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Quantum Annealing for Air Traffic Management

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(Dated: May 16, 2017)

I. INTRODUCTION

Efficiently automating air traffic management is increasingly important (increased volume and diversity, environmental concerns, etc.).

Quantum annealing is a promising computational method.

We investigate the feasibility of applying quantum annealing to a particular problem in air traffic management known as "deconflicting", in which the goal is to modify a set of independently optimal trajectories in a way that removes conflicts between them while minimizing the cost of doing so.

II. PROBLEM SPECIFICATION

The basic input of the deconflicting problem is a set of ideal flight trajectories (space-time paths). These ideal trajectories are specified by the individual flight operators. Each ideal trajectory represents some independent optimization from the operator's perspective, especially minimizing fuel costs given expected wind conditions between the desired origin and destination at the desired times; for this reason, they are called the "wind-optimal" trajectories. Because of the number of such trajectories and the correlation between them, these trajectories are likely to conflict; that is, two or more aircraft are likely to get dangerously close to each other if their ideal trajectories are followed without modification. The goal thus is to modify the trajectories to avoid such conflicts.

In theory, the configuration space consists of all physically realistic trajectories; in practice, computational bounds constrain us to consider perturbations of the ideal trajectories. The simplest such perturbation is a departure delay, which is the main focus of the present work. Previous work [1] additionally considered a global perturbation by which a trajectory is sinusoidally shifted parallel to the Earth's surface. We focus instead on local perturbations to the trajectories, in which a modification to the trajectory is parameterized by some choice of active maneuvers near a potential conflict; such a modification does not affect the preceding part of the trajectory and only affects the subsequent part by the additional delay it introduces.

A full accounting of the cost of such modifications would take into account the cost of departure delays, the change in fuel cost due to perturbing the trajectories, the relative importance of each flight, and many other factors. As in previous work, we consider only the total, unweighted arrival delay, aggregated equally over all of the flights.

Formally, each ideal trajectory $\mathbf{x}_i = (x_{i,t})_{t=t_{i,0}}^{t_{i,1}}$ is specified as a time-discretized path from the departure point $x_{i,t_{i,0}}$ at time $t_{i,0}$ to the arrival point $x_{i,t_{i,1}}$ at time $t_{i,1}$. For each flight i, the geographical coordinates $x_{i,t}$ (as latitude, longitude, and altitude) are specified at every unit of time (i.e. one minute) between $t_{i,0}$ and $t_{i,1}$; we call this interval $T_i = (t_{i,0}, t_{i,0}+1, \ldots, t_{i,1})$.

For notational simplicity, suppose momentarily that each trajectory $\mathbf{x}_i$ is modified only by introducing delays between time steps. Let $\delta_{i,t}$ be the accumulated delay of flight i at the time that it reaches the point $x_{i,t}$, and let $\delta^*_{i,t}$ be the maximum such delay.

A pair of flights (i,j) are in conflict with each other if any pair of points from their respective trajectories is in conflict. (The trajectories are reasonably assumed to be sufficiently time-resolved so that if the continuously interpolated trajectories conflict than there is a pair of discrete trajectory points that conflict.) A pair of trajectory points $(x_{i,s}, x_{j,t})$ conflict if their spatial and temporal separations are both within the respective mandatory separation standards Δ_x and Δ_t (i.e. 3 nautical miles and 3 minutes):

$$\|x_{i,s} - x_{j,t}\| < \Delta_x \text{ and } |(s+\delta_{i,s}) - (t+\delta_{j,t})| \quad (1)$$

. The latter condition can be met for some $(\delta_{i,s}, \delta_{j,t}) \in [0, \delta^*_{i,s}] \times [0, \delta^*_{j,t}]$ if and only if

$$\max\left\{\delta^*_{i,s}, \delta^*_{j,t}\right\} + \Delta_t > |s-t|, \quad (2)$$

in which case we call the pair of trajectory points *potentially* conflicting. The set C of such pairs of potentially conflicting trajectory points contains strongly correlated clusters. To simplify the constraints, we enumerate all such clusters and refer to them simply as *the* conflicts. That is, we partition the potentially conflicting pairs of trajectory points into disjoint sets,

$$C = \bigcup_k C_k, \quad (3)$$

such that if $\{(i,s),(j,t)\}, \{(i',s'),(j',t')\} \in C_k$ for some k then $i = i' < j = j'$ and for all $s'' \in [\min\{s,s'\}, \max\{s,s'\}]$ there exists some $t'' \in [\min\{t,t'\}, \max\{t,t'\}]$ such that

