Określone tylko dla tych $\omega \in \Omega$, dla których $X(\omega)\ != 0$, ale $P(X = 0) = 0$

$$f_X(x) = \frac{1}{\pi} \cdot \frac{1}{1+x^2}, \quad Y = \frac{1}{X}$$

$f_X(x)$ jest symetryczna względem OY

$$\boxed{P(X \geq 1/t) = 1 - P(X < 1/t)}$$

$$F_Y(t) = P(Y \leq t) = P\left(\frac{1}{X} \leq t\right) = \begin{cases} P(X<0) + P(X \geq 1/t), & t>0 \\ P(1/t \leq X < 0), & t<0 \\ P(X<0), & t=0 \end{cases} = \begin{cases} 1/2 + P(X \geq 1/t), & t>0 \\ P(1/t \leq X<0), & t<0 \\ 1/2, & t=0 \end{cases} = \begin{cases} 3/2 - P(X \leq 1/t), & t>0 \\ P(X \leq 0) - P(X \leq 1/t), & t<0 \\ 1/2, & t=0 \end{cases} = \begin{cases} \frac{3}{2} - F_X\left(\frac{1}{t}\right), & t>0 \\ F_X(0) - F_X\left(\frac{1}{t}\right), & t<0 \\ 1/2, & t=0 \end{cases}$$

$$f_Y(t) = F_Y'(t) = \begin{cases} \left(\frac{3}{2} - F_X\left(\frac{1}{t}\right)\right)' & t>0 \\ \left(F_X(0) - F_X\left(\frac{1}{t}\right)\right)' & t<0 \\ 0 & t=0 \end{cases} = \begin{cases} -f_X\left(\frac{1}{t}\right) \cdot \frac{-1}{t^2} & t>0 \\ -f_X\left(\frac{1}{t}\right) \cdot \frac{-1}{t^2} & t<0 \\ 0 & t=0 \end{cases} = \begin{cases} f_X\left(\frac{1}{t}\right) \cdot \frac{1}{t^2} & t \neq 0 \\ 0 & t=0 \end{cases} = \begin{cases} \frac{1}{t^2} \cdot \frac{1}{\pi} \cdot \frac{1}{1+\left(\frac{1}{t}\right)^2} & t \neq 0 \\ 0 & t=0 \end{cases} = \begin{cases} \frac{1}{\pi} \cdot \frac{1}{t^2+1} & t \neq 0 \\ 0 & t=0 \end{cases} = \frac{1}{\pi} \cdot \frac{1}{t^2+1}$$

Pojedynczy punkt nie zmienia rozkładu