Rozkład geometryczny

$$X \sim geom(p) \iff P(X=k)=(1-p)^{k-1}p$$

$$(k=1,2,3,\dots)$$

Wariancja

$$V(X)=\sum_{i\in N}\left(x_i - E(X)^2\right)p_i = \overset{(2)}{E}(X^2) - \left(\overset{(1)}{E}(X)\right)^2$$

$$\boxed{\sum_{k=0}^{\infty} q^k = \frac{1}{1-q}, \quad |q|<1}$$

(1)
$$E(X)=\sum_{k=1}^{\infty} k\cdot P(X=k) = \sum_{k=1}^{\infty} k\cdot(1-p)^{k-1}p = -p\left(\sum_{k=1}^{\infty}(1-p)^k\right)' = -p\left(\frac{1}{1-1+p}\right)' = -p\,(-p^{-1})' = \frac{1}{p}$$

(2)
$$E(X^2)=\sum_{k=1}^{\infty} k^2\cdot P(X=k) = \sum_{k=1}^{\infty} k^2\cdot(1-p)^{k-1}p = -p\left(\sum_{k=1}^{\infty} k\,(1-p)^k\right)' = -p\left((1-p)\sum_{k=1}^{\infty} k\,(1-p)^{k-1}\right)'$$

$$= \cancel{-}p\left((1-p)\underbrace{\left(\sum_{k=1}^{\infty} +(1-p)^k\right)'}_{\frac{E(X)}{-p}}\right)' = p\left((1-p)\cdot\frac{-1}{p^2}\right)' = \frac{2}{p^2}-\frac{1}{p}$$

$$V(X)=E(X^2)-\left(E(X)\right)^2 \overset{(1),(2)}{=} \frac{2}{p^2}-\frac{1}{p}-\frac{1}{p^2} = \frac{1}{p^2}-\frac{1}{p} = \frac{1-p}{p^2}$$

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