Date	Event	Event description
Between 15/09/20 and 17/09/20	Emailed Prof. Korshunov to ask about getting started on the dissertation and about the general nature of dissertation	-Aim of dissertation is to produce review on some topic and show skill in understanding mathematics. -Recommended Grimmett&Stirzaker Probability and Random Processes and J. Norris, Markov chains for basic reading.
Between 18/09/20 and 19/09/20	Independent study: skim-read sections 6.1 and 6.2 of Grimmett&Stirzaker Probability and Random Processes	-Refreshed all the basic terminology from MATH332. -Some definition were presented in a different way to MATH332, useful to get a different point of view. -A lot of shorthand notation was constantly used and it took some getting used to at first. E.g. $p_{ij} = P\left(X_{n+1} = j \middle\vert X_n = i\right)$ and P is the transition matrix with entries p_{ij} and later on, $p_{ij}(m,m+n) = P\left(X_{m+n} = j \middle\vert X_m = i\right)$ was defined, which we later showed didn't depend on m, by the assumption of homogeneity, so we also defined $p_{ij}(n) = p_{ij}(m,m+n)$. Once can see how confusion may arise.
05/10/20	Independent study: in-depth look at section 6.1, with note-taking	-Strictly defined the aforementioned basic terminology in my own words. It was useful to write things out in the way that is easiest for me to read. I feel like this greatly improves my ability to talk about the terminology, because it forces me to re-model things slightly so that it's closer to the way I fundamentally understand math. -Condensed Section 6.1 without losing important information. Very useful pragmatically as if I forget the meaning of a term, I look back to the way I defined it, with my handwriting and it inevitably brings back memories of me writing it down and understanding it in the moment of writing it.
07/10/20	Dissertation talk by Jan Grabowski	-Good rule of thumb regarding aim of the dissertation and presentation: "Imagine you're giving a lecture about a 4th year topic to a 4th year student". -Expected Length 30-50 pages including title pages at LaTeX font size 12. -Get started on logbook as soon as possible.
07/10/20	First meeting with Prof. Korshunov	-The meeting was focused around dissertation generalities. -Rule of thumb: at least 5 sources, 10 is about average. -After inquiring Prof. Korshunov about it, we agreed that theorem 6.1.5(a) can be strengthened from $p_{ij} \geq 0$ to $0 \leq p_{ij} \leq 1$ since p_{ij} is a probability. -Talked generally about the timeline of the dissertation. -Scheduled regular meetings every Thursday for the next 4 weeks (starting from the 15th).
14/10/20	Organised my notes	
14/10/20	Dissertation talk by math librarian Lesley English	-Useful information on where to search for texts, e.g. One Search, Google Scholar, Web of science.

15/10/20	Second meeting with Prof. Korshunov	-Question regarding title of dissertation: should aim to have **some** title in mind in the coming weeks, just for direction, can always be changed later on. -Question regarding importance of examples (encountered many in Grimmett&Stirzaker Probability and Random Processes): useful when considering very abstract or complicated concepts; do not copy from books. -Question regarding nature of source texts: mostly books, not too many papers as they are beyond my expertise in the area of probability in particular. -Asked whether we can strengthen particular lemmas/propositions, in various places. Answer was no for two of those cases and yes for the third: pg.221 Section 6.2, Corollary 4a and 4b from "if" to "if and only if".
15/10/20	Started logbook	-The earlier the better! -Helps me as something to look at for reference. -Helps to write while events are still relatively fresh in my memory.
21/10/20	Independent study: covered sections 6.2 and 6.3 of Grimmett&Stirzaker Probability and Random Processes	
22/10/20	Third meeting with Prof. Korshunov	-The topic of Polya's theorem for random walks came into discussion, it impressed me and I decided to make it one of the focal points of my dissertation. -Prof. Korshunov recommended me the book 'Probability Theory' by Borovkov, where the theorem is stated and proven.
22/10/20	Independent study	-I found a short 7-page text by Jonathan Novak which was focused around proving Polya's theorem for random walks.
28/10/20	Independent study	-I found an interesting YouTube video which talks about unintuitive and bizarre geometric properties of shapes as we increase dimensions: let us have a ball fully contained inside of a cube in 3 dimension; as we increase the dimensions and keep the shapes' lengths the same, at some dimension the length of the ball will surpass the length of the cube. -This is not directly related to Polya's theorem, so likely it won't find a good place in my dissertation.
29/10/20	Fourth meeting with Prof. Korshunov	-Should start reading and then writing out the proof to see what gaps of knowledge should be filled in to build up to the theorem. -I wasn't convinced that this theorem applies to 2-D Brownian motion, say, since at the n^{th} step there are uncountably many possible values the walk can take at the $(n+1)^{th}$ step, and hence the chance of ever reaching any particular point is 0 and hence the same applies for the origin. Prof. Korshunov explained that rather than looking at the specific origin point, we consider an ε-radius 2-D ball around the origin and consider the ball as ε tends to 0.

