

Circuit Analysis 1

Wire in a circuit allows any level of current to move without any voltage drop.
Current is a flow rate of charge through an element. Think of it like water, charges being its molecules.

Voltage is an electric potential difference between two points. Imagine gravitational potential differences of an object, the object here being the charge.

3 basic components of a circuit are resistor, capacitor and inductor. Often called as RLC.
Resistor is an element whose voltage is proportional to current, also known as Ohm's law.
Capacitor is an element that stores electrical energy in an electrical field. It also opposes sudden change in voltage.

Inductor is an element that stores energy in a magnetic field, this allows the element to oppose sudden changes in current.

$$\text{QSCST} \quad V = IR \quad I = C \frac{dU}{dt}, \quad \Rightarrow \quad V = L \frac{di}{dt}$$

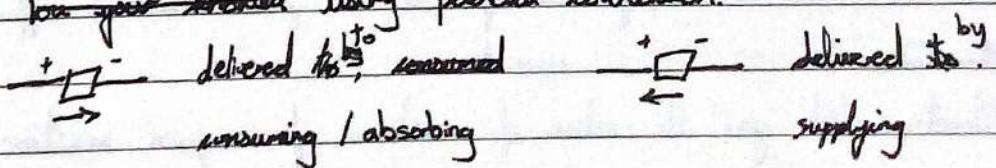
$$Q = CV$$

As with any other objects, the elements consume or produce energy. The unit J, represents the energy delivered to 1C charge by e-field with 1V voltage.

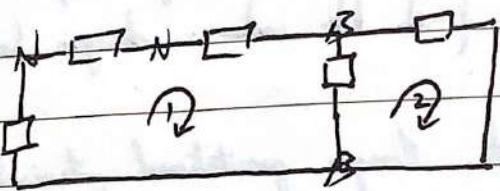
Power is the energy per time of. And thus the power of an element in circuit can be derived.

$$P = IV \quad \text{How do we know if power is consumed or produced?}$$

You ~~your~~ calculate using potential convention.



Geometry of circuit consists of :



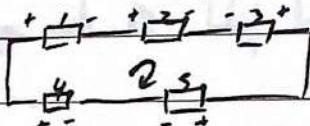
Node, point of connection.

Branch, a place connecting two node.

loop, node \rightarrow branches \rightarrow original node.

Two main laws governing circuit analysis are :

Kirchhoff's voltage law, where sum of voltage in a given loop is zero.

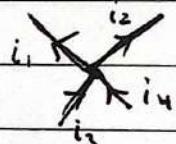


$$-V_1 - V_2 + V_3 - V_5 + V_4 = 0$$

* note your polarity.

use potential convention if possible.

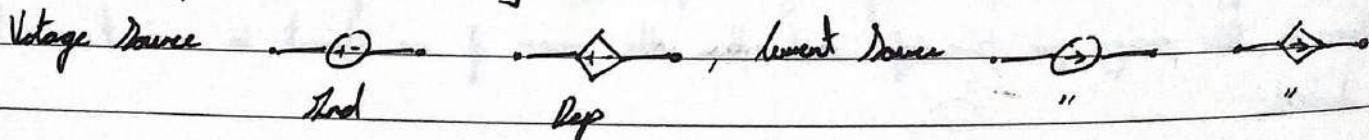
Kirchhoff's current law, sum of current in & out is zero, in a given node / branch.



$$-i_1 - i_2 + i_3 + i_4 = 0$$

* in \rightarrow pos out \rightarrow neg.

More elements of the circuit, mainly sources:



Voltmeter tells the voltage voltage at a place without effecting its current.
(btw two terminal)

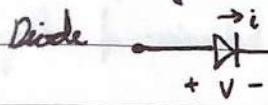
Ammeter

current

voltage.

Ohmster tells you the value of resistance of a given resistor.

Simple overview of electronics:

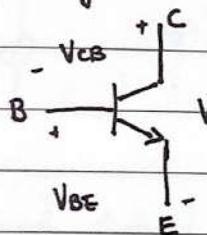


is a device that allows the current to flow only in one direction. When applied small $+V$, current is allowed to pass, however, when $-V$ is applied, no current is allowed to pass.

$$V \approx 0 \quad i > 0$$

$$V < 0 \quad i = 0$$

Bipolar Junction Transistor is a device that in which current through collector (C) & emitter (E) is controlled by the base (B).

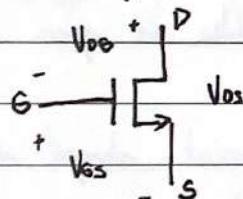


$$i_C = \beta i_B \quad i_E = i_C + i_B$$

↳ current gain

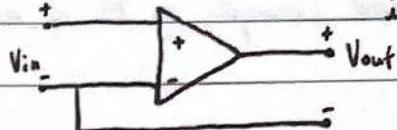
Field - Effect Transistor is a device in which the current flowing in Source (S) &

Drain (D) is controlled by the voltage applied in Gate (G)



$$i_D \rightarrow \text{depends on } V_{GS} \quad i_S = i_D + i_G$$

Operational Amplifier is a device that amplifies the difference in voltage between its two terminals.



Introduction to 1st-order circuit:

1st order time dependent circuit is a circuit in which the behavior of voltage and current is described by 1st order linear ODE.

As a systematic approach to 1st order circuit

1. KVL KCL element, before switching. Get desired variable.
2. Determine how the variable of interest behaves \Rightarrow with switching action.
3. KVL KCL element, after switching. (a new circuit)
4. Eliminate variable to get 1st-order ODE
5. Use the info. on step 2 to solve the ODE.

Behaviors of C & L

— II — $i = C \frac{dv}{dt}$ During switching, capacitor will resist abrupt changes in voltage.

In DC-ss, capacitor will act like an open circuit.

— III — $v = L \frac{di}{dt}$ During switching, inductor will resist abrupt change in current.

In DC-ss, inductor \Rightarrow will act like a short circuit, like on a wire.

Growth & decay of both components can be derived (example pg 25 of e-note).

Plan

τ is a time constant which the exponentially decaying function dropped to ≈ 0.36 to its original value.

$\forall t > 0$ the general equation $V(t) = (V_{\text{cap}} - V_{\text{final}})e^{-t/\tau} + V_{\text{final}}$ is valid.

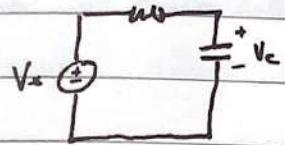
General solution for $V_c(t) = Ae^{-t/\tau} + B$ where $\tau = RC$.

$$V_L(t) = Ae^{-t/(L/R)} + B \text{ where } \tau = \frac{L}{R}$$

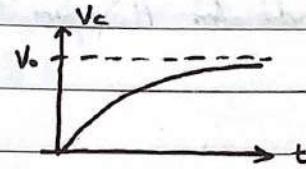
$\therefore V(t) = Ae^{-t/\tau} + B$ is the general solution for all first order circuit components.

Circuit Responses

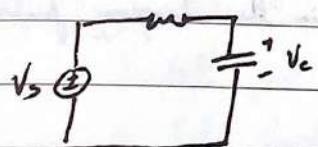
Step response is the response of a circuit to a step function.



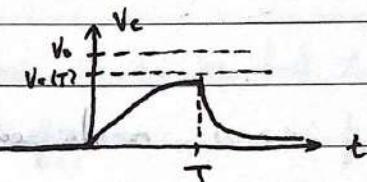
$$V_s = V_o u(t)$$



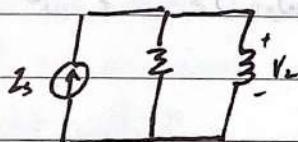
Pulse response is the response of a circuit to a pulse function.



$$V_s = V_o [u(t) - u(t-T)]$$



Numerical Approach to TD circuit (finite difference time domain)



$$I_L = I_o u(t)$$

$$V_L = V_R = R i_R = R i_o$$

$$V_L' = -\frac{1}{L/R} V_L$$

$$V_L' + \frac{1}{L/R} V_L = 0$$

using differentiation

$$V_L' \approx \frac{\Delta V_L}{\Delta t} = V_L(t + \Delta t) - V_L(t) = -\frac{1}{L/R} V_L(t)$$

$$V_L(t + \Delta t) = V_L(t) - \frac{\Delta t}{L/R} V_L(t)$$

Introduction to 2nd Order circuit:

2nd order time dependent circuit, unlike the first, consists of both component P.C. & L . The combination, then, provides an oscillating factor that exhibits a damping property.

comes about when

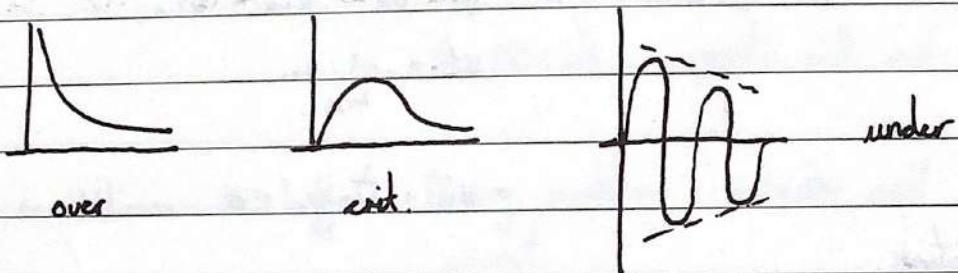
Damping

From the equation $\frac{d^2x}{dt^2} + 2\alpha \frac{dx}{dt} + \omega^2 x = f$ where ω is the natural frequency
 α is the damping factor

if $\alpha > \omega$, over damped. $x(t) = m_1 e^{s_1 t} + m_2 e^{s_2 t} + \frac{f}{\omega^2}$
 $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega^2}$

if $\alpha = \omega$, critically damped $x(t) = (m_1 t + m_2) e^{-\alpha t} + \frac{f}{\omega^2}$

if $\alpha < \omega$, under damped $x(t) = (m_1 \cos(\omega_d t) + m_2 \sin(\omega_d t)) e^{-\alpha t} + \frac{f}{\omega^2}$



Linear circuit:

Linear circuit is a circuit where all the components hold linear relationships. All σ voltage and current will have linear dependence on all independent sources.

$$V_j = a_{1j}S_1 + a_{2j}S_2 \dots + b_{kj}i_k S_j$$

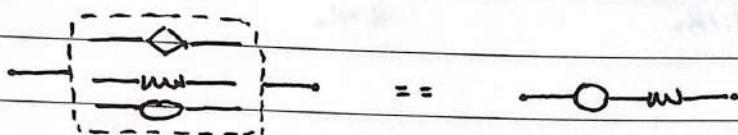
Superposition:

Superposition in a circuit describes that in a linear circuit you are able compute the sum of all independent source by computing individual parts and adding them.

