Quantum Annealing for Air Traffic Management

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Content

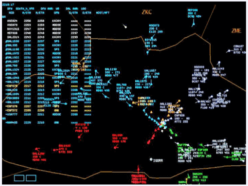
- The problem of optimizing flight trajectories
- Classical preprocessing
- Mapping to Quadratic Unconstrained Binary Optimization (QUBO)
- Quantum annealing experiments (Work in progress)

Motivation

Today

- Manual conflict avoidance
- Computer support for humans

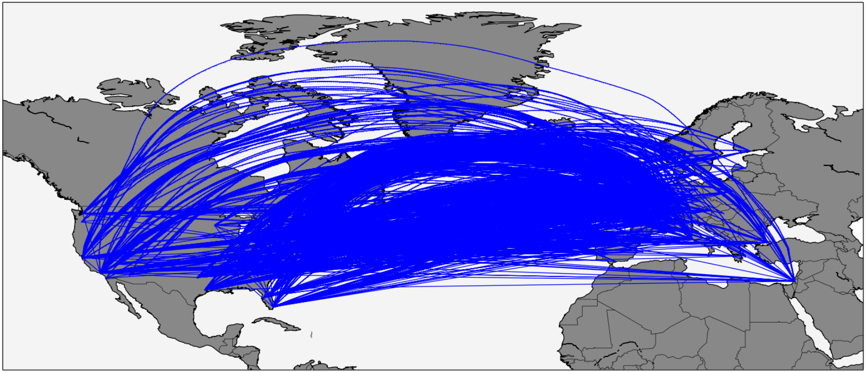
Future



- Increase in computer support for humans
- Precomputation of trajectories: Scheduling

Wind-Optimal Trajectories

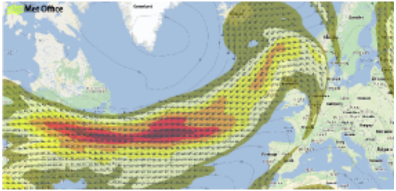
- 984 transatlantic flights on a single day



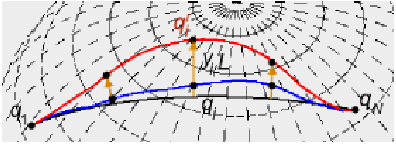
Data from O. Rodionova

Classical Optimization Based on Wind-Optimal Trajectories

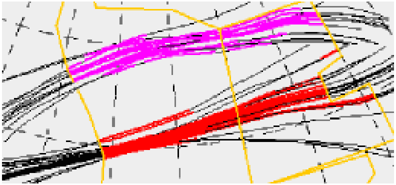
- Jetstream lead to narrow *highways*
- Clusters of conflicting flights
- Classical algorithm: Continuous deviation of trajectories



Woollings et. al. (2010)



Rodionova et. al. (2016)

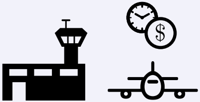


Rodionova et. al. (2016)

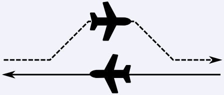
Optimization Problem Formulation

Variables

- Departure delays d_i for each flight i



- Maneuver of flight i to avoid conflict k introduce delay d_{ik}

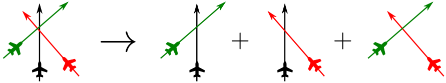


Cost function contribution

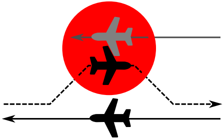
Total delay: $C = \sum_i d_i + \sum_{ik} d_{ik}$

Optimization Problem Formulation - Simplifications

- Only pairwise conflicts

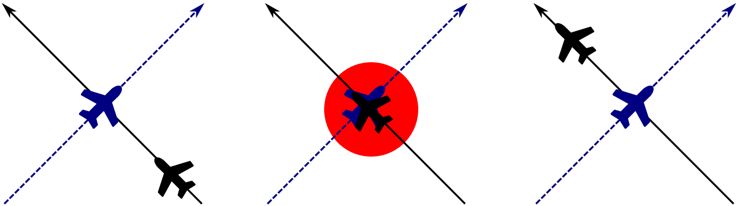


- Conflict avoiding maneuvers impact **only** on delay.



