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Chapter 85

Comparisons between the Markowitz Model and the Black–Litterman Model*

Huei-Wen Teng

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Abstract

Model risk is critical in constructing a portfolio. To avoid model risk, the Black–Litterman model is an approach that allows to adjust the original estimated parameters using the implied market equilibrium returns and investors' views (Black and Litterman, 1991). This chapter contrasts the standard approach of Markowitz (1952) with the Black–Litterman model and reviews different investment philosophies by Longo (2021). For empirical demonstrations, we consider a predictive regression to form investors' views, where asset returns are regressed against their lagged values and the market return. Motivated by stylized features of historical returns, we employ heteroscedastic time-series models. Empirical analysis using five industry indexes in the Taiwan stock market shows that the proposed Black–Litterman portfolio outperforms the $1/N$ portfolio and Markowitz portfolio.

Keywords

Markowitz modern portfolio theory • Black–Litterman model • GARCH model • Investor's views • Volatility clustering

85.1 Introduction

Markowitz (1952) provided a framework for modern portfolio theory, in which the asset returns can be assumed as identical and independently distributed multivariate normal vector with mean μ and covariance Σ. In practice, investment theory seeks portfolios to diversify risk. However, a direct implementation of the Markowitz model is challenging as the optimal weight is extremely sensitive to the estimation of the assets means and covariances and may be a corner solution, which is however against the intuition about portfolio diversification. For example, Chopra *et al.* (1993) and Brianton (1998) have revealed that estimation errors in the parameters severely affect the optimized portfolio and efficiency frontier. In addition, Michaud (1989) points out that Markowitz model may lead to maximization of the effect of errors on the input parameter assumptions.

Black and Litterman (1991) improve the Markowitz model by combining the implied market equilibrium return and investors' views so that the estimated means and covariances are adjusted to reflect the market behavior more realistically. Using a Bayes' formula, the return, given investors' views, is a weighted average between the implied market equilibrium returns and investors' views. When the investors have a higher confidence level in their views, the weighted average return given investors' views tend to be closer to the investors' views, and vice versa.

Detailed expositions to the Black–Litterman (BL) model have been extensively studied in the literature. For example, Christodoulakis (2002) and Walters (2014) provide detailed instructions on how to use Bayes formulas to derive the BL model; Idzorek (2004) summarizes the step-by-step guide to derive the BL model's parameters; the standard BL model is further extended by Black and Litterman (1992) and He and Litterman (1999). On the other hand, O'toole (2013) revisits the BL model from a risk budgeting perspective to demystify the BL model in an insightful and intuitive manner. Similar to the BL model, Jurczenko and Teiletche (2018) introduce an analytic framework that allows investors to add active views on top of a risk-based investing solution.

The employment of the BL model requires an assignment of investors' views, which are subject to investors and hence problem dependent. Examples include a simple statistical method by Meucci (2010) and a generalized autoregressive conditional heteroscedastic model by Beach and Orlov (2007). Therefore, in order to provide a simple and useful scheme for the BL model application, we first provide a predictive regression to form investors' views, where asset returns are regressed against their lagged values and the market return. AQ01

Because the estimation of the covariance matrix for the asset returns is critical, various methods to estimate the covariance matrix have been studied (Guo *et al*., 2017). Simple estimators, such as diagonal estimators or constant correlation estimators, are usually used as benchmarks for comparison. The former assumes that asset returns pairwise uncorrelated, whereas the former assumes that every pair of asset returns has the same correlation coefficients (Elton and Gruber, 1973). Covariance matrix can be estimated via linear factor models, which help to reduce the number of model parameters. Examples include studies by Avellaneda and Lee (2010), Chen *et al.* (1986), Sharpe (1963), and Torun, Akansu, and Avellaneda *et al.* (2011).

In addition to the above methods, Litterman and Winkelmann (1998) use Goldman Sachs decay rate covariance matrix model for portfolio optimization and risk management. Ledoit and Wolf (2003b) estimate the covariance matrix of stock returns by an optimally weighted average of the sample covariance matrix and the single-index covariance matrix. Ledoit and Wolf (2003a, 2003b) consider a shrinkage estimator for the covariance matrix which combines the sample covariance and any covariance estimator, where the mixing weight can be obtained through cross validation. But Disatnik and Benninga (2007) show that there is no statistically significant gain from using more sophisticated shrinkage estimators, in terms of *ex-post* standard deviation of the global minimum variance portfolio.

To estimate the covariance, we implement a careful exploratory data analysis of five industry indexes in the Taiwan stock market. It is shown that returns exhibit stylized features, such as volatility clustering, heavy-tail distributions, and leverage effects. For these reasons, we estimate a time-varying covariance matrices using heteroscedastic models.

The rest of this chapter is organized as follows. Section 85.2 contrasts the Markowitz model and the BL model and briefly reviews both academics' and practitioners' views from Longo (2021). In addition, both the absolute and relative views are considered for investors. Section 85.3 presents a methodology to apply time-series modeling to incorporate heteroscedasticity in the return as an objective view and compares the portfolio of the Black–Scholes model with the standard $1/N$ portfolio and the Markowitz portfolio. Section 85.4 summarizes empirical analysis, and Section 85.5 concludes.

85.2 Preliminaries

We first review the Markowitz and BL models. Then, we present procedures for deciding investors' views. Finally, we summarize different investment philosophies across the academic and practitioners from Longo (2021).

85.2.1 *The Markowitz model*

Consider a market of m securities or asset classes. In the standard Markowitz model, it is assumed that they have normally distributed returns $r_t = (r_{1t}, \ldots, r_{mt})'$:

$$r \sim N(\mu, \Sigma) \tag{85.2a}$$

The normality is not a necessary distribution assumption, and the model parameters, μ and Σ, are estimated by the sample mean and sample covariance from historical returns, respectively. Focusing on the mean-variance portfolio, the optimal weight is a solution of the following constrained optimization problem:

$$\begin{aligned} \max_{\omega} \quad & \omega'\mu - \tfrac{\lambda}{2}\omega'\Sigma\omega, \\ s.t. \quad & \sum_{i=1}^{m} \omega_i = 1, \omega_i \geq 0. \end{aligned} \tag{85.2b}$$

85.2.2 *The Black–Litterman model*

To avoid the model risk, the BL model uses a Bayes' formula to combine the investors' subjective views regarding the expected returns of one or more assets with the implied market equilibrium returns to form a new mixed estimate of expected returns. For detailed expositions, please see Meucci (2010), Satchell and Scowcroft (2000), Idzorek (2004), Walters (2014), and references therein.