Remember setting V_S to zero \Rightarrow short, Using this we are able to compute individual CS to zero \Rightarrow open sources V & I .

Rhevin Equivalent :

This theorem says that if we have a circuit comprising of n independent & dependent sources, in a linear relationship, we can represent any two point in a circuit by an independent voltage source and in series with R .

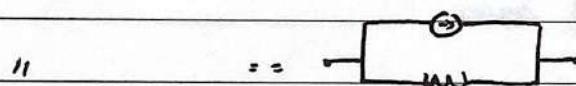


Steps you take

1. Get your open circuit voltage (V_{oc}). This will be equivalent to V_{Th} .
2. Get short circuit current (I_{sc}). Note $R_{Th} = \frac{V_{Th}}{I_{sc}}$.

Norton Equivalent :

Under the same assumption & with Rhevin, this theorem suggests that any two point in a circuit can be represented by independent current source in series parallel with R .



Steps also are identical.

The inter-change between the two representation, use source transformation.

$$V_{Th} = R_N I_{sc} \rightarrow R_N = R_{Th} \quad \therefore V_{Th} = R_{Th} I_{sc}$$

$$\frac{V_{Th}}{R_{Th}} = I_{sc} \quad = R_N I_N$$

Equivalent resistance is similar to Rhevin & Norton equivalent. R_{Th} .

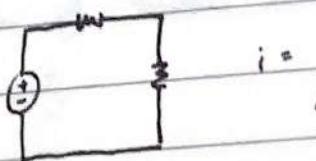
When ~~there's~~ no independent source, apply a voltage or current and compute the rest, divide the two to get R .

$$R_s = R_1 + R_2 + \dots + R_n \quad (\text{same current})$$

$$R_{Th} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} \right)^{-1} \quad (\text{same voltage})$$

Circuit Division :

Voltage division allow a analysis of two R's in series.

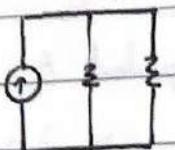


$$i = \frac{V}{R_1 + R_2}$$

$$V_1 = R_1 i = \frac{R_1}{R_1 + R_2} V$$

$$V_2 = R_2 i = \frac{R_2}{R_1 + R_2} V$$

Current division allow a analysis of two R's in parallel.



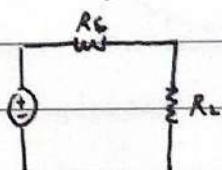
$$V = \frac{I}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$i_1 = \frac{V}{R_1} = \frac{R_2}{R_1 + R_2} I$$

$$i_2 = \frac{V}{R_2} = \frac{R_1}{R_1 + R_2} I$$

Maximum Power :

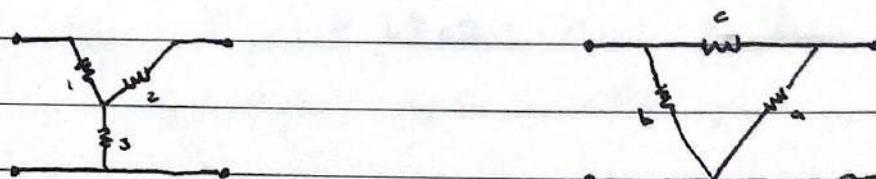
Maximum power theorem states that to draw a maximum power, resistance of load must equal resistance of source.



$$P_{\max} \rightarrow R_s = R_L$$

$$P_{\max} = \frac{V_{TH}^2}{4R_{TH}}$$

Wye - Delta Transformation :



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

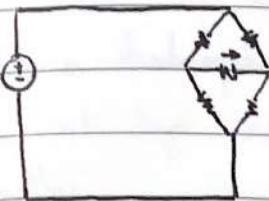
$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

The Wheatstone Bridge :

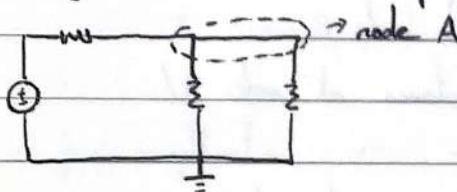


current through the middle is 0.

Nodal Analysis :

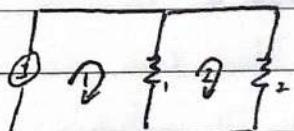
Node voltage is the voltage difference between a given node and a fixed reference node.

Defining any one of the nodes as a reference, with nodal voltages, we can compute any information in a given circuit.



Mesh Analysis :

Use the understanding that in a given loop, ^{Voltage} current must add up. And the current within two loops through a element ~~can~~ co-exists in both must equal zero.



$$i_1 = I_1 - I_2$$

$$N_1 : -V + V_1 = 0$$

$$V_1 = I_1 R_1$$

$$i_2 = I_2$$

$$N_2 : -V_1 + V_2 = 0$$

$$V_2 = I_2 R_2$$

Super-position in larger circuits :

To find V_{oc} , solve V by killing independent voltage source and add the V you get killing the independent current source.

Also, utilize source transformations in getting the equivalent circuits.

Mesh Analysis Inspection:

Analysis Inspection:

Nodal inspection can be made when all sources in a circuit is independent current source.

Mesh inspection

source.

"

voltage

$G_{11} \dots G_{1N}$	V_1	I_1
$\vdots \quad \vdots$	\vdots	\vdots
$G_{NN} \dots G_{NN}$	V_N	I_N

Nodal - G_{kk} , sum of conductance ($\frac{1}{R}$) at node k.

G_{kl} , neg. of sum of the conductance at node kl.

V_k , unknown voltage at node k.

I_k , sum of indep. current source at node k.

Mesh - R_{kk} , " resistance " mesh k

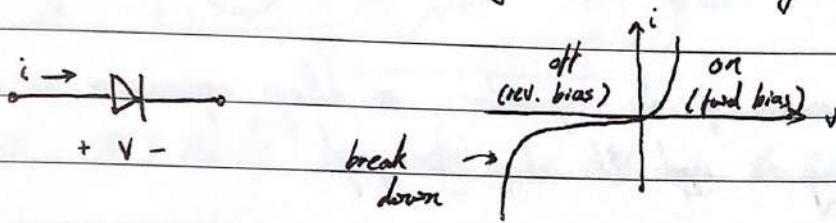
G_{kk} , " resistance " mesh kk

I_k , " current " mesh k

V_k , " voltage " mesh k.

Introduction to Electronics :

Diodes are elements in which at a positive input voltage, it behaves with a low resistance allowing the current to go through and at high ~~resistance~~^{neg. voltage}, it behaves with high resistive resistance, allowing no current to go through.



$$i = I_s (e^{\frac{V}{nV_{TH}}} - 1)$$

n , ideality factor

~~V_{TH}~~ , ~~kT~~ $V_{TH} = kT$, the Boltzmann constant · Absolute temp.
 I_s , reverse saturation current.

few simplifications,

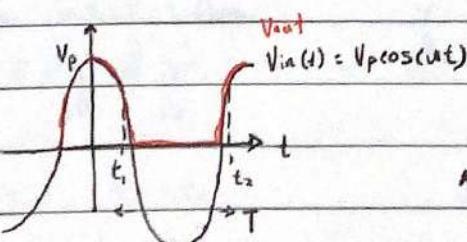
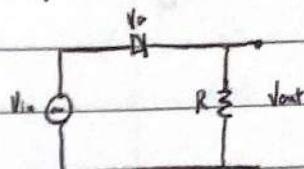
$$\frac{V_0}{V_0 + i} \approx 1$$

- assume reverse bias current is 0. And 0 until pos. voltage reaches $\sim 0.7V$ of threshold value. Beyond that, a linear increase.
- neglect resistance at fwd bias.
- turn-on voltage is 0.

How do we know in a given circuit that a diode is fwd or rev?

We can't until we try every possibilities and find the right one with computing V 's or on assumptions.

Rectification,



* V_{out} has pos. avg.

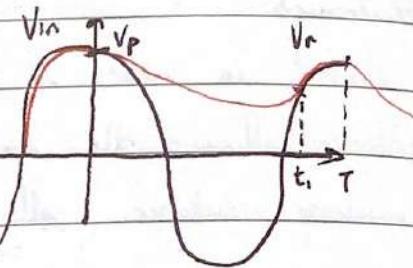
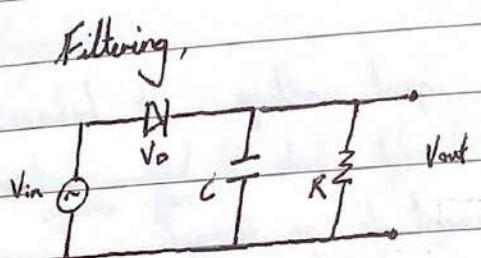
$$V_{in} = V_o + V_{out}$$

$$V_{in} > 0, V_o = 0$$

$$V_{in} < 0, V_{out} = 0$$

$$T = \frac{1}{f}$$

$$\omega = 2\pi f$$



Due to the presence of the capacitor, the voltage experiences an exponential decay, creating a ripple-like shape in graph.

when $V_r \ll V_p$,

$$(T \gg RC) \quad V_r \approx V_p(1 - e^{-\frac{T}{RC}})$$

$T \ll RC$,

$$V_r \approx \frac{I}{RC} V_p$$

$$V_{avg} = V_p - \frac{I}{2RC} V_p \quad i_{avg} = \frac{V_{avg}}{R}$$

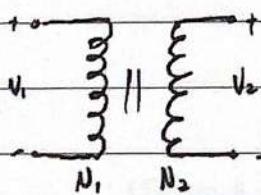
The conduction interval $T - t_1 = \Delta t$?

$$V_p(\cos(\omega_0 t)) = V_p - V_r \quad \cos(\omega_0 t) \approx 1 \approx 1 - \frac{1}{2} (\omega_0 t)^2$$

$$V_p(1 - \frac{1}{2} (\omega_0 t)^2) \approx V_p - V_r$$

$$\Delta t \approx \frac{I}{2n} \left(\frac{2T V_p}{RC} \frac{1}{V_p} \right)^{\frac{1}{2}}$$

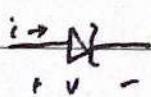
Transformer,



Device that transfers electric energy from one circuit to another, either increasing or decreasing the circuit voltage.

