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Quantum Annealing for Air Traffic Management

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I. INTRODUCTION

II. PROBLEM DESCRIPTION

The problem at hand is the deconflicting of transatlantic wind-optimal trajectories. As it was done in [1] we are using the same wind-optimal trajectories of a single day, July 29 2012. These wind-optimal trajectories are given as $(\mathbf{x}_i)_{i=1}^n$, where $\mathbf{x}_i = (x_{i,t})_{t=t_{i,0}}^{t_{i,1}}$ and $x_{i,t}$ is the location (as latitude, longitude, and altitude) of the ith flight at time t. The times $t_{i,0}$ and $t_{i,1}$ are the times at which the wind-optimal trajectory for the ith flight begins and ends, respectively. Furthermore, the times are given in units of one minutes $T_i = (t_{i,0}, t_{i,0}+1, \ldots, t_{i,1})$. Each flight i is at a constant speed v_i, to within (classical) machine precision.

A conflict between two flights is defined as a pair of trajectory points which are too close to each other in space and time.

$$\{(x_{i,t}, x_{j,t'}) \mid \mathcal{D}(x_{i,t}, x_{j,t'}) < \Delta_x, |t-t'| < \Delta_t\} \ , \quad (1)$$

where $\mathcal{D}(x, y)$ is the spatial distance between two points x and y given as latitude, longitude and altitude. Following [1], the space threshold is $\Delta_x = 3$ nautical miles and the time threshold is $\Delta_t = 3$ minutes. In this paper, we consider the following means to deconflict the trajectories: First, we can delay each flight i at departure time by a departure delay d_i

$$x_{i,t} \to x_{i,t+d_i} \quad \forall\, t \in T_i$$

Second, we can avoid a conflict by maneuvers of both involved flights. We assume, however, that the maneuvers will not introduce new conflicts. In doing so, these maneuvers can be view as resulting in time shifts only.

A. Classical Preprocessing

It is beneficial to reduce the data to conflicting regions in space and decoupling the spacial and temporal components of the problem. As a first step, we detect all pairs of trajectory points which are separated by a spacial distance below Δ_x

$$\{(x_{i,t}, x_{j,t'}) \mid \mathcal{D}(x_{i,t}, x_{j,t'}) < \Delta_x\} \ ,$$

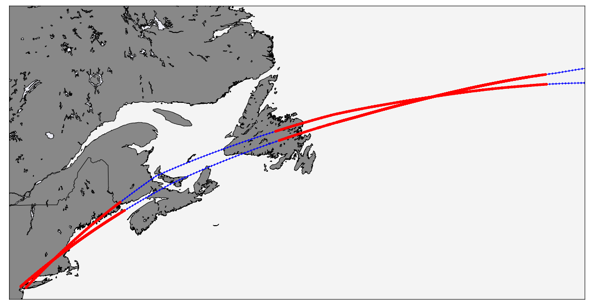

FIG. 1. Example of two parallel potential conflicts between two transatlantic flights starting from the east cost of the USA.

Two spatially conflicting trajectory points might never become conflicting in time if the corresponding times are far apart. By introducing a constant maximum delay D_{max} we can dismiss all spatial conflicts which can never become conflicting in time

$$\{(x_{i,t}, x_{j,t'}) \mid \mathcal{D}(x_{i,t}, x_{j,t'}) < \Delta_x, |t-t'| \geq \Delta_t + D_{\max}\} \ .$$

With this, we are left with a set of potentially conflicting pairs of trajectory points

$$C_0^{ij} = \{(x_{i,t}, x_{j,t'}) \mid \mathcal{D}(x_{i,t}, x_{j,t'}) < \Delta_x, |t-t'| < \Delta_t + D_{\max}\} \ .$$

As a next step, we group together conflicting trajectory point pairs which are subsequent in time

$$\begin{aligned} C_{\parallel}^{ij} = \{((x_{i,t}, x_{j,t'}), (x_{i,s}, x_{j,s'})) \mid\ & (x_{i,t}, x_{j,t'}) \in C_0^{ij}, \\ & (x_{i,s}, x_{j,s'}) \in C_0^{ij}, \\ & |t-s| < \Delta_t', \\ & |t'-s'| < \Delta_t'\} \end{aligned}$$

where we set $\Delta_t' = 2$ minutes.

For a given pair of flights (i, j) there might be multiple "disjoint" subsets in $C_{\parallel}^{ij}$

$$\bigcup_n C_{\parallel n}^{ij} = C_{\parallel}^{ij}$$

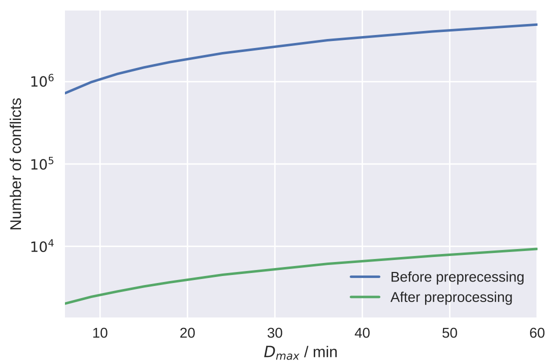

FIG. 2. Preprocessing: Reduction in the number of potential conflicts for various upper delay bounds $D_{\max}$.