In addition to (85.2a), the BL model assumes that the expected return μ follows

$$\mu \sim N(\pi, \Sigma_\pi = \tau\Sigma). \tag{85.2c}$$

Furthermore, the covariance matrix of μ is proportional to the covariance of the returns Σ, i.e., $\Sigma_\pi = \tau\Sigma$. For the selection of τ, please refer to Black and Litterman (1992), He and Litterman (1999), Satchell and Scowcroft (2000), and Allaj (2013). In this chapter, τ is set as 0.025 (Idzorek, 2004).

The BL model inversely employs assets market weight to obtain the implied market equilibrium return:

$$\pi = \lambda\Sigma\omega^M \tag{85.2d}$$

where λ is a constant denoting the risk aversion coefficient, Σ is an $m \times m$ covariance matrix of the assets, and the market weight $\omega^M = (\omega_1^M, \ldots, \omega_m^M)'$ in a portfolio is a $m \times 1$ vector calculated by the following equation:

$$\omega_i^M = \frac{P_i Q_i}{\sum_{i=1}^m P_i Q_i}, \tag{85.2e}$$

for $i = 1, \ldots, m$. Here, P_i is the market price of the ith asset and Q_i is the outstanding shares of the ith asset.

To construct the views for investors' expected returns, the BL model considers that the expectation of the views are related to the parameter μ. Thus, the uncertainty with the views, v, is a random vector that follows

$$v|\mu \sim N(P\mu, \Omega) \tag{85.2f}$$

where P is a $K \times m$ matrix to represent K views, and the covariance matrix Ω quantifies investors' confidence level toward their views. For mathematical simplicity, it is assumed that

$$\Omega = \tau P \Sigma P', \tag{85.2g}$$

where the constant τ characterizes the uncertainty about the confidence of the views. We remark that there is no rule to set up the value of τ. For example, Almadi *et al.* (2014) set τ to be unity and Michaud and Esch (2013) set τ to be $1/T$, where T is the number of return observations. In this chapter, we set $\tau = 0.025$ to indicate a relatively high level of confidence toward investors' views (Idzorek, 2004).

Given the expected returns μ and investors' views v, Bayes' formula yields the following critical formulas for the return:

$$r|v, \Omega \sim N(\mu_{BL}, \Sigma_{BL}) \tag{85.2h}$$

where

$$\mu_{BL} = \pi + \tau\Sigma P'(\tau P\Sigma P' + \Omega)^{-1}(v - P\pi) \tag{85.2i}$$

$$\Sigma_{BL} = (1 + \tau)\Sigma - \tau^2\Sigma P'(\tau P\Sigma P' + \Omega)^{-1}P\Sigma \tag{85.2j}$$

The return given investors' views v, and the confidence toward the views Ω remains a normal distribution with mean μ_{BL} and covariance Σ_{BL}. Here, μ_{BL} and Σ_{BL} modify coefficients for the return in the reference model $r \sim N(\mu\Sigma)$ by incorporating the views $P\mu \sim N(v\Omega)$. Derivations of these formulas can be found in the work by Meucci (2010). Finally, the optimal portfolio weight in the BL model solves the following constrained optimization problem:

$$\begin{aligned} \max_{\omega} \quad & \omega'\mu_{BL} - \frac{\lambda}{2}\omega'\Sigma_{BL}\omega \\ s.t. \quad & \sum_{i=1}^{m} \omega_i = 1,\ \omega_i \geq 0. \end{aligned} \tag{85.2k}$$

85.2.3 *Views selection*

P is a $K \times m$ matrix of the asset weights within each view that shows investors' K views on m assets (Walters, 2014). In this chapter, we consider two types of views, the absolute view and the relative view. In addition, different authors compute the various matrix to represent weights; He and Litterman (1999) and Idzorek (2004) have used market capitalization weighed scheme, whereas Satchell and Scowcroft (2000) used an equally weighted scheme in their examples.

Now, a predictive regression model is proposed as a part to establish investors' views on the return for each asset. In the predictive regression model, the asset return at time t is the dependent variable and explanatory variables include the asset return at time $t - 1$ and the market return at time t.

Let r_{it} denote the ith asset return at time t and r_t^M denote the market return at time t. Then, r_{it} is modeled as the following prediction regression:

$$\begin{aligned} & r_{it} = \alpha_i + \beta_{i1}r_{i(t-1)} + \beta_{i2}r_t^M + \varepsilon_{it}, \\ & \varepsilon_{it} \sim i.i.d.N(0, \sigma^2), \quad i = 1, \ldots, m,\ t = 1, \ldots, T. \end{aligned} \tag{85.2l}$$

Recall that the capital asset pricing model (CAPM) assumes

$$r_{it} = \alpha_i + \beta_i r_t^M + \varepsilon_{it} \tag{85.2m}$$

where $\varepsilon_{it} \sim i.i.d.N(0, \sigma^2)$. Through empirical analysis, we generalize CAPM by including a lagged dependent variable as in Eq. (85.2k) in the chapter. Indeed, the prediction regression in Eq. (85.2k) coincides with the standard market model with a lagged dependent variable in Cartwright and Lee (1987), where 49 US stocks are considered.

The expected returns for the ith asset at time t, denoted by $\hat{r}_{i,t}$, is calculated via the model in Eq. (85.2k):

$$\hat{r}_{it} = \hat{\alpha}_i + \hat{\beta}_{i1} r_{i(t-1)} + \hat{\beta}_{i2} r_t^M, \quad i = 1, \ldots, m, \ t = 1, \ldots, T. \tag{85.2n}$$

Investors' views v are obtained via

$$v = P\hat{r}_t \tag{85.2o}$$

where $\hat{r}_t = (\hat{r}_{1t}, \ldots, \hat{r}_{mt})'$ with $\hat{r}_{i,t}$ is given in Eq. (85.2m).

85.2.3.1 *The absolute view*

For the absolute view, P is defined as the k-dimensional identity matrix, which displays the weights of each asset in each of the m views. Specifically, P is

$$P_{m\times m} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \tag{85.2p}$$

With the absolute view, investors have views toward the expected return of each single asset.

85.2.3.2 *The relative view*

Another approach for defining P, called the relative view, is a $1 \times m$ vector and is given in the following. We first use the prediction regression model in Eq. (85.2k) to establish the expected return of each asset. Then, these assets are sorted based on their expected returns from the lowest to the highest. Suppose the ith asset has the highest expected return and the jth variate has the lowest expected return. P is everywhere zeros but with value 1 at the ith variate and with value -1 at the jth variate.