Conflict Avoidance - Arrival Times

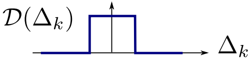
- Difference of arrival times at the conflict between flights i and j, $\Delta_k = T_{ik} - T_{jk}$



$\Delta_k \ll 0$ $\qquad$ $\Delta_k \approx 0$ $\qquad$ $\Delta_k \gg 0$

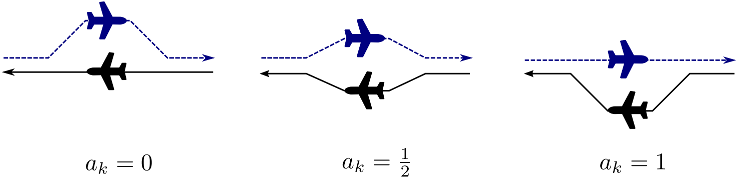
- Delay resulting from conflict avoidance is function of $\Delta_k = T_{ik} - T_{jk}$:

$$d_{ik} = \mathcal{D}_{ik}(\Delta_k)$$



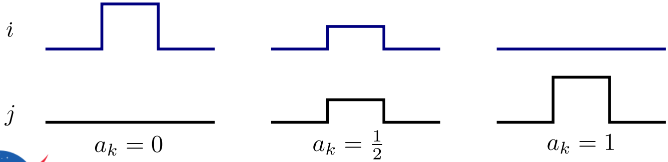
Conflict Avoidance - Maneuver Parameter

- Maneuver parameter a_k, e.g. $a_k \in [0, 1]$



- Delay resulting from conflict avoidance depends on maneuver:

$$d_{ik} = \mathcal{D}_{ik}(\Delta_k, a_k)$$



Optimization Problem Formulation

- Arrival time of flight i at conflict k is delayed by preceding conflicts

$$T_{ik} = t_{ik} + d_i + \sum_{p<k} d_{ip} \qquad t_{ik}\text{: Wind-optimal arrival time}$$

- Optimization problem

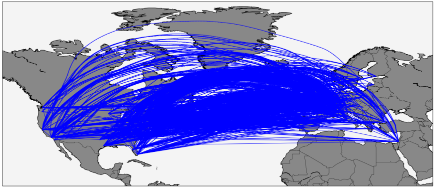
$$\underset{d_i, d_{ik}, a_k}{\text{minimize}} \sum_i d_i + \sum_{ik} d_{ik}$$

$$\text{subject to} \qquad \Delta_k = t_{ik} + d_i + \sum_{p<k} d_{ip} - t_{jk} - d_j - \sum_{q<k} d_{jq}$$

$$d_{ik} = \mathcal{D}_{ik}(\Delta_k, a_k)$$

Precalculating Conflicts

Given the trajectories of all flights i



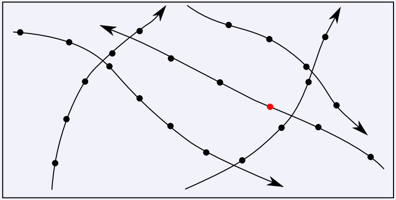
$\Rightarrow$ How to calculate the *potential* conflicts k?

Spatial Conflict Detection

- Spatial conflict, if trajectory points are close (3 nautical miles) to each other.

Brute force algorithm

- Check distance between nearly **all** trajectory points



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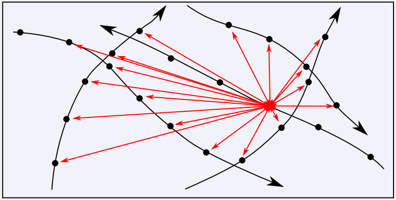

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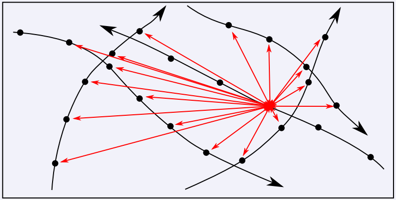
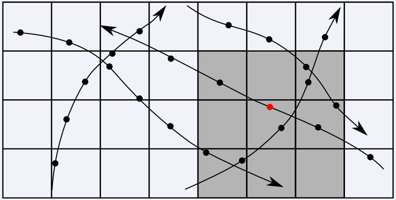