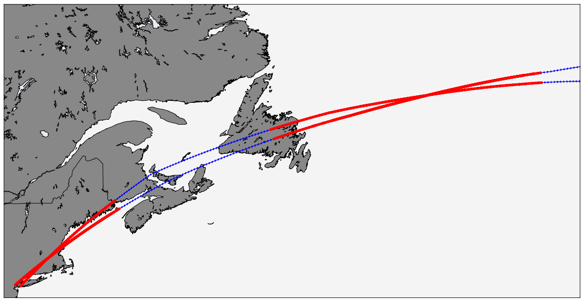

FIG. 1. Example of two parallel potential conflicts between two transatlantic flights starting from the east cost of the USA.

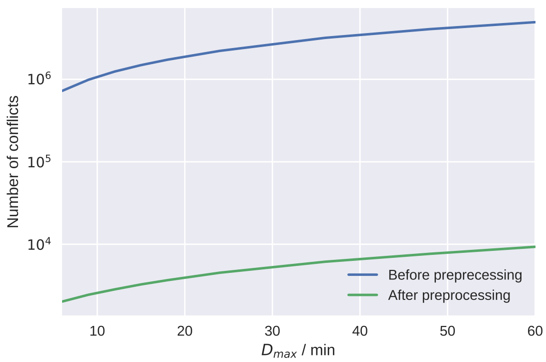

FIG. 2. Preprocessing: Reduction in the number of potential conflicts for various upper delay bounds $D_{\max}$.

$\{(i,s''),(j,t'')\} \in C_k$. Thus every conflict k is associated with a pair of flights $I_k = \{i,j\}$. Let $K_i = \{k | i \in I_k\}$ be the set of conflicts to which flight i is associated.

Having identified disjoint sets of conflicts, we relax the supposition that the trajectory modifications only introduce delays between time steps. Instead, we consider modifications to the trajectories that introduce delays local to particular conflicts. Specifically, the configuration space consists of the departure delays $\mathbf{d} = (d_i)_{i=1}^n$ and the set of local maneuvers $\mathbf{a_k} = (\mathbf{a}_k)_k$, where $\mathbf{a}_k$ represents some parameterization of the local maneuvers used to avoid conflict k. Let $d_{i,k}(\mathbf{d}, \mathbf{a_k})$ be the delay introduced to flight i at conflict k, as a function of the departure delays and local maneuvers. With this notation, we can write the total delay as

$$D = \sum_{i=1}^{n} \left(d_i + \sum_{k \in K_i} d_{i,k} \right). \tag{4}$$

This is the quantity we wish to minimize subject to avoiding all potential conflicts.

A conflict can be avoided locally by introducing earlier delays differentially, thereby increasing the temporal separation; by some active maneuver of one or both of the flights; or by some combination thereof. We focus on the former case. Let

$$D_{i,k} = d_i + \sum_{k' \in K_i | k' < k} d_{i,k'} \tag{5}$$

be the accumulated delay of flight i by the time it reaches conflict k. We assume that the set of conflicts K_i associated with flight i is indexed in temporal order, i.e. if $k' < k$ and $k, k' \in K_i$, then flight i reaches conflict k' before conflict k. The pairs of conflicting trajectory points associated with conflict k are given by

$$T_k = \{(s,t) | \{(i,s),(j,t)\} \in C_k, i < j\}. \tag{6}$$

Thus the potential conflict is avoided only if

$$D_{i,k} - D_{j,k} \notin D_k \tag{7}$$

where

$$D_k = \bigcup_{(s,t) \in T_k} (-\Delta_t + t - s, \Delta_t + t - s) = [\Delta_k^{\min}, \Delta_k^{\max}], \tag{8}$$

$$\Delta_k^{\min} = 1 - \Delta_t + \min_{(s,t) \in T_k} \{t - s\}, \tag{9}$$

$$\Delta_k^{\max} = \Delta_t - 1 + \max_{(s,t) \in T_k} \{t - s\}. \tag{10}$$

III. INSTANCES

To assess our methods on realistic instances of the problem, we use the actual wind-optimal trajectories for transatlantic flights on July 29, 2012, as was done in previous work [1]. In these trajectories, each flight i has a constant (cruising) altitude and constant speed, to within (classical) machine precision, though our methods generalize to instances without these special properties.