For example, five assets are included in a portfolio; they are sorted from the lowest to the largest based on their expected returns. Suppose that the first asset has the highest expected return, whereas the fifth asset has the lowest expected return. Then, P is set to

$$P_{m\times m} = [1\ 0\ 0\ 0\ \ -1] \tag{85.2q}$$

Therefore, investors have a relative view simply to the first and fifth assets in the portfolio. The relative view indicates that the investors only focus on assets with the highest or lowest returns.

85.2.4 *Summary of philosophies of great investors of Longo (2021)*

Recently, Longo (2021) summarized divergence on risk, return, and portfolio construction between academics' and practitioners' views, and suggested that the Black–Litterman model should be revisited and elevated by both academics and practitioners. We briefly review them in the following.

From the academic aspect, the earlier work starts with the discounted cash flows, where the fair stock price is the discounted expected future cash flow. Then, Markowitz portfolio theory is good for asset allocation and is a pioneer that uses quantitative method to objectively quantify the idea of "risk" using standard deviation. However, its implementation is difficult and unstable for real implementation. The major reasons rely on the statistical issues. The simplest form of the Markowitz portfolio theory uses the sample mean of historical returns to estimate the parameter μ and the sample covariance to estimate the covariance parameter Σ.

The CAPM connects the market excess expected return to the excess expected return of an asset. The arbitrage pricing theory (APT), also known as the factor model, extends CAPM by including additional explanatory variables (or factors). The popular factors that help to explain the cross section of stock price returns include size and style, momentum, liquidity, quality, and accruals. Recently, neural networks have been applied to include hundreds of macro and micro factors (Gu *et al.*, 2020).

Last but not the least, behavioral finance is a subset of behavioral economics and focuses on how psychological finance can affect market outcomes. One key aspect of behavioral finance studies is the influence of psychological biases.

Now, we proceed to summarize practitioners' views as follows. Benjamin Graham is known as a value investing pioneer and as a mentor to Warrant Buffett. In addition, Graham is also pioneer of hedge funds, active investing, arbitrage, behavior finance, and quantitative investing. Warren Buffett believes that a company's business, history, management, profit margins all matter. In contrast to the idea of diversifying risk in classic portfolio management, Buffett (1993) said "Diversification is projection against ignorance."

Jesse Livemore focuses mostly on technical analysis and viewed long-term investing as riskier than speculation. Based on his philosophical background, George Soros developed a "Reflexivity" theory that aims to explain the relationship between market participants and prices and proposed a novel idea that prices impact fundamentals.

James Simons is perhaps the most successful hedge fund manager ever. He is the founder of Renaissance Technologies, which is a quantitative hedge fund and hires mostly mathematicians and scientists. Co-founded in 1994 in New York City by Jim and Marilyn Simons, the Simon Foundation's mission is to advance the frontiers of research in mathematics and basic science.

John Templeton pioneers international investing with his famous quote, "If you search worldwide, you will find more bargains and better bargains than by studying only one nation." Cathie Wood became a superstar in 2020 when her ARK INOVATION exchange traded fund (ETF) grew an incredible 152%. Modifying Schumpeters's (1942) creative disruption hypothesis, Wood follows a disruptive innovation strategy.

Car Icahn, Bill Ackman, Paul Singer, and Dan Loeb are among the best known activist investors. They take a sizeable stake in a firm and aim to affect the firms' stock price by pressuring the firm to change their stock buyback program, dividend policy, cut costs, among others. David Swensen managed Yale's Endowment from 1985 until shortly before his death in 2021. He pioneered the "endowment model," which emphasizes on about 80% or more to illiquid assets, such as hedge funds, private equity, real estate, and natural resources.

It is noted that most practitioners' views are motivated by their individual profound knowledge and investment belief and are difficult to replicate. On the other hand, one simple and feasible approach to form a view is through time-series modeling. This approach is objective and replicable. Moreover, a proper back test could be obtained. In the next section, we will demonstrate a procedure to form a BL portfolio through time-series models.

85.3 Our Methodology

Section 85.3.1 explains that statistical features of the data, which motivate the incorporation of a GARCH-typed model to adjust the standard BL model. Section 85.3.2 implements portfolio allocation using heteroskedastic information on the volatility.

85.3.1 *Data and motivations*

The return at time t is defined as $r_t = \ln(p_t) - \ln(p_{t-1})$, where p_t is the closing price at time t. In this chapter, five industry indexes in Taiwan are examined as research samples, retrieved from the Taiwan Economic Journal. The study period is from January 6, 2010 to December 31, 2014, and a dataset of 1,240 daily returns is used.

Figure 85.1 shows that these five returns have no significant trend and gives empirical evidence of volatility clustering and leverage effects.

Table 85.1 provides summary statistics, including mean, standard deviation, maximum, minimum, skewness, kurtosis, R^2 of the predictive regression in Eq. (85.2k), Ljung–Box Q-test for heteroskedasticity, and Engle's ARCH test for the log-returns of these five industry indexes.

The expected return of food industry index in the observed period is the highest and that of the steel industry index is the lowest. The textile industry index has the highest standard deviation, food industry index has the third highest, and the lowest is in the steel industry index. The finance industry index has the largest difference between the highest and the lowest returns, whereas the steel industry index has the lowest difference.

From the skewness and kurtosis, it is shown that these index returns appear to be skewed and heavy tailed. The Ljung–Box Q-test and Engle's ARCH test show that these index returns exhibit heteroscedasticity.

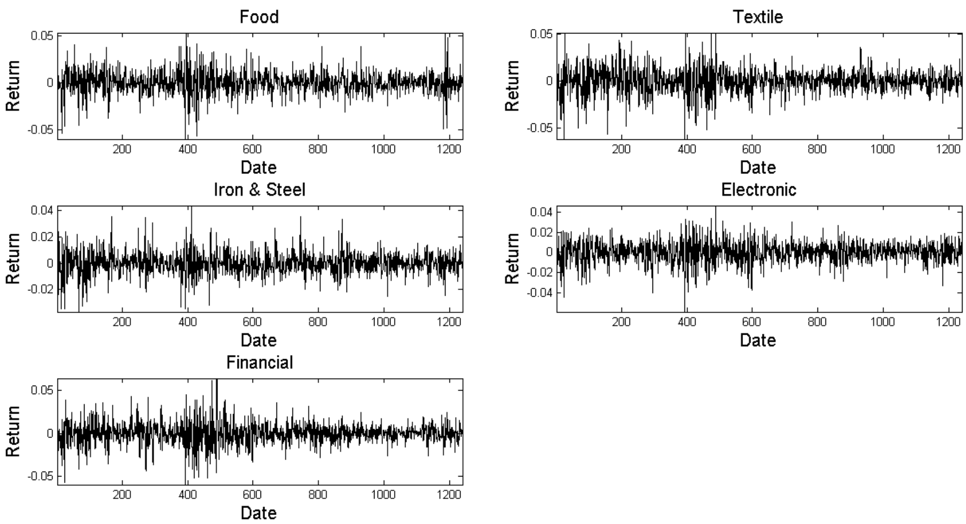

Figure 85.1. Time-series plots for returns of five industry indexes.

