.947		0.5372	
	Absolute	Relative	Absolute	Relative	Absolute	Relative	Absolute	Relative
BL	0.1493	0.2052	0.1572	0.165	0.8134	0.8271	0.8935	1.1909
BL-Gn	0.1578	0.2098	0.1599	0.1652	0.8357	0.8253	0.9316	1.2165
BL-Gt	0.1611	0.2128	0.16	0.1655	0.8362	0.8256	0.9514	1.2328
BL-Egt	0.1721	0.2211	0.1604	0.1679	0.8438	0.8364	1.0178	1.2643
λ =10								
1/N	0.1115		0.1422		0.8708		0.7225	
Markowitz	0.1041		0.1738		0.9163		0.5483	
	Absolute	Relative	Absolute	Relative	Absolute	Relative	Absolute	Relative
BL	0.1735	0.1591	0.1536	0.1584	0.8073	0.7947	1.0719	0.9485
BL-Gn	0.0011	-0.0142	0.1758	0.1807	0.9271	0.9072	-0.0437	-0.1274
BL-Gt	0.185	0.1598	0.1558	0.158	0.831	0.7962	1.131	0.9556
BL-Egt	0.2056	0.1776	0.1547	0.1601	0.8342	0.8021	1.2727	1.0544

Figure 14.4 plots cumulative returns for ten portfolios at every sixty days for N_{train} to be 124 days.

