Exploration[1] of the Potential of Quantum Annealing for Hard Scheduling Problems in Air Traffic Management: Report, April-July 2016 (3.5 months)

Introduction

Given encouraging of early results in the planning domain [1, 2] and the expertise of our team in such sector, aim of the project is to explore the potential use of the Quantum Annealing (QA), and in particular of the state-of-art D-Wave 2X^{TM} quantum annealer hosted at NASA Ames, as a metaheuristic for solving computational challenging problems in the context of Air Traffic Management (ATM) [3, 4]. More precisely, the project has been focused on the problem to reduce the number of potential enroute conflicts of wind-optimal trajectories. In this project, we limit our attention to the North Atlantic oceanic airspace (NAT) for which we have wind-optimal trajectories for two consecutive days (July 28th-29th 2012). The project has been conducted by experts of the QuAIL team (Tobias Stollenwerk, Bryan O'Gorman, Salvatore Mandrà, Davide Venturelli and Eleanor G. Rieffel), in close collaboration with ATM experts (Olga Rodionova, Hok K. Ng and Banavar Sridhar).

Since quantum annealer like the D-Wave 2X^{TM} quantum chip can only optimize problems expressed in terms of binary variables, in the first part of the project we mainly focused on the formulation of the ATM problem in terms of discrete variables. See Table 1 for a complete overview of the tasks/milestones that have been completed and the future steps of the project.

Summary of completed tasks (3.5 Months):

- Devised code to analyze wind-optimal trajectories.
- Identified all the potential conflicts.
- Devised a "delay model" which uses delays as variables to optimize.
- Delay model has been discretized to be expressed in terms of binary variables (PUBO).
- Delay model has been reduced from PUBO to QUBO (partially completed).

Next steps (remaining 8.5 months):

- Identify a set of benchmark ATM problems.
- Analyze the performance of state-of-art classical QUBO solvers on the benchmark set.
- Analyze the performance of the D-Wave 2X^{TM} quantum chip on the benchmark set.
- Compare classical/quantum performance for signals of potential quantum speedup.
- Outlook on different architectures/hardwares.

Approach and description of completed tasks

NAT dataset consists in 984 wind-optimal trajectories in (3+1)-dimensions. Since wind-optimal trajectories have been computed independently from each others, two or more trajectories can intersect in space creating a "conflict". A conflict becomes "potential" if two or more flights can reach at the *same* time the spatial conflict. The ATM problem consists in finding minimal changes of the wind-optimal trajectories to avoid all the potential conflicts. Given the fixed architecture and the limited amount of physical resources of the current quantum hardware, it is impossible to directly solve ATMs problems on quantum annealers. For this reason, we have designed a series of transformation of the original ATM formulation to be able to optimize ATM problems on state-of-art quantum optimizers like the D-Wave 2X^{TM} quantum chip. Figure 1 summarizes the transformation we developed in this project (green/light-green and orange/dark-green represent respectively the completed and partially completed tasks). Here follows a detailed description of the work that has been completed in the period April-June 2016 (3.5 months):

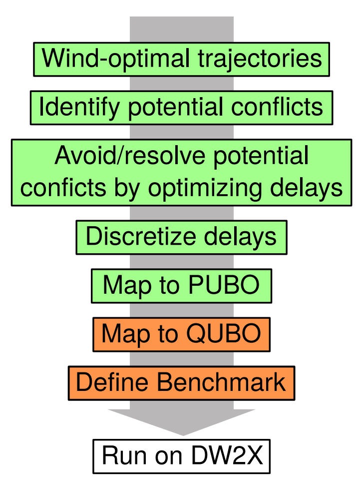

Figure 1: Schematic flow diagram to map ATM problems onto the D-Wave 2X^{TM} quantum chip. Green/light-gray and orange/dark-gray boxes represent respectively completed and partially completed tasks.

- **Identify potential conflict.** In principle, any conflict is a potential conflict: indeed, flights can accumulate delay and therefore arrive at the same to a spatial conflict. Nevertheless, the number of spatial conflicts is prohibitive. To limit the number of potential conflicts, we pre-processed the wind-optimal trajectories in the following way:

[1] Feel free to shuffle the order of the authors

1. Pool together potential conflicts which are subsequent in time and space.
2. Assume that the flights can not have a delay which exceeds 1 hour and then eliminate all the potential conflicts that do not respect the bound.
3. Reduce the number of of potential conflict even further by the following self-consistent algorithm:
   (a) For each flight, order the potential conflicts in time.
   (b) For each flight, calculate the maximal delay which can be picked up due to anteceding conflict avoiding maneuvers.
   (c) For each potential conflict, check if a real conflict is possible considering the maximal delays of the involved flights calculated in step 3b.
   (d) Remove the potential conflicts which can not become real.
   (e) Repeat the above steps 3a to 3d until convergence (i.e. the number of potential conflicts is invariant).

