Reminder: Preprocessing of the Trajectory Data

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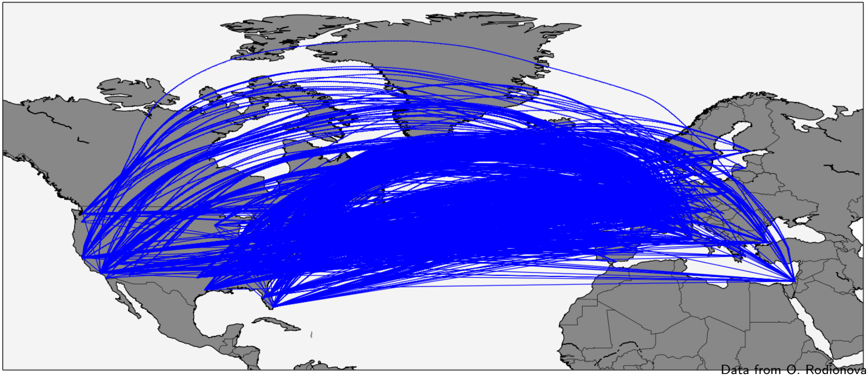
Wind-Optimal Trajectories

- 984 transatlantic flights on a single day

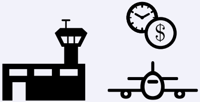


Data from O. Rodionova

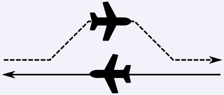
Optimization Problem Formulation

Variables

- Departure delays d_i for each flight i



- Maneuver of flight i to avoid conflict k introduce delay d_{ik}

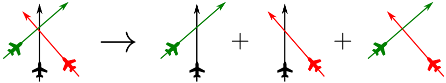


Cost function contribution

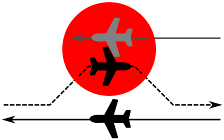
Total delay: $C = \sum_i d_i + \sum_{ik} d_{ik}$

Optimization Problem Formulation - Simplifications

- Only pairwise conflicts

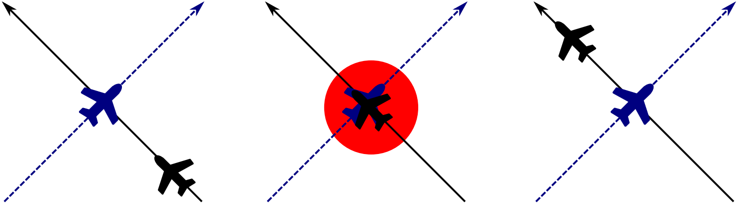


- Conflict avoiding maneuvers impact **only** on delay.



Conflict Avoidance - Arrival Times

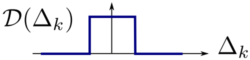
- Difference of arrival times at the conflict between flights i and j, $\Delta_k = T_{ik} - T_{jk}$



$\Delta_k \ll 0$ $\Delta_k \approx 0$ $\Delta_k \gg 0$

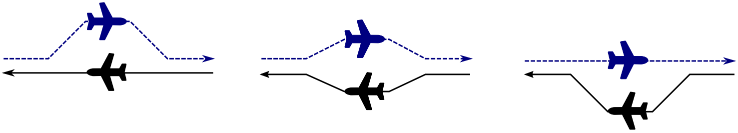
- Delay resulting from conflict avoidance is function of $\Delta_k = T_{ik} - T_{jk}$:

$$d_{ik} = \mathcal{D}_{ik}(\Delta_k)$$



Conflict Avoidance - Maneuver Parameter

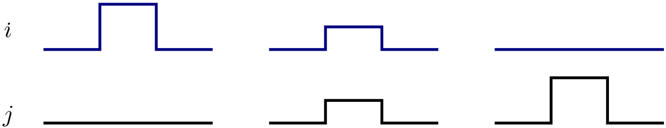
- Maneuver parameter a_k, e.g. $a_k \in [0, 1]$



$a_k = 0$ $a_k = \frac{1}{2}$ $a_k = 1$

- Delay resulting from conflict avoidance depends on maneuver:

$$d_{ik} = \mathcal{D}_{ik}(\Delta_k, a_k)$$



$a_k = 0$ $a_k = \frac{1}{2}$ $a_k = 1$

Optimization Problem Formulation

- Arrival time of flight i at conflict k is delayed by preceding conflicts

$$T_{ik} = t_{ik} + d_i + \sum_{p<k} d_{ip} \qquad t_{ik}\text{: Wind-optimal arrival time}$$

- Optimization problem

$$\underset{d_i, d_{ik}, a_k}{\text{minimize}} \sum_i d_i + \sum_{ik} \mathcal{D}_{ik}(\Delta_k, a_k)$$

$$\text{subject to} \qquad \Delta_k = t_{ik} + d_i + \sum_{p<k} d_{ip} - t_{jk} - d_j - \sum_{q<k} d_{jq}$$

$$d_{ik} = \mathcal{D}_{ik}(\Delta_k, a_k)$$

Simplification: Delay-Only Model

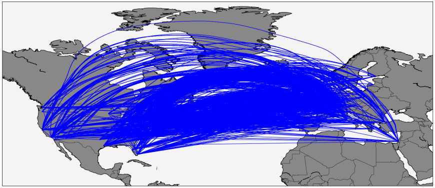
Optimization problem

$$\underset{d_i}{\text{minimize}} \sum_i d_i$$

$$\text{subject to} \qquad |t_{ik} + d_i - t_{jk} - d_j| < 3 \text{ minutes} \quad \forall k$$

Precalculating Conflicts

Given the trajectories of all flights i



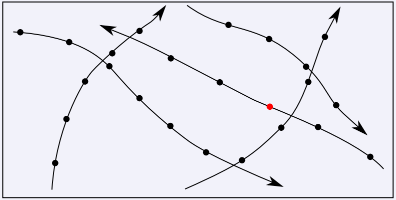
⇒ How to calculate the *potential* conflicts k?