Table 85.1. This table lists basic summary statistics of return for five industry indexes, the R^2 of fitting the predictive regression in Eq. (85.2k), and test statistics of the Ljung–Box Q-test for heteroscedasticity at lag 10 and Engle's ARCH test at lag 10.

Industry	Market	Food	Textile	Iron & Steel	Electronic	Financial
Mean	0.0002	0.0005	0.0003	−0.0001	0.0001	0.0002
Std	0.0099	0.0121	0.131	0.0089	0.0107	0.0129
Max	0.0456	0.0551	0.0526	0.045	0.0477	0.0666
Min	−0.0558	−0.0579	−0.0595	−0.0366	−0.0574	−0.0578
Skewness	−0.4085	−0.0135	−0.177	0.1212	−0.3229	−0.0455
Kurtosis	5.5593	5.9805	5.3989	5.3694	4.8625	6.0743
R^2		0.5003	0.5954	0.4966	0.9172	0.7575
Ljung–Box Q(10)		175.1303*	176.3671*	123.0557*	202.9977*	219.5338*
ARCH test(10)		89.5639*	87.287*	72.6929*	102.7902*	104.7472*

Note: * represents significance at the 0.05 level.

Various empirical analysis has shown that asset returns feature conditional variance that changes with time, volatility clustering, and heavy-tailed distributions (Mandelbrot, 1963; Morgan, 1976). A better understanding of the volatility facilitates investment decision-making and market stabilization. Compared with constant volatility models, GARCH models allows to model with features such as time-varying volatility and volatility clustering.

In addition to these informal plots, the mean-adjusted residuals are used to test for ARCH effects through the Ljung–Box test and Engle's test at lag 10. The Ljung–Box test considers the null hypothesis that the squared residuals of the returns are not autocorrelated. The Engle's ARCH test is a Lagrange multiplier test with the null hypothesis that no conditional heteroscedasticity exists. The testing results in Table 85.1 show that all the returns for these five indexes exhibit GARCH effects based on the Ljung–Box test and Engle's ARCH test.

Now, we consider three GARCH models. For a return series r_t, we say that it follows a GARCH(1,1) model if

$$r_t = \mu_t + a_t \tag{85.3a}$$

$$a_t = \sigma_t \varepsilon_t \tag{85.3b}$$

$$\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + \alpha_1 a_{t-1}^2 \tag{85.3c}$$

where ε_t is a sequence of independently and identically distributed random variables with zero mean and unit variance. If ε_t is normal distribution, we call the model a GARCH(1,1) model with normal innovations, and abbreviate it as Gn model. If ε_t is a studentized-t distribution, we call the model a GARCH(1,1) model with t innovations and abbreviate this model as Gt.

To incorporate the phenomenon of asymmetric shocks in the return (leverage effect), the EGARCH model is also considered (Nelson, 1991). The EGARCH model identifies markets with more downward movements and higher volatilities than upward movements under the same conditions. A sequence of data follows the EGARCH(1,1) model if

$$r_t = \mu_t + a_t \tag{85.3d}$$

$$a_t = \sigma_t \varepsilon_t \tag{85.3e}$$

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \frac{|a_{t-1}| + \gamma_1 a_{t-1}}{\sigma_{t-1}} \beta_1 \ln(\sigma_{t-1}^2) \tag{85.3f}$$

where α_0 is a constant and ε_t is a sequence of independently and identically distributed random variables with zero mean and unit variance. Here, we assume ε_t to follow the student's t distribution. We call this model an EGARCH(1,1) model with t innovations and abbreviate the model as EGt.

A positive at a_{t-1} contributes $\alpha_1(1+\gamma_1)|\varepsilon_{t-1}|$ to the log volatility, whereas a negative at a_{t-1} provides $\alpha_1(1-\gamma_1)|\varepsilon_{t-1}|$, where $\varepsilon_{t-1} = a_{t-1}/\sigma_{t-1}$. Thus, the γ_1 parameter signifies a leverage effect of a_{t-1}. Using the alternative form and noting that $\varepsilon_t = a_t/\sigma_t$, EGARCH(1,1) is written as

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1(|\varepsilon_{t-1}| + \gamma_1 \varepsilon_{t-1}) + \beta_1 \ln(\sigma_{t-1}^2) \tag{85.3g}$$

Since it is difficult to select the orders with the GARCH models, we choose GARCH(1,1) and EGARCH(1,1) because they are parsimonious yet provide a reasonable fit to the data.

85.3.2 *Dynamic covariance matrices estimation using GARCH models*

We follow Tsay (2010) to estimate the covariance matrix using the conditional covariance matrix with GARCH models, where applications are shown in time-varying correlation between returns of two stocks and time-varying betas under the CAPM. This chapter adopts this method for time-varying covariance estimation and applies it for portfolio allocation using the BL model. This estimation method is simpler than the commonly used exponentially weighted moving average (EWMA) method because it does not need to estimate the decay factor, yet the estimated time-varying volatility and correlation is of a similar pattern as those using the EWMA method.

Let $r_t = (r_{1t}, \ldots, r_{mt})'$ denote the return vector of m assets at time t. It is shown by Tsay (2010) that a multivariate GARCH model can be used to estimate the conditional covariance matrix, denoted by

$$\Sigma_t = [\Sigma_{ijt}] = Cov(r_t | F_{t-1}) \tag{85.3h}$$

where Σ_{ijt} is the ith row and jth column of Σ_t, F_{t-1} is the information available up to time $t-1$, and Cov is the covariance operator. For simplicity, we follow Tsay (2013) to estimate the conditional covariance matrix at time t with univariate GARCH models.