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

Zener Diode,

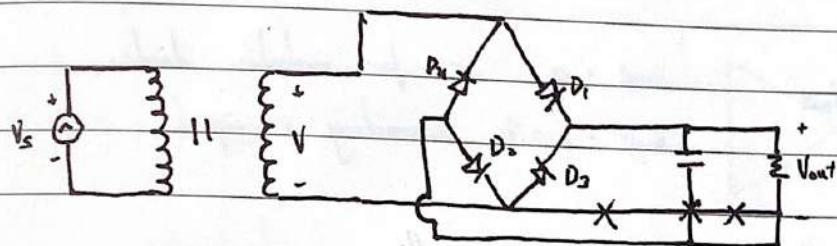


Allow forward flow in a similar manner with ideal diode but also permit reverse flow once above a value known as breakdown voltage, where the diode provides a constant output voltage regardless on the changes in the input.

$$V_o = V_{D0} + r_o I_o$$

$$\text{At } V_o = 8V \quad r_o = 20\Omega \quad \text{for } 1mA$$

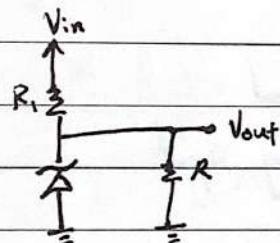
Rectification, Filtering & Transforming:



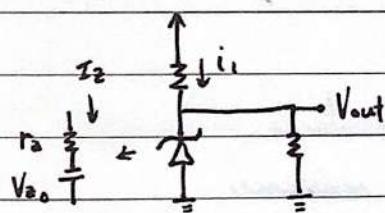
$V > 0$, D_1 & D_2 are on making $V_{out} = V$ as V_{D1} & $V_{D2} = 0$.

$V < 0$, D_3 & D_4 are on making $V_{out} = -V$ as V_{D3} & $V_{D4} = 0$

Regulation:



Want to deliver a certain voltage, with V_{in} being a sinusoidal input. This means we need to know what R_1 should be.



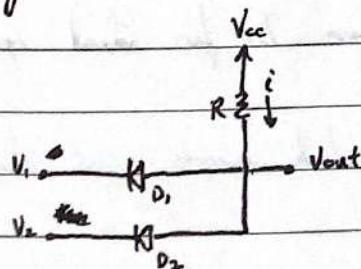
We must ensure $R_{2K} < R_2 < R_{1max}$.

Compute i_1 when V_{in} is at min and get R_{1max} .

for $R_2 = R_{1max}$,

compute i_1 when V_{in} is at max and get R_{1min} .

Logic Gates:



If $V_1 = V_{cc}$ $V_2 = V_{cc}$,

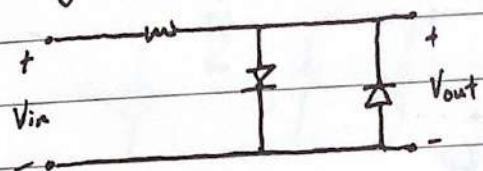
D_1 & D_2 off $\Rightarrow i = 0 \Rightarrow V_{out} = V_{cc}$

If $V_1 = 0$ $V_2 = V_{cc}$

D_1 on $\Rightarrow V_1 = V_{out} \Rightarrow V_{out} = 0$

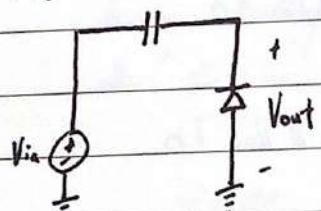
\therefore AND gate.

Limiting :



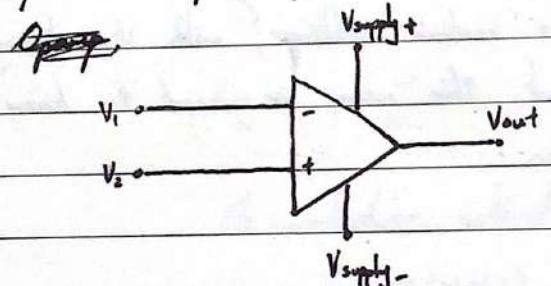
$V_{out} = 0$ or for realistic diode,
 $V_{out} = \pm 0.7$ according to input.

Clamping :

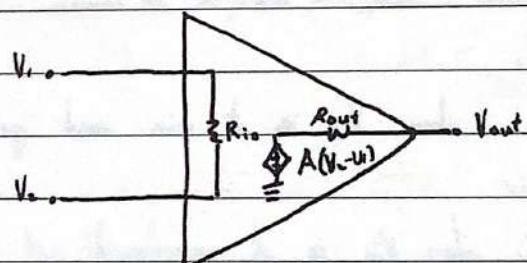


Adding DC value to AC input
(shifting waves in y-axis)

Operation Amplifier,



V_1 - Inverting input
 V_2 - Non-Inverting input



R_i - Input resistance
 R_{out} - Output resistance
 A - Open loop gain

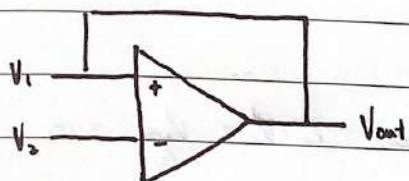
Operation amplifier amplifies voltages.

We want high R_{in} & low R_{out} for optimal amplification. So for ideal opamp,

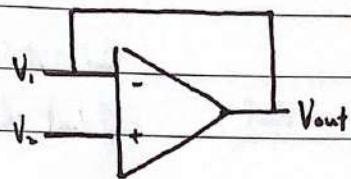
$$R_i = \infty, R_{out} = 0, A = \infty$$

We also want V_{out} stay within linear region, which means V_{out} should be bounded to V_{Supply} 's.

positive & negative feedback :



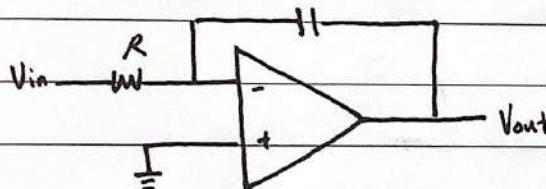
pos feedback



neg feedback ($V_2 = V_-$)

Other applications of opamps :

Integrator



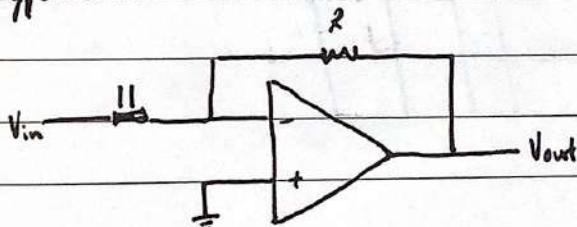
$$i_C = C \frac{dV_C}{dt} \quad i = \frac{V_{in}}{R}$$

$$V_C = \int_0^t dV_C = \frac{1}{C} \int_0^t i_C dt$$

↳ initially unchanged.

$$V_{out} = -\frac{1}{RC} \int_0^t V_{in} dt$$

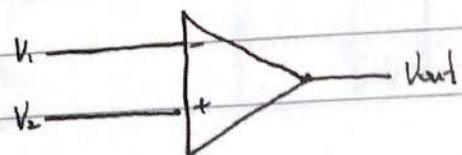
Differentiator :



$$i_C = C \frac{dV_C}{dt} = C \frac{dV_{in}}{dt}$$

$$V_{out} = -RC \frac{dV_{in}}{dt}$$

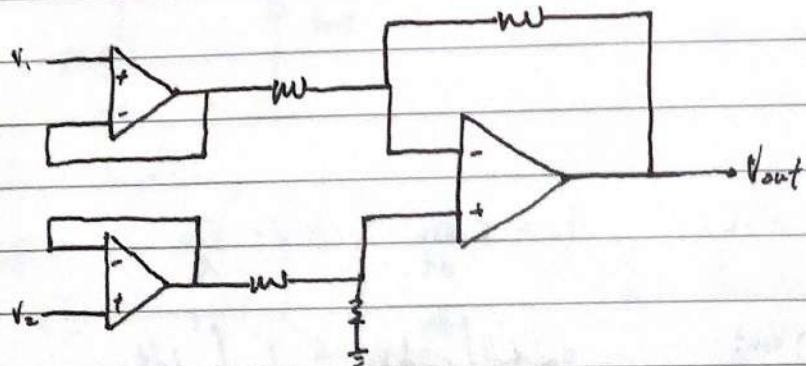
Ideal Op-Amp :



$$V_{out} = A(V_2 - V_1)$$

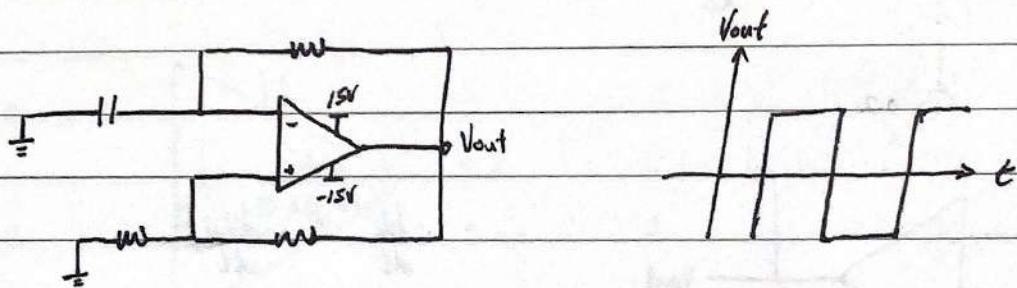
If at linear region $V_2 - V_1 = \frac{V_{out}}{A} = 0$

Instrumental Amp :

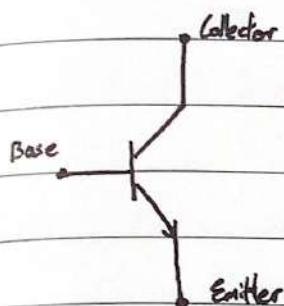


Latching op-amp circuits.

Oscillator :



Bipolar Junction Transistor



$$i_E = i_C + i_B$$

$$i_C = B i_B$$

$$\alpha = \frac{B}{B+1}$$

$$= \alpha i_E$$

NPN

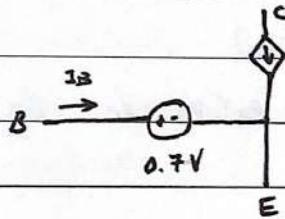
$$i_C \approx I_{SCE} \left(\frac{V_{BE}}{V_T} \right)$$

↳ saturation current

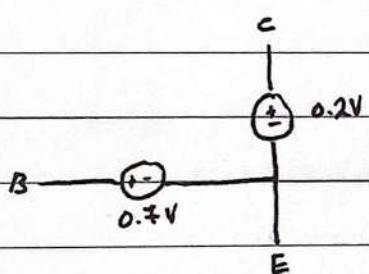
3 models to represent BJT depending on I_B & V_{CE} .

