

$$\mathbb{Z}_1$$

$$A(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + \dots$$

$$s_0 = a_0$$

$$s_1 = a_0 + a_1$$

$$s_2 = a_0 + a_1 + a_2$$

$$\vdots$$

$$s_n = a_0 + a_1 + a_2 + \dots + a_n$$

$$S(x) = a_0 x^0 + (a_0 + a_1) x^1 + (a_0 + a_1 + a_2) x^2 + \dots$$

$$= a_0 (x^0 + x^1 + x^2 + \dots) +$$

$$a_1 (x^1 + x^2 + x^3 + \dots) +$$

$$a_2 (x^2 + x^3 + x^4 + \dots) + \dots =$$

$$= a_0 \left( \frac{1}{1-x} \right) + a_1 \left( \frac{x}{1-x} \right) + a_2 \left( \frac{x^2}{1-x} \right) + \dots$$

Wyciągam  $a_i$  przed sumę tak jak powyżej

$$S(x) = \sum_{n=0}^{\infty} s_n x^n = \sum_{n=0}^{\infty} \sum_{i=0}^n a_i x^n = \sum_{n=0}^{\infty} a_n \sum_{i=n}^{\infty} x^i =$$

$$= \sum_{n=0}^{\infty} a_n \left( \frac{x^n}{1-x} \right) = \frac{1}{1-x} \sum_{n=0}^{\infty} a_n x^n = \frac{A(x)}{1-x}$$