The diagonal element of the conditional covariance matrix Σ_t is the conditional variance of the ith return at time t. To estimate Σ_{iit}, we first estimate the conditional volatility of the ith return at time t with a specific GARCH model, denoted by σ_{it}. Here, we assume that

$$r_{it} = a_{it} \tag{85.3i}$$

$$a_{it} = \sigma_{it}\varepsilon_{it} \tag{85.3j}$$

where ε_{it} are innovations for $i = 1, \ldots, m$ and $t = 1, \ldots, T$. We estimate σ_{it} by maximum likelihood estimation using the series $\{r_{it}\}$ under the GARCH model, denoted by $\hat{\sigma}_{it}$. Then, we estimate Σ_{iit} by

$$\hat{\Sigma}_{iit} = \hat{\sigma}_{it}^2 \tag{85.3k}$$

To estimate Σ_{ijt}, note that

$$\begin{aligned} V(r_{it} + r_{jt}) &= V(r_{it}) + 2\,Cov(r_{it}, r_{jt}) + V(r_{jt}) \\ &= \Sigma_{iit} + 2\Sigma_{ijt} + \Sigma_{jjt} \end{aligned} \tag{85.3l}$$

$$\begin{aligned} V(r_{it} - r_{jt}) &= V(r_{it}) - 2\,Cov(r_{it}, r_{jt}) + V(r_{jt}) \\ &= \Sigma_{iit} - 2\Sigma_{ijt} + \Sigma_{jjt} \end{aligned} \tag{85.3m}$$

where $V(\cdot)$ is the variance operator. Therefore, we obtain

$$\Sigma_{ijt} = \frac{V(r_{it} + r_{jt}) - V(r_{it} - r_{jt})}{4}. \tag{85.3n}$$

Now, $V(r_{it} + r_{jt})$ and $V(r_{it} - r_{jt})$ are estimated using two artificial time series $r^{+}_{ijt} = r_{it} + r_{jt}$ and $r^{-}_{ijt} = r_{it} - r_{jt}$. Similarly, conditional volatilities for r^{+}_{ijt} and r^{-}_{ijt} are estimated, denoted by $\hat{\sigma}^{+}_{ijt}$ and $\hat{\sigma}^{-}_{ijt}$, respectively. Then, we estimate $V(r_{it} + r_{jt})$ and $V(r_{it} - r_{jt})$ by

$$V(r_{it} + r_{jt}) = \hat{\sigma}^{+}_{ijt} \tag{85.3o}$$

$$V(r_{it} - r_{jt}) = \hat{\sigma}^{-}_{ijt} \tag{85.3p}$$

Finally, we estimate $\hat{\Sigma}_{iit}$ by

$$\hat{\Sigma}_{ijt} = \frac{\hat{V}(r_{it} + r_{jt}) - \hat{V}(r_{it} - r_{jt})}{4} \tag{85.3q}$$

85.4 Empirical Analysis

In this section, the daily closing values of five industry indexes in Taiwan are used to demonstrate the superiority of our proposed heteroscedastic BL model. Recall that in Section 85.1, we collected daily log-returns using closing indexes for five major industry indexes, including food, textile, iron and steel, electronic, and financial sectors in Taiwan stock markets from January 6, 2010 to December 31, 2014, a total of 1,024 observations. According to Taiwan Stock Exchange (TSE) on January 23, 2018, 74% of market capitalization is composed of these five industries: electronic sector (56%), financial sector (13%), food sector (2%), textile sector (2%), and iron and steel sector (2%), as summarized in Figure 85.2. Consequently, Taiwan stock markets are mainly composed of these major five industries, and they approximately span the whole stock market and make the reverse optimization possible.

85.4.1 *Study plan*

To begin with, we consider three categories of risk preferences for investors: mildly risk averse ($\lambda = 1$), moderately risk averse ($\lambda = 5$), and severely risk averse ($\lambda = 10$).

Now, we propose a dynamic rolling window scheme as follows. A rolling window consists of two parts: the training period of size N_{train} days and the holding period of size N_{hold} days. Hence, the size of the rolling window is of $(N_{train} + N_{hold})$ days. Given a rolling window, we use the data in the training period to estimate parameters and obtain optimal portfolio weight, then we form the portfolio and hold it during the holding period to measure

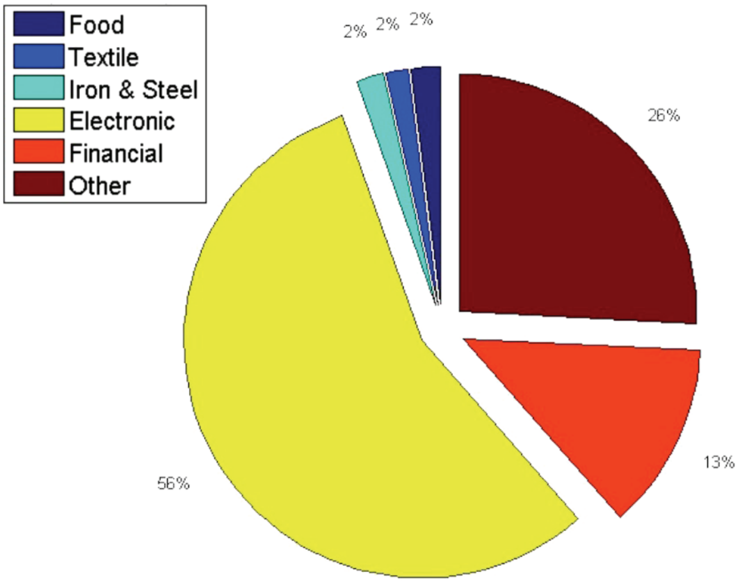

Figure 85.2. Percentage of market capitalization per industry.
Source: Taiwan Stock Exchange.

its performance in terms of various strategy summary statistics. Then, we move the rolling window N_{hold} days forward and repeat the above procedures till the end of the study period.

In our study, we set N_{train} as 124 days and N_{hold} as 60 days to measure out-of-sample performance. In detail, when N_{train} is 124 and N_{hold} is 60, the parameter is estimated using samples from days 1–124 to obtain the portfolio weight, and the portfolio is reallocated with the optimal portfolio weight and held for the next 60 days, i.e., days 125–184. Then, we move the rolling window 60 days further. The next iteration uses samples from days 61–184 to estimate the parameter and obtain the optimal portfolio weight, and the portfolio is held with the updated portfolio weight for the next 60 days, i.e., days 185–234.