		-Prof. Korshunov mentioned that n-dimensional walks are useful for modelling, say, wait times for a store with n tills. Since Polya's theorem for random walks implies a fundamental difference between walks in 2 dimensions and walks in further dimensions, I wonder how that difference manifests in real life scenarios, e.g. a store with n tills, say.
4/11/20	Independent study	-I came up with a, clearly wrong, argument which seemed to disprove Polya's theorem for random walks, but I can't spot the mistake. Its logic was that since we can prove that the walk is persistent in 2-dimensions at the 2-D origin, then it crosses that point infinitely many times. Then for a particular 4-D walk, which satisfies Polya's conditions, it can be treated as 2 i.i.d. 2-D walks, and we can just take the value of one 2-D walk whenever the other is at the origin. As this happens infinitely many times, and as the other 2-D walk is also persistent, then we must with certainty, and infinitely many times, return to the other 2-D origin and hence the 4-D origin. -I read the proof of Polya's theorem in 'Probability Theory' by Borovkov and understood its gist. There are still steps I am not certain of but I now have an idea of what needs to be introduced before I can discuss the proof.
5/11/20	Fifth meeting with Prof. Korshunov	-I brought up my argument and asked where my mistake is; it turns out that the two 2-D walks cannot be treated as independent. In fact, when we write out their relation and correct my argument using it, we prove Polya's theorem. -We discussed the viability of showing this as a second proof, since this theorem will be a focal point for my dissertation. We decided that this might indeed be a good idea.
9/11/20	Independent study: starting to write up the dissertation itself on LaTeX	-I often mistakenly used terms like "stochastic process", "Markov process" and "Markov chain" interchangeably without much thought; I decided to outright define them and point out the difference in the dissertation.
12/11/20	Independent study: work on LaTeX dissertation document	-Added more subsections, restructured some things, fiddled with some words.
12/11/20	Sixth meeting with Prof. Korshunov	-Talked about whether it was a problem to copy an author's short-hand notation if it was convenient – he said that notation isn't very important and to just choose a style and be consistent with it. -I explained to Prof. Korshunov that in my dissertation I disambiguated between stochastic processes, Markov processes, Markov chains and discrete Markov chains. However it turns out that my definition of these terms is in accordance with an older standard, and the modern standard is more widely used, so I will fix up my definitions. -Asked about a few other small formalities. -Prof. Korshunov mentioned that it's not a good idea to have, "theorem", "definition", "lemma", etc. in bold-face and then also put newly defined words in bold-face too. He suggested I use italics

		for newly defined words instead. -Is it better to use logic symbols like => (implies), ∀ (for all), etc. or full sentences to explain arguments. He explained full arguments are preferable. -He suggested that I send him a draft of my dissertation so he can look over it and give me feedback in the next meeting.
12/11/20	Independent study: work on LaTeX dissertation document	I implemented all the changes we talked about (mentioned above) in the meeting while they were still fresh in my memory.
17/11/20	Independent study: work on LaTeX dissertation document	-I introduced some short-hand notation. I then also introduced a definition, theorem, proof and corollary, which all made use of this notation. -I cleaned up a few things that I'd written before. -I had some problems with the enumeration of my definitions, theorems, lemmas, etc. and couldn't figure out how to fix it.
17/11/20	I sent a draft of my dissertation to Prof. Korshunov	We had agreed in our last meeting that I would send Prof. Korshunov a draft of my dissertation for him to look over.
19/11/20	Seventh meeting with Prof. Korshunov	Check document where meetings seven and onward are detailed.
19/11/20	Independent study: work on LaTeX dissertation document	I implemented all the changes (including the renumeration, which was quite minor in the end) we discussed with Prof. Korshunov right away, so that everything would be fresh in my mind. That is all changes, except the disambiguation for def 1.2.. I will do this at a later point, since it seems like it will be a little more time-consuming.
23/11/20	Independent study: work on LaTeX dissertation document	I implemented the distinction for definition 1.2, but it was a bit tricky since def. 1.2 and def. 1.4 depend on each other.
26/11/20	Eighth meeting with Prof. Korshunov	Check document where meetings are detailed.
26/11/20	Independent study: work on LaTeX dissertation document	I fixed everything we talked about while it was fresh in my mind, except for example 1.9, since I couldn't see how Prof. Korshunov's example was suitable in my case. I also re-numerated (for the second time!) my dissertation so that 1.X. became 1.Y.Z.; clearly this is a much more major change than before. This is the numeration I will refer to in here from now on.
1/12/20	Independent study: work on LaTeX dissertation document	I spent several hours thinking about Prof. Korshunov's example but it felt as if I was bashing my head against a wall as I couldn't figure out how it suited my scenario.
9/12/20	Analysis on Polya's theorem proof from Borovkov's book,	-When I first began working on the dissertation, I had a vague idea of the path I wanted to take in order to get to the proof of Polya's theorem. It started with introducing the foundational concepts. But