Steps:

represent BJT as a linear model

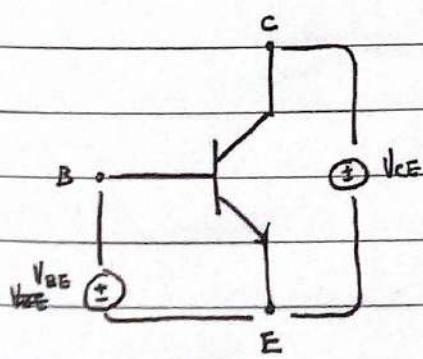


if $I_B < 0$, cutoff model
if $I_B > 0$, test for saturation

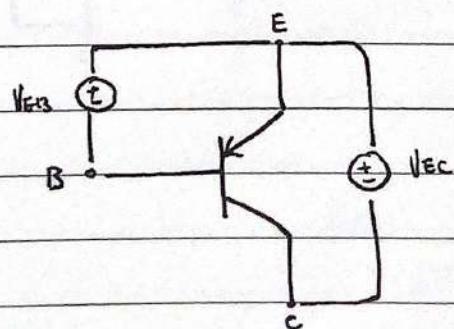


if $V_{CE} \leq 0.2$, saturation model
if $V_{CE} > 0.2$, linear model

PNP & NPN

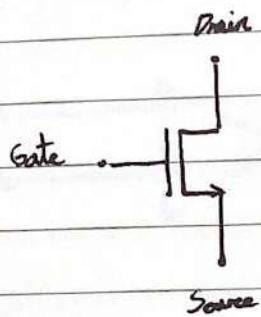


NPN



PNP

Metal Oxide Semiconductor Field Effect Transistor



$$k_n = \frac{w}{l} \mu C_{ox}$$

width oxide cap per unit area.
channel length mobility

n-channel

3 ways to represent MOSFET V_{DS} & V_{GS}

If $V_{GS} < V_t$, cut off so $I_D = 0A$

If $V_{GS} > V_t$, compute $V_{DS} = V_{GS} - V_t$

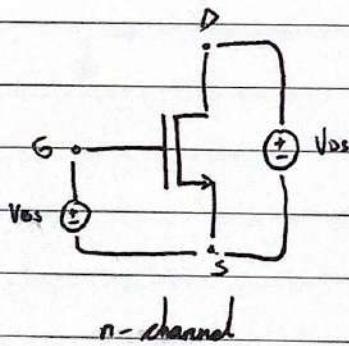
If $V_{GS} > V_{DS}$, triode (variable resistor)

$$g_{DS} = k_n (V_{GS} - V_t - \frac{V_{DS}}{2}), \quad I_D = k_n (V_{GS} - V_t - \frac{V_{DS}}{2}) V_{DS}$$

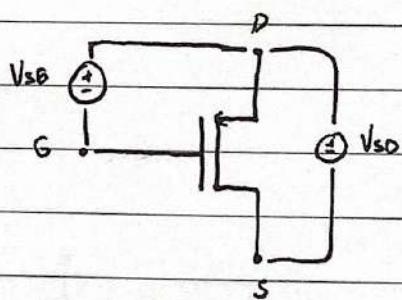
If $V_{GS} \geq V_{DS}$, saturation (linear)

$$I_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$$

n & p channels



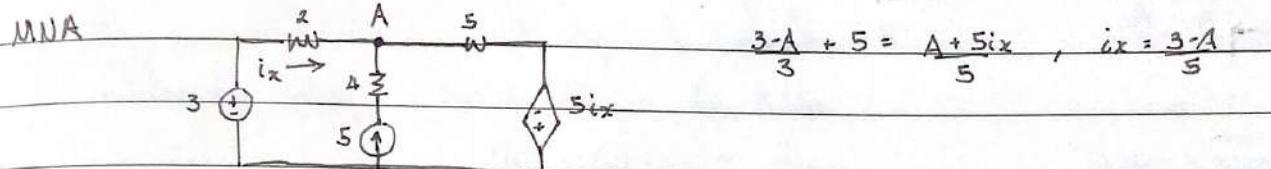
n-channel



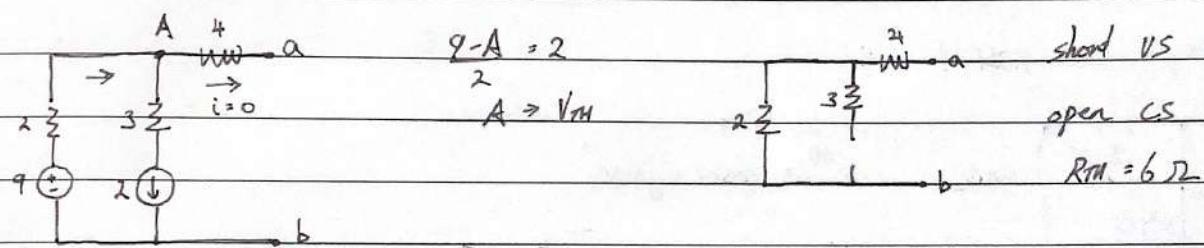
$V_{GS} - V_{tP}$ shift to this.

Circuit Analysis 21

Few fundamentals from the previous :

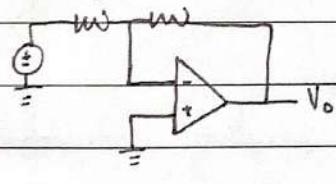


Therein

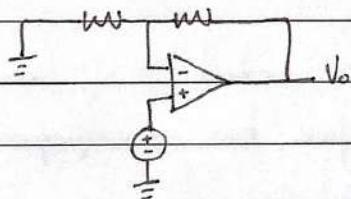


Op-amps (basic configuration)

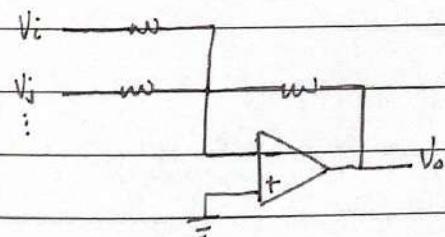
Inverting



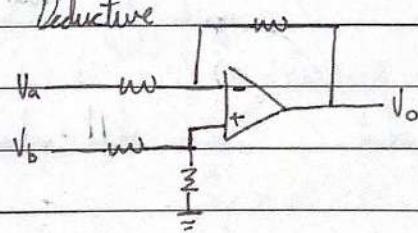
Non-inverting



Additive



Differential



Inductors & Capacitors

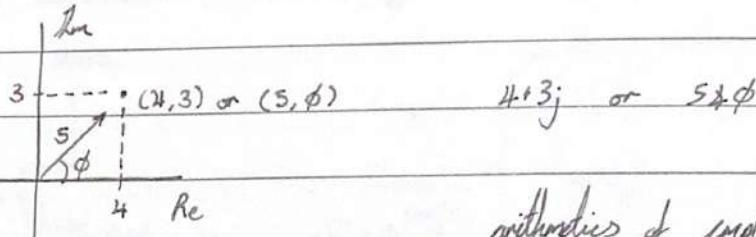
$$3L \quad V_L = L \frac{di}{dt}$$

$$\frac{1}{C} C \quad I_C = C \frac{dv}{dt}$$

Power

$$P = IV$$

Complex Numbers :

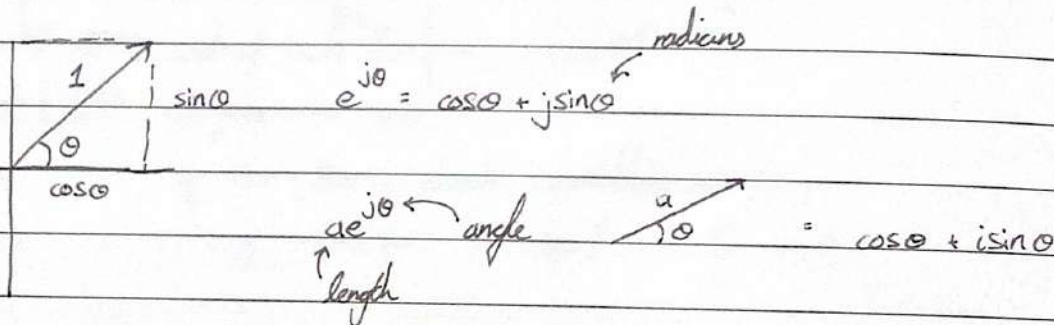


arithmetics of complex #'s

add & sub \rightarrow real & imaginary, vector representation
mult. $\rightarrow (a+jb)(c+jd)$

$$\text{div. } \rightarrow \frac{a+jb}{c+jd} = \frac{(a+jb)}{(c+jd)} \cdot \frac{(c-jd)}{(c-jd)} = \frac{(a+jb)(c-jd)}{(c^2+d^2)}$$

Euler's



Steady State :

DC, long-time (DC) \rightarrow everything is constant \rightarrow L short C open

AC, " \rightarrow sinusoidal func. have same frequency \rightarrow everything sinusoidal \rightarrow LRC is represented by ohms (ω)

The sine function, $y = A \sin(\omega t + \phi)$
 amplitude \downarrow angular freq. (rad/s)
 \uparrow phase shift (radians)

Stimulus Idea (of add & sub sinusoidal functions)

$$\underbrace{A \cos(\omega t + \alpha) + B \sin(\omega t + \beta)}_{\text{in}} = C \cos(\omega t + \gamma)$$

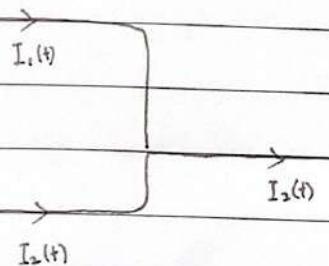
$$\text{Adding } \operatorname{Re}(Ae^{j(\omega t + \alpha)}) + \operatorname{Re}(Be^{j(\omega t + \beta)}) = \operatorname{Re}(Ce^{j(\omega t + \gamma)})$$

$$Ae^{j(\omega t + \alpha)} + Be^{j(\omega t + \beta)} = Ce^{j(\omega t + \gamma)}$$

$$\underbrace{e^{j\omega t} (Ae^{j\alpha} + Be^{j\beta})}_{\text{or } A \cos \alpha + B \sin \beta} = e^{j\omega t} (Ce^{j\gamma}) \leftarrow \begin{array}{l} \text{complex # representation of sinusoid} \\ \text{Phasors.} \end{array}$$

KVL & KCL

Just like in DC circuit, we can use the circuit analysis skill like MNA.



$$i_1(t) = 5 \cos(300t + 30^\circ) \rightarrow i_2(t) = 5 \angle 30^\circ + 7 \angle -20^\circ$$

$$i_2(t) = 7 \cos(300t - 20^\circ) = 10.908 \angle 0.556^\circ$$

or

$$10.907 + 0.105i$$

$$= 10.91 \cos(300t + 0.556^\circ)$$

$$\frac{0.556\pi}{180}$$

Impedance :

Resistance (R) of components in AC circuit.

to compute instantaneous i_2 .

