

hand written notes university ELEC 202 101 2022WI Circuit WebWork : 2022WI\_ELEC\_201\_2022WI/CPX/1?effectiveUser=OC68NRJB700

Problem 1 User Settings Grades Problems Problem 1 Problem 2 Problem 3 Problem 4 Problem 5 Problem 6

Let  $z_1 = 8.1 + j \cdot 3$ ,  $z_2 = 3.9 + j \cdot 3$ ,  $z_3 = 11.6e^{j(-16)^\circ}$  and  $z_4 = 16.5e^{j(101)^\circ}$  (Hint: these values will be repeatedly used in this exercise so it may help to save these as variables in your calculator). Compute the following quantities in both Cartesian form and polar form:

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a.  $z_1 + z_2 =$  [ ] + j [ ] = [ ]  $\angle$  [ ] °  
b.  $z_3 + z_4 =$  [ ] + j [ ] = [ ]  $\angle$  [ ] °  
c.  $\frac{1}{z_1} =$  [ ] + j [ ] = [ ]  $\angle$  [ ] °  
d.  $\frac{1}{z_3} =$  [ ] + j [ ] = [ ]  $\angle$  [ ] °  
e.  $\sqrt{z_3} =$  [ ] + j [ ] = [ ]  $\angle$  [ ] °  
f.  $\frac{z_1}{z_2} =$  [ ] + j [ ] = [ ]  $\angle$  [ ] °  
g.  $\frac{z_3}{z_4} =$  [ ] + j [ ] = [ ]  $\angle$  [ ] °  
h.  $z_3 z_4^* =$  [ ] + j [ ] = [ ]  $\angle$  [ ] °

[Note: The objectives of this exercise are firstly, to gain experience using your calculator for some simple complex number entry and operations, and secondly to practice visualizing numbers in the complex plane. For each of the exercises (a)-(h), you should visualize both the operand(s) and the result in the complex plane and recognize the procedure of using either the polar or Cartesian forms for each operation; you should develop enough proficiency that each of those parts can be estimated in less than 10 seconds.]

Note: You can earn partial credit on this problem.

MacBook Pro

Let  $z_1 = 1 + j \cdot 1.2$ ,  $z_2 = 10 + j \cdot 6.2$ ,  $z_3 = 10.3e^{j(-123)^\circ}$  and  $z_4 = 14.1e^{j(145)^\circ}$  (Hint: these values will be repeatedly used in this exercise so it may help to save these as variables in your calculator). Compute the following quantities in the form indicated (Cartesian or polar):

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a.  $(\sqrt{3} \angle 30^\circ)z_2 - \frac{z_1^*}{z_4} e^{j(\frac{1}{3}\pi)} =$  [ ] + j [ ]  $\angle$  [ ] °  
b.  $(\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3})^{-1} =$  [ ] + j [ ] °  
c.  $\frac{z_1^* + z_3}{z_4 z_3} + z_2 =$  [ ] + j [ ] °  
d.  $\sqrt{z_3} - \frac{z_1 z_2}{z_4 - \frac{j}{1-z_3}} =$  [ ] + j [ ]  $\angle$  [ ] °

[Note: For each of the exercises (a) to (d), you are primarily gaining practice using the calculator.]

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a.  $\begin{cases} z_1 + z_2 = 10.8e^{j(-10)^\circ} \\ 2z_1 + 3z_2 = 9.6e^{j(11)^\circ} \end{cases}$

$e_1 := z_1 + z_2 = 10.8e^{j(-10)^\circ}$  solve ( $\{e_1, e_2, z_1, z_2\}$ )  
 $e_2 := \dots$

$z_1 =$  [ ] + j [ ]  
 $z_2 =$  [ ] + j [ ]

b.  $\begin{bmatrix} \frac{1}{3} + \frac{1}{j10} + \frac{1}{-j2} & \frac{-1}{j10} & 0 \\ \frac{-1}{j10} & \frac{1}{j10} + \frac{1}{11} & 8 \\ \frac{1}{j2} & 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} -9 + \frac{16}{3} \\ 9 \\ 0 \end{bmatrix}$

$z_1 =$  [ ] + j [ ]  
 $z_2 =$  [ ] + j [ ]  
 $z_3 =$  [ ] + j [ ]

c.  $\begin{cases} \frac{z_1 - z_2}{6} + \frac{z_1}{-j8} + z_3 = 0 \\ \frac{z_2}{12} + \frac{z_2 - 12}{j4} - z_3 = 0 \\ z_3 = 0.2(12 - z_1) \end{cases}$

$z_1 =$  [ ] + j [ ]  
 $z_2 =$  [ ] + j [ ]  
 $z_3 =$  [ ] + j [ ]

For any pair of numbers  $(A, B)$  ( $A \in \mathbb{R}$  and  $B \in \mathbb{R}$ ), one can find another pair of numbers  $(C, \theta)$  ( $C \in \mathbb{R}_+$  and  $-180^\circ < \theta \leq 180^\circ$ ), such that  $A\cos(\omega t) + B\sin(\omega t) = C\cos(\omega t + \theta)$ . Find the relationship between the two pairs of numbers (i.e., find the functions  $A(C, \theta), B(C, \theta), C(A, B)$ , and  $\theta(A, B)$  and use this to fill in the blanks in the following equations.)

**Note:** In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a.  $8.9\cos(\omega t) + 0.8\sin(\omega t) = \boxed{\quad} \cos(\omega t + \boxed{\quad})^\circ$

b.  $12.4\cos(\omega t + 44^\circ) = \boxed{\quad} \cos(\omega t + \boxed{\quad}) \sin(\omega t)$

$$A\cos(\omega t) + B\sin(\omega t) = C\cos(\omega t - \theta)$$

$$C = \sqrt{A^2 + B^2}^{1/2} \quad \theta = \arctan\left(\frac{B}{A}\right)$$

$$\tan(-24^\circ) = \frac{b}{a}$$

calc.

$$8.9\cos(\omega t) + 0.8\sin(\omega t)$$

A B

Use triangle congruence theorems to solve the following problems:

**Note:** In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a. Given  $z_1 = 7.9\angle 0^\circ$ ,  $|z_2| = 6$ ,  $\angle(z_1 + z_2) = -41^\circ$ , determine the two possible values for  $z_2$ . (Hint: This is an SSA problem.)

$$z_2 = \boxed{\quad} + j \boxed{\quad} \text{ or }$$

$$z_2 = \boxed{\quad} + j \boxed{\quad} \text{ or }$$

b. Given  $z_1 = 3.1\angle 0^\circ$ ,  $|z_2| = 8.6$ ,  $|z_3| = 8.5$ , and  $z_3 = z_1 + z_2$  determine the two possible values for  $z_2$ . (Hint: This is an SSS problem.)

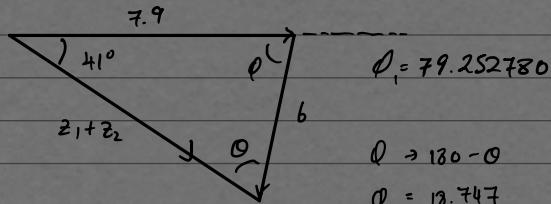
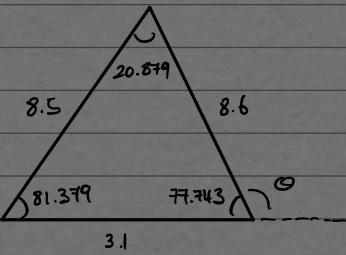
$$z_2 = \boxed{\quad} + j \boxed{\quad} \text{ or }$$

$$z_2 = \boxed{\quad} + j \boxed{\quad}$$

a)  $z_1 = 7.9\angle 0^\circ \quad |z_2| = 6 \quad \angle(z_1 + z_2) = -41^\circ$

$$\angle z_1 = 0^\circ$$

b)



$$\theta = 79.252780$$

$$\theta \rightarrow 180 - \theta$$

$$\theta = 18.747$$

$$6 \angle (180 - 79.252) = -5.681 - 1.928i$$

$$6 \angle (180 - 18.747) = -1.1188 - 5.8947i$$

$$8.6 \angle (180 - 77.743) = -1.8257 + 8.404i$$

or

$$-1.8257 - 8.404i$$

as triangle has a set shape due to length being pre-defined.

For each expression, find the positive real value of  $\omega$  that would cause  $R_{eq}$  to be purely real, as well as the resulting  $R_{eq}$ . Note that // is the binary operator that finds the equivalent resistance of two resistors in parallel (i.e.,  $a//b = \frac{ab}{(a+b)}$ ).

[Hint: For (a), try setting the imaginary coordinate to be zero. For (b), try equating the numerator and denominator angles].

**Note:** In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a.  $R_{eq} = \frac{1}{j \cdot 18.9\omega} + (j \cdot 158.876\omega) // 8.7$

$\omega = \boxed{\quad}$  and  $R_{eq} = \boxed{\quad}$

b.  $R_{eq} = (2.2 + \frac{1}{j \cdot 18.9\omega}) // (j \cdot 158.876\omega)$

$\omega = \boxed{\quad}$  and  $R_{eq} = \boxed{\quad}$

$$a) R_{eq} = \frac{1}{j \cdot 18.9\omega} + (j \cdot 158.876\omega) // 8.7$$

$$= \frac{1}{18.9\omega j} + \frac{158.876\omega j (8.7)}{158.876\omega j + 8.7}$$

↓

$$\frac{1382\omega j}{158.876\omega j + 8.7}$$

↓

$$\frac{1382\omega j (-158.876\omega j + 8.7)}{(158.876\omega j + 8.7)(158.876\omega j + 8.7)}$$

↓

$$\frac{219566.632\omega^2 + 12023.4\omega j}{-25241.58\omega^2 + 75.69} \Rightarrow I_m = \frac{1}{18.9\omega j} + \frac{12023.4\omega j}{-25241.58\omega^2 + 75.69}$$