	and minor LaTeX document additions	as they themselves are very loosely connected to the proof itself (i.e. they're definitions which are used in definitions, which are themselves used in definitions, which are themselves used in definitions… and so on), it hadn't felt like I was getting anywhere near Polya's theorem. I decided to delve deep into the theorem and highlight all the missing pieces, see how many of them I already understand, and decide which (if any) I should omit from my dissertation, which to include, and which (if any) besides those in Borovkov's book I should include. -I then implemented the 'one-liner' definitions (read: definition) into my dissertation. -Reading all these definitions, theorems and respective proofs gave rise to a lot of questions, all of which are noted in our 10th meeting with Prof. Korshunov.
10/12/20	Ninth meeting with Prof. Korshunov	Check document where meetings are detailed.
14-15/12/20	Independent study: work on LaTeX dissertation document	I briefly introduced generating functions and proved a theorem and corollary.
17/12/20	Tenth meeting with Prof. Korshunov	Check document where meetings are detailed.
19/12/20	Independent study: work on LaTeX dissertation document	Introduced walks.
20/12/20	Independent study: work on LaTeX dissertation document	Stated and proved local limit theorem and other relevant results.
29/12/20	Independent study: work on LaTeX dissertation document	Introduced reducibility, which is relevant to a theorem which we'll state and prove later.
2/1/21	Independent study: work on LaTeX dissertation document	Proved a theorem relating to reducibility.
3/1/21 and 4/1/21	Independent study: work on LaTeX dissertation document	Proved that in 1-dimension, symmetricity is equivalent to recurrence.
5/1/21	Independent study: work on LaTeX dissertation document	Proved Polya's theorem on random walks.
5/1/21	Submitted the draft	

	for Review to Prof. Korshunov	
8/1/21	Received draft feedback	
19/1/21	Reviewed all feedback from Prof. Korshunov	The majority of the feedback focused on typesetting, grammar and bad notation. This was useful, as most of his corrections were as a result of lack of knowledge as opposed to typos. The rest of his feedback revolved around mathematical errors; I will discuss those in our meeting.
21/1/21	Eleventh meeting with Prof. Korshunov	I began by asking for clarification on typesetting, grammar and notational mistakes he'd pointed out, since those were fairly straight forward. The rest of his feedback was twofold: firstly, regarding a slight formality where I should have defined something as a constant with restrictions in the theorem rather than assuming it to be such in the proof. Secondly, I gave the wrong definition of a walk, which invalidates the proof I have presented for Polya's theorem on walks in n dimensions. This is not catastrophic however, since all of the build up is just as relevant and I only need to reformulate my argument (and definition for walk). Unfortunately the hour was only enough for us to discuss his feedback, and I couldn't ask the questions I had prepared independently from his feedback.
28/1/21	Twelfth meeting with Prof. Korshunov	Check document where meetings are detailed.
2/2/21	Independent study: work on LaTeX dissertation document	I ran through all the correction we discussed in the past weeks, except for re-defining my walk definition and re-writing Polya's theorem, as these will be lengthier tasks.
5/2/21	Thirteenth meeting with Prof. Korshunov	The meeting was focused on Borovkov's proof of Polya's result; it turns out his proof is for a less-common definition of the walk. Talked about alternative proof, and whether it was viable.
7/2/21	Independent study: work on LaTeX dissertation document	Cleaning up, re-ordering. Also introduced a vague example for null recurrence; might have to be tweaked.
10/2/21	Independent study: work on LaTeX dissertation document	Began working on alternate proof of Polya's theorem. Read and understood it. Requires some combinatoric results. Introduced multinomials.
12/2/21	Fourteenth meeting with Prof. Korshunov	Talked about null recurrence of 1 and 2 dimensional walks, and talked about real-life applications of Polya