Derivation

$$\text{say } i(t) = A \cos(\omega t + \alpha) \leftrightarrow A \angle \alpha$$

$$i'(t) = -\omega A \sin(\omega t + \alpha)$$

$$= \omega A \sin(\omega t + \alpha + 90^\circ) \rightarrow \omega A \angle 90^\circ$$

$$i(t) = \frac{A}{\omega} \cos(\omega t + \alpha - 90^\circ) \rightarrow \frac{A}{\omega} \angle -90^\circ$$

or

$$j\omega(A \angle \alpha)$$

$$\frac{A \angle \alpha}{j\omega}$$

Inductor

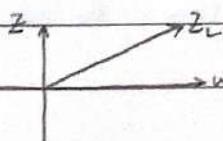
$$V_L = L \frac{di}{dt}$$

$$= L j\omega(A \angle \alpha)$$

$$\frac{\bar{V}_L}{V} = \frac{j\omega L}{R_L A}$$

$$Z_L = j\omega L$$

impedance reactance



Capacitor

$$i_C = C \frac{dV}{dt}$$

$$= C j\omega(A \angle \alpha)$$

$$= j\omega C \bar{V}_C$$

$$\bar{V}_C = \frac{1}{j\omega C} i_C = -j \frac{1}{\omega C} i_C$$

$$Z_C = R_C \times \frac{1}{\omega C}$$

Resistor

$$\bar{V}_R = R \bar{I}_R$$

Ohm's Law

$$\bar{V} = \bar{Z} \bar{I}_R, \text{ where } Z_L = j\omega L, Z_C = -j \frac{1}{\omega C}, Z_R = R$$

Phase Implication of Ohm's law

Inductor 'stretches' current phasor by ωL times.

Capacitor 'shrink's " ωC times.

Power in AC :

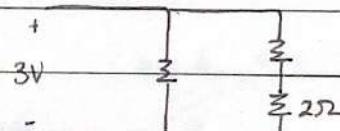
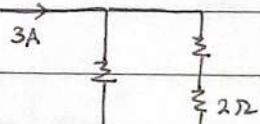
Power for AC circuit requires different implication to DC circuit.

DC Review

$$P = VI$$

Power in Resistor (only resistor!)

$$P_R = I^2 R = \frac{V^2}{R}$$



$$P_R = 2(3)^2 \text{ as } I_{2\Omega} = 3A$$

$$P_R = \frac{3^2}{2} \text{ as } V_{2\Omega} = 3V$$

Root Mean Square

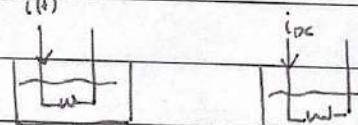
DC (1) reads I_{avg} \Rightarrow for $I(t)_{avg} = 0$ as $\sin \theta \neq \cos \theta$.

To make the reading work \rightarrow take square & \sqrt

$$\therefore \text{at AC, } I_{AC} = \left(\frac{1}{T} \int_T f(t)^2 dt \right)^{1/2}, \text{ AC (2) } \rightarrow I_{ACMS} \quad \text{AC (3) } \rightarrow V_{ACMS}$$

Effective Value

Analogy,



where heating = P_{avg} dissipated.

when $i(t)$ & i_{DC} cause the water to be same temp at the same time, the value of i is denoted as I_{eff} (or I_{AC}).

$$I_{DC} \rightarrow P = RI_{DC}^2$$

$$I_{AC} \rightarrow P = \frac{1}{T} \int_0^T R i(t)^2 dt$$

Effective Value for Sinusoids

$$i(t) = I_{\text{peak}} \sin(\omega t + \phi) \Rightarrow I_{\text{eff}} \text{ or } I_{\text{ac}} = \sqrt{\frac{1}{T} \int_0^T I_{\text{peak}}^2 \sin^2(\omega t + \phi) dt}$$

$$= \frac{I_{\text{peak}}}{\sqrt{2}}$$

From now on ... $\bar{I} = I_{\text{peak}} \pm 0^\circ \Rightarrow \bar{I} = I_{\text{rms}} \pm 0^\circ$
rms angle

Power in AC Steady State :

Generalization of I_{eff}

$i(t) = A \sin(\omega t + \phi)$ heat a resistor the same as I_{ac} with a value of $\frac{A}{\sqrt{2}}$.

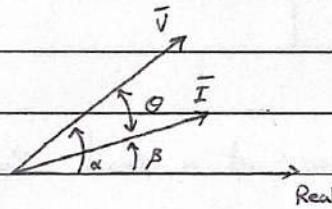
If we call 'M' effective value of $i(t)$, $M = \frac{A}{\sqrt{2}}$. Then, $i(t) = \sqrt{2} M \sin(\omega t + \phi)$

Active & Reactive & Apparent

Voltage in generic load is $v(t) = \sqrt{2} V \sin(\omega t + \alpha)$ if $i(t) = \sqrt{2} I \sin(\omega t + \beta)$

$$\bar{V} = V_{\text{rms}} \pm \alpha^\circ$$

$$\bar{I} = I_{\text{rms}} \pm \beta^\circ$$



$$P(t) = i(t)v(t) = 2VI \sin(\omega t + \alpha) \sin(\omega t + \beta) \text{ simplifying ...}$$

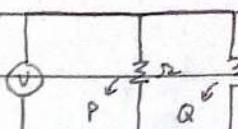
$$= VI \cos(\alpha - \beta) + VI \cos(2\omega t + \alpha + \beta)$$

$$P_{\text{avg}} = VI \cos(\alpha - \beta) - 0$$

$$= V_{\text{rms}} I_{\text{rms}} \cos(\underbrace{\alpha - \beta}_{\phi, \text{ power factor angle}}) \quad \left. \begin{array}{l} \text{power factor (\%)} \\ \text{inductive \& capacitive} \\ \text{(lagging) \& (leading)} \end{array} \right\} \text{Active Power (W), positive}$$

$$\text{source} \rightarrow \text{load}$$

$$(Q = VI \sin(\alpha - \beta)) \quad \left. \begin{array}{l} \text{Reactive Power (VAr), load} \rightarrow \text{source (phenomenon in all non-linear)} \\ \text{power that goes back \& forth without doing} \\ \text{meaningful work.} \end{array} \right.$$

 Apparent Power $S = VI$ (VA)

$$R \rightarrow \phi = 0^\circ \quad P = S \quad Q = 0$$

$$L \rightarrow \phi = 90^\circ \quad P = 0 \quad Q = S$$

$$C \rightarrow \phi = -90^\circ \quad P = 0 \quad Q = -S$$

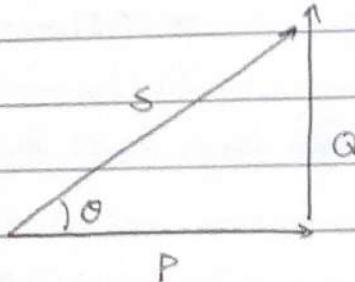
$$S = P + Qj$$

$$Q = (S^2 + P^2)^{1/2} \text{ for power triangle}$$

$$P = S \cos \theta = S \cdot \text{pf}$$

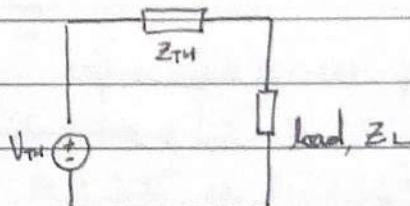
$Q = S \sin \theta$; Powers can also be computed using

$$\frac{ZI^2}{Z} \text{ & } \frac{V^2 I^2}{Z}$$



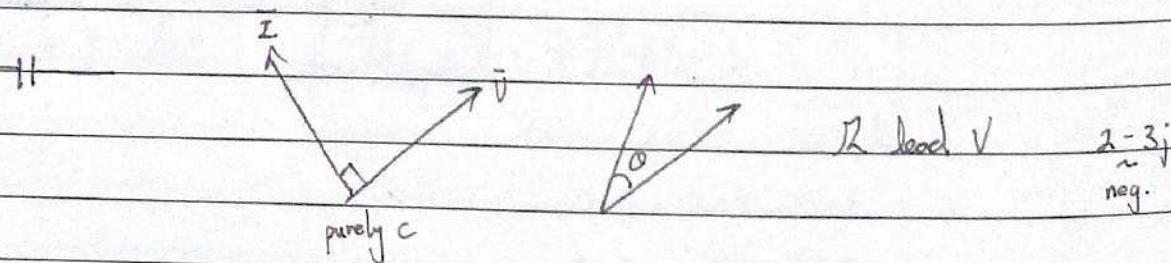
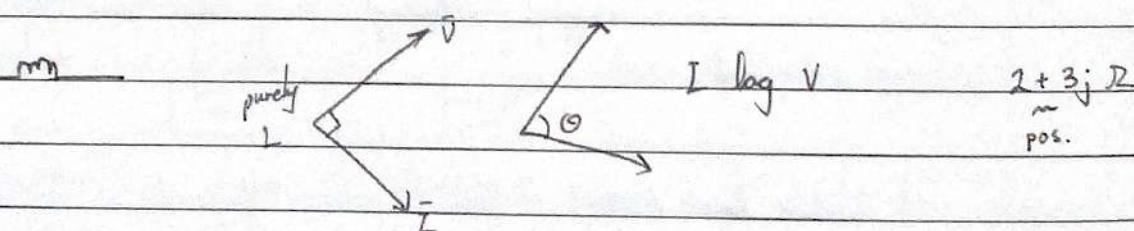
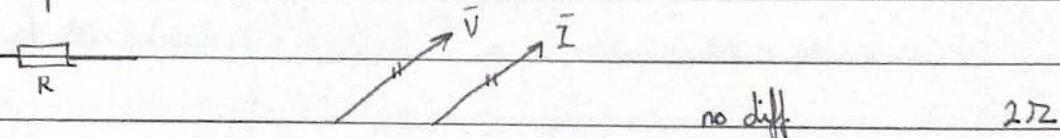
Maximum Power