This process drastically reduced the number of potential conflicts from 33878 to 4168. The code to pre-process the wind-optimal trajectories has been developed by Tobias Stollenwerk.

• **Avoid/resolve potential conflicts by optimizing delays/Discretize delays.** Even after the reduction of potential conflicts, it would be impossible to directly map the (3+1)-dimensional trajectories onto the D-Wave 2X™ quantum chip. Instead, we formulate the ATMs in terms of a "delay" problem. More precisely, let us define the time T_{ik} of a flight i to reach a potential conflict k, that is:

$$T_{ik}(d_i, \{d_{ip}\}) = t_{ik} + d_i + \sum_{p \in \mathcal{P}_i^{<k}} d_{ip}, \tag{1}$$

with t_{ik} the wind-optimal time to reach the conflict k, d_i the delay at the departure, $\mathcal{P}_i^{<k}$ the set of potential conflicts p that antecede the conflict k and d_{ip} the delay of flight i at the potential conflict p. In the above formulation, it is assumed that all the maneuvers to avoid potential conflicts consist in small and local changes of the wind-optimal trajectories that do not create a cascade of new potential conflicts and do not change the location of the existing potential conflicts. Therefore, we can express the cost function to optimize as

$$f(\{d_i\}, \{d_{ip}\}) = \sum_i d_i + \sum_{ip} d_{ip}, \tag{2}$$

where all the $\{d_i\}$ and $\{d_{ip}\}$ are constrained so that two flights i (j) must have a different arrival time T_{ik} (T_{jk}) to the same potential conflict k, namely

$$\Theta_{a_{ijk}}(|T_{ik} - T_{jk}|) = 0, \tag{3}$$

with Θ a certain function which ensure that the two flights are sufficiently far apart to complete their maneuvers. Here, a_{ijk} represent some extra-parameters that can be added to the model to have more control on the maneuvers.

The delay model is completely defined by the Equations (1)-(3). The successive step consists in the discretization of the variables involved in the delay model. The discretization is achieved by expressing each variable x in terms of a discrete set of values $\{x_\alpha\}$

$$x = \sum_\alpha x_\alpha \sigma_\alpha, \tag{4}$$

with $\sigma_\alpha = \{0, 1\}$, and then enforcing that only one among all the $\{\sigma_\alpha\}$ is one, namely

$$\sum_\alpha \sigma_\alpha = 1. \tag{5}$$

The delay model formulation has been devised by Bryan O'Gorman, in collaboration with Tobias Stollenwerk, Salvatore Mandrà, Davide Venturelli and Eleanor G. Rieffel.

• **Map to PUBO/QUBO.** The delay model, as expressed in Equation (2), is a polynomial unconstrained binary optimization (PUBO) because some of the terms involved are polynomials of order higher than 2. However, most of the quantum annealers, including the D-Wave 2X™ quantum chip, can only natively solve quadratic unconstrained binary optimization (QUBO) problems, where only quadratic terms are present. In order to reduce the PUBO formulation of the delay model to a QUBO formulation, we are devising two different approaches. The first approach is to use gadgets to reduce higher order polynomials to quadratic terms [5]. The second approach consists in using Lagrange multipliers [6] to enforce constraints as in Equation (5) and then reduce the number of higher order polynomials. The two approaches can be either used independently or mixed together. Salvatore Mandrà, Davide Venturelli, Bryan O'Gorman and Eleanor G. Rieffel are working in finding a suitable reduction for ATM problems.

• **Define benchmark.** Given the limited amount of resources of the current quantum technology, it is necessary to identify benchmark problems to explore the potentiality of quantum annealing. Instances in the benchmark ensemble must be small enough to be optimized by the current quantum hardware, like the D-Wave 2X™ quantum chip, but they must be still representative of the hardness of typical size ATM problems. We are devising benchmark instances by using two different approaches. On one hand, we are looking at ATM-like instances with a reduced number of potential conflicts (e.g. considering only a subset of the total wind-optimal trajectories), so that the instances can be optimized by the current architecture of quantum chips. On the other hand, we are looking at archetypal spin glass problems [7] which are not ATM problems but share with them common structures and hardness. The benchmark analysis is conducted by Salvatore Mandrà, Davide Venturelli and Eleanor G. Rieffel.