By repeating this procedure, a dynamically adjusted portfolio allocation for a total of 1,240 days is constructed. We remark that the selection of N_{train} to be 60 is subjective. However, our empirical analysis shows that it can reasonably avoid frequent trading and control transaction cost. How to select N_{train} and N_{hold} is beyond the scope of this chapter.

For comparison, we first consider the $1/N$ portfolio as a benchmark. The $1/N$ portfolio uses equal weight for each assets. Furthermore, we consider the Markowitz portfolio. The Markowitz model is also called mean-variance model due to the fact that its portfolio weight is selected based on expected returns (mean) and the risk (standard deviation) of the various assets. The weight of the Markowitz portfolio solves the following constrained optimization problem:

$$\begin{aligned} \max_{\omega} \quad & \omega' \mu_{BL} - \frac{\lambda}{2} \omega' \Sigma \omega \\ s.t. \quad & \sum_{i=1}^{m} \omega_i = 1, \ \omega_i \geq 0. \end{aligned} \tag{85.4a}$$

To avoid short selling, a positive constraint is placed on the portfolio weight. Here, μ and Σ are estimated as sample mean and sample covariance matrix using data in the training period.

For the BL portfolio, we consider four models to estimate the covariance matrix parameter: i.i.d. normal, Gn, Gt, and EGt. The later three models present volatility clustering, volatility clustering with a heavy-tailed distribution, and volatility clustering with a heavy-tailed distribution and asymmetric leverage effects, respectively. For notations, we abbreviate these BL portfolios by BL, BL-Gn, BL-Gt, and BL-EGt, respectively. In addition, the absolute and relative views are employed separately as investors' views. Consequently, there are eight versions of BL portfolios. For BL portfolio, the covariance matrix Σ is estimated using sample covariance. For BL-G, BL-EG, and BL-EGt portfolios, the covariance matrix is estimated with univariate GARCH models, as described in Section 85.3.2. We recall that the weight of the BL portfolio solves the constrained optimization problem in Eq. (85.2j).

Portfolio performance is compared in terms of cumulative returns, standard deviations, averaged returns, Sharpe ratios, and beta values. Note that the beta value is the covariance between portfolio returns and market returns divided by the variance of market returns.

85.4.2 *Empirical comparisons*

Figure 85.3 summarizes average weights in different portfolios: Markowitz portfolio, BL portfolio, BL-Gn portfolio, BL-Gt portfolio, and BL-EGt portfolio, at three levels of risk aversion $\lambda = 1, 5$, and 10. For BL portfolios, the absolute and relative views are separately considered. Compared with the Markowitz portfolio, it appears that BL portfolios in general prefer the iron and steel sector to the textile sector.

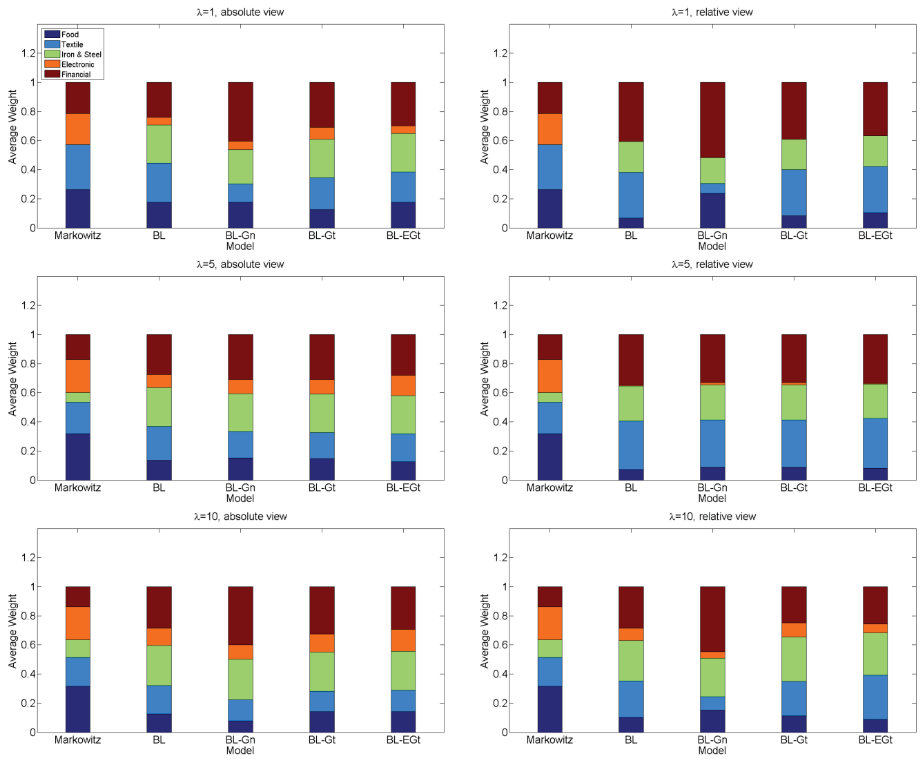

Figure 85.3. Average weight of the Markowitz portfolio, BL portfolio, BL-Gn portfolio, BL-Gt portfolio, and BL- EGt portfolio, during a total of 1,240 days at various levels of risk aversion ($\lambda = 1, 5$, and 10) with $N_{\text{train}} = 124$. For BL portfolios, the absolute and relative views are separately considered.

Table 85.2 summarizes the empirical results in terms of cumulative returns, standard deviations, average returns, beta values, and ratios of the averaged return to the standard deviation.

Figure 85.4 plots cumulative returns for 10 portfolios at every 60 days for N_{train} to be 124 days.

In terms of investors' views, it is shown that, in general, BL portfolios with relative view outperforms BL with absolute views in terms of average returns and Sharpe ratios. When risk-aversion coefficient is neutral, the BL-EGt portfolio with the relative view has the highest average return and Sharpe ratio (22.11% and 1.26, respectively).

The average return and Sharpe ratio of the BL-EGt portfolio is slightly higher than that of the BL-Gt and BL-Gn portfolios but significantly higher than the BL portfolio. The beta values for all portfolios are less than 1. This indicates that all these portfolio returns are less volatile compared with the market return.

Table 85.2. Strategy summary statistics of the $1/N$ portfolio, Markowitz portfolio, BL portfolio, BL-Gn portfolio, BL-Gt portfolio, and BL-EGt portfolio, at three levels of risk aversion ($\lambda = 1, 5$, and 10) with $N_{\text{train}} = 124$. For BL portfolios, the absolute and relative views are separately considered.

