Z2

$$B_k^n(t) = \binom{n}{k} t^k (1-t)^{n-k}$$

a) $\boxed{B_k^n(t) \geq 0 \;, t \in [0,1] \;, \text{ jedno maksimum}}$

$$B_k^n(t) = \underbrace{\overset{>0}{\binom{n}{k}}}_{\geq 0} \underbrace{t^k}_{\geq 0} \underbrace{(1-t)^{n-k}}_{\geq 0}$$

$t \in [0,1]$

$$\left(B_k^n(t)\right)' = \left(\binom{n}{k} t^k (1-t)^{n-k}\right)' = \binom{n}{k}\left(t^k (1-t)^{n-k}\right)'$$

$$= \binom{n}{k} x^{k-1} (1-x)^{n-k-1} \left(k(1-x) - (n-k)x\right)$$

$$= \binom{n}{k} x^{k-1} (1-x)^{n-k-1} (k - nx) = 0$$

$x=0$ $\quad$ $x=1$ $\quad$ $x = \frac{k}{n}$

Miejsca zerowe

$B(0) = B(1) = 0 \Rightarrow 0$ i 1 to nie maksima, gdy $\frac{k}{n} \notin \{0,1\}$

$\frac{k}{n}$ to ekstremum

Wiemy, że $B_k^n(x) \geq 0$, czyli $\frac{k}{n}$ to maksimum

b) $\boxed{\sum_{k=0}^{n} B_k^n(x) \equiv 1}$

$$B_k^n(x) = \binom{n}{k} x^k (1-x)^{n-k}$$

$$\sum_{k=0}^{n} B_k^n(x) = \boxed{\sum_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k}} = (x + (1-x))^n = 1$$

dwumian Newtona

c) $\boxed{B_k^n(x) = \underbrace{(1-x) B_k^{n-1}}_{\alpha} + \underbrace{x B_{k-1}^{n-1}(x)}_{\beta}}$

$$\alpha = (1-x)\binom{n-1}{k} x^k (1-x)^{n-k+1}$$

$$= \frac{(n-1)!}{k!(n-1-k)!} \cdot x^k (1-x)^{n-k}$$

$$\beta = \frac{(n-1)!}{(k-1)!(n-k)!} x^k (1-x)^{n-k}$$

$$B_k^n(x) = \frac{(n-1)!}{k!(n-1-k)!} \cdot x^k (1-x)^{n-k} + \frac{(n-1)!}{(k-1)!(n-k)!} x^k (1-x)^{n-k}$$

$$= \left(\frac{(n-1)!}{k!(n-1-k)!} + \frac{(n-1)!}{(k-1)!(n-k)!}\right) x^k (1-x)^{n-k}$$

$$= \binom{n}{k} x^k (1-x)^{n-k}$$

d) $$B_k^n(x) = \frac{n+1-k}{n+1} B_k^{n+1}(x) + \frac{k+1}{n+1} B_{k+1}^{n+1}(x)$$

$$= \frac{n+1-k}{n+1}\binom{n+1}{k} x^k (1-x)^{n-k+1} + \frac{k+1}{n+1}\binom{n+1}{k+1} x^{k+1}(1-x)^{n-k}$$

$$= \frac{n+1-k}{n+1} \cdot \frac{(n+1)!}{k!(n+1-k)!} x^k (1-x)^{n-k+1} + \frac{k+1}{n+1} \frac{(n+1)!}{(k+1)!(n-k)!} x^{k+1}(1-x)^{n-k}$$

$$= \binom{n}{k} x^k (1-x)^{n-k+1} + \binom{n}{k} x^{k+1} (1-x)^{n-k}$$

$$= \binom{n}{k} x^k (1-x)^{n-k} (1-x+x)$$

$$= \binom{n}{k} x^k (1-x)^{n-k}$$

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