SVCJ Model
(Stochastic Volatility with Correlated Jumps)

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SV Model

Expected log return

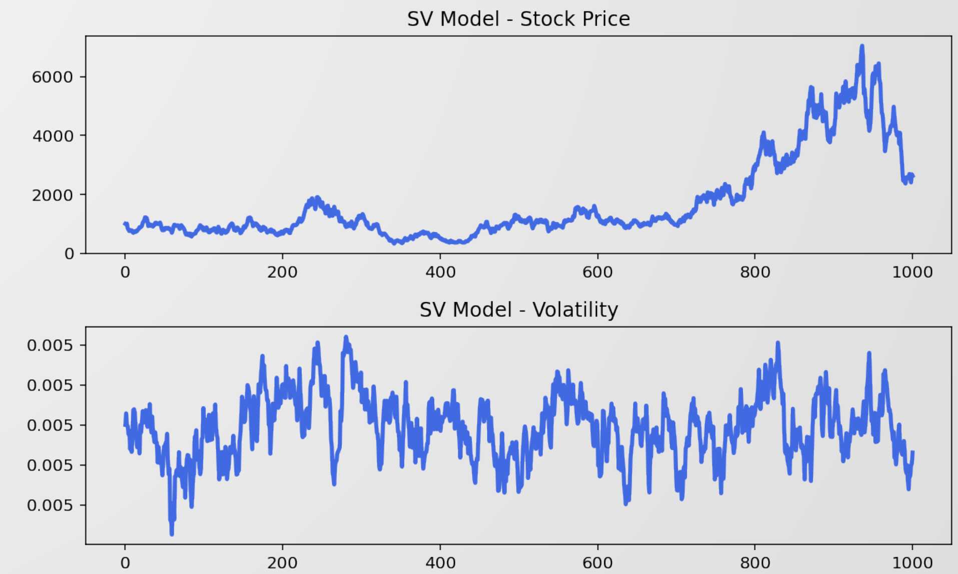
$$\frac{dS_t}{S_t} = \mu dt + \sqrt{V_t} dW_t^s$$

Brownian motion
$dW \sim N(0, 1)$

$$dV_t = \kappa \left(\theta - V_t\right) dt + \sigma_v \sqrt{V_t}\, dW_t^v$$

CIR model

$$\text{Cov}\left(dW_t^s, dW_t^v\right) = \rho dt$$



Parameter: $\left(\mu, \rho, \kappa, \theta, V_0, \sigma_v\right)$

SVJ Model

Jump size

$$\frac{dS_t}{S_t} = \mu dt + \sqrt{V_t} dW_t^s + Z_t^y dN_t$$

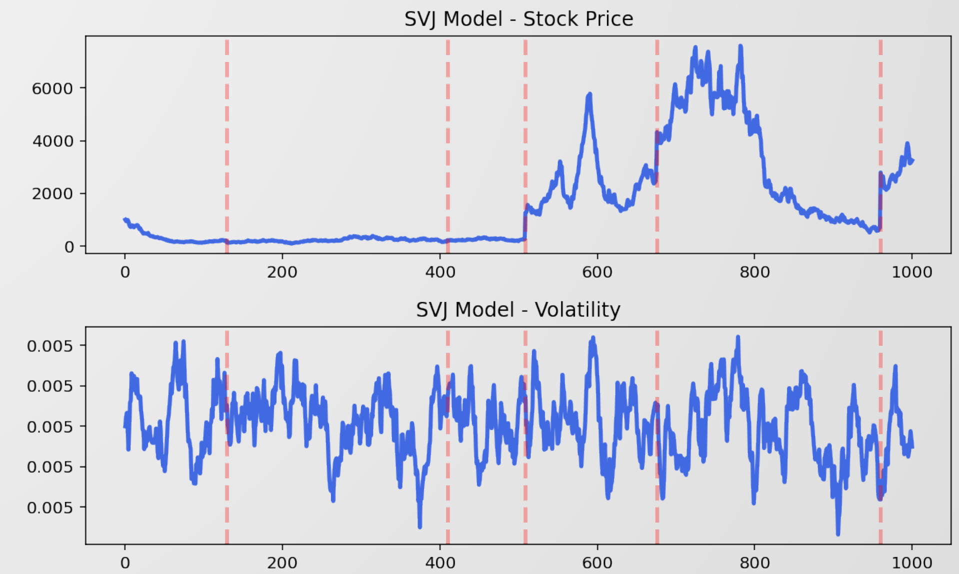
Jump process

$$dV_t = \kappa \left(\theta - V_t\right) dt + \sigma_v \sqrt{V_t}\, dW_t^v$$

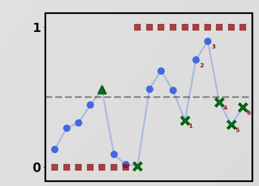
$$\text{Cov}\left(dW_t^s, dW_t^v\right) = \rho dt$$

$$P\left(dN_t = 1\right) = \lambda dt$$

$$Z_t^y \sim N\left(\mu_y, \sigma_y^2\right)$$



Parameter: $\left(\mu, \rho, \kappa, \theta, V_0, \sigma_v, \lambda, \mu_y, \sigma_y\right)$