↓

$$\frac{1}{18.9\omega} - \frac{1}{-25241.58\omega^2 + 75.69} = 0$$

$$\omega = \pm 0.019357$$

$$\frac{-1}{18.9\omega} + \frac{12023.4\omega j}{-25241.58\omega^2 + 75.69}$$

$$j \left( \frac{-1}{18.9\omega} + \frac{12023.4\omega j}{-25241.58\omega^2 + 75.69} \right)$$

$$R_{eq} = \text{Re}(")$$

$$= \frac{219566.632\omega}{\omega = 0.019357}$$

$$= 0.965 + j$$

b)  $\left[ 2.2 + \left( \frac{1}{18.9\omega j} \right) \right] [158.876\omega j]$

$$\left[ 2.2 + \left( \frac{1}{18.9\omega j} \right) \right] + [158.876\omega j]$$

↓

$$(349.5272\omega j + 8.406) (2.2 - j(158.876\omega j + \frac{1}{18.9\omega}))$$

$$2.2 + j(158.876\omega j + \frac{1}{18.9\omega}) (2.2 - j(158.876\omega j + \frac{1}{18.9\omega}))$$

$$\frac{1}{18.9\omega j} = -\frac{j}{18.9\omega}$$

$$349.5272\omega j \cdot 2.2 - 349.5272\omega j (158.876\omega j + \frac{1}{18.9\omega}) + 8.406(2.2) - 8.406j(158.876\omega j)$$

$$2.2^2 - (158.876\omega j + \frac{1}{18.9\omega})^2$$

$$\therefore I_m(") = 349.5272(2.2)\omega - 8.406(158.876\omega - \frac{1}{18.9\omega}) - \frac{1}{2.2^2 + (158.876\omega + \frac{1}{18.9\omega})^2}$$

$$0 =$$

$$\therefore \omega \approx 0.028$$

$$= 0.028$$

$$R_{eq} = \text{Re}(") | \omega = 0.028$$

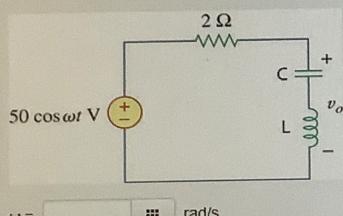
$$= 349.5272\omega (158.876\omega + 8.406(2.2))$$

$$= 3.823\omega$$

1.

In the circuit shown  $C = 10 \text{ millifarads}$  and  $L = 18 \text{ millihenrys}$ . If the steady state voltage  $V_o$  across the two reactive elements in series is zero, what is the frequency of the source in rad/s?

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)



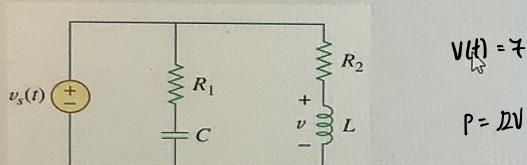
$$\omega = \boxed{\quad} \text{ rad/s}$$

$$Z = \frac{V_s}{Z_C + Z_L}, \text{ but } I = 0 \text{ as } V_o = 0$$

$$\therefore Z_C + Z_L = 0 \rightarrow Z_C = -Z_L \rightarrow \frac{1}{j\omega C} = -j\omega L \Rightarrow \omega = 74.53559 \text{ rad/s}$$

The voltage in the inductor is known from an oscilloscope as  $v(t) = 7 \cos(25t + 40^\circ)$  volts. If the complex power in any element is the product of the voltage phasor by the conjugate of the current phasor DIVIDED BY TWO, What is the real part of the power delivered by the voltage source on the left of the circuit, if  $L = 330 \text{ millihenrys}$  and  $C = 2 \text{ millifarads}$ ,  $R_1 = 65 \text{ ohms}$  and  $R_2 = 80 \text{ ohms}$ .

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)



$$P_{ave} = \boxed{\quad} \text{ watts}$$

$$v(t) = 7 \cos(25t + 40^\circ) = 7 \angle 40^\circ, \quad \omega = 25 \text{ rad/s}$$

$$X_C = \frac{1}{j\omega L} = -j \frac{1}{\omega L} = -j \frac{1}{25(2 \times 10^{-3})} = -20j \rightarrow 65 - 20j$$

$$X_L = j\omega L = j(25)(330 \times 10^{-3}) = 8.25j$$

$$\rightarrow Z_{eq} = \frac{(65-20j)(80+8.25j)}{(65-20j)+(80+8.25j)} = 37.3492 - 4.3096i \Omega$$

$$V(t) = \frac{X_L}{R_2 + X_L} \cdot V_s \rightarrow V_s = \frac{V(t)}{\left(\frac{X_L}{R_2 + X_L}\right)} = \frac{8.25j}{80 + 8.25j} = 68.238 \angle -14.11218^\circ V$$

$$i(t) = \frac{V_s}{Z_{eq}} = 1.815 \angle -37.53^\circ A$$

$$P = \frac{|V||I|^2}{2} = \frac{(68.238 \angle -14.11218^\circ V)(1.815 \angle -37.53^\circ A)^2}{2}$$

$$= 61.92680 \angle -6.5821$$

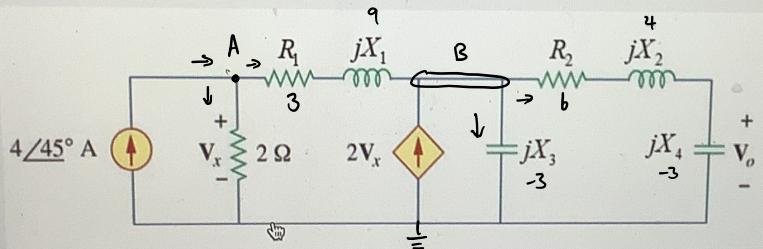
$$= 61.5186 - 7.0984i$$

$$P_{ave} = 61.5186 \text{ W}$$

The current source on the left is  $i_s(t) = 4 \cos(377t + 45^\circ)$  volts. We know the impedance of every element in the network. N.B. The X is a real number and is called the "reactance" in ohms. So,  $jX$  is the impedance, X is the reactance, a real number, in ohms. The reactance of the inductor is  $\omega L$  and the reactance of the capacitor is a negative real number  $-1/(\omega C)$  ohms. Find the value of  $v_o(t)$  in volts at  $t = 0$ , at  $t = 4$  ms, and at  $t = 12$  ms. If the complex power in any element is the product of the voltage phasor by the conjugate of the current phasor DIVIDED BY TWO, compute also the real part of the complex power delivered by the source on the left.

$$R_1 = 3 \Omega, R_2 = 6 \Omega, X_1 = 9 \Omega, X_2 = 4 \Omega, X_3 = -3 \Omega, X_4 = -3 \Omega$$

**Note:** In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)



$$V_o(0) = \boxed{\quad} \text{ volts}$$

$$\omega = 377 \text{ rad/s}$$

$$V_o(0.004) = \boxed{\quad} \text{ volts}$$

$$\sqrt{x} := A$$

$$V_o(0.012) = \boxed{\quad} \text{ volts}$$

$$P_{ave} = \boxed{\quad} \text{ watts}$$

$$4\angle 45^\circ = \frac{A}{2} + \frac{A - B}{3 + 9j} \quad \frac{A - B}{3 + 9j} + 2A = \frac{B}{-3j} + \frac{B}{6 + 4j - 3j}$$

$$A = 3.61682 + 0.919i \quad B = 21.8733 - 15.3657i$$

$$V_o = B - \frac{jX_4}{R_2 + jX_2 + jX_4}$$

$$= 13.18368 - 134.5499 \rightarrow 13.18368 \cos(377t - 134.5499)$$

$$V_o(0) = -9.2487 \text{ V}$$

$$V_o(0.004) = 13.18368 \cos(377t - 2.3483(0.004)) = 8.7962$$

$$V_o(0.012) = -7.4969$$

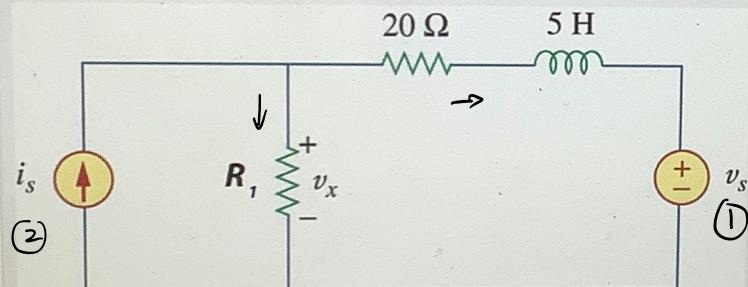
$$P_{ave} = \frac{V_m I_m^*}{2} = \frac{V_x (4\angle 45)}{2} = \frac{(3.61682 + 0.919i)(4\angle 45)}{2} = 7.4639 - 44.75 \text{ W}$$

$$= 6.4146 - 3.815i$$

$$P_{ave} = 6.4146 \text{ W}$$

We have insisted on the fact that phasor analysis is predicated on all the sources having the same frequency. If they don't, we can use superposition. Solve the circuit for all the sources with the same frequency  $\omega_1$ , then for all the sources with frequency  $\omega_2$ , etc., and then we superimpose the responses. Here is your opportunity. Use superposition to find the voltage  $v_x(t)$  at  $t = 0$ , and at  $t = 0.8$  seconds.  $R_1 = 20$  ohms.  $v_s(t) = 42\sin(2t)$  volts,  $i_s(t) = 12\cos(6t + 10^\circ)$  amps.

**Note:** In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)



$$V_x(0) = \boxed{\quad} \text{ volts}$$

$$V_x(0.8) = \boxed{\quad} \text{ volts}$$

$$\text{short } i_s \Rightarrow V_x = \frac{R_1}{R_1 + 20 + jX_L} \cdot V_s$$

$$= \frac{20}{20 + 20 + 2(5)j} \cdot 42 \angle 0^\circ$$

$$= 19.7647 - 4.9411i$$

$$|V_x| = 20.372 \angle -14.0362^\circ$$

$$\text{short } V_s \Rightarrow i_s = \frac{V_x}{R_1} + \frac{V_x}{20 + j\omega L}$$

$$12\cos(6t + 10^\circ) = \frac{V_x}{20} + \frac{V_x}{20 + 30j}$$

$$12 \angle 10^\circ = V_x \left( \frac{1}{20} + \frac{1}{20 + 30j} \right)$$

$$\frac{V_x - 42}{20 + 10j} = -\frac{V_x}{20} \quad // \text{ same ans.}$$

$$V_x = 173.066 \angle 29.44^\circ$$

$$V_x \left( \frac{1}{20 + 10j} + \frac{1}{20} \right) = \frac{42}{20 + 10j}$$

$$V_x = 20.372 \sin(2t - 14.0362^\circ) + 173.066 \cos(2t + 29.44^\circ)$$

$$V_x(0) = 20.372 \sin(-14.0362^\circ) + 173.066 \cos(29.44^\circ) = 150.631$$

$$V_x(0.8) = 20.372 \sin(2(0.8) - 14.0362^\circ) + 173.066 \cos(2(0.8) + 29.44^\circ) = 118.292$$

Find the absolute value of the current in each inductor at  $t = 0$ , and the absolute value of the voltage in the capacitor at  $t = 0$ .