Outlook and next steps

In the first part of the project, we mainly focused on the formulation of ATM problems in terms of binary optimization problems. More precisely, We have devised the delay model to reduce ATM problems to a quadratic binary optimization problem (QUBO), which are natively solved by typical quantum annealers like the D-Wave 2X™ quantum chip. Despite its simplicity, the model can still capture many of the properties of ATM problems.

Next step of the project will be to devise a benchmark set of ATM instances and analyze the quality/variety of the solutions of the delay model using state-of-art classical QUBO optimizers. This will give us a bottom line for the performance of quantum annealers. Successively, we we will run the ATM benchmark set on the current D-Wave 2X™ quantum chip and compare the quality of results with the classical counterpart. Given the limited amount of resources for the D-Wave 2X™ quantum chip, the potential quantum enhancement for large system size will be extrapolate from the data. The results of the D-Wave 2X™ quantum chip will be used to identify advantages and bottlenecks of the current quantum architecture. Finally, driven by the results from both classical and quantum optimizers, we will explore different quantum architectures and hardware changes to achieve better performances for quantum optimizers.

For a complete overview of completed tasks and future milestones, see Table 1.

Project outputs

- Code to analyze the wind-optimal trajectories and identify potential conflicts within given a maximum delay time.
- Mapping of the ATM problem to PUBO.
- Reduction from PUBO to QUBO.

References

[1] Eleanor G Rieffel, Davide Venturelli, Bryan O'Gorman, Minh B Do, Elicia M Prystay, and Vadim N Smelyanskiy. A case study in programming a quantum annealer for hard operational planning problems. *Quantum Information Processing*, 14(1):1–36, 2015.

[2] Davide Venturelli, Dominic JJ Marchand, and Galo Rojo. Quantum annealing implementation of job-shop scheduling. *arXiv preprint arXiv:1506.08479*, 2015.

[3] Olga Rodionova, Daniel Delahaye, Banavar Sridhar, and Hok K Ng. Deconflicting wind-optimal aircraft trajectories in north atlantic oceanic airspace. In *AEGATS '16, Advanced Aircraft Efficiency in a Global Air Transport System*, 2016.

[4] Olga Rodionova. *Aircraft trajectory optimization in North Atlantic oceanic airspace*. PhD thesis, Université de Toulouse, Paul Sabatier, 2015.

Task/Milestone	Performance Metric	Expected completion from start of project
Devise a code to analyze spatial conflicts and identify potential conflicts within a certain amount of delay.	Largest set of trajectories that the code can analyze. Overall performance of the code.	1.5 month
Analyze the wind-optimal NAT trajectories to identify potential conflicts.	Number of potential conflicts.	2 month
Map the ATM problem to a suitable polynomial binary optimization problem (PUBO).	Number of required qubits. Largest degree of the polynomials. Connectivity of the underlying coupling graph.	3 month
Identify suitable approaches to reduce the degree of the polynomials in the PUBO formulation	Overhead of the reduction.	4 month
Map the ATM problem to a suitable quadratic binary optimization problem (QUBO).	Number of required qubits. Connectivity of the underlying coupling graph.	5 month
Identify a set of benchmark ATM problems.	Hardness as a function of size.	6.5 month
Analyze the results/performance of classical QUBO solvers on ATM problems.	Assess the quality/variety of the solutions after discretization. Find the bottom line for classical computation.	8 month
Compile the ATM benchmark ensemble for the D-Wave 2X™ chip.	Scaling of expected time to solution vs. size compared to classical code. Variety of different acceptable solutions.	10 month
Compare results/quality of solutions of the D-Wave 2X™ chip against classical solvers	Potential quantum enhancement	11 month
Outlook on different architectures and annealing strategies, hardware changes	Potential quantum enhancement.	12 month

Table 1: Breakdown of the project effort into milestones, including suggested performance metric and completion dates (green/light-grey and orange/dark-grey represent respectively completed and partially completed tasks).

[5] Ryan Babbush, Bryan O'Gorman, and Alán Aspuru-Guzik. Resource efficient gadgets for compiling adiabatic quantum optimization problems. *Annalen der Physik*, 525(10-11):877–888, 2013.

[6] Pooya Ronagh, Brad Woods, and Ehsan Iranmanesh. Solving constrained quadratic binary problems via quantum adiabatic evolution. *arXiv preprint arXiv:1509.05001*, 2015.

[7] Davide Venturelli, Salvatore Mandrà, Sergey Knysh, Bryan O'Gorman, Rupak Biswas, and Vadim Smelyanskiy. Quantum optimization of fully connected spin glasses. *Physical Review X*, 5(3):031040, 2015.