To investigate the problem we consider each flight as a vertex of a graph and each conflict between two flights as an edge of this graph. The connected components of this *conflict graph* represent natural subsets of the problem.

IV. DISCRETIZING THE CONFIGURATION SPACE

It is important to understand how the restrictions to the configuration space by (**??**) influence the solution quality. Therefore we solve (**??**) with a constraint programming solver [2] for various delay discretizations and upper bounds as well as for the continuous problem. As problem instances we used most of the connected components of the conflict graph for $D_{\max} = 18$ minutes with up to $N_f = 50$ flights and $N_c = 104$ conflicts.

In figure 3 one can see the results for a problem instance extracted from a connected component of the conflict graph with $N_f = 19$ flights and $N_c = 47$ conflicts.

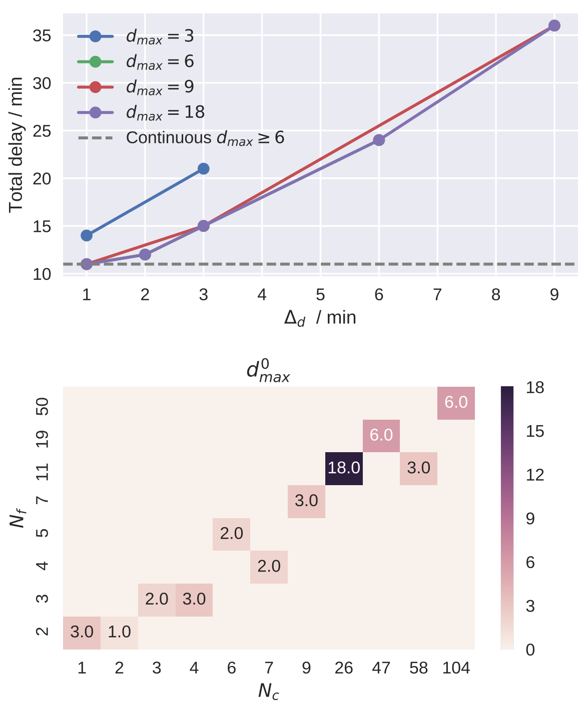

FIG. 3. Top: Total delay of constraint programming solutions for a problem instance with $N_f = 19$ flights and $N_c = 47$ conflicts for various discretization parameters. Bottom: Minimum $d_{\max}$ which yield optimal solution in continuous problem for various problem instances. For all problem instances we used $D_{\max} = 18$ minutes.

With the exception of the small maximum delay $d_{\max} = 3$ min, the total delay of the solutions is nearly independent of the maximum delay. Moreover it is monotonically increasing with the coarseness of the discretization. Since the original data is discretized in time in units of 1 minute, $\Delta_t = 1$ yield the same result as a continuous variable with the same upper bound. Obviously the total delay for the continuous solution decreases monotonically with $d_{\max}$. Above a certain value $d^0_{\max}$ the total delay stays the same. With one exception, we found that for all the investigated problem instances $d^0_{\max} \leq 6$ minutes (see figure 3). Therefore we conclude, that a moderate maximum delay is sufficient even for larger problem instances. On the other hand, the delay discretization should be as fine as possible to obtain a high quality solutions.