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

$$v_s(t) = 10 \cos(100t - 45^\circ) u(-t) \text{ V}$$

$$i_s(t) = 3 \cos(100t - 30^\circ) u(-t) \text{ A}$$

$$R_1 = 50 \Omega$$

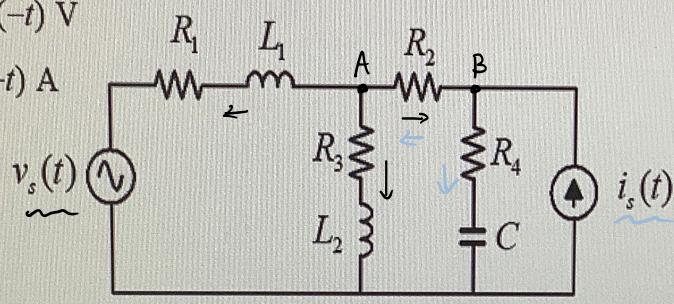
$$R_2 = 20 \Omega$$

$$R_3 = 70 \Omega$$

$$R_4 = 100 \Omega$$

$$L_1 = 2 \text{ H}$$

$$C = 50 \mu\text{F}$$



$$|I_{L1}(0)| = -0.875 \text{ amps}$$

$$|I_{L2}(0)| = 0.9867 \text{ amps}$$

$$|V_C(0)| = 342.467 \text{ volts}$$

$$\frac{A - V_s}{R_1 + X_{L1}} + \frac{A - B}{R_2} + \frac{A}{R_3 + X_{L2}} = 0 \quad i_s = \frac{B}{R_4 + X_C} + \frac{B - A}{R_2}$$

$$V_s = 10 \angle -45^\circ$$

$$X_{L1} = 100(2)\text{j}$$

$$X_{L2} = 100(2)\text{j}$$

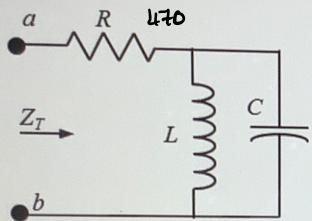
$$i_s = 3 \angle -30$$

$$X_C = -(100)(50 \times 10^{-6})\text{j}$$

$$A = 173.981 \angle 8.4968 \quad B = 186.71839 \angle -1.09839$$

|I<sub>L1</sub>(0)| is 0.8737, not the answer there.

Let  $R = 470\Omega$ ,  $C = 1.7\mu F$  and  $L = 45mH$ . Determine the impedance,  $Z_T$  seen at terminals  $a - b$ , for the different operating frequencies:



$$Z_L = (1 \times 10^3)(45 \times 10^{-3}) j \Omega$$

$$Z_C = \frac{1}{j(1 \times 10^3)(1.7 \times 10^{-6})} \Omega$$

$$\frac{1}{j\omega C} \quad j\omega L$$

$$(Z_L \parallel Z_C) + R$$

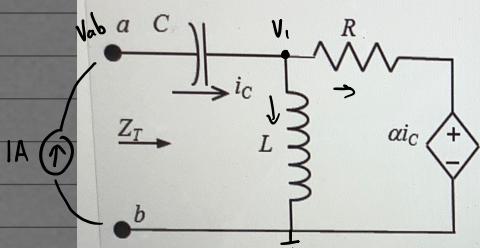
Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a.  $\omega = 1k\text{rad/s}$  :  $Z_T = \boxed{\quad} \angle \boxed{\quad}^\circ \Omega$

b.  $\omega = 10k\text{rad/s}$  :  $Z_T = \boxed{\quad} \angle \boxed{\quad}^\circ \Omega$

c.  $f = 1\text{kHz}$  :  $Z_T = \boxed{\quad} \angle \boxed{\quad}^\circ \Omega$

Let  $R = 7\Omega$ ,  $C = \frac{1}{3}\text{F}$ ,  $L = 9\text{H}$  and  $\alpha = 4V/A$ . Determine the requested quantities seen at terminals  $a - b$  for an operating frequency of  $2\text{rad/s}$ : The overall impedance of this circuit seen at the port on the left can be represented either as a resistor  $R_s$  in series with a reactor  $X_s$ , or by a (different) resistor  $R_p$  in parallel with a (different) reactor  $X_p$ .



$$Z_C = \frac{1}{j\omega C} \quad Z_L = j\omega L \quad \omega = 2$$

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a. Impedance:  $Z_T = \boxed{\quad} \angle \boxed{\quad}^\circ \Omega$

b. Series-Resistance:  $R_s = \boxed{\quad} \Omega$

c. Series-Reactance:  $X_s = \boxed{\quad} \Omega$

d. Parallel-Resistance:  $R_p = \boxed{\quad} \Omega$

e. Parallel-Reactance:  $X_p = \boxed{\quad} \Omega$

$$Z_C = \frac{1}{j(2)(\frac{1}{3})} \quad Z_L = j(2)(9) \quad \alpha = 4$$

$$R_p \parallel X_p \Rightarrow \frac{1}{R_p} + \frac{1}{X_p} = \frac{1}{Z_p}$$

$$i_C = 1 \Rightarrow I = \frac{V_1}{Z_L} + \frac{V_1 - \alpha}{R}$$

$$= 9.8 \angle 13.05^\circ$$

$$V_1 = 10.25 \angle 21.25^\circ$$

$$Z_p = \frac{R_s j}{R + Z_p j} \quad \text{where } Z_p = 9.8 \angle 13.05^\circ \Omega$$

$$\frac{V_{ab} - V_1}{j \frac{1}{2(CV_1)}} = 1$$

$$= \frac{R \cdot \alpha j}{R + \alpha j} \cdot \frac{R - \alpha j}{R - \alpha j}$$

$$V_{ab} = 9.8 \angle 13.05^\circ$$

$$= \frac{R^2 \alpha j - R \alpha^2 (-1)}{R^2 - \alpha^2 (-1)}$$

$$\text{or } 9.5549 + 2.2158i$$

$$= \frac{R^2 \alpha j + R \alpha^2}{R^2 + \alpha^2}$$

$$= \underbrace{\frac{R \alpha^2}{R^2 + \alpha^2}}_a + \underbrace{\frac{R^2 \alpha j}{R^2 + \alpha^2}}_b \quad Z_p = a + bj$$

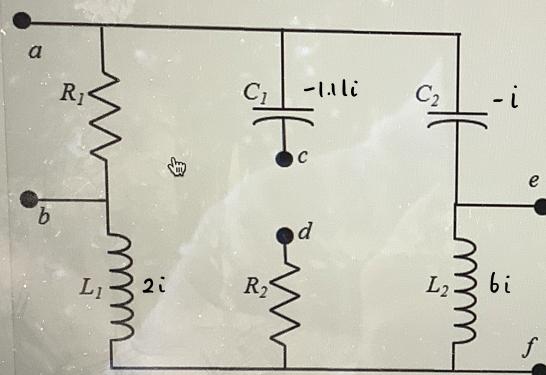
$$= 9.5549 + 2.2158i$$

$$9.5549 = \frac{R \alpha^2}{R^2 + \alpha^2} \quad 2.2158 = \frac{R^2 \alpha}{R^2 + \alpha^2} \Rightarrow R = 10.059 \Omega$$

$$\alpha = 43.401 \Omega$$

Let  $R_1 = 9\Omega$ ,  $R_2 = 4\Omega$ ,  $C_1 = 0.9F$ ,  $C_2 = 1F$ ,  $L_1 = 2H$  and  $L_2 = 6H$ . Determine the impedance that would be seen at the requested terminals for an operating frequency of  $1\text{rad/s}$ :

1.12842 1.73077 3.65385 1.73077+3.65385j



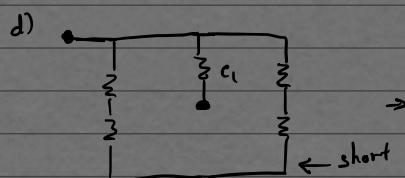
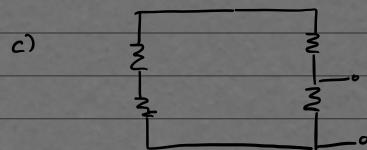
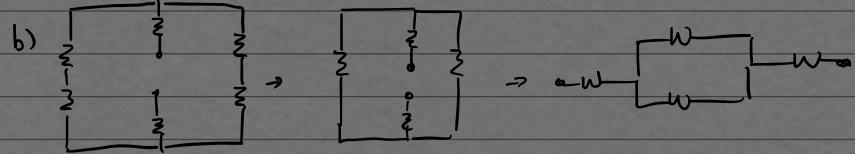
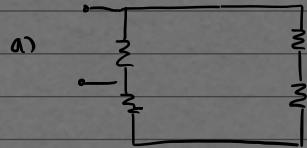
Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a.  $a - b : Z_T = \boxed{\quad} \angle \boxed{\quad}^\circ \Omega$

b.  $c - d : Z_T = \boxed{\quad} \angle \boxed{\quad}^\circ \Omega$

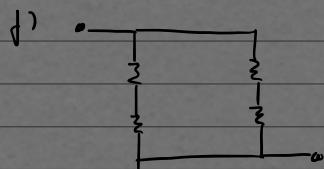
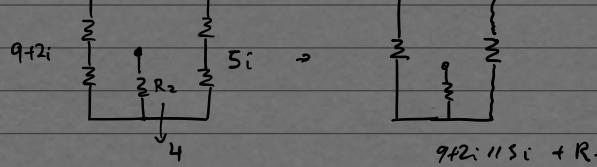
c.  $e - f : Z_T = \boxed{\quad} \angle \boxed{\quad}^\circ \Omega$