Portfolio	Averaged return		Std		Beta		Sharpe ratio	
$\lambda = 1$								
$1/N$	0.1115		0.1422		0.8708		0.7225	
Markowitz	0.0656		0.1859		0.9544		0.3057	
	Absolute	Relative	Absolute	Relative	Absolute	Relative	Absolute	Relative
BL	0.1289	0.1436	0.163	0.175	0.8077	0.9031	0.7363	0.7705
BL-Gn	−0.0075	0.0353	0.1969	0.1978	0.9517	0.9372	−0.0827	0.1341
BL-Gt	0.1606	0.1655	0.1613	0.1735	0.8341	0.8779	0.9412	0.9032
BL-Egt	0.1336	0.1919	0.1638	0.1743	0.8269	0.8469	0.762	1.0507
$\lambda = 5$								
$1/N$	0.1115		0.1422		0.8708		0.7225	

(*Continued*)

Table 85.2. (*Continued.*)

Markowitz	0.1062		0.1813		0.947		0.5372	
	Absolute	Relative	Absolute	Relative	Absolute	Relative	Absolute	Relative
BL	0.1493	0.2052	0.1572	0.165	0.8134	0.8271	0.8935	1.1909
BL-Gn	0.1578	0.2098	0.1599	0.1652	0.8357	0.8253	0.9316	1.2165
BL-Gt	0.1611	0.2128	0.16	0.1655	0.8362	0.8256	0.9514	1.2328
BL-Egt	0.1721	0.2211	0.1604	0.1679	0.8438	0.8364	1.0178	1.2643
$\lambda = 10$								
1/N	0.1115		0.1422		0.8708		0.7225	
Markowitz	0.1041		0.1738		0.9163		0.5483	
	Absolute	Relative	Absolute	Relative	Absolute	Relative	Absolute	Relative
BL	0.1735	0.1591	0.1536	0.1584	0.8073	0.7947	1.0719	0.9485
BL-Gn	0.0011	−0.0142	0.1758	0.1807	0.9271	0.9072	−0.0437	−0.1274
BL-Gt	0.185	0.1598	0.1558	0.158	0.831	0.7962	1.131	0.9556
BL-Egt	0.2056	0.1776	0.1547	0.1601	0.8342	0.8021	1.2727	1.0544

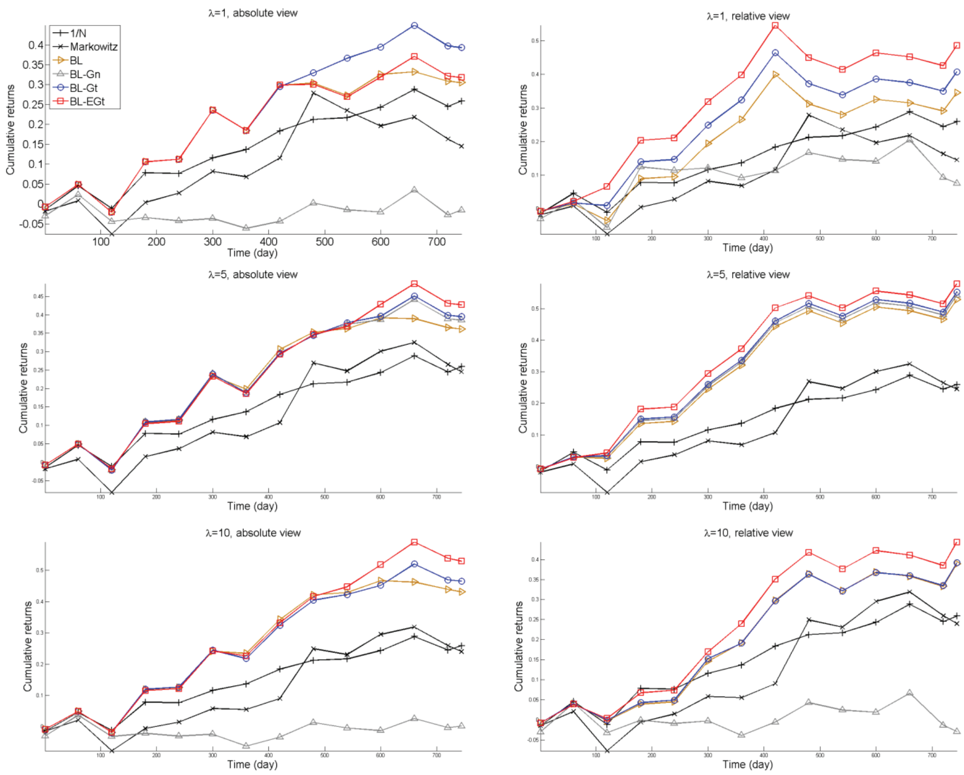

Figure 85.4. Cumulative returns of the $1/N$ portfolio, Markowitz portfolio, BL portfolio, BL-Gn portfolio, BL-Gt portfolio, and BL-EGt portfolio, at every 60 days at various levels of risk aversion ($\lambda = 1, 5$, and 10) with $N_{\text{train}} = 124$. For BL portfolios, the absolute and relative views are separately considered.

Finally, both BL-EGt and BL-Gt portfolios outperform the $1/N$ and Markowitz portfolios in terms of average returns and Sharpe ratios. Specifically, BL-EGt portfolio dominates the BL-Gt portfolio.

Based on the predictive regression in Eq. (85.2k), investors with the absolute view pose m views, where each view assigns an expected return to each asset. In this case, each asset return is adjusted by the investors views, and it is likely that such adjustment does not yield higher predictability. By contrast, the relative view only poses one view: This view represents the difference between the asset of the highest expected return and that of the lowest return. Our empirical analysis shows that portfolios with the BL model with the relative view outperform those with the absolute view in terms of averaged returns in many cases, which highlights the benefits of using the relative view.

The relative view is in spirit similar to the momentum strategies (Chan *et al.*, 1996), where stocks are first ranked by their returns into five return levels and a portfolio is composed of a long position in stocks of the highest return level and a short position in stocks of lowest return level. It is hence possible that prediction for the difference between the highest and lowest returns (as in the relative view) is more accurate than prediction for all returns (as in the absolute view).

85.5 Conclusion

Portfolio management is an active and exciting area for both academics and practitioners. It has been rapidly improved together with the growing data science and application programming interfaces (APIs) economy in today's digital economy. This chapter improves the standard BL portfolios by considering a more sophisticated estimation scheme for the covariance matrix with heteroscedastic models under the framework of the BL framework. For empirical demonstration, this chapter compared the $1/N$ portfolio, Markowitz portfolio, and eight versions of BL portfolios. Empirical analysis is applied to five industry indexes in Taiwan.