V. MAPPING TO QUBO

A. Binary encoding

1. *Departure delays*

To apply quantum annealing to the deconflicting problem, we must encode the configuration space $\mathbf{d}$ in binary-valued variables. To do so, we must first discretize and bound the allowed values. Let Δ_d be the resolution of the allowed delays and $d_{\max} = N_d \Delta_d$ the maximum allowed delay, so that $d_i \in \{\Delta_d l | l \in [0, 1, \ldots, N_d]\}$. The value of d_i is encoded in $N_d + 1$ variables $d_{i,0}, \ldots, d_{i,N_d+1} \in \{0, 1\}$ using a one-hot encoding:

$$d_{i,\alpha} = \begin{cases} 1, & d_i = \alpha, \\ 0, & d_i \neq \alpha; \end{cases} \qquad d_i = \Delta_d \sum_{l=0}^{N_d} d_{i,l}. \tag{11}$$

To enforce this encoding, we add the penalty function

$$f_{\text{encoding}} = \lambda_{\text{encoding}} \sum_{i=1}^{n} \left(\sum_{l=0}^{N_d} d_{i,l} - 1 \right)^2, \tag{12}$$

where $\lambda_{\text{encoding}}$ is a penalty weight sufficiently large to ensure that any cost minimizing state satisfies $f_{\text{encoding}} = 0$. In terms of these binary variables, the cost function is

$$f_{\text{delay}} = \Delta_d \sum_{i=1}^{n} \sum_{l=0}^{N_d} d_{i,l}, \tag{13}$$

Lastly, actualized conflicts are penalized by

$$f_{\text{conflict}} = \lambda_{\text{conflict}} \sum_{k} \sum_{\substack{l,l' | \Delta_d(l-l') \in D_k \\ i,j \in I_k | i<j}} d_{i,l} d_{j,l}, \tag{14}$$

where again $\lambda_{\text{conflict}}$ is a sufficiently large penalty weight. The overall cost function to be minimized is

$$f = f_{\text{encoding}} + f_{\text{delay}} + f_{\text{conflict}}. \tag{15}$$

B. Softening the constraints

The contributions from (**??**) and (**??**) to the QUBO for the departure delay model of section V A 1 are hard constraints. This means a solution to the QUBO is only valid if both (**??**) and (**??**) vanish. Therefore, the penalty weights λ_{unique} and $\lambda_{\text{conflict}}$ must be chosen sufficiently large to ensure that the hard constraints are fulfilled for the solution to the problem. On the other hand, large penalty weights lead to large differences between the largest and smallest non-vanishing coefficients in the QUBO. Since the D-Wave quantum annealers have a limited resolution for the specification of the QUBO, this can lead to a misspecification of the problem. [3] Hence, it is desirable to find a sweet spot of the smallest penalty weights which still yield valid solutions.

In order to find these optimal penalty weights, we employed an exact solver [4] to explore the validity of a solution in dependence on the penalty weights. We investigated problem instances with up to $N_f = 7$ flights and $N_c = 9$ conflicts. For all these problem instances we found a box like shape of the boundary between valid and invalid solutions as it is depicted in figure 4.

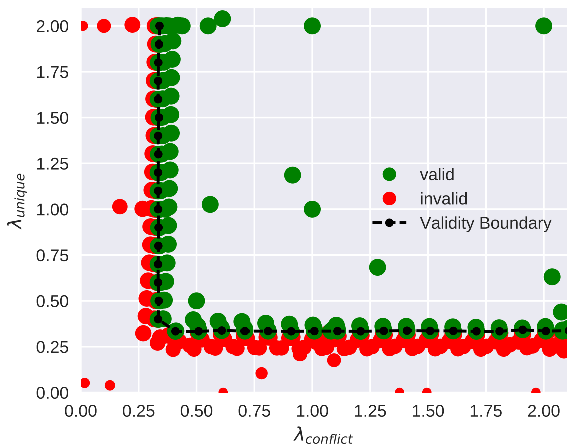

FIG. 4. Validity of exact solution to a QUBO extracted from a problem instance with $N_f = 7$ flights and $N_c = 9$ conflicts in dependence on the choice of the penalty weights, λ_{unique} and $\lambda_{\text{conflict}}$.