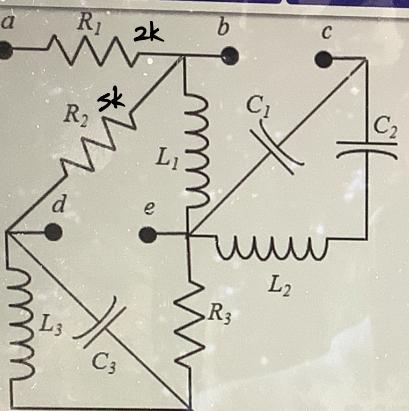
d.  $a - c : Z_T = \boxed{\quad} \angle \boxed{\quad}^\circ \Omega$



$$c_i \rightarrow$$

e)





$$r_1 = 2k \quad r_2 = 5k$$

$$Z_{L1} = j(2\pi \cdot 50 \times 10^3)(27 \times 10^{-3}) =$$

$$Z_{L2} = j(2\pi \cdot 50 \times 10^3)(29 \times 10^{-3}) =$$

$$Z_{L3} = j(2\pi \cdot 50 \times 10^3)(18 \times 10^{-3}) =$$

$$Z_{C1} = -j(2\pi \cdot 50 \times 10^3)(550 \times 10^{-12})$$

$$Z_{C2} = -j(2\pi \cdot 50 \times 10^3)(850 \times 10^{-12})$$

$$Z_{C3} = -j(2\pi \cdot 50 \times 10^3)(500 \times 10^{-12})$$

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a.  $a - b : Z_T = \boxed{\phantom{00}} + j\boxed{\phantom{00}} \angle \boxed{\phantom{00}}^\circ k\Omega$

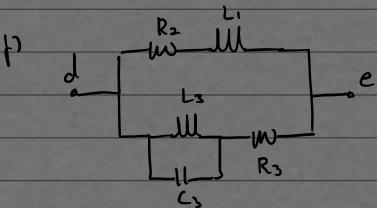
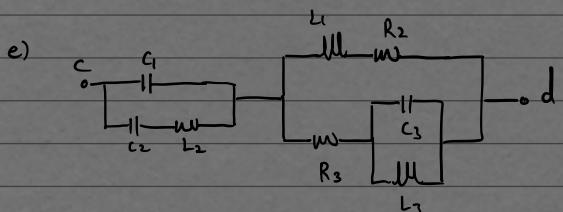
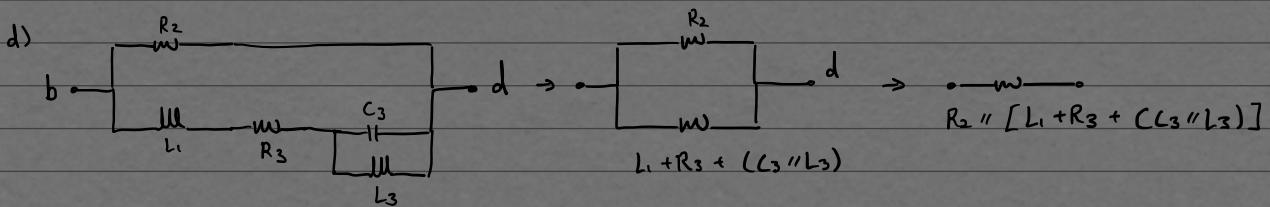
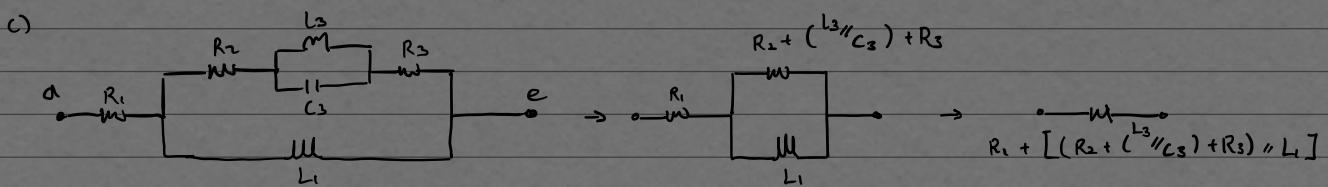
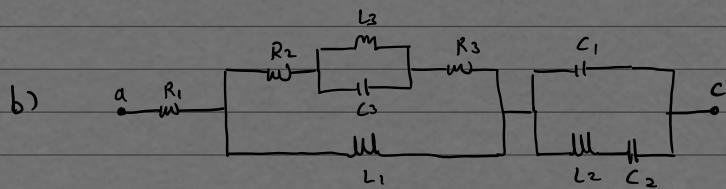
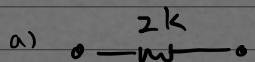
b.  $a - c : Z_T = \boxed{\phantom{00}} + j\boxed{\phantom{00}} \angle \boxed{\phantom{00}}^\circ k\Omega$

c.  $a - e : Z_T = \boxed{\phantom{00}} + j\boxed{\phantom{00}} \angle \boxed{\phantom{00}}^\circ k\Omega$

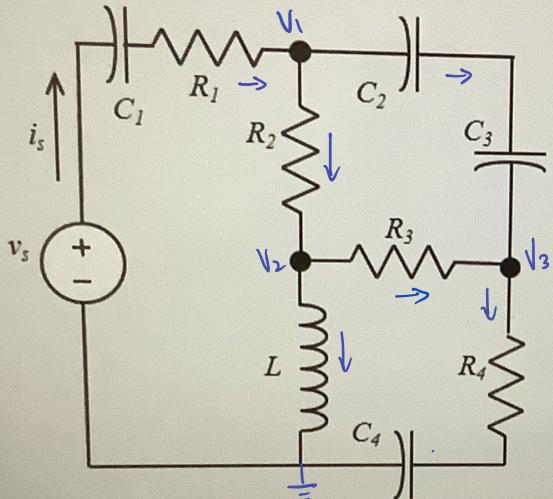
d.  $b - d : Z_T = \boxed{\phantom{00}} + j\boxed{\phantom{00}} \angle \boxed{\phantom{00}}^\circ k\Omega$

e.  $c - d : Z_T = \boxed{\phantom{00}} + j\boxed{\phantom{00}} \angle \boxed{\phantom{00}}^\circ k\Omega$

f.  $d - e : Z_T = \boxed{\phantom{00}} + j\boxed{\phantom{00}} \angle \boxed{\phantom{00}}^\circ k\Omega$



In the circuit shown,  $R_1 = 6k\Omega$ ,  $R_2 = 9k\Omega$ ,  $R_3 = 10k\Omega$ ,  $R_4 = 4k\Omega$ ,  $C_1 = 450nF$ ,  $C_2 = 200nF$ ,  $C_3 = 750nF$ ,  $C_4 = 200nF$ , and  $L_1 = 23H$ . Determine  $i_s(t)$  under different values of  $v_s(t)$ :



Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a.  $v_s(t) = 12\cos(377t)V : i_s(t) = \boxed{\quad} \cos(377t + \boxed{\quad})A$

b.  $v_s(t) = 60\cos(377t + 100^\circ)V : i_s(t) = \boxed{\quad} \cos(377t + \boxed{\quad})A$

$$Z_{C_1} = -j \frac{1}{377(450 \times 10^{-9})} \quad R_1 = 6k \quad R_2 = 9k \quad R_3 = 10k$$

$$Z_{C_2} = -j \frac{1}{377(200 \times 10^{-9})} \quad A: \quad \frac{V_s - V_1}{C_1 + R_1} = \frac{V_1 - V_2}{R_2} + \frac{V_1 - V_3}{C_2 + C_3} \quad V_s = 12\cos(377t)$$

$$Z_{C_3} = -j \frac{1}{377(750 \times 10^{-9})} \quad B: \quad \frac{V_1 - V_2}{R_2} = \frac{V_2 - V_3}{R_3} + \frac{V_2}{L} \quad V_s = 12 \angle 0^\circ$$

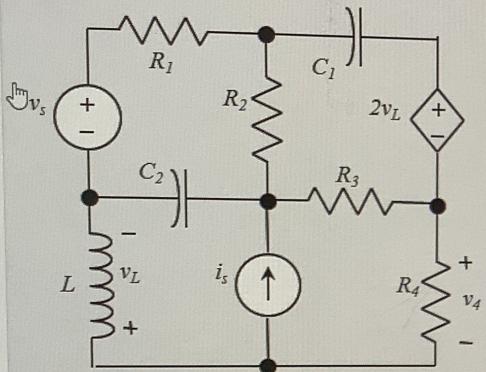
$$Z_{C_4} = -j \frac{1}{377(200 \times 10^{-9})} \quad C: \quad \frac{V_1 - V_3}{C_2 + C_3} + \frac{V_2 - V_3}{R_3} = \frac{V_3}{R_4 + C_4} \quad V_s = 60\cos(377t + 100^\circ)$$

$$Z_{L_1} = j(377)(23) \quad = 60 \angle 100^\circ$$

$$i_s = \frac{V_s - V_L}{C_1 + R_1}$$

$$= 2.682 \times 10^{-5} \approx 95.698$$

In the circuit shown,  $R_1 = 9k\Omega$ ,  $R_2 = 8k\Omega$ ,  $R_3 = 9k\Omega$ ,  $R_4 = 6k\Omega$ ,  $C_1 = 400nF$ ,  $C_2 = 250nF$  and  $L = 3H$ . Given that  $v_s(t) = 2\cos(500t)V$  and  $i_s(t) = 8\sin(500t)mA$ , determine the Thevenin equivalent seen by resistor  $R_4$  (assume the lower node as the reference) and use this to determine the signal  $v_A(t)$ .

