$$\overline{s} := \sum_{i=1}^{n} \frac{s_i}{n}$$

a) $$\sum_{k=1}^{n} (x_k - \overline{x})^2 = \sum_{k=1}^{n} x_k^2 + \sum_{k=1}^{n} (\overline{x}^2 - 2x_k\overline{x}) = \sum_{k=1}^{n} x_k^2 - \sum_{k=1}^{n} (2x_k\overline{x} - \overline{x}^2) \overset{(1)}{=} \sum_{k=1}^{n} x_k^2 - n\overline{x}^2$$

(1) $$\sum_{k=1}^{n} (2x_k\overline{x} - \overline{x}^2) = \overline{x} \sum_{k=1}^{n} (2x_k - \overline{x}) \overset{(2)}{=} \overline{x} \cdot (n \cdot \overline{x}) = n\overline{x}^2$$

(2) $$\sum_{k=1}^{n} \left(2x_k - \sum_{i=1}^{n} \frac{x_i}{n}\right) = \sum_{k=1}^{n} 2x_k - \cancel{n} \sum_{i=1}^{n} \frac{x_i}{\cancel{n}} = \sum_{k=1}^{n} 2x_k - x_k = \sum_{k=1}^{n} x_k = n \cdot \overline{x}$$

$i \to k$

b) $$\sum_{k=1}^{n} (x_k - \overline{x})(y_k - \overline{y}) = \sum_{k=1}^{n} x_k y_k - \sum_{k=1}^{n} (x_k\overline{y} + y_k\overline{x} - \overline{x}\,\overline{y}) \overset{(3)}{=} \sum_{k=1}^{n} x_k y_k - n\overline{x}\,\overline{y}$$

(3) $$\sum_{k=1}^{n} (x_k\overline{y} + y_k\overline{x} - \overline{x}\,\overline{y}) = \sum_{k=1}^{n} (y_k\overline{x} + \overline{y}(x_k - \overline{x})) = \sum_{k=1}^{n} y_k\overline{x} + \overline{y} \sum_{k=1}^{n} (x_k - \overline{x}) =$$

$$\sum_{k=1}^{n} y_k\overline{x} + \overline{y} \sum_{k=1}^{n} x_k - n\overline{x}\overline{y} = \sum_{k=1}^{n} y_k\overline{x} + n\overline{x}\overline{y} - n\overline{x}\overline{y} = \overline{x} \sum_{k=1}^{n} y_k = n\overline{x}\,\overline{y}$$