Coarse grid algorithm

- Map trajectory points to coarse grid



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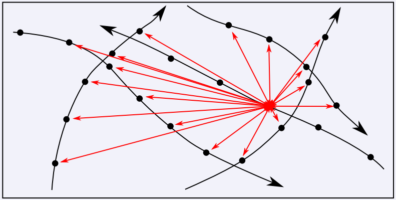

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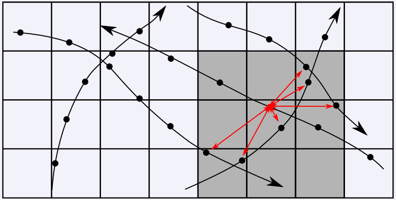
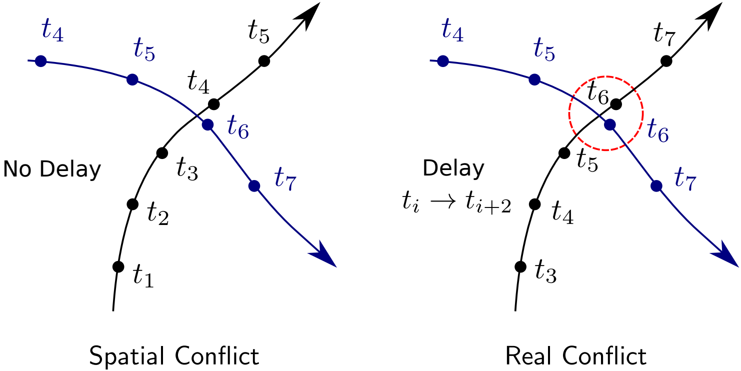


- Check distance only with neighboring cells

Potential Conflicts

- Potential conflict: Spatial conflict which can become real conflict



- First step: Potential conflict, if difference in wind-optimal arrival times $t_{ik} - t_{jk} < 2$ hours.

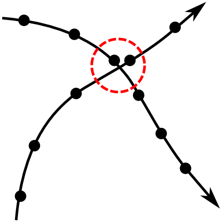
Potential Conflicts

- How to reduce the huge number of potential conflicts: $N_{\text{conflict}} = 43251$?

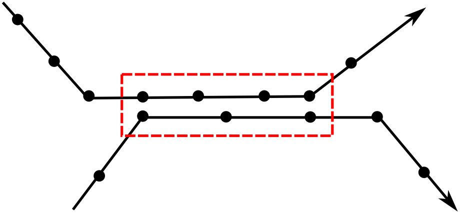
Potential Conflicts - Classification

Reduce the vast number of potential conflicts by categorizing:

- Point Conflict: Isolated in time $N_{\text{point}} = 3573$
- Parallel conflict: Point conflicts consecutive in time $N_{\text{parallel}} = 2510$
- Reduction of 86%



Point Conflict

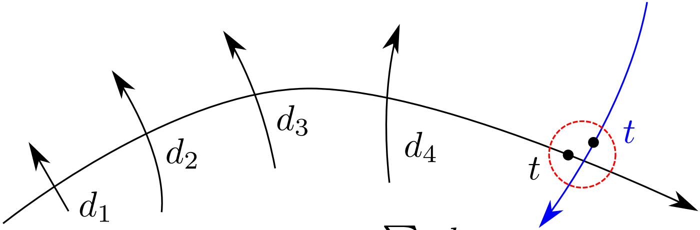


Parallel Conflict

Potential Conflicts - Reduction

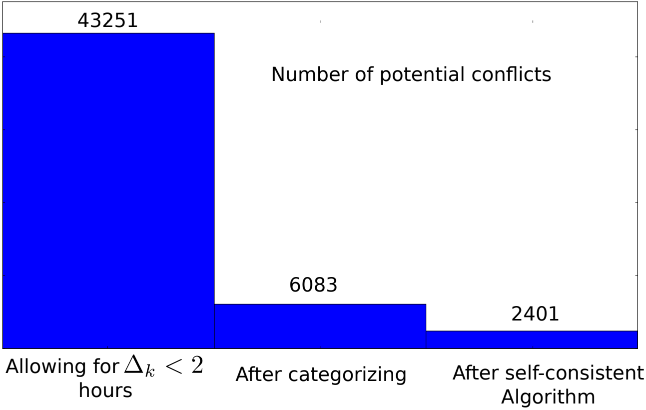
Self-consistent algorithm:

- For each flight, order conflicts in time
- For each potential conflict, calculate the maximal delay of both flights
- Remove potential conflicts which can not become real conflicts
- Repeat the above steps until convergence (N_{conflict} invariant)

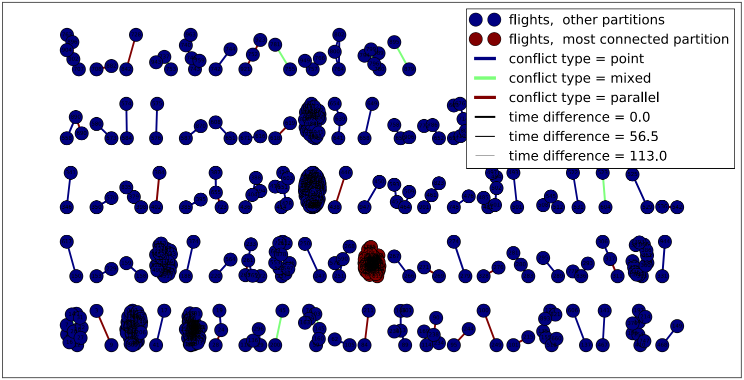


$$t \to t + \sum_i d_i$$

Potential Conflicts - Reduction



Potential Conflicts - Disconnected Subsets



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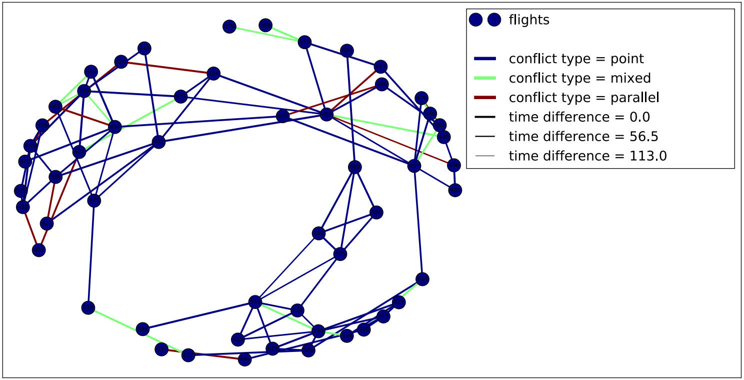


Potential Conflicts - Disconnected Subsets

Use disconnected subsets

- As starting point for hybrid classical-quantum approaches
- For solving small problems on the D-Wave quantum annealer

Mapping to PUBO

Optimization problem

$$\underset{d_i, d_{ik}, a_k}{\text{minimize}} \sum_i d_i + \sum_{ik} \mathcal{D}_{ik}(\Delta_k, a_k)$$

$$\begin{aligned} \text{subject to} \qquad & \Delta_k = t_{ik} + d_i + \sum_{p<k} d_{ip} - t_{jk} - d_j - \sum_{q<k} d_{jq} \\ & d_{ik} = \mathcal{D}_{ik}(\Delta_k, a_k) \end{aligned}$$

- Restrict all variables to integers
- Express integers as binary variables. E.g.

$$d_i = \sum_\alpha \alpha d_{i\alpha} \quad \text{with } d_{i\alpha} \in \{0, 1\}$$