To obtain Power more, V_m & Z_m must be established where $Z_L = Z_{TH}^*$

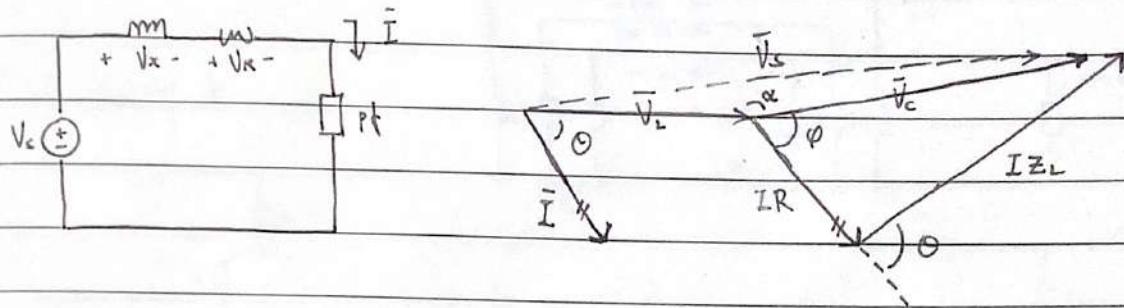


The maximum complex power at a load $P_{max} = \frac{1}{8} \frac{V_m^2}{Z_{TH}}$

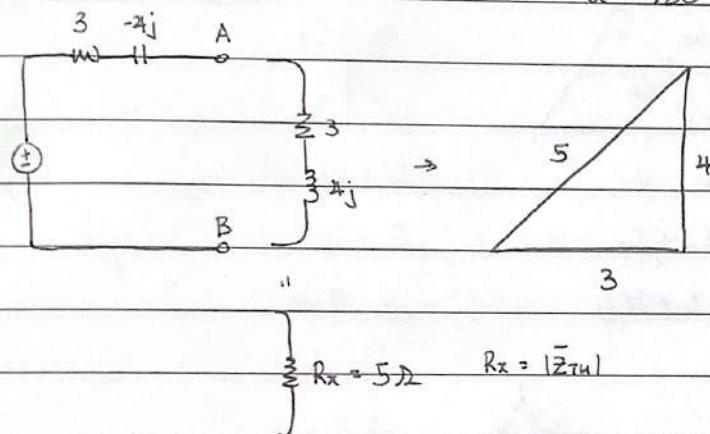
Graphic Representation



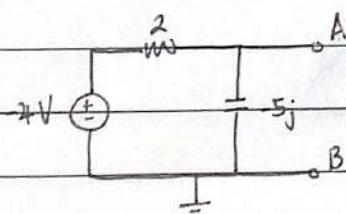
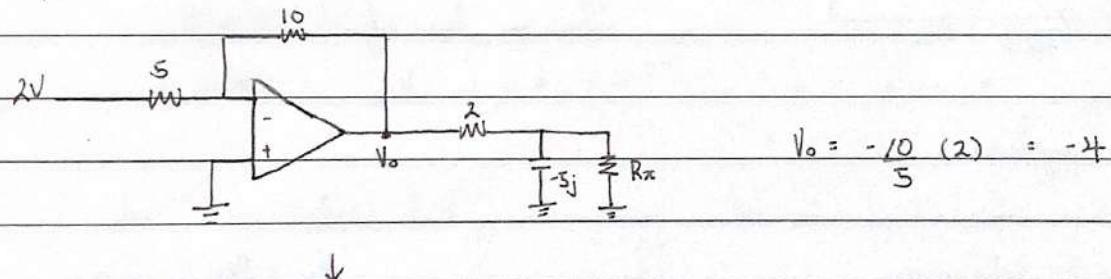
One Common Circuit



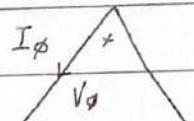
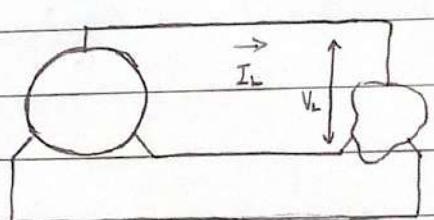
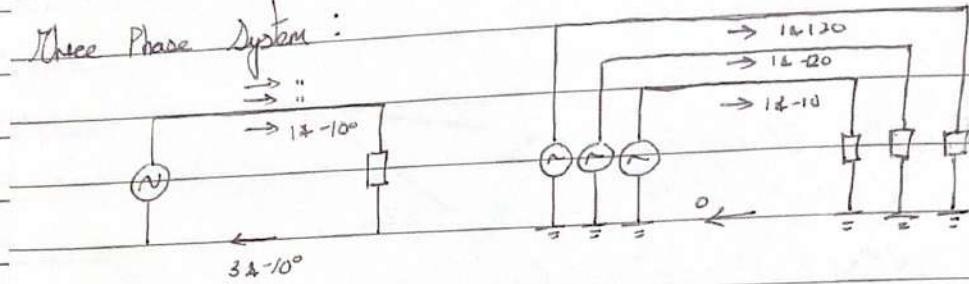
$$\alpha = 180 - (\phi - \theta) \quad \text{where } \phi = \tan^{-1}(Z_L/R)$$



$\Delta_p - \Delta_{np}$



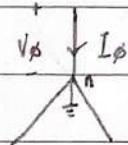
Three Phase System :



$$V_L = V_\phi$$

$$I_L \neq I_\phi$$

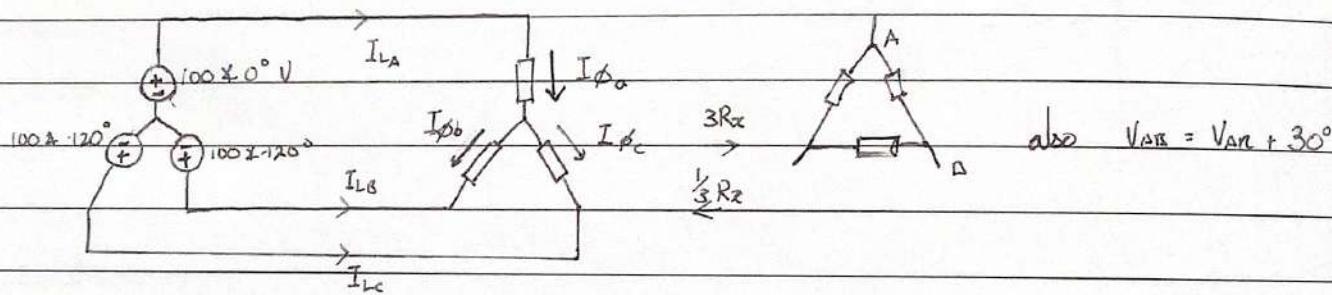
$$I_L = \sqrt{3} I_\phi$$



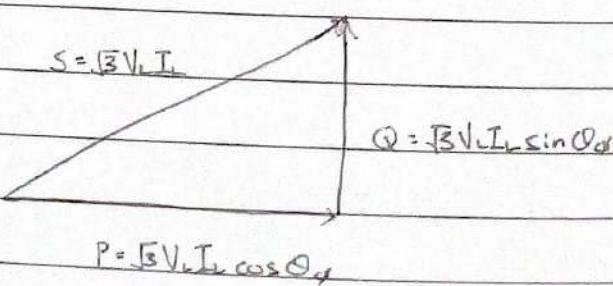
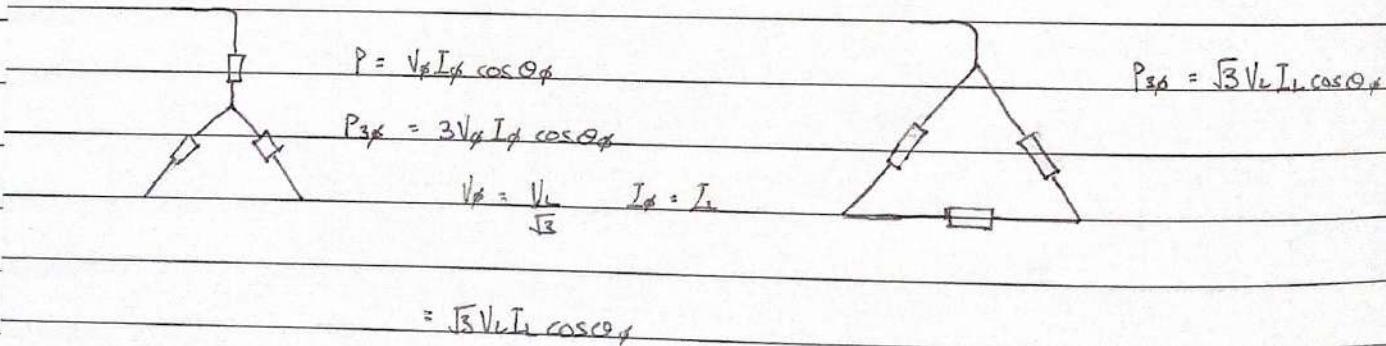
$$V_L \neq V_\phi$$

$$Z_L = I_\phi$$

$$V_L = \sqrt{3} V_\phi$$

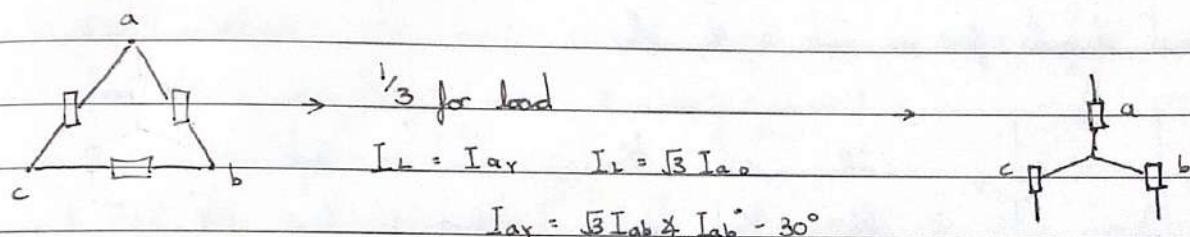
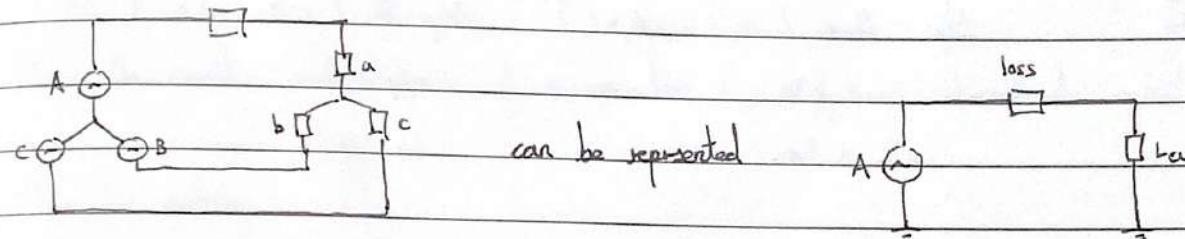


Power in 3 phase systems



$$\text{Power at the load} \Rightarrow \bar{S} = |\bar{I}|^2 \bar{Z}$$

Conversion and Simplification



$$V_L = \sqrt{3} V_{ab} \quad V_L = V_{ab}$$

$$V_{ab} = \frac{V_{ab}}{\sqrt{3}} \angle V_{ab}^\circ - 30^\circ$$

Power, $S = \sqrt{3} V_L I_L$ is for real rms value.