For the BL models, the standard i.i.d. normal model and three types of GARCH model are used to estimate the covariance matrix. In addition, the absolute and relative views are separately induced in the BL model. Our empirical analysis shows that the BL portfolios with the relative view outperform those with the absolute view for mildly and moderately risk averse investors. In addition, the BL portfolio with EGARCH model, with both absolute and relative views, outperform the other portfolios.

A successful application of the BL model is indeed difficult because it needs a good estimation of the investor's views and their precision matrix, which is unfortunately not easier than the original estimation problem. With the advanced developments in data science and summarized views from the great investors reported by Longo (2021), the BL model is likely to obtain more advanced views and would definitely deserve further research.

References

Allaj, E. (2013). The Black-Litterman model: A consistent estimation of the parameter tau. *Financial Markets and Portfolio Management*, 27, 217–251.

Almadi, H., D. E. Rapach, and A. Suri (2014). Return predictability and dynamic asset allocation: How often should investors rebalance? *Journal of Portfolio Management*, 40, 16–27.

Avellaneda, M. and J. H. Lee (2010). Statistical arbitrage in the US equities market. *Quantitative Finance*, 10, 761–782.

Beach, S. L. and A. G. Orlov (2007). An application of the Black-Litterman model with EGARCH-M-derived views for international portfolio management. *Financial Market Portfolio Management*, 21, 147–166.

Black, F. and R. Litterman (1991). Asset allocation: Combing investors view with market equilibrium. *Journal of Fixed Income*, 1, 7–18.

Black, F. and R. Litterman (1992). Global portfolio optimization. *Financial Analysts Journal*, 48, 28–43.

Brianton, G. (1998). Portfolio optimization. *Risk Management and Financial Derivatives: A Guide to the Mathematics, 1st edition, Palgrave (trade)*, 4, 39–44.

Cartwright, P. and C. F. Lee (1987). Time aggregation and the estimation of the market model: Empirical evidence. *Journal of Business and Economic Statistics*, 5, 131–143.

Chan, L. K. C., N. Jeqadeesh, and J. Lakonishok (1996). Momentum strategies. *Journal of Finance*, 51, 1681–1713.

Chen, N., R. Roll, and S. Ross (1986). Economic forces and the stock market. *Journal of Business*, 59, 383–403.

Chopra, V. K., C. R. Hensel, and A. L. Turner (1993). Massaging mean-variance inputs: Returns from alternative global investment strategies in the 1980s. *Management Science*, 39, 845–855.

Christodoulakis, G. A. (2002). Bayesian optimal portfolio selection: The Black-Litterman approach. *Goldman Sachs Fixed Income Research Paper*.

Disatnik, D. and S. Benninga (2007). Shrinking the covariance Matrix-Simpler is better. *Journal of Portfolio Management*, 33, 56–63.

Elton, E. and M. Gruber (1973). Estimating the dependence structure of share prices. *Journal of Finance*, 28, 1203–1232.

Guo, X., T. Lai, H. Shek, and S. Wong (2017). *Quantitative Trading: Algorithms, Analytics, Data, Models, Optimization.* Chapman and Hall/CRC.

Gu, S., B. Kelly, and D. Xiu (2020). Empirical asset pricing via machine learning. *The Review of Financial Studies*, 33(5), 2223–2273.

He, G. and R. Litterman (1999). The intuition behind Black-Litterman Model Portfolios. *Goldman Sachs Fixed Income Research paper*.

Idzorek, T. (2004). A step-by-step guide to the Black-Litterman model. Working paper.

Jurczenko, E. and J. Teiletche (2018). Active risk-based investing. *Journal of Portfolio Management*, 44, 56–65.

Ledoit, O. and M. Wolf (2003a). Honey, I shrunk the sample covariance matrix. *Journal of Portfolio Management*, 30, 110–119.

Ledoit, O. and M. Wolf (2003b). Improved estimation of the covariance matrix of stock returns with an application to portfolio selection. *Journal of Empirical Finance*, 10, 603–621.

Litterman, R. and K. Winkelmann (1998). Estimating covariance matrices. *Goldman Sachs, Risk Management Series*.

AQ02

Longo, J. M. (2021). Lessons on risk, returns, and portfolio construction from the great investors. Cheng Few Lee, Alice Lee, and John Lee, editors, Chapter 64. Handbook of Investment Analysis, Portfolio Management, and Financial Derivatives, forthcoming.

Mandelbrot, B. (1963). The variation of certain speculative prices. *Journal of Business*, 36, 394–419.

Markowitz, H. (1952). Portfolio selection. *Journal of Finance*, 7, 77–91.

AQ03

Meucci, A. (2010). The Black-Litterman approach: Original model and extensions. *The Encyclopedia of Quantitative Finance.*

Michaud, R. O. (1989). The Markowitz optimization enigma: Is 'optimized' optimal? *Financial Analysts Journal*, 45, 31–42.

Michaud, R. O. and D. N. Esch (2013). Deconstructing Black-Litterman: How to get the portfolio you already knew you wanted. *Journal of Investment Management*, 11, 6–20.

Morgan, I. G. (1976). Stock prices and heteroscedasticity. *Journal of Business*, 49, 496–508.

Nelson, B. (1991). Conditional heteroskedasticity in asset return: A new approach. *Econometrica: Journal of the Econometric Society*, 59, 347–370.

O'toole, R. (2013). The Black–Litterman model: A risk budgeting perspective. *Journal of Asset Management*, 14, 2–13.

Satchell, S. and A. Scowcroft (2000). A demystification of the Black-Litterman model: Managing quantitative and traditional portfolio construction. *Journal of Asset Management*, 1, 138–150.

Shumpeter, J. (1942). *Capitalism, Socialism, and Democracy.* New York: Harper & Bros.

Sharpe, W. (1963). A simplified model for portfolio analysis. *Management Science*, 9, 277–293.

Torun, M., A. Akansu, and M. Avellaneda (2011). Portfolio risk in multiple frequencies. *IEEE Signal Processing Magazine*, 28, 61–71.

Tsay, R. S. (2010). *Analysis of Financial Time Series.* 3 ed. Hoboken, NJ: Wiley.

Tsay, R. S. (2013). *An Introduction to Analysis of Financial Data with R.* 3 ed. Hoboken, NJ: Wiley.

Walters, J. (2014). The Black-Litterman model in detail. C. F. A.