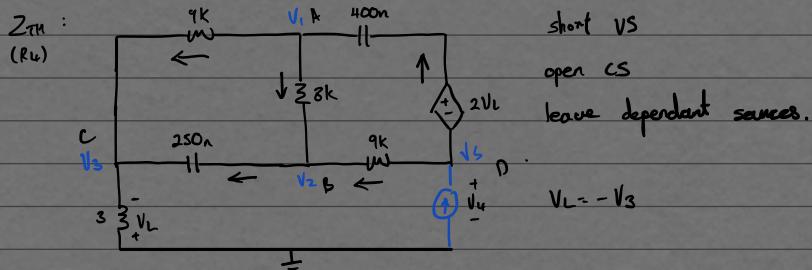


Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a.  $Z_{Th} = \boxed{\quad} \angle \boxed{\quad}^{\circ} k\Omega$

b.  $V_{Th} = \boxed{\quad} \angle \boxed{\quad}^{\circ} V$

c.  $v_A(t) = \boxed{\quad} \cos(500t + \boxed{\quad})^{\circ} V$



$$A: \frac{V_s - 2V_3 - V_1}{j \frac{1}{500(400n)}} = \frac{V_1 - V_3}{9k} + \frac{V_1 - V_2}{8k}$$

$$B: \frac{V_s - V_2}{9k} + \frac{V_1 - V_2}{8k} = \frac{V_2 - V_3}{j \frac{1}{500(250n)}}$$

$$C: \frac{V_1 - V_3}{9k} + \frac{V_2 - V_3}{j \frac{1}{500(250n)}} = \frac{V_3}{j \frac{1}{500(3)}}$$

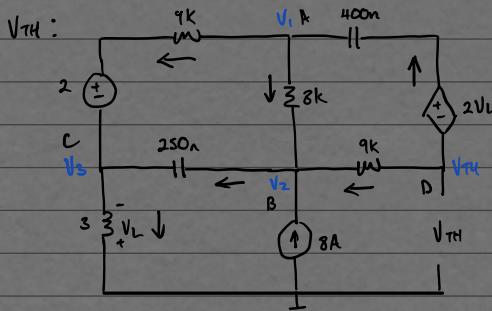
where  $V = RI$

$$V_s = Z_{Th} I$$

$$V_s = Z_{Th}(I)$$

$$= 14.509 \angle -24.4264$$

$$D: I = \frac{V_s - 2V_3 - V_1}{j \frac{1}{500(400n)}} + \frac{V_s - V_2}{9k}$$



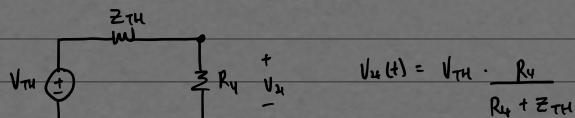
$$A: \frac{V_{Th} - 2V_3 - V_1}{j \frac{1}{500(400n)}} = \frac{V_1 - V_3}{8k} + \frac{V_1 - 2 - V_3}{9k}$$

$$B: \frac{V_1 - V_3}{8k} + \frac{V_{Th} - V_2}{9k} + 8A \angle -90^\circ = \frac{V_2 - V_3}{j \frac{1}{500(250n)}}$$

$$C: \frac{V_1 - 2 - V_3}{9k} + \frac{V_2 - V_3}{j \frac{1}{500(250n)}} = \frac{V_3}{j \frac{1}{500(3)}}$$

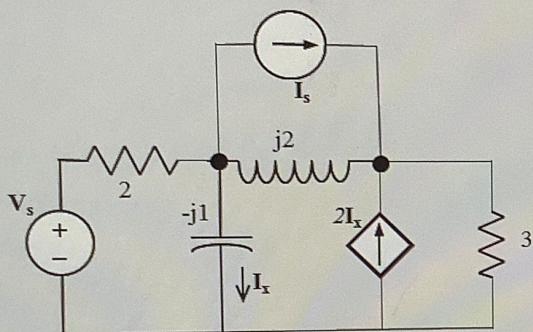
$$V_{Th} = 13.56033 \angle -118.8789^\circ$$

$$D: \frac{V_{Th} - 2V_3 - V_1}{j \frac{1}{500(400n)}} + \frac{V_{Th} - V_2}{9k} = 0$$



$$V_{Th}(t) = V_{Th} \cdot \frac{R_4}{R_4 + Z_{Th}}$$

For the circuit shown, the voltage and current sources have respective phasors  $V_s = 4\angle 70^\circ V$  and  $I_s = 8\angle 85^\circ A$ . Compute the phasor  $I_x$  flowing through the capacitor as well as the complex power ( $S_c$ ) received by the capacitor. Compute also the complex power ( $S_{DS}$ ) supplied by the dependent source. Note: All phasors in this question, as is common in the Electric Power Industry, are RMS/IEEE Phasors.

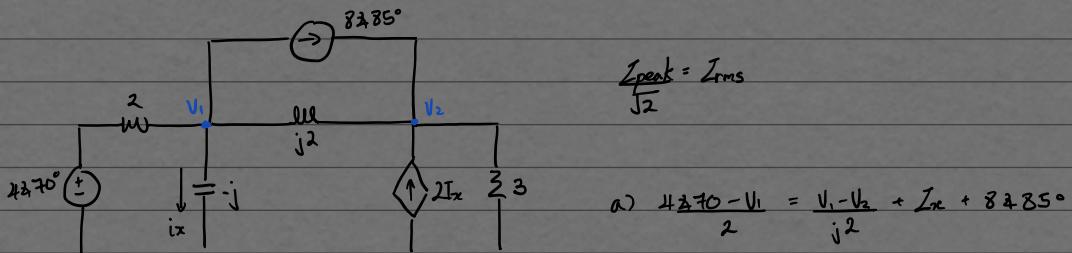


Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a.  $I_x = \boxed{\quad} \angle \boxed{\quad} {}^\circ A$

b.  $S_c = \boxed{\quad} \angle \boxed{\quad} {}^\circ VA$

c.  $S_{DS} = \boxed{\quad} \angle \boxed{\quad} {}^\circ VA$



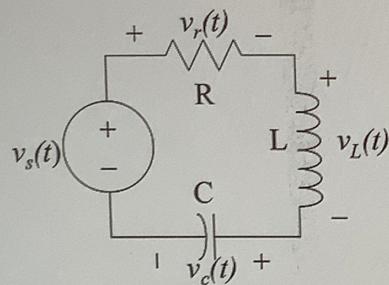
$$I_x = \frac{V_1}{-j}$$

$$b) S_c = Z_{rms}^* V_1$$

$$= Z_x^* V_1$$

$$c) S_{DS} = Z_x^*(2) \cdot V_2$$

Consider the RLC series circuit shown with  $v_s(t) = 17\cos(\omega t)V$ ,  $R = 2\Omega$ ,  $L = 13mH$  and  $C = 5\mu F$ . Determine the resonant frequency by  $\omega_r = (L \cdot C)^{-1/2}$ . Compute the complex power received by each circuit component (the subscripts "R", "L", "C" and "S" respectively refer to the resistor, inductor, capacitor, and source) under the following source frequencies:



$$17\cos(\omega t) \Rightarrow \frac{17}{\sqrt{2}} \& 0^\circ$$

Why?

Note: Use "j" when submitting complex and imaginary numbers (e.g. submit 4+j3 for a complex number with real component 4 and imaginary component 3)

a.  $\omega = 0.1\omega_r$

$S_R = \boxed{\quad} \text{ VA}$

$S_L = \boxed{\quad} \text{ VA}$

$S_C = \boxed{\quad} \text{ VA}$

$S_S = \boxed{\quad} \text{ VA}$

b.  $\omega = \omega_r$

$S_R = \boxed{\quad} \text{ VA}$

$S_L = \boxed{\quad} \text{ VA}$

$S_C = \boxed{\quad} \text{ VA}$

$S_S = \boxed{\quad} \text{ VA}$

c.  $\omega = 10\omega_r$

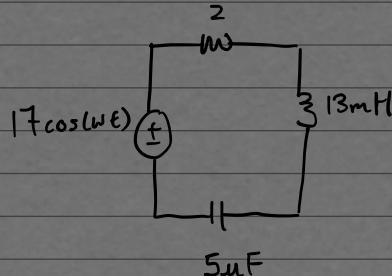
$S_R = \boxed{\quad} \text{ VA}$

$S_L = \boxed{\quad} \text{ VA}$

$S_C = \boxed{\quad} \text{ VA}$

$S_S = \boxed{\quad} \text{ VA}$

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$\omega_r = (L \cdot C)^{-1/2}$

$V = \frac{17}{\sqrt{2}}$

$I = \frac{\frac{17}{\sqrt{2}}}{2 + j\omega(13m) + \frac{1}{j\omega(5\mu)}}$

$\omega = 0.1\omega_r$

$: \bar{Z} = 9.434 \times 10^{-5} + 0.023i$

$= 392.230227$

$V_R = \bar{Z} \cdot R$

$= 2.668 \times 10^{-4} + 6.73519 \times 10^{-2}i$

$S = \bar{Z}^* V$

$S_R = \bar{Z}^* V_R = 1.134 \times 10^{-3}$

$V_C = \bar{Z} \cdot \bar{Z}_C$

$S_L = \bar{Z}^* V_L = 0.00289j$

$17.17 - 6.8 \times 10^{-2}i$

$S_C = \bar{Z}^* V_C = -0.289j$

$V_L = \bar{Z} \cdot \bar{Z}_L$

$S_S = \bar{Z}_{rms} V_S =$

$= -0.1717 + 6.8 \times 10^{-4}i$

$S_R = 72.25 \quad \checkmark$

$S_S = \bar{Z}_{rms} V_S$

$S_L = 1842.0207i \quad \checkmark$

$= (6.010407 \times 2.9245 \times 10^{-6}i)(\frac{17}{\sqrt{2}})$

$\omega = \omega_r : i = 6.010407 + 2.9245 \times 10^{-6}i \Rightarrow S_C = -1842.0207i \quad \checkmark$

$= 72.25 \times$

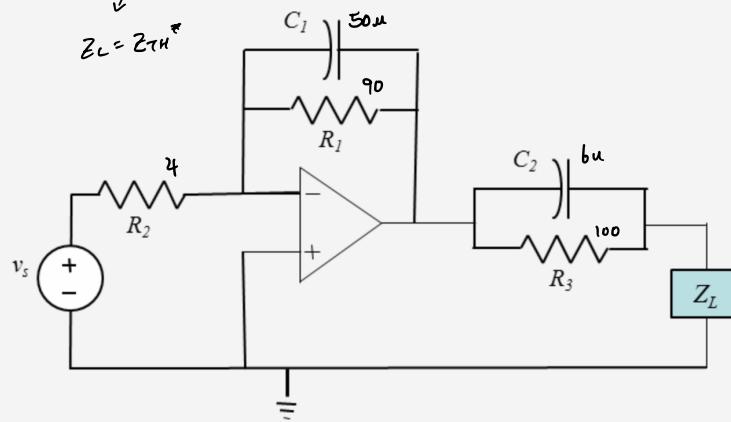
$\omega = 10\omega_r : S_R = 1.134 \times 10^{-3}$

$S_L = 0.289i$

$S_C = -2.891 \times 10^{-3}i$

$S_S = 1.134 \times 10^{-3} + 0.289i$

Let  $v_s(t) = \cos(377t)V$ ,  $R_1 = 90\Omega$ ,  $R_2 = 4\Omega$ ,  $R_3 = 100\Omega$ ,  $C_1 = 50\mu F$  and  $C_2 = 6\mu F$ . Determine the complex load  $Z_L$  to maximize the average power it receives and compute the resulting values for the load: When writing the power as a function of time, write the phase as an angle between -180 and 180 degrees.