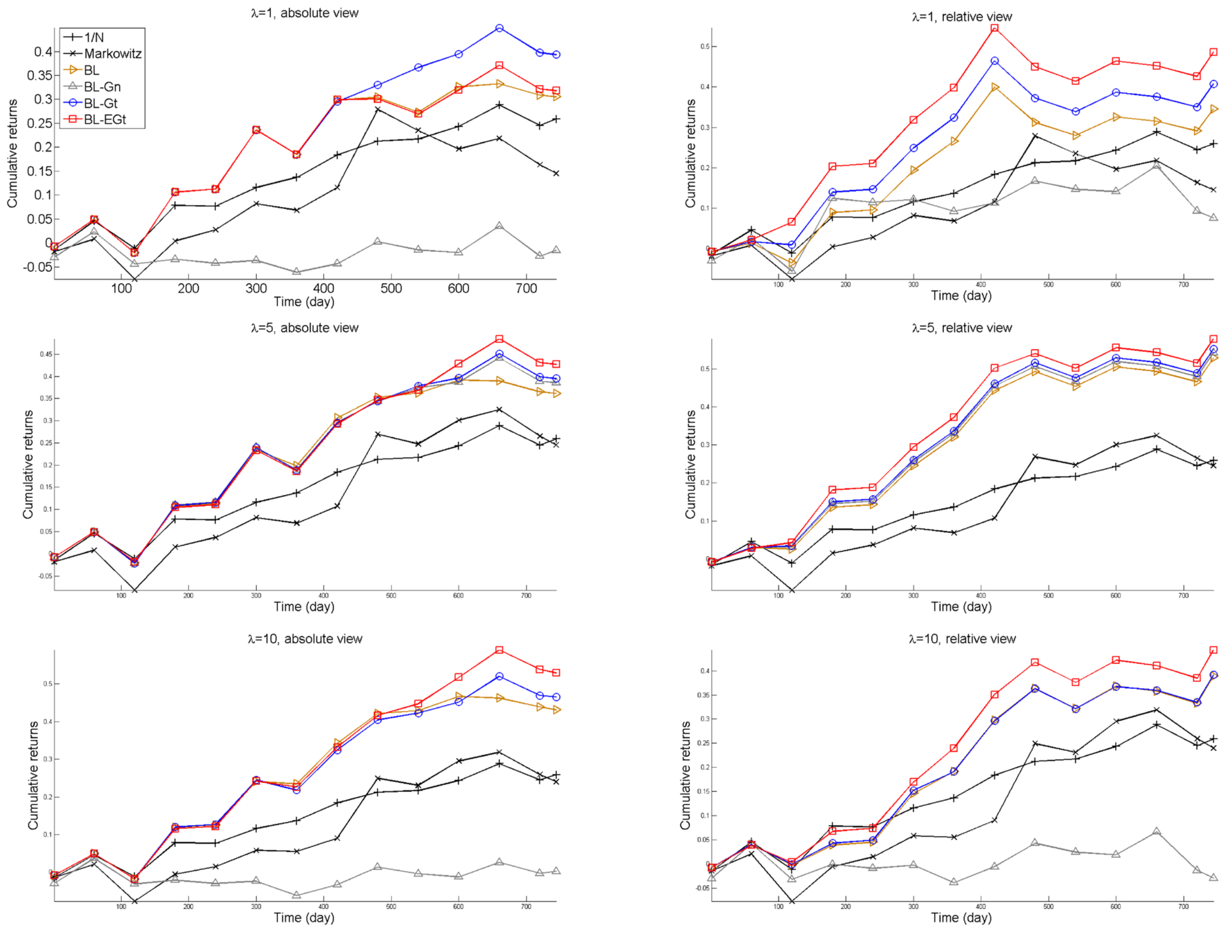

Figure 14.4: Cumulative returns of the 1/N portfolio, Markowitz portfolio, BL portfolio, BL-Gn portfolio, BL- Gt portfolio, and BL-EGt portfolio, at every sixty days at various levels of risk aversion (λ = 1, 5, 10) with $N_{\text{train}} = 124$. For Black-Litterman portfolios, the absolute and relative views are separately considered.

In terms of investors' views, it is shown that, in general, BL portfolios with relative view outperforms BL with absolute views in terms of average returns and Sharpe ratios. When risk-aversion coefficient is neutral, the BL-EGt portfolio with the relative view has the highest average return and Sharpe ratio (22.11 % and 1.26 respectively).

The average return and Sharpe ratio of the BL-EGt portfolio is slightly higher than that of the BL-Gt and BL-Gn portfolios but significantly higher than the BL portfolio. The beta values for all portfolios are less than 1. This indicates that all these portfolio returns are less volatile compared with the market return.

Finally, both BL-EGt and BL-Gt portfolios outperform the 1/N and Markowtiz portfolios in terms of average returns and Sharpe ratios. Specifically, BL-EGt portfolio dominates the BL-Gt portfolio.

Based on the predictive regression in Eq. (14.2.k), investors with the absolute view pose *m* views, where each view assigns an expected return to each asset. In this case, each asset return is adjusted by the investors views, and it is likely that such adjustment does not yield higher predictability. In contrast, the relative view only poses one view: this view represents the difference between the asset of the highest expected return and that of the lowest return. Our empirical analysis shows that portfolios with the BL model with the relative view outperform those with the absolute view in terms of averaged returns in many cases, and highlight the benefits of using the relative view.

The relative view is in spirit similar to the momentum strategies (Chan et al., 1996), where stocks are first ranked by their returns into five return levels and a portfolio is composed of a long position in stocks of the highest return level and a short position in stocks of lowest return level. It is hence possible that prediction for the difference between the highest and lowest returns (as in the relative view) is more accurate than prediction for all returns (as in the absolute view).

14.5 CONCLUSION

In earlier work, portfolio weight is determined to match Markowitz's portfolio efficiency. Recently, investors' forecast to market returns has been employed to form a more compatitive portfolio. For example, Black and Litterman employed a new model and Idzorek (2004) added an optimistic view of expected return to increase portfolio efficiency. This paper aims at improving the standard Black-Litterman portfolios by proposing a more sophisticated estimation scheme for the covariance matrix via heteroscedastic models.

This article compared ten portfolios, including the 1/N portfolio, Markowitz portfolio, and eight versions of Black-Litterman portfolios. Empirical analysis is applied to five industry indexes in Taiwan.

For the Black-Litterman models, the standard i.i.d. normal model and three types of GARCH model are used to estimate the covariance matrix. In addition, the absolute and relative views are separately induced in the Black-Litterman model. Our empirical analysis shows that Black-Litterman portfolios with the relative view outperform those with the absolute view for mildly and moderately risk averse investors. In addition, Black-Litterman portfolio with EGARCH model, with both absolute and relative views, outperform the standard Black-Litterman portfolio.

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