NASIC problem specification

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1 Introduction

Notation:

- I: num. airfields
- J: num. tasks
- L: num. aircraft types
- $b(i,l)$: num. l-type aircraft available at airfield i
- $B(i)$: max. num. aircraft able to be dispatched from airfield i
- $A(l)$: total num. l-type aircraft available
- $m(j,l)$: how much l-type aircraft contribute to task j
- $M(j)$: how much task j needs
- $r(l,l',j)$: how much coverage l'-type aircraft provide for l-type aircraft executing task j
- $R(i,j,l)$: how much coverage each l-type aircraft from airfield i executing task j needs
- $C(i,j,l)$: Range of l-type aircraft from airfield i executing task l
- $h(l)$: Range of l-type aircraft
- $N(i,j)$: set of airfields from which an aircraft can cover another aircraft executing task j from airfield i
- $Q(i,j,l,i')$: set of aircraft types in airfield i' that can cover an l-type aircraft from airfield i executing task j

2 Constraints

$y_{i,l,i',j',l'}$ only for $i \in N(i',j')$ and $l \in Q(i',j',l',i)$

$\forall i,l$:

$$\sum_j \left(x_{i,j,l} + \sum_{i',j',l'} y_{i,l,i',j',l'} \right) \leq b(i,l) \tag{1}$$

$\forall i$:

$$\sum_{j,l} x_{i,j,l} + \sum_{l,i',j',l'} y_{i,l,i',j',l'} \leq B(i) \tag{2}$$

$\forall l$:

$$\sum_{i,j} x_{i,j,l} + \sum_{i,i',j',l'} y_{i,l,i',j',l'} \leq A(l) \tag{3}$$

$\forall j$:

$$\sum_{i,l} m_{j,l} x_{i,j,l} \geq M(j) \tag{4}$$

$\forall i, j, l$:

$$h(l) < C(i,j,l) \Rightarrow x_{i,j,l} = 0 \tag{5}$$

$\forall i, j, l$:

$$\sum_{i',l'} r(l,l',j) y_{i',l',i,j,l} \geq R_{i,j,l} x_{i,j,l} \tag{6}$$

3 Instance ensembles

Simplifications:

- Set $A(l) = \sum_i b(i,l)$
- Set $B(i) = \sum_l b(i,l)$
- Have $y_{i,l,i',j',l'}$ only when $(i,l) \in K_2(i',j',l')$
- Have $x_{i,j,l}$ only when $l \in K_1(i,j)$
- $r(l,l',j) = r(j) = 1$
- $R(i,j,l) = R(i,j) = 1$

Oversimplifications:

- $L = 1$
- I small
- J small
- $m(j,l) = 1$
- $M(j)$ small
- $b(i,l) = b(i)$
- (Maybe) $K_2(i',j',l') = I \times L = I \times \{1\}$
- (Maybe) $K_1(i,j) = L$

Remaining:

- I, J, L
- $b(i,l)$
- $m(j,l)$, $M(j)$
- $r(l,l',j)$, $R(i,j,l)$
- $N(i,j)$, $C(i,j,l)$, $h(l)$, $Q(i,j,l,i') \Rightarrow K_1, K_2$

Reduced problem: $\mathbf{x} = \{x_{i,j}\}$, $\mathbf{y} = \{y_{i,i',j'}\}$

- $\forall i$: $\sum_j \left(x_{i,j} + \sum_{i',j'} y_{i,i',j'} \right) \leq b(i,l)$
- $\forall i$: $\sum_j x_{i,j} + \sum_{i',j'} y_{i,i',j'} \leq B(i)$

- (3) implies (1)
- $\forall j$: $\sum_i x_{i,j} \geq M(j)$
- $\forall i, j$: $\sum_{i'} y_{i',i,j} \geq x_{i,j}$

(For all sums, $j \in [J]$, $i \in [I]$.)

- $I = 2$, 3, 4
- $J = I$, $2I$, $3I$
- $M(j) = 1$, 2, 3
- $b(i) = 1, 2, \ldots, 5$

Is this hard?