One can give an upper bound for the sufficiently large penalty weights by the following considerations. A minimal violation of the hard constraints yield an additional contribution to the QUBO cost function of λ_{unique} or $\lambda_{\text{conflict}}$, respectively. As a result of this violation, the contribution from (**??**) can be reduced maximally by

$$\min_{\{d_{i\alpha}\}} \left\{ \frac{1}{d_{\max}} \sum_{i\alpha} \alpha d_{i\alpha} - \frac{1}{d_{\max}} \sum_{j\beta} \beta d_{j\beta} \right\} = -1$$

Therefore an upper bound for sufficiently large penalty weights is given by

$$\lambda_{\text{unique}} > 1$$
$$\lambda_{\text{conflict}} > 1$$

VI. RESULTS FROM ICM

VII. D-WAVE

A. Embedding

B. Quantum Annealing Results

VIII. CONCLUSIONS

TODO

IX. ACKNOWLEDGEMENTS

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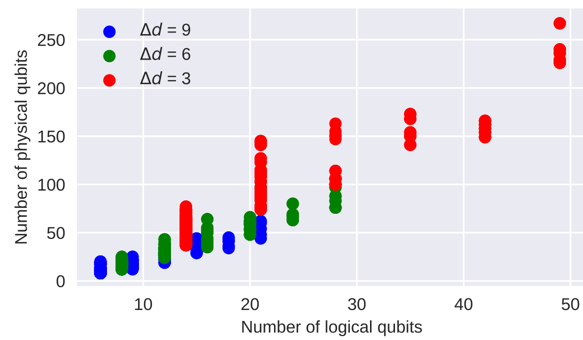

FIG. 5. Number of physical qubits versus the number of logical qubits after embedding of QUBO instances with up to $N_f = 50$ and $N_c = 104$

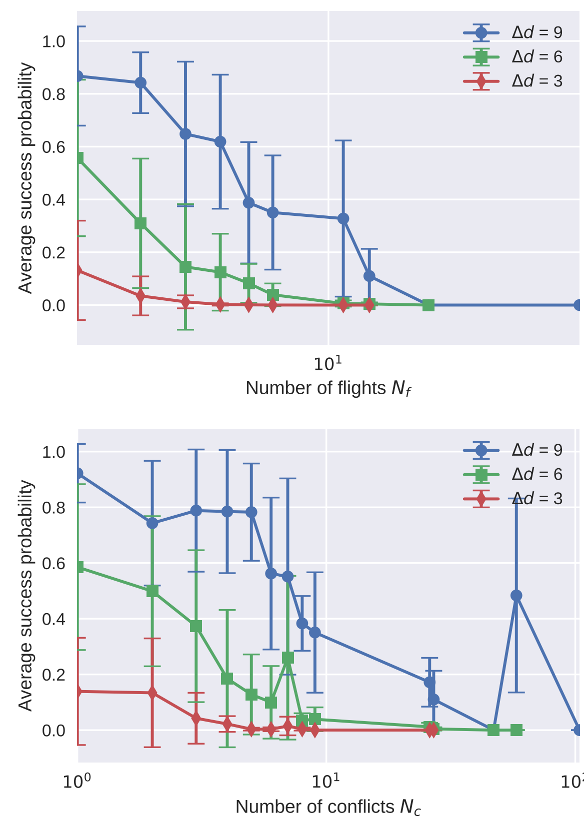

FIG. 6. Success probability for QUBO instances in dependence of the number of flights N_f and the number of conflicts N_c. The error bars indicate the standard deviation. We used 10000 annealing runs for each instance and penalty weights $\lambda = \lambda_{\text{conflict}} = \lambda_{\text{unique}} \in \{0.5, 1, 2\}$.

a. Maneuvers

A more realistic model of the problem can be created by including maneuvers. As mentioned above the maneuvers enter our formulation as additional delays d_{ik} at the conflict time. In the course of mapping to a QUBO formulation, we need to make sure to retain the combinatorial nature of the problem. We do this by restricting the vast realm of maneuvers to two distinct choices: Only one of the two involved flights is delayed while leaving the other flight untouched

$$\text{if } d_{ik} \neq 0 \Rightarrow d_{jk} = 0 \qquad \forall (i,j) \in I_k \; \forall k \; . \tag{16}$$

Moreover, we set the resulting maneuver delays to a constant value d_M large enough to capture all kinds of real maneuvers. A natural choice for this is the temporal conflict threshold $d_M = \Delta_t$.