\rightarrow but often we don't have this info TRL.

$$\therefore \bar{S} = \sqrt{3} V_L I_L \text{ with } \angle V_L^\circ - I_L^\circ \quad \text{or} \quad S = 3 V_{ph} I_{ph}^*$$

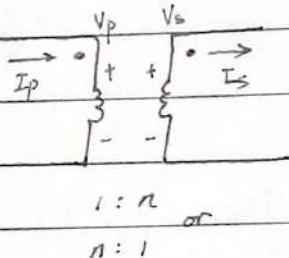
also, S_L where load is in series, S_L seen by the source = $3 S_L$

S_L	" " " " " " "	$= S_L$
$\stackrel{(Y)}{\text{parallel}}$	$\stackrel{(\Delta)}{\text{parallel}}$	

$$S_L = |\bar{I}_L|^2 Z_L \quad \text{or} \quad V_L I_L \quad \bar{S}_L = V_L I_L^* \dots$$

1) Transformers:

A device that adjust the $V \cdot Z \cdot P$ according to the device's coefficient (transformation ratio)



Step - Down [$n:1$, $n > 1$]

$$V_p = n V_s$$

$$1:n \text{ or } n:1$$

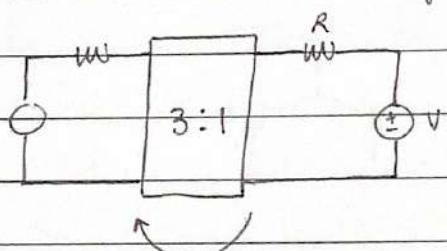
$$n I_p = I_s$$

Step - Up [$1:n$, $n > 1$]

$$n V_p = V_s$$

$$I_p = n I_s$$

We can move circuit elements from one side to the other.



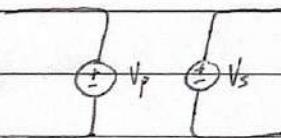
$$zR, aV, \frac{1}{a}$$

Low - to - high $n:1$

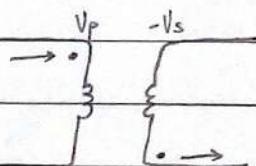
$$Ra^2, Ya, aZ$$

High - to - low $1:n$

Treating each transformer winding as a voltage source, simplifies many analytical steps.



Dkt. - Convection:



current goes in at one side, must come out at other

Notice that power equation stays the same. $S = V_{rms} I^2_{rms}$

* $I_p = \sum I_s$ Primary current is the combination of sum of all secondary currents.

2nd - Order Circuit :

The order in a circuit represents # reactive elements in a circuit.

∴ circuit with 2 reactive element

Solving the circuit, we make use of p-operator ($p = \frac{d}{dt}$) & characteristic equations for ODE.

Steps for solving :

1. Find conditions $t \leq 0$, using usual analysis tools
2. Find conditions $t > 0^+$, using $i_L'(0^+) = V_L(0^+)$ $V_C'(0^+) = i_C(0^+)$ with reactive values
from step 1 as $V_C(0^+) = V_C(0^-)$ & $i_L(0^+) = i_C(0^-)$.
 $V - \text{Src}$ $I - \text{Src}$
3. Find the equation for asked variable by converting the $t \geq 0$ circuit to p-operable
using $R = R$ $L \Rightarrow L_p$ $C \Rightarrow C_p$.
4. Take the characteristic equation from the denominator of the eq. at step 3, and get eigen-values. This allows us to know which solution to ODE is most fit, and
the time-to-steady state = $5\tau_{\text{larger}}$
 $\hookrightarrow z_1 = -\frac{1}{b_1}$ $z_2 = -\frac{1}{b_2}$
5. Use ODE solution, b_1 & b_2

$$b^2 > 4ac \quad b_1, b_2 \in \mathbb{R} \quad (\text{Over-damped}) \quad - y(t) = k_1 e^{b_1 t} + k_2 e^{b_2 t} + k_3$$

$$b^2 < 4ac \quad b_1, b_2 \notin \mathbb{R} \quad (\text{Under-damped}) \quad - y(t) = k_1 e^{\alpha t} \cos(\omega t + \phi) + k_3, \quad s = \alpha + \omega j$$

$$b^2 = 4ac \quad b_1 = b_2 \quad (\text{Critically-damped}) \quad - y(t) = (k_1 t + k_2) e^{b_1 t} + k_3$$

and the IC's, $y(0)$ & $y'(0)$, to solve the coefficients k_1, k_2 & k_3 .

Note,

$$\cdot (0.1p^2 + 17p + 100)V_C = 12p^2 + 600 \quad \& \quad k_3 = \frac{600}{100} = 6.$$

• For $V-D$ case, one can also use

$$y(t) = e^{\alpha t} [k_1 \cos(\omega t) + k_2 \sin(\omega t)]$$

$$y'(t) = e^{\alpha t} [(\omega k_2 - \alpha k_1) \cos(\omega t) - (\omega k_1 + \alpha k_2) \sin(\omega t)]$$

In laplace:

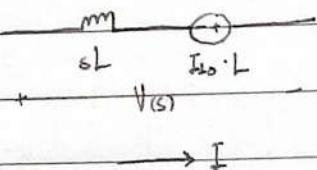
$$A \cos(\omega t + \phi) \rightarrow \boxed{\text{circuit}} \rightarrow k A \cos(\omega t + (\theta + \phi))$$

$$k A \cos(\theta + \phi) = \underbrace{k_2 \phi}_{\sim} \cdot \underbrace{A \cos \theta}_{\sim}$$

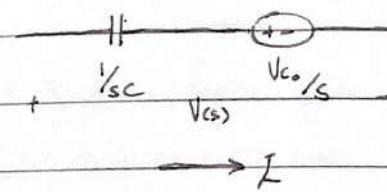
↳ effect of the circuit to signal.

With initial conditions, we can transform the circuit in laplace domain $t > 0$.

Inductor



Capacitance



Solve the circuit using 'normal' & 'ilaplace' function in HP Prime by solving for the right equation.
An equation for a given parameter can be solved by taking the inverse-laplace.

Note that maximum of something can be solved using graph, or taking the 0's of derivative.

Transfer - Function :

$$Y(s) = H(s)X(s) \rightarrow \text{input function in } s\text{-domain}$$

↓
output \hookrightarrow tf, behavior of which $H(s) = \frac{Y(s)}{X(s)}$ at energy 0 [$V_{in} & T_{in} = 0$, V_{out} short]
Isac Open

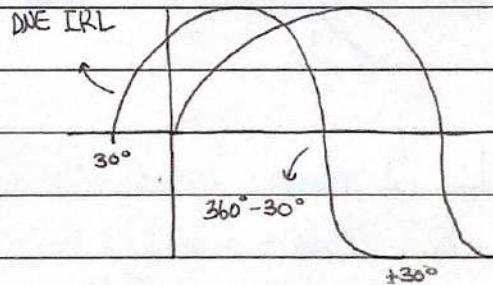
$$|H(s)| = k \quad H(s)^\circ = \delta, \quad k(\text{dB}) = 20 \log_{10}(k) \quad \delta(\text{rad}) = \frac{\delta \cdot 180}{\pi}$$

Notice if $X(s)$, the input is dirac-delta $\delta(x)$, $H(s) = Y(s)$

Time delay of the signal can be computed using δ , $(\delta(\text{rad}) - 2\pi) \cdot \frac{2\pi}{\omega}$ or $(\delta(\text{deg}) - 360) \cdot \frac{360}{\omega}$

* if pole @ RHS \rightarrow unstable.

* but s here must come before the output, $\therefore -2\pi$ (if rad) or -360 (if $^\circ$) from the circuit signal $H(s)$



7. Bode Plot:

We simulate the frequency response of a circuit using Bode plot (both amplitude & degrees).
 (dB) (deg)

Amplitude Plot

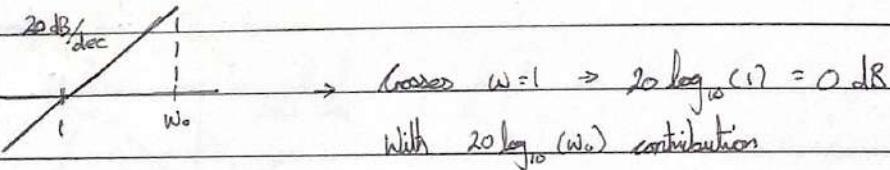
Decibel of any given frequency can be computed using $20 \log_{10}(w)$ where $w (\text{rad/s}) = 2\pi f$.

$$\text{Given a } H(s) = k \cdot \frac{s(s+a)(s+b)^2}{(s+c)(s+d)}$$

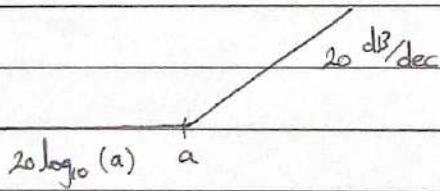
Contribution of a constant k ,

$$20 \log_{10} \left(\frac{k \cdot 1 \cdot a \cdot b^2}{c \cdot d} \right) \rightarrow \text{shifting the graph up / down.}$$

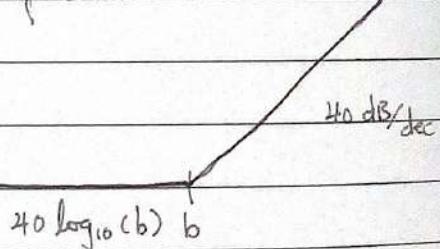
Contribution of $s (0 @ 0)$



Contribution of $(s+a)$



Contribution of $(s+b)^2$



For denormalization, just flip the graph upside-down.

