$P_k \quad (1 \leq k \leq N)$

$$(f, g)_N = \sum_{i=0}^{N} f(x_i)\, g(x_i)$$

$$\boxed{\text{Cel: } \forall_{w \in \Pi_{k-1}} (w, P_k)_N = 0}$$

$$(P_i, P_j)_N \overset{*}{=} 0 \qquad (i \neq j)$$

$$(w, P_k)_N = \sum_{i=0}^{N} w(x_i) \cdot P_k(x_i)$$

Wiemy, że $\operatorname{Lin}\{P_0, P_1, P_2, \dots, P_{k-1}\} = \Pi_{k-1}$

Skoro $w \in \Pi_{k-1}$ to w jest kombinacją liniową $P_0, P_1, P_2, \dots, P_{k-1}$:

$$w = \sum_{j=0}^{k-1} \alpha_j P_j$$

Wtedy:

$$(w, P_k)_N