13/2/21 – 17/2/21	Independent study: work on LaTeX dissertation document	Introduced Chu-Vandermonde identity, and a Corollary arising form it. Proved walk's recurrence in 2D. Wrote out Corollary of Polya's theorem, showing 1D walk's recurrence – it was simpler than I thought. Proving Polya's theorem for 3D walks' transience quite tricky; I tried comparing with a larger summation and showing that larger summation is finite, but the larger summation turned out to diverge. Have to find a smaller upper-bound summation; need to prove that multinomial holds its maximal value when all bottom values are close to each other.
18/2/21	Fifteenth meeting with Prof. Korshunov	Discussed a variety of topics, see questions sheet.
20-25/2/21	Independent study: work on Beamer dissertation presentation document	Planned out what to include and omit. Began by including plenty of topics; presenting took too long, had to cut parts out. Practiced presentation, trying to cut down as much time; multiple presentations in a row took right around 10 minutes.
9/3/21 – 10/3/21	Independent study: work on LaTeX dissertation document	Generalised Example 2.2.8. as discussed with Prof. Korshunov. Proved that multinomial holds its maximal value when all bottom values are close to each other on my own. Began work on proving 3D case's transience.
11/3/21	Sixenteenth meeting with Prof. Korshunov	Discussed sourcing extensively; see questions sheet for details.
15/3/21 – 17/3/21	Independent study: work on LaTeX dissertation document	Extensive work on proving 3D case, a lot of nasty algebra, but almost there.
25/3/21	Seventeenth meeting with Prof. Korshunov	Discussed whether Cauchy condensation test for convergence and closed form of geometric series need to be proved/ sourced; only Cauchy condensation test. Prof. Korshunov spotted mistake affecting most of my algebra for 3D case.
25/3/21	Independent study: work on LaTeX dissertation document	Looked into mistake with 3D case, my line of thinking was correct, need to change around a few things.
26/3/21	Independent study: work on LaTeX	Found larger upper bound summation for 3D case which is finite and much simpler than previous approach.

	dissertation document	Finished proof of 3D transience, proof shorter by about 2 pages.
29/3/21 – 30/3/21	Independent study: work on LaTeX dissertation document	Proof of corollary about null recurrence of 1D walk needs to be expanded. Proved null recurrence of 2D walk as well. Decided to re-order dissertation; subsections not needed, removed them and have only sections as separators.
1/4/21 – 5/4/21	Independent study: work on LaTeX dissertation document	After reading proof for Theorem 2.11., and its Corollary 2.12, I decided to omit the proof; will instead source where it is proved. As a Remark, I added a false argument which attempts to prove that the 3D walk is recurrent, and pointed out the logical flaws. Useful to point out, since argument somewhat plays to intuition, but is false. Re-wrote some things at the start of the dissertation. Added in a lot of examples to help clarify definitions and results. Introduced 1D walk early on as an example, since walks are central to the dissertation. Used Python to program diagrams to visualise random walks and used Python to program an algorithm which terminates 1D and 2D walks after their first recurrence, and notes the number of steps this took. Finishing touches, fixing ugly type-setting, i.e. equations split across pages, etc.
5/4/21	Sent dissertation draft for review to Prof. Korshunov	
9/4/21	Received feedback on dissertation draft	I reviewed all comments, and changed those I agreed with, noted all others to discuss in next meeting
13/4/21	Eighteenth meeting with Prof. Korshunov	Talked about dissertation feedback and asked some other questions. See questions document for details.
13/4/21 – 14/4/21	Independent study: work on LaTeX dissertation document	Acted on feedback discussed in meeting, mostly to do with citations.
15/4/21	Final nineteenth meeting with Prof. Korshunov	Mainly regarding referencing and style. See questions document for details.
15/4/21	Independent study: work on LaTeX dissertation	Acted on feedback discussed in the meeting – relatively quick. Also added final touches.

	document	
15/4/21	Submitted Dissertation .pdf file and all other files	
17/4/21	Re-submitted Dissertation .pdf file and all other files	Corrected a few mistakes and instances of incorrect typesetting.
22/4/21 - 23/4/21	Re-submitted Dissertation .pdf file and logbook file	-Added abstract, conclusion and acknowledgments of sources at the start. -Improved formatting. Added more spacing above and below theorems, definitions, etc. to stand out more, moved a lot of cluttered in-line expressions to math environment. -Corrected mistakes