Note: Use "j" when submitting complex and imaginary numbers (e.g. submit 4+j3 for a complex number with real component 4 and imaginary component 3)

a. Impedance,  $Z_L = \boxed{\quad} \Omega$

b. Apparent power received,  $S = \boxed{\quad} \text{? } \diamond$

c. Average power received,  $P = \boxed{\quad} \text{? } \diamond$

d. Reactive power received,  $Q = \boxed{\quad} \text{? } \diamond$

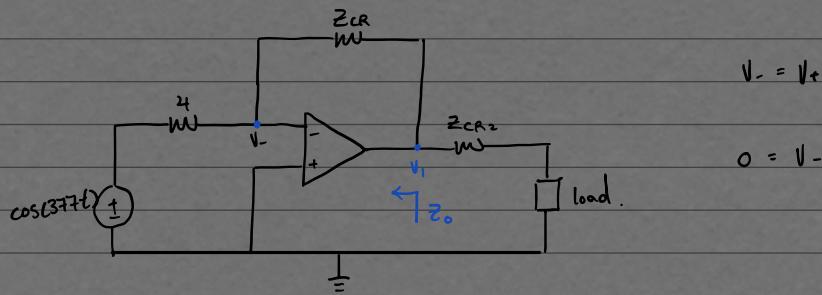
e. Complex power received,  $S = \boxed{\quad} \text{? } \diamond$

f. Instantaneous power received  $p(t) = \boxed{\quad} + \boxed{\quad} \cos(754t + \boxed{\quad}) \text{? } \diamond$

$$Z_{C1} = \frac{1}{j(377)(50 \times 10^{-6})} \parallel R_1 = 9\Omega \quad \downarrow \quad Z_{CR1} = \frac{1}{j(377)(6 \times 10^{-6})} \parallel R_3 = 100\Omega$$

$Z_{CR1}$

$Z_{CR2}$



Output impedance of ideal op-amp is 0.

$$\text{a)} Z_{TH} = Z_o + (Z_{CR2}) \\ = 0 + 95.132 - 21.5189$$

$$Z_L = Z_{TH}^*$$

$$\text{b)} \frac{V_o}{V_s} = -\frac{R_1}{R_2} \quad I = \frac{V_o}{Z_{CR2} + Z_L} \quad V_L = J_L Z_L \\ = -\frac{(R_1 \parallel C_1) \cdot 180^\circ}{R_2} = -3.04932 \times 10^{-2} + 0.0517i \\ = -5.80179 + 9.8427i \quad = 6.005 \times 10^{-2} + 120.517^\circ \\ = 5.8703 + 133.2629^\circ$$

$$S = V_{L\text{rms}} I_{\text{rms}} \\ = \frac{6 \times 10^{-2}}{\sqrt{2}} \cdot \frac{5.857}{\sqrt{2}}$$

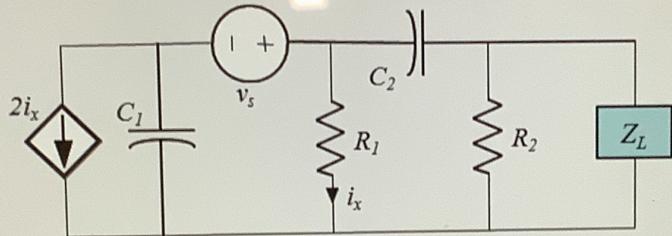
$$\text{c)} P = V_{L\text{rms}} I_{\text{rms}} \cos(\theta_L - \phi_i) \\ = 0.17138 \text{ W}$$

$$\text{d)} Q = V_{L\text{rms}} I_{\text{rms}} \sin(\theta_L - \phi_i) \\ = 3.87 \times 10^{-2} \text{ VAr}$$

$\frac{360}{1000} = 0.36 \text{ per VAr}$

$$\text{e)} \bar{S} = P + Qj = 0.17138 + 0.0387j \quad \text{f)} p_{inst}(t) = V_L I_L \cos(\theta) + V_L I_L \cos(\phi_i) \\ = P + 2\omega t + Q \sin(\theta_i)$$

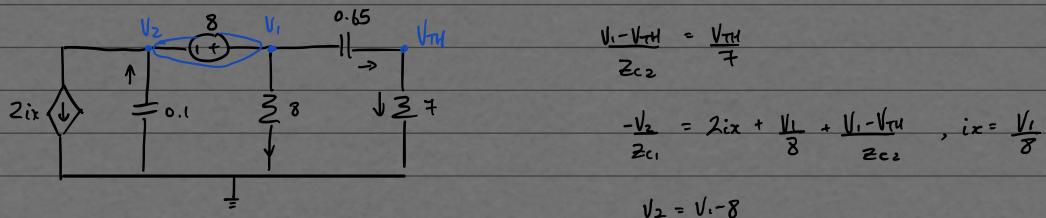
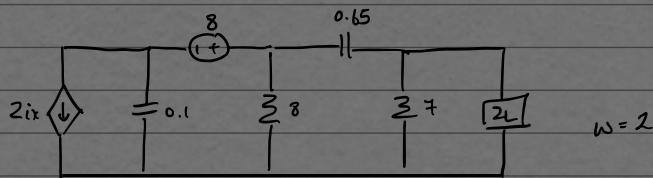
Let  $v_s(t) = 8\cos(2t)V$ ,  $R_1 = 8\Omega$ ,  $R_2 = 7\Omega$ ,  $C_1 = 100mF$  and  $C_2 = 650mF$ .



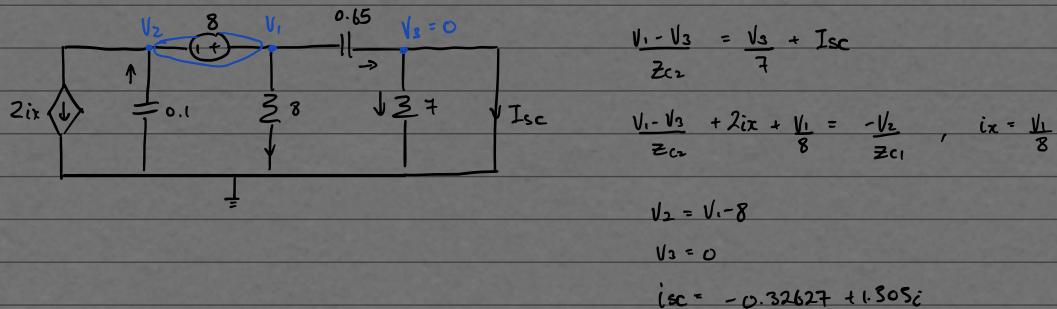
Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a. If  $Z_L$  is a purely resistive load chosen to maximize the average power it receives, the resistor value is  $Z_L = \boxed{\quad} \Omega$   
and the resulting average power is  $P = \boxed{\quad} W$ .

b. If  $Z_L$  is a complex load chosen to maximize the average power it receives, the load value is  $Z_L = \boxed{\quad} + j \boxed{\quad} \Omega$   
and the resulting average power is  $P = \boxed{\quad} W$ .



$$V_{TH} = 0.8023 + 2.427i$$



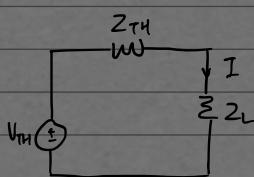
$$Z_{TH} = \frac{V_{TH}}{I_{SC}} = 1.82264 - 1.0705i$$

Purely R:

$$Z_L = |Z_{TH}| = \sqrt{1.82264^2 + 1.0705^2} \\ = 2.1129$$

Complex R:

$$Z_L = Z_{TH}^* = 1.82264 + 1.0705i \\ P = \frac{|V_{TH}|^2}{8R_{TH}}$$



$$I_{rms} = \frac{|V_{TH}|}{Z_{TH} + Z_L}$$

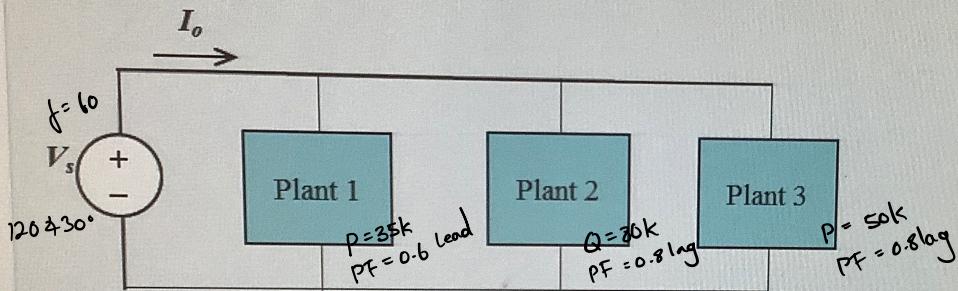
$$P_L = |I_{rms}|^2 Z_L \\ = \left[ \frac{(0.018^2 + 0.69^2)^{1/2}}{\sqrt{2}} \right]^2 (2.1129) \\ I_{rms} = \frac{I}{\sqrt{2}}$$

$$I = 1.4323 \times 10^{-2} + 0.69607i$$

$$P_L = \frac{|V_{TH}|^2}{8R_L} \quad \text{where } R_L = 1.82264 \\ = 0.5544$$

$$= 0.5132$$

In the diagram below, the source voltage phasor is  $V_s = 120 \angle 30^\circ V$  (rms) and measurements show that Plant 1 receives  $35kW$  with PF 0.6 leading, plant 2 receives  $80kVA$  with PF 0.8 lagging and Plant 3 receives  $50kW$  with PF 0.8 lagging. Compute the current phasor  $I_o$  and the overall power factor unity?