Spatial Conflict Detection

- Spatial conflict, if trajectory points are close (30 NM) to each other.

Brute force algorithm

- Check distance between nearly **all** trajectory points



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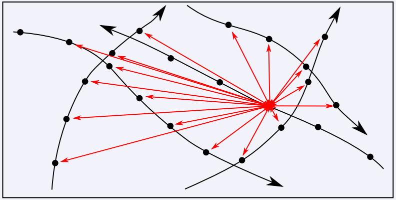

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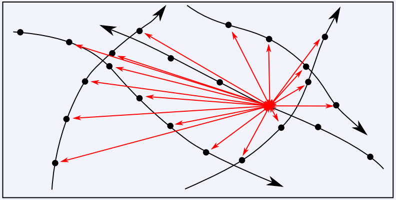
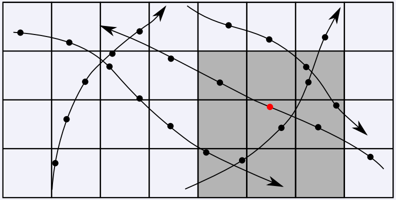


Coarse grid algorithm

- Map trajectory points to coarse grid



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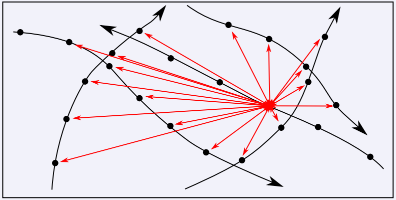

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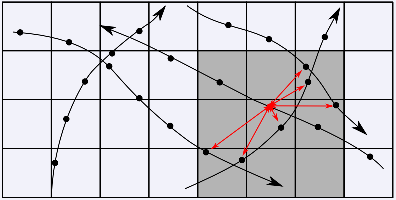
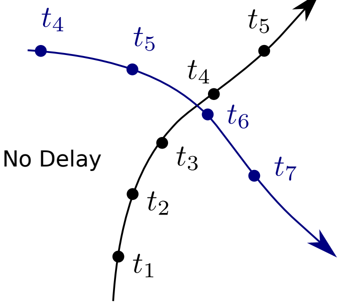


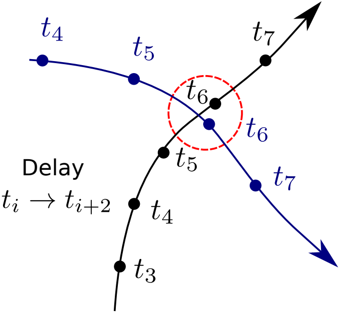
- Check distance only with neighboring cells

Potential Conflicts

- Potential conflict: Spatial conflict which can become real conflict



Spatial Conflict



Real Conflict

- First step: Potential conflict, if difference in wind-optimal arrival times $t_{ik} - t_{jk} < 2$ hours.

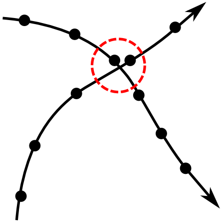
Potential Conflicts

- How to reduce the huge number of potential conflicts: $6,609,623$?

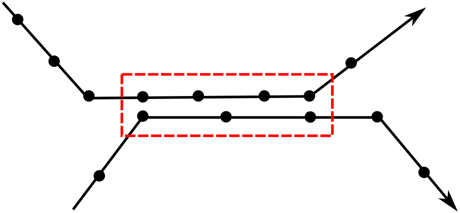
Potential Conflicts - Classification

Reduce the vast number of potential conflicts by categorizing:

- Point Conflict: Isolated in time $N_{\text{point}} = 265$
- Parallel conflict: Point conflicts consecutive in time $N_{\text{parallel}} = 20867$
- Reduction of 99.7%



Point Conflict

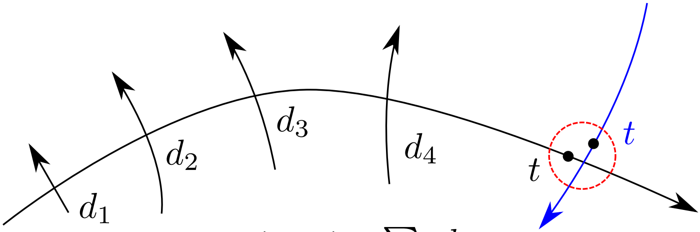


Parallel Conflict

Potential Conflicts - Reduction

Self-consistent algorithm:

- For each flight, order conflicts in time
- For each potential conflict, calculate the maximal delay of both flights
- Remove potential conflicts which can not become real conflicts
- Repeat the above steps until convergence (N_{spatial} invariant)



$$t \to t + \sum_i d_i$$

Potential Conflicts - Reduction