SVCJ Model

$$\frac{dS_t}{S_t} = \mu dt + \sqrt{V_t} dW_t^s + Z_t^y dN_t$$

Jump size

$$dV_t = \kappa\left(\theta - V_t\right) dt + \sigma_v \sqrt{V_t}\, dW_t^v + Z_t^v dN_t$$

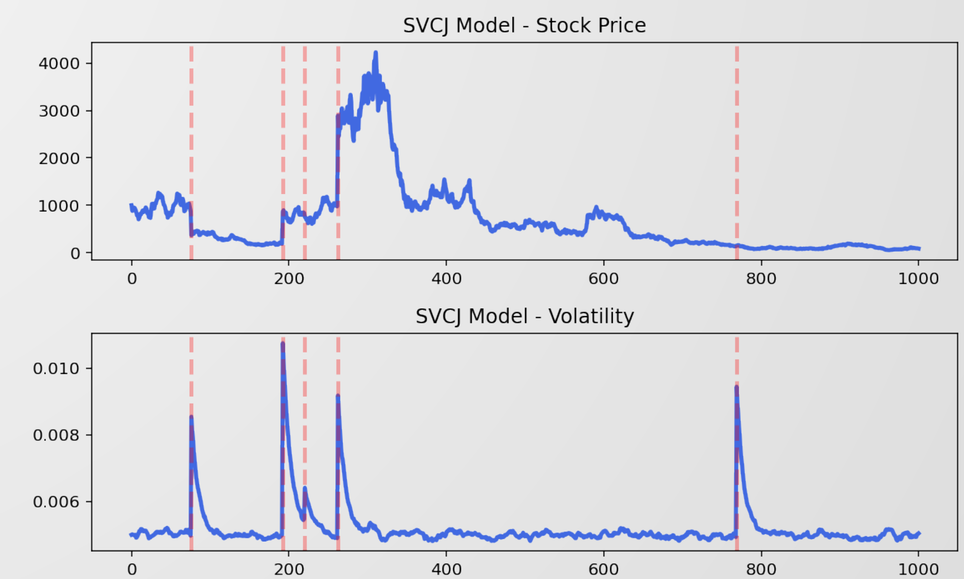
Jump process

$$\text{Cov}\left(dW_t^s, dW_t^v\right) = \rho dt$$

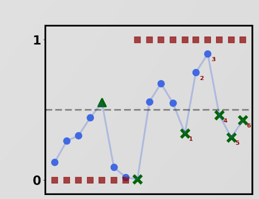
$$P\left(dN_t = 1\right) = \lambda dt$$

$$Z_t^y \Big| Z_t^v \sim N\left(\mu_y + \rho_j Z_t^v, \sigma_y^2\right)$$

$$Z_t^v \sim \text{Exp}\left(\mu_v\right)$$



Parameter: $\left(\mu, \rho, \kappa, \theta, V_0, \sigma_v, \lambda, \mu_y, \sigma_y, \rho_j, \mu_v\right)$



Euler-Maruyama discretization

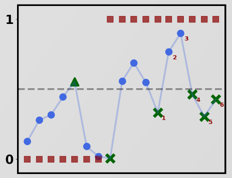
- Define $Y_{t+1} = \log\left(\frac{S_{t+1}}{S_t}\right), t = 1, \cdots, T$

$$\Longrightarrow S_t = S_0 \exp\left(Y_1 + \cdots + Y_t\right), t = 1, \cdots, T$$

- Set $\alpha = \kappa\theta$, $\beta = 1 - \kappa$
- Then,

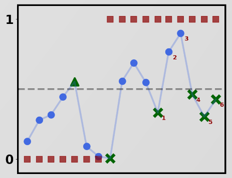
$$Y_t = \mu + \sqrt{V_{t-1}}\varepsilon_t^y + Z_t^y J_t,$$
$$V_t = \alpha + \beta V_{t-1} + \sigma_v \sqrt{V_{t-1}}\varepsilon_t^v + Z_t^v J_t$$

- where $\varepsilon_t^t, \varepsilon_t^v \sim N(0, 1), P\left(J_t = 1\right) = \lambda$



