$$\sum_{n=0}^{\infty} (2^{n}-1) x^{n} = 0 \cdot x^{0} + 1 \cdot x^{1} + 3 x^{2} + \dots$$

$$\sum_{n=0}^{\infty} (2^{n}-1) x^{n+2} = 0 \cdot x^{0} \cdot 0 \cdot x^{1} + 0 \cdot x^{2} + 1 \cdot x^{3} + 3 \cdot x^{5} + \dots$$

$$\sum_{m=0}^{\infty} 2^{n} x^{m+2} - \sum_{m=0}^{\infty} x^{m+2} = \frac{x^{2}}{1-2x} - \frac{x^{2}}{1-x}$$