where

$$
\begin{aligned}
&|t-s| \geq \Delta_t' \wedge |t'-s'| \geq \Delta_t' \\
&\forall (x_{i,t}, x_{j,t'}) \in C^{ij}_{\| n}, \\
&\forall (x_{i,s}, x_{j,s'}) \in C^{ij}_{\| n'}, \\
&n \neq n' \ .
\end{aligned}
$$

In figure 1 an example of two separated clusters are shown. Together with the remaining, spatially isolated, conflicting trajectory points

$$C^{ij}_{\times} = C^{ij}_0 \setminus C^{ij}_{\|} \ ,$$

these subsets of trajectory point clusters are called *potential conflicts*.

$$C_k \in C = \{C^{ij}_{\| n} | \forall i, j, n\} \cup \{C^{ij}_{\times} | \forall i, j\}$$

Here, we introduced a conflict index $k \in \{1, \ldots, N_C\}$, with $N_C = |C|$. For each conflict index k, we will denote the pair of involved flights by $I_k = (i_k, j_k)$.

Before the preprocessing, the number of conflicts was given by $N_C^{\text{before}} = \sum_{ij} |C^{ij}_0|$. As one can see in figure 2 the preprocessing reduces the number of conflicts by orders of magnitude.

B. Conflict Avoidance

In order to avoid conflicts, a flight i can be either delayed at departure time by d_i or by maneuver introduced delays d_{ik} for each conflict k the flight is involved in. With this, the trajectory times of flight i are shifted

$$
\begin{aligned}
t_i \to \tau_i = t_i + D_i(t) \ , \\
D_i(t) = d_i + \sum_{k \in K_i(t)} d_{ik}
\end{aligned}
$$

where $D_i(t)$ is the delay of flight i at time t and the sum runs over all the conflicts which are before t

$$K_i(t) = \left\{ k | \max_{t'} x_{i,t'} \in C_k < t \right\}$$

We introduce the pairs of times of spatially conflicting points inside a conflict k:

$$T_k = \{(t, t') \mid |x_{i,t} - x_{j,t'}| < \Delta_x, (i, j) \in I_k\}$$

A conflicts k between two flights i and j occurs if the delays are chosen such that a temporal difference between spatially conflicting points is below Δ_t:

$$
\begin{aligned}
&|t + D_i(t) - t' - D_j(t')| < \Delta_t \\
\Rightarrow &- \Delta_t - (t - t') < D_i(t) - D_j(t') < \Delta_t - (t - t')
\end{aligned}
$$

for any $(t, t') \in T_k$. Hence a conflict k is avoided if

$$D_i(t) - D_j(t') \notin D_k \qquad \forall\, (t, t') \in T_k$$

with

$$D_k = \left(-\Delta_t - \max_{(t,t') \in T_k} (t - t'), \Delta_t - \min_{(t,t') \in T_k} (t - t') \right)$$

III. DEPARTURE DELAY MODEL

In this section we describe a simplified version of the above problem. We restrict ourselves to departure delays and neglect maneuvers. Hence the optimization problem can be written as

$$
\begin{aligned}
&\arg, \min_{d_i} \sum_i d_i \\
&\text{s.t.} \quad d_i - d_j \notin D_k \qquad \forall\, (t, t') \in T_k
\end{aligned}
\tag{2}
$$

The problem needs to be mapped to a quadratic binary optimization problem (QUBO) in order to be solvable by a D-Wave quantum annealer. As a first step, we introduce a discretization and upper bound for the departure delay variables

$$d_i \in \{0, \Delta_d, 2\Delta_d, \ldots, d_{\max}\} \, .$$

With this we can write the departure delay variables in terms of new binary variables $d_{i\alpha}$ by

$$
\begin{aligned}
&d_i = \sum_\alpha \alpha d_{i\alpha} \\
&\alpha \in \{0, \Delta_d, 2\Delta_d, \ldots, d_{\max}\} \, .
\end{aligned}
$$

However, this approach requires $\sum_\alpha d_{i\alpha} = 1$ to have a unique representation of d_i by $d_{i\alpha}$. This is achieved by adding the following contribution to the QUBO

$$Q_{\text{unique}} = \lambda_{\text{unique}} \sum_i \left(\sum_\alpha d_{i\alpha} - 1 \right)^2 \, .$$

where λ_{unique} is a penalty weight sufficiently large to ensure $Q_{\text{unique}} = 0$ for the solution. The cost function in (2) yields contribution

$$Q_{\text{delay}} = \frac{1}{d_{\max}} \sum_{i\alpha} d_{i\alpha} \ ,$$

where we have chosen the prefactor for convenience. The last contribution to the QUBO represents the conflict avoidance

$$Q_{\text{conflict}} = \lambda_{\text{conflict}} \sum_{k} \sum_{(\alpha,\beta)\in K_k} d_{i\alpha} d_{j\alpha} \bigg|_{i,j\in I_k}$$

where K_k is the set of all (α, β) which correspond to a conflict

$$K_k = \{(\alpha,\beta) | \alpha - \beta \in D_k\}$$

Again, $\lambda_{\text{conflict}}$ is the penalty weight for this contribution. The total QUBO of the departure delay model then reads

$$Q_{\text{DDM}} = Q_{\text{delay}} + Q_{\text{unique}} + Q_{\text{conflict}}$$

A. Configuration Space Restrictions

B. Embedding to D-Wave

C. Quantum Annealing Results

IV. MANEUVER MODEL

[1] O. Rodionova, D. Delahaye, B. Sridhar, and H. Ng., Proceedings of Advanced Aircraft Efficiency in a Global Air Transport System (AEGATS'16) Conference (2016).