With (**??**) we can introduce the delay a flight i at the conflict k as

$$D_{ik} = d_i + \sum_{k'<k} d_{ik} \; , \tag{17}$$

where we have defined a temporal ordering of the conflicts for each flight i by

$$\begin{aligned} k < p \text{ if } t < t' \\ \text{for } t = \min_s x_{i,s} \in C_k \; , \\ t' = \min_s x_{i,s} \in C_p \end{aligned}$$

The departure delay variables are represented by binary variables as it was done in Section V A 1. The maneuver delays are given by

$$d_{ik} = d_M a_{ik} \qquad a_{ik} \in \{0,1\}$$

Since the total delay is given by $\sum_{ik} D_{ik}$, we can write the corresponding QUBO contribution as

$$\tilde{Q}_{\text{delay}} = \sum_{i\alpha} \alpha d_{i\alpha} + \sum_{ik} d_M a_{ik} \; ,$$

For the conflict avoidance, we need to introduce another variable representing the delay at a given conflict

$$D_{ik} = \sum_{\delta} \delta \Delta_{ik\delta} \qquad \Delta_{ik\delta} \in \{0,1\} \; .$$

By restricting ourselves to $\Delta_d = \Delta_t$ the values of δ in the above equation are given as

$$\delta \in \{0, \Delta_t, 2\Delta_t, \ldots, (N_d + M_{ik})\Delta_t\} \; .$$

Here, M_{ik} is the number of conflicts the flight i is involved in before k. In order to fulfill (17) we add the following contribution to the QUBO

$$\tilde{Q}_{\Delta} = \lambda_{\Delta} \sum_{ik} \left(\sum_{\alpha} \alpha d_{i\alpha} + \sum_{k'<k} d_M a_{ik'} - \sum_{\delta} \delta \Delta_{ik\delta} \right)^2 \Bigg|_{i,j\in I_k}$$

For unique representation of the variables we add

$$\tilde{Q}_{\text{unique}} = \lambda_{\text{unique}} \left\{ \sum_i \left(\sum_{\alpha} d_{i\alpha} - 1 \right)^2 + \sum_{ik} \left(\sum_{\delta} \Delta_{ik\delta} - 1 \right)^2 \right\} \; .$$

Conflicts are avoided if $D_{ik} - D_{jk} \notin D_k$, $(i,j) \in I_k$. The corresponding QUBO contribution reads

$$\tilde{Q}_{\text{conflict}} = \lambda_{\text{conflict}} \sum_k \sum_{(\delta,\delta')\in B_k} \Delta_{ik\delta} \Delta_{jk\delta'} \Big|_{i,j\in I_k}$$

where B_k is the set of all (δ, δ') which correspond to a conflict

$$B_k = \{(\delta, \delta') \mid \delta - \delta' \in D_k\}$$

The penalty weights λ_{Δ}, λ_{unique} and $\lambda_{\text{conflict}}$ must be chosen large enough to ensure vanishing contributions from the corresponding QUBO terms for the solution.

Finally, the maneuver decision described by (16) is incorporated by a antiferromagnetic coupling between the two maneuver delay variables

$$\tilde{Q}_{\text{maneuver}} = J \sum_k (s_{ik} s_{jk} + 1)_{i,j\in I_k} \; .$$

with

$$s_{ik} = 2a_{ik} - 1 \in \{-1, 1\}$$

and $J > 0$ has to be chosen large enough. A solution is considered to be valid only if $\tilde{Q}_{\text{maneuver}} = 0$. Hence, the total QUBO for the maneuver model reads

$$Q_{\text{MM}} = \tilde{Q}_{\text{delay}} + \tilde{Q}_{\Delta} + \tilde{Q}_{\text{unique}} + \tilde{Q}_{\text{conflict}} + \tilde{Q}_{\text{maneuver}}$$

[1] O. Rodionova, D. Delahaye, B. Sridhar, and H. Ng., Proceedings of Advanced Aircraft Efficiency in a Global Air Transport System (AEGATS'16) Conference (2016).

[2] E. Hebrard, E. O'Mahony, and B. O'Sullivan, "Constraint programming and combinatorial optimisation in numberjack," in *Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems: 7th International Conference, CPAIOR 2010, Bologna, Italy, June 14-18, 2010. Proceedings*, edited by A. Lodi, M. Milano, and P. Toth (Springer Berlin Heidelberg, Berlin, Heidelberg, 2010) pp. 181–185.

[3] Reference to D-Wave limited resolution.

[4] Reference to mapping from QUBO to max sat and to akmaxsat solver.