For initial contribution (starting point), sum all the contributions from first $w (w_0)$ that is not 1. For $H(s)$ above $\text{dB}(w_0) = 20 \log_{10} \left(\frac{k \cdot 1 \cdot a \cdot b^2}{c \cdot d} \right) + 20 \log_{10}(w_0)$

Note, decade = $\log_{10} \left(\frac{w_2}{w_1} \right)$

Phasor Plot

$$\text{Given } H(s) = \frac{k \cdot s(s+a)(s+b)^2}{(s+c)(s+d)}$$

$$k \rightarrow 0^\circ$$

$$s \rightarrow 90^\circ/\text{dec}$$

$$0.1a \rightarrow 45^\circ/\text{dec}$$

$$0.1b \rightarrow 90^\circ/\text{dec}$$

$$s^2 \rightarrow 180^\circ/\text{dec}$$

$$10a \rightarrow -45^\circ/\text{dec}$$

$$10b \rightarrow -90^\circ/\text{dec}$$

$$0.1c \rightarrow -225^\circ/\text{dec}$$

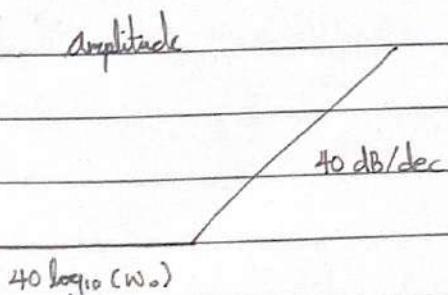
$$10c \rightarrow 45^\circ/\text{dec}$$

Bode - Martini Plot

When $H(s)$ has a two complex conj., instead of using Bode approx, we make use of Martini approx.

$$at+bi \& a-bi \rightarrow [s + (at+bi)][s + (a-bi)] = s^2 + 2as + a^2 + b^2 \\ = s^2 + 2\xi\omega_0 s + \omega_0^2, \quad a = \xi\omega_0$$

ξ (damping freq), ω_0 (undamped freq.)



$$\log_{10}(\frac{\omega_0}{\omega}) = \xi \quad \log_{10}(\frac{\omega_2}{\omega_0}) = \xi$$

$$\omega_1 \rightarrow \frac{90}{\xi} \text{ deg/deg} \quad \omega_2 \rightarrow \frac{90}{\xi} \text{ deg/deg}$$

* watch for denom or numer. Above is numer example.

- RL Filters:

Resonance, is a freq where $L \times C$ cancels out (no reactive element) such that V_{out} only sees resistive component. (Z_r is real only)

Quality Factor, number _____ that determines the selectivity of the circuit (filter)

Half-Power Freq, ω where the signal is 3dB off of filtered value.

$$W_0 = \frac{1}{\sqrt{LC}}$$

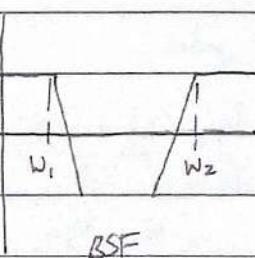
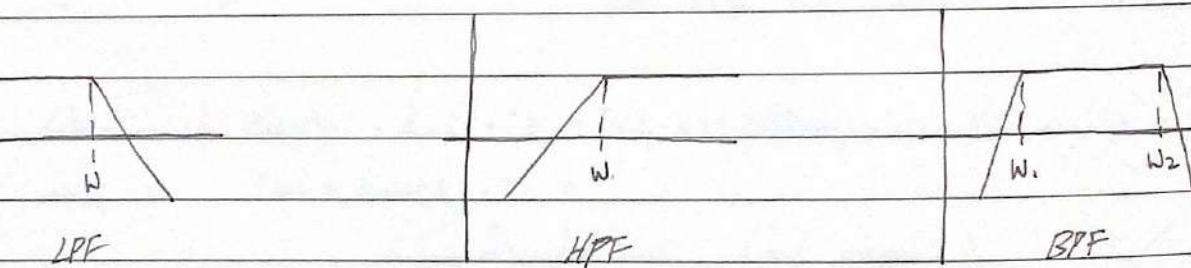
$$Q_s = \frac{W_0 L}{R} \text{ or } \frac{1}{W_0 R C}$$

$$\text{HPF} = W_0 \left[\sqrt{1 + \left(\frac{1}{2Q} \right)^2} \pm \frac{1}{2Q} \right] (W_1, W_2)$$

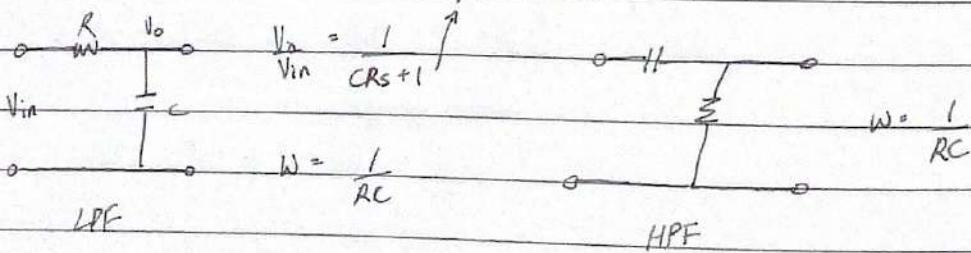
$$BW = \frac{W_0}{Q}$$

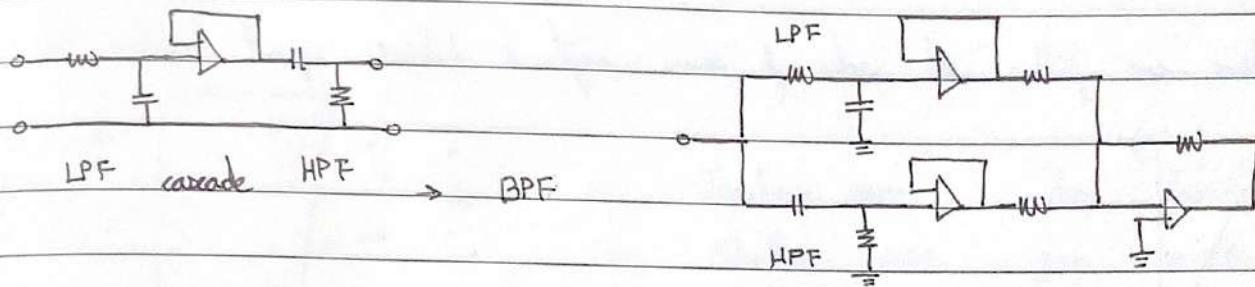
$$Q_s = \frac{R}{W_0 L} \text{ or } W_0 R C$$

$$Q \gg 10 \rightarrow \text{highly selective}, (W_1, W_2) = \frac{W_0 \pm BW}{2}$$



pole @ $-1/R_C$



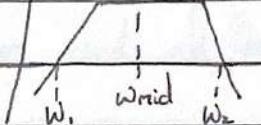


additive op-amp \rightarrow BSF

Order of filters are determined by the slope of its poles.

Mid-freq. is where filter hits its peak among BPF.

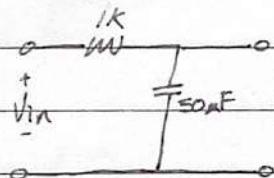
$$\omega_{mid} = (\omega_1 \omega_2)^{1/2}$$



A_{max} & A_{min} ,

A_{max} is the max level of signal it rejects & A_{min} is the min level of signal it rejects.

example.



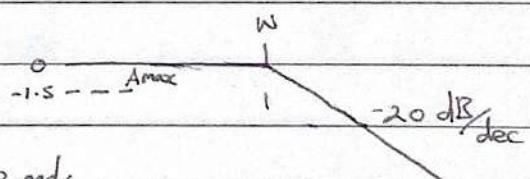
$$H(s) = \frac{1/sC}{R + 1/sC} = \frac{1/RC}{s + 1/RC} \quad \omega = \frac{1}{RC}$$

$$f = 1 \text{ Hz}$$

$$A_{min} = 1.5$$

$$A_{max} = 15$$

$$\omega = \frac{1}{RC} = 20 \text{ rad/s}$$



* solve.

$$H(s) = \frac{20}{s+20} \rightarrow 20 \log_{10} \left(\frac{20}{j\omega + 20} \right) = 0 - 1.5$$

$$\omega_p = 12.85$$

$$20 \log_{10} \left(\frac{20}{j\omega_p + 20} \right) = 0 - 1.5$$

$$\omega_s = 110.672$$

Sensitivity factor measures the changes a filter experience with a change of variable. Sensitivity = $\frac{\omega_p}{\omega_s} < 1$

Filter cookbook allows a scaling computation with a given filter.

$$R' = k_m R$$

$$R' = R$$

$$R' = k_n R$$

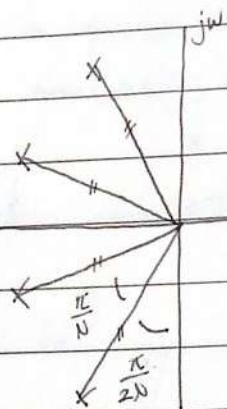
$$L' = k_m L \quad \& \quad L' = \frac{L}{k_p} \rightarrow L' = L \frac{k_m}{k_p}$$

$$C' = C/k_m$$

$$C' = \frac{C}{k_p}$$

$$C' = C \frac{1}{k_m k_p}$$

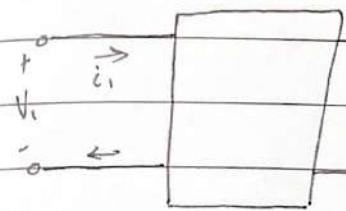
16 F Butterworth Filters are filters with poles of same angle & distance apart.



N - order

x's are point which are equidistant from the origin.

Two-Port Network :



$$V_1 = Z_{11}i_1 + Z_{12}i_2$$

$$i_1 = Y_{11}V_1 + Y_{12}V_2$$

$$V_2 = Z_{21}i_1 + Z_{22}i_2$$

$$i_2 = Y_{21}V_1 + Y_{22}V_2$$

Z_{xy} - Impedance param

h_{xy} - hybrid param

Y_{xy} - Admittance param

g_{xy} - inverse param.

Use param. chart and convert params.

$$V_1 = h_{11}i_1 + h_{12}V_2$$

$$V_1 = A V_2 + B(i_2)$$

$$i_2 = h_{21}i_1 + h_{22}V_2$$

$$i_1 = C V_2 + D(i_2) \quad \{ABCD \Rightarrow \text{transmission params}\}$$