Mapping to PUBO

Polynomial Unconstrained Binary Optimization (PUBO)

$$
\begin{aligned}
&\underset{d_{i\alpha}, d_{ik\beta}, \Delta_{k\delta}, a_{k\gamma}}{\text{minimize}} \sum_{i\alpha} \alpha d_{i\alpha} + \sum_{ik\beta} \beta d_{ik\beta} \\
&+ \sum_{k} \left(\sum_{\delta} \delta \Delta_{k\delta} - t_{ik} - \sum_{\alpha} \alpha d_{i\alpha} - \sum_{\substack{p<k \\ \beta}} \beta d_{ip\beta} + t_{jk} + \sum_{\alpha} \alpha d_{j\alpha} + \sum_{\substack{q<k \\ \beta}} \beta d_{jq\beta} \right)^2 \\
&+ \sum_{ik} \left(\sum_{\beta} \beta d_{ik\beta} - \sum_{\delta\gamma} \mathcal{D}_{ik}(\delta, \gamma) \Delta_{k\delta} a_{k\gamma} \right)^2 + \sum_{i} \left(\sum_{\alpha} d_{i\alpha} - 1 \right)^2 + \dots
\end{aligned}
$$

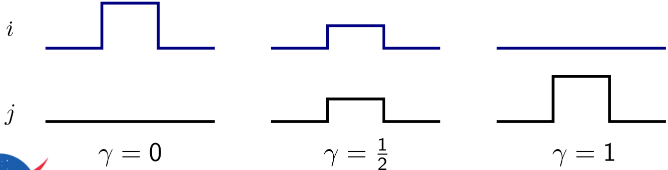
Mapping to PUBO

- Binary variable for difference in arrival times $\Delta_k = \sum_\delta \delta \Delta_{k\delta}$
- Conflict avoiding term

$$\sum_{ik} \left(\sum_\beta \beta d_{ik\beta} - \sum_{\delta\gamma} \mathcal{D}_{ik}(\delta,\gamma) \Delta_{k\delta} a_{k\gamma} \right)^2$$

where

$$\mathcal{D}_{ik}(\delta,\gamma) = \begin{cases} D & \text{if } |\delta| < 3 \text{ minutes} \\ 0 & \text{else} \end{cases}$$



Reduction to QUBO

- Reduce quartic and cubic terms to quadratic terms by degree reduction
- Drawback: More variables

Simplified Model: Only Departure Delays

- Simplification: No maneuver delays d_{ik}
- Penalize conflicts with

$$\mathcal{P}_k(\Delta_k) = \begin{cases} P & \text{if } |\Delta_k| < 3 \text{ minutes} \\ 0 & \text{else} \end{cases}$$

where P =const.

Optimization problem

$$\underset{d_i}{\text{minimize}} \sum_i d_i + \sum_k \mathcal{P}_k(\Delta_k)$$

Simplified Model - Mapping to QUBO

Difference of conflict arrival times becomes

$$\Delta_k = t_{ik} - t_{jk} + d_i - d_j$$

QUBO

$$\begin{aligned}
\underset{d_{i\alpha},\Delta_{k\delta}}{\text{minimize}}\ & p_1 \sum_{i\alpha} \alpha d_{i\alpha} + p_2 \sum_{k} \sum_{\substack{\delta \\ |\delta|<3\text{min}}} \Delta_{k\delta} \\
& + p_3 \sum_k \left(\sum_\delta \delta \Delta_{k\delta} - t_{ik} - \sum_\alpha \alpha d_{i\alpha} + t_{jk} + \sum_\alpha \alpha d_{j\alpha} \right)^2 \\
& + p_4 \sum_i \left(\sum_\alpha d_{i\alpha} - 1 \right)^2 + p_5 \sum_k \left(\sum_\delta \Delta_{k\delta} - 1 \right)^2
\end{aligned}$$

Simplified Model - Small Problem Instances

Generate random instances with

- Number of flights: 5, 10, 20, 50
- Number of random conflicts: 5, 10, 20, 50
- Random conflict arrival times
- Departure delays: $0, 3, \ldots, 18$ minutes
- Different sets of penalty weights p_i

Preliminary quantum annealing results from D-Wave 2X at NASA Ames (Work in progress)

- So far, no valid solutions
- Possible reason: Precision issue

Summary and Outlook

Summary

- Air traffic management problems can be mapped to PUBO
- Significant classical preprocessing for real data

Next steps

- Map PUBO to QUBO
- Investigate alternative approaches, like Jop Shop Scheduling
- Quantum annealing runs