Parameter of SV, SVJ, SVCJ Models

- SV : $\Theta = \left(\mu, \rho, \alpha, \beta, V_0, \sigma_v\right), \ \left|\Theta\right| = 6$
- SVJ : $\Theta = \left(\mu, \rho, \alpha, \beta, V_0, \sigma_v, \lambda, \mu_y, \sigma_y\right), \ \left|\Theta\right| = 9$
- SVCJ : $\Theta = \left(\mu, \rho, \alpha, \beta, V_0, \sigma_v, \lambda, \mu_y, \sigma_y, \rho_j, \mu_v\right), \ \left|\Theta\right| = 11$



Estimate the parameters of SVCJ

- Use p to denote the market price; m to denote model price
- Market price: $p(\omega, \tau, K)$
- Model price: $m(\Theta; \omega, \tau, K)$
- Define the loss function as

$$\mathscr{L}(\Theta) = \sum_i \left\{ p(\omega_i, T_i, K_i) - m(\Theta; \omega_i, T_i, K_i) \right\}^2$$

$$\approx \sum_i \left\{ p(\omega_i, T_i, K_i) - \widehat{m}(\Theta; \omega_i, T_i, K_i) \right\}^2$$

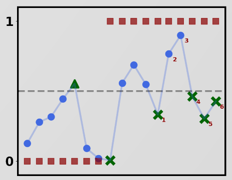
Approximated by Monte Carlo Simulation

Estimate the parameters of SVCJ

- The parameters are calibrated by minimizing the loss function

$$\Theta^* = \arg\min_{\Theta \in \mathscr{A}} \mathscr{L}(\Theta)$$

- $\mathscr{A}$ is defined to guarantee the positiveness of the CIR models.
  - Feller condition: $2\kappa\theta > \sigma_v^2$, or, $2\alpha > \sigma_v^2$.
  - Non-negativity Conditions:
    $\kappa > 0,\ \theta > 0,\ V_0 > 0$, or, $\alpha > 0,\ \beta < 1,\ V_0 > 0$.
  - In the SVCJ model, there should be another restriction $\mu_v \geq 0$



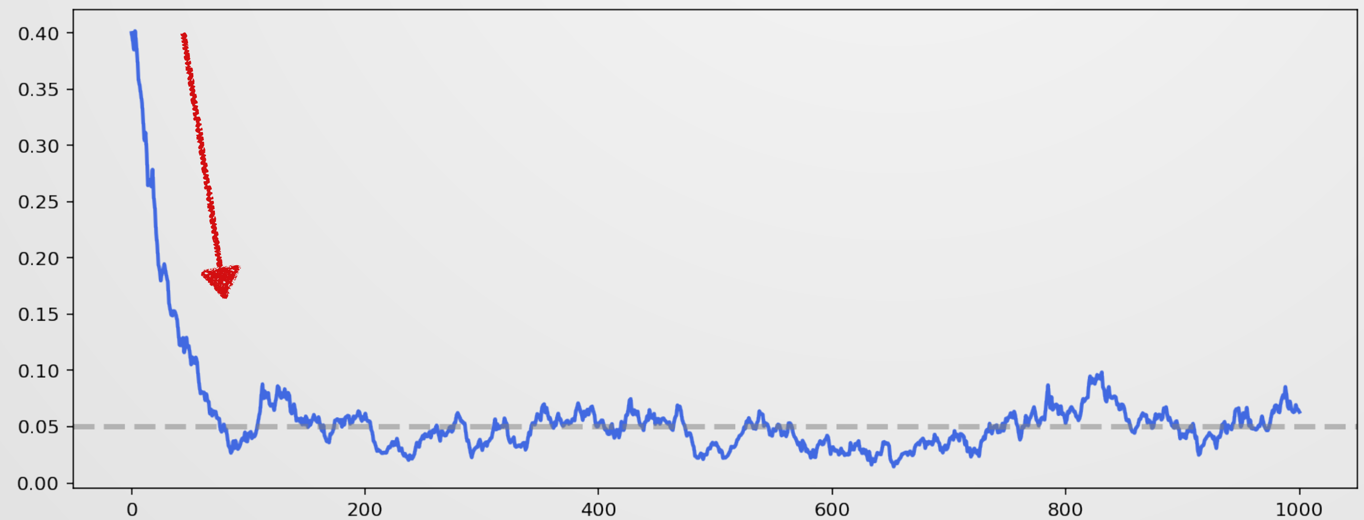
CIR (Cox-Ingersoll-Ross) Model

Back

- Originally describes the evolution of interest rates
- Also suitable for describing non-negative variables such as volatility
- One-factor model (driven by standard Wiener's process)

$$dV_t = \kappa\left(\theta - V_t\right)dt + \sigma_v\sqrt{V_t}\,dW_t^v$$

- Applying the Feller condition $2\kappa\theta > \sigma_v^2$ to make sure it won't reach 0



$$\begin{aligned} \kappa &= 0.2 \\ \theta &= 0.05 \\ \sigma_v &= 0.02 \\ V_0 &= 0.4 \\ T &= 1000 \end{aligned}$$