$$S = P \cdot \text{pf}$$

$$\begin{aligned} S_1 + S_2 + S_3 &= S_{\text{total}} \\ &= V_{\text{rms}} I_{\text{rms}}^* \\ &= 120 \angle 30^\circ (I_o^*) \end{aligned}$$

**Note:** In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

$$I_o = \boxed{\quad} \angle \boxed{\quad} {}^\circ A \text{ (rms)}$$

$$\text{Overall PF: } \boxed{\quad} \quad ? \quad \downarrow$$

$$\text{Component: } \boxed{\quad} \quad ? \quad \downarrow$$

$$S = VI \quad P = VI \cos(\phi)$$

$$P_1 + P_2 + P_3$$

$$VI = \frac{P}{\cos(\phi)} = \frac{P}{\text{pf}}$$

$$S = \frac{P}{\cos\phi}$$

$$\text{pf} = \cos(\phi)$$

$$\phi = \cos^{-1}(\text{pf})$$

$$0.00715338$$

$$S_1 = 58.333.33 \angle -53.13$$

$$S_2 = \underline{80kVAr} 36.869$$

$$S_3 = 62.5k \angle 36.869$$

leading

$$S_{\text{total}} = S_1 + S_2 + S_3$$

$$I_o^* = \frac{S_{\text{total}}}{V_s} = \frac{153977.36 \angle 14.6078}{120 \angle 30}$$

$$Z_o^* = 1283.144 \angle -15.392$$

$$Z_o = 1283.144 \angle 15.392$$

b)  $\cos(30^\circ - 14.607) = 0.964$  since  $\text{pf} > 0$  its lagging

c)  $S_{\text{total}} = \frac{149000 + 38833.3i}{P + Qj} \rightarrow$  to make unity, we need leading (-)  $\therefore$  capacitor

$$Q = \frac{|V_s|^2}{X_C} = \frac{|V_s|^2}{\frac{1}{\omega C}}$$

$$\ln(S_I) = \text{abs} \left( \frac{\text{abs}(V_s)}{\frac{1}{\omega C}} \right)^2$$

$$C = \frac{Q}{|V_s|^2} = \frac{38833.3}{120^2 \cdot 2\pi 60} = 7.1533 \times 10^{-3}$$

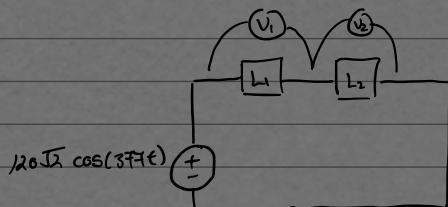
Consider a voltage source,  $v_s(t) = 120\sqrt{2}\cos(377t)V$ , connected across two loads in series ( $v_s = v_1 + v_2$ ). If the load voltmeter RMS readings measure 100V and 46V, respectively, with the voltage of the first load leading that of the second (by an angle between 0 and 180 degrees), determine the phases for  $v_1$  and  $v_2$ . If you also know that load 1 has PF 0.9 lagging, compute the PF of load 2.

**Note:** In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a. Phase of  $v_1(t)$ :   °

b. Phase of  $v_2(t)$ :   °

c. Load 2 PF:   ?



$$V_1 = 100V \quad V_2 = 46V$$

$$120\sqrt{2}\cos(377t) = 100\cos(\alpha + \theta) + 46\cos(377t + \beta)$$

$$120\sqrt{2} = 100\cos\alpha + 46\cos\beta$$

$$100\cos\alpha \rightarrow (\alpha^2 + b^2)^{1/2} = 100$$

$$46\cos\beta \rightarrow (\beta^2 + d^2)^{1/2} = 46$$

$$\tan^{-1}\left(\frac{b}{a}\right) = \alpha$$

$$\tan^{-1}\left(\frac{d}{c}\right) = \beta$$

$$\tan^{-1}\left(\frac{d}{c}\right) = \beta$$

$$a + c = 120 \quad b + d = 0$$

$$a = 92.85$$

$$c = 27.15$$

$$b = 37.13323$$

$$d = -37.13323$$

$$\rightarrow 100\angle 21.797$$

$$\rightarrow 46\angle -53.827$$

$$P_{L1} = 0.9 \text{ lag}$$

$$P_{L2} = \cos(\beta - \theta_i)$$

$$0.9 = \cos(\alpha - \theta_i)$$

$$= \cos(-53.827 - (-4.044))$$

$$P_{L1} = 25.849$$

$$= 0.145 \text{ since}$$

$$\theta_i = \alpha - 25.849$$

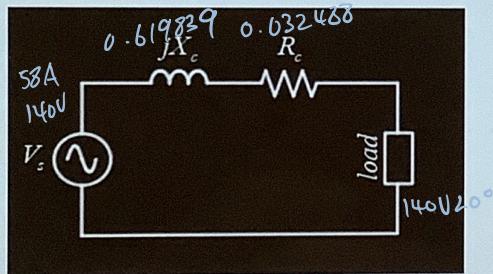
pf angle < 0 .. leading

$$= -4.044$$

In the quiescent electric power system in the figure, a voltmeter reads the same at the source and at the load, 140 volts. At the source, an ammeter reads 58 amps. The source sees an inductive circuit. The cable (feeder) has a resistance of 0.032484 ohms and a reactance of 0.619839 ohms. (a) What is the power factor angle, in degrees, at the source; (b) What is the active power delivered by the source; (c) What is the reactive power delivered by the source; (d) What is the power factor angle at the load, in degrees; (e) What is the active power absorbed by the load; (f) What is the reactive power absorbed by the load.

**Note:** In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

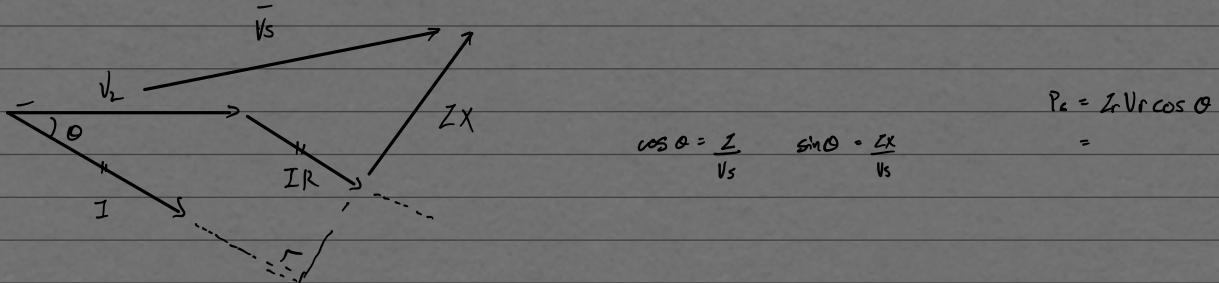
### Simple Power System



(a)  $\theta_s = \boxed{3} \deg$

(b)  $P_s = \boxed{\quad} W$

(c)  $Q_s = \boxed{\quad} VAr$



$$\begin{aligned} V_s^2 &= (\cos\theta V_L + jR)^2 + (\sin\theta V_L + jX)^2 \\ 140^2 &= (140\cos\theta + 58(0.032484)) + (140\sin\theta + 58(0.619839))^2 \\ 19600 &= (140\cos\theta + 1.8240952)^2 + (140\sin\theta + 35.950)^2 \\ &= 19600\cos^2\theta + 2.263.77\cos\theta + 3.54981 + 19600\sin^2\theta + 2 \cdot 5033.09 \sin\theta + 1292.45 \\ &= 19600\cos^2\theta + 19600\sin^2\theta + 527.5466\cos(\theta) + 10D.66.2\sin(\theta) + 1296 \end{aligned}$$

$$\theta = -175.612^\circ \text{ or } -10.387^\circ$$

$$I = 58.3 \angle 10.387^\circ$$

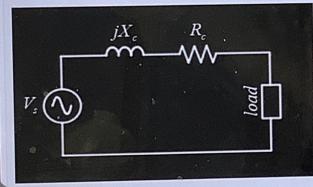
$$\begin{aligned} V_s &= 140 \angle 0^\circ + (0.032484 + 0.619839j)(58.3 \angle 10.387^\circ) \\ &= 140 \angle 14.77^\circ \end{aligned}$$

$$\begin{aligned} \text{(a) } \theta_s &= \theta_s - \theta_L \\ &= 14.77 - 10.387^\circ \end{aligned}$$

In the quiescent electric power system in the figure, a voltmeter reads the same at the source and at the load, 146 volts. At the source, an ammeter reads 52 amps. The source sees a capacitive circuit. The cable (feeder) has a resistance of 0.550543 ohms and a reactance of 0.704663 ohms. (a) What is the power factor angle, in degrees, at the source; (b) What is the active power delivered by the source; (c) What is the reactive power delivered by the source; (d) What is the power factor angle at the load, in degrees; (e) What is the active power absorbed by the load; (f) What is the reactive power absorbed by the load.

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

#### Simple Power System



$$Z = 52 \angle 47.168^\circ$$

$$V_s = 146 \angle 0^\circ + (52 \angle -47.168^\circ)(0.550543 + 0.704663j)$$

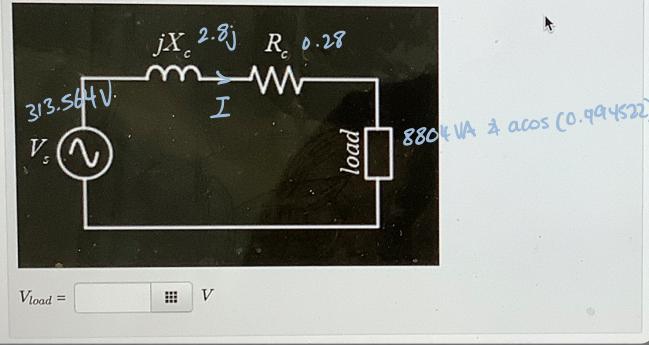
$$= 146 \angle 18.3263^\circ$$

$$\theta_s = \theta_v - \theta_i \\ = 18.3263 - 47.058 \\ = -28.837^\circ$$

In the figure, the load absorbs 8804 kVA at a power factor 0.994522 inductive. For the feeder (cable)  $R = 0.28$  ohms, and  $X = 10R$ . The voltage at the source is 313.564 volts. What is the voltage at the load, in volts?

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

#### Simple Power System



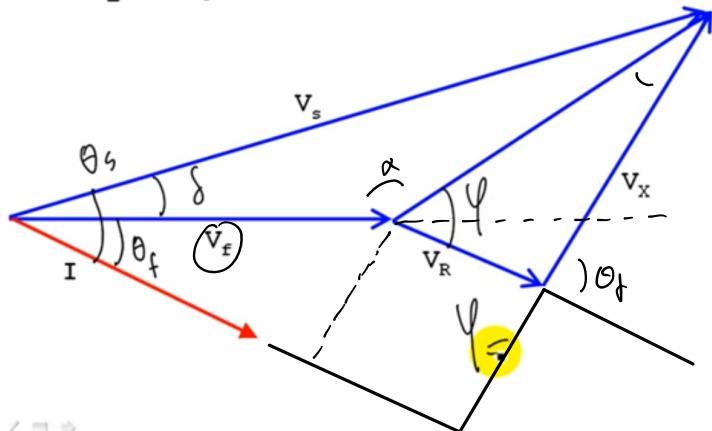
$$S_L = 8804 \quad P_f = 0.994522 \rightarrow S_L = 8804 \angle \cos^{-1}(P_f)$$

$$\frac{V_s - V_L}{jX_c + R_c} = \left( \frac{S}{V_s} \right)^* \quad \bar{S} = \bar{V} \bar{I}^*$$

$$V_{load} = \boxed{\quad} \quad \boxed{\quad} \quad V$$

## If only $I_{rms}$ changes

- If  $V_f$  and pf at the load are constant...



$$\sin \cos \theta_s = \frac{V_x + \cos \phi V_L}{V_s}$$

$$\cos \theta_s = \frac{I + \sin \phi V_L}{V_s}$$

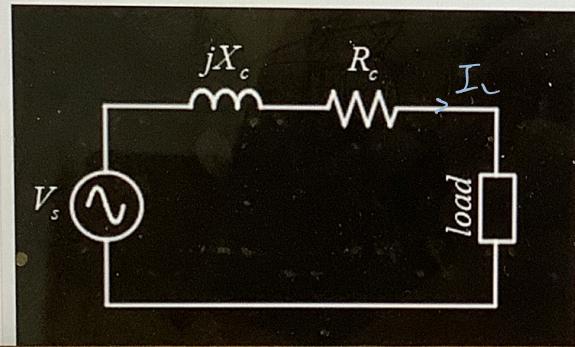
In the figure, a 247.574 volts source delivers 3.96118 kVA at a power factor 0.972502 inductive through a cable with  $R = 0.07$  ohms and  $X = 6R$  to a load. Compute: (a) the voltage at the load; (b) the active power of the load; (c) the reactive power of the load and (d) the power factor angle of the load.

**Note:** In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

24S.012

12°

### Simple Power System



$$I_L = \frac{S}{V_s}$$

$$\theta = \cos^{-1}(\rho_f)$$

$$\text{b) } P = ZV \cos(\phi_L) \\ =$$

$$\phi_L = (\phi_s - \phi_L) \\ = 12^\circ$$

$$Z_{TH} = \frac{V_s}{I_L \angle (-\theta)} \rightarrow \text{lag}$$

$$\bar{S} = S \angle \theta = \bar{V} \bar{I}^*$$

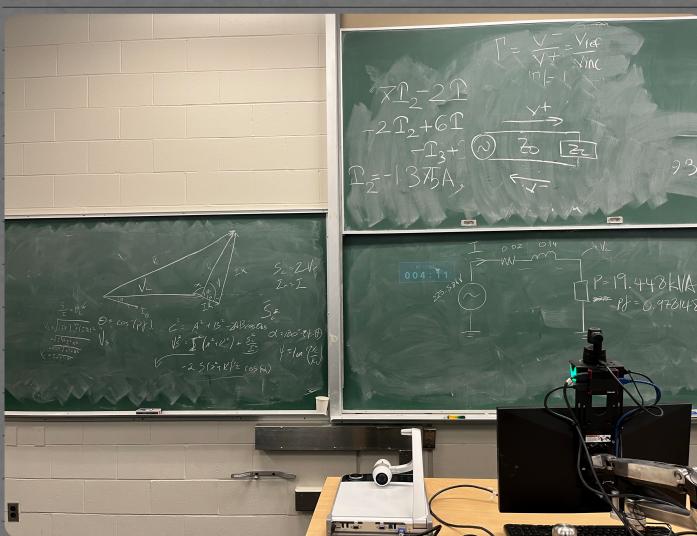
$$Z_{TH} = Z_R + Z_c = Z_L$$

$$\bar{I} = \left( \frac{S \angle \theta}{V} \right)^*$$

$$V_L = I_L Z_L$$

$$= 15.56 - 3.72i$$

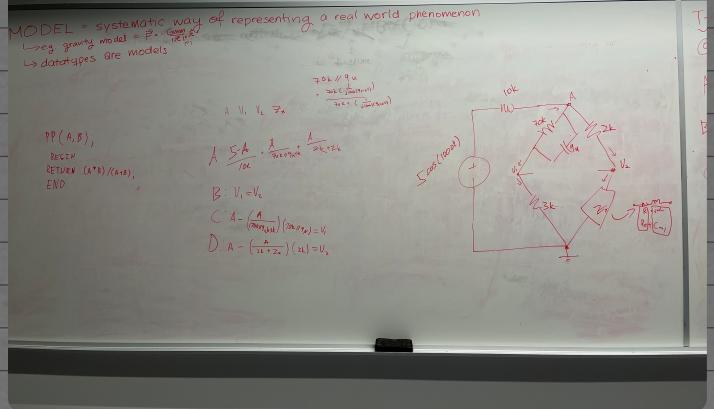
$$\begin{aligned} V_L &= V_s - V_2 \\ &= V_s - \bar{I}(Z_L + R) \\ &= 253.65 - 44.61i \end{aligned}$$



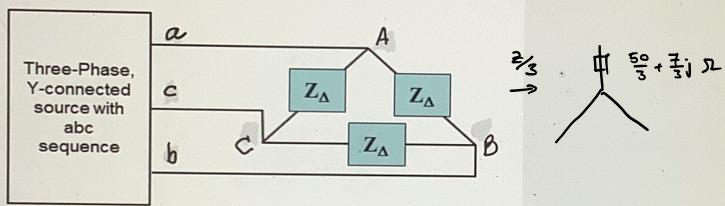
MODEL = systematic way of representing a real world phenomenon  
↳ eng. model =  $P = \frac{V^2}{Z}$   
↳ data types are models

```
PP(A,B),
BEGIN
RETURN (A+B)/(A*B);
END;
```

$$\begin{aligned} A &= 5A + \frac{1}{10000} \cdot \frac{1}{2k} \\ B &= V_1 \\ C &= A - \frac{(1 + 10000)}{(2k + 10000)} = 0 \\ D &= A - \frac{1}{(2k + 10000)}(2kV_1) = V_1 \end{aligned}$$



For the balanced three phase circuit below, take  $V_{an} = 165\angle 0^\circ V$ ,  $V_{bn} = 165\angle -120^\circ V$ ,  $V_{cn} = 165\angle 120^\circ V$  and  $Z_\Delta = 50 + j7\Omega$ . Compute the requested line voltage, line current, phase current and total complex power supplied to the load. Compute the load PF (specify leading or lagging) and determine the value of a component that can be added in parallel to each branch of the delta load (i.e., three identical components) to raise the PF to unity; to do the latter, assume the frequency of operation is 60 Hz.



Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

$$V_{ab} = \boxed{\quad} \ \boxed{\quad} \angle \boxed{\quad} \ \boxed{\quad} {}^\circ V$$

$$I_a = \boxed{\quad} \ \boxed{\quad} \angle \boxed{\quad} \ \boxed{\quad} {}^\circ A$$

$$I_{AB} = \boxed{\quad} \ \boxed{\quad} \angle \boxed{\quad} \ \boxed{\quad} {}^\circ A$$

$$S_L = \boxed{\quad} \ \boxed{\quad} \angle \boxed{\quad} \ \boxed{\quad} {}^\circ VA$$

$$PF = \boxed{\quad} \ \boxed{\quad} \ ? \ \downarrow$$

$$V_{an} = 165\angle 0^\circ V, V_{bn} = 165\angle -120^\circ V, V_{cn} = 165\angle 120^\circ V \text{ and } Z_\Delta = 50 + j7\Omega.$$

$$V_{an} = 165 \angle 0^\circ V \rightarrow V_{ab} = 165\sqrt{3} \angle 30^\circ = V_L$$

$$I_a = \frac{V_{an}}{(Z_\Delta)} = \frac{165}{(50 + j7)} = 9.804 \angle -7.969^\circ$$

$$I_{AB} = \frac{V_{ab}}{Z_\Delta} = \frac{165\sqrt{3} \angle 30^\circ}{50 + j7} = 5.66 \angle 22.03^\circ$$

$$S_L = 3V_a I_a \text{ with } \angle V_a^\circ - I_a^\circ \text{ or } \sqrt{3} V_{an} I_a$$

$$= 3V_a R I_{AB} \angle 30 - 22.03$$

$$PF = \cos(30 - 22.03)$$

$$= 0.9903$$

$$I_m(Z_0 \parallel Z_x) = 0$$

$$I_m((50 + j7) \parallel \frac{1}{j\omega C}) = 0$$

$$I_m(50 + j7 \parallel \frac{1}{2\pi f \cdot C \cdot j}) = 0$$

$$C = 7.2844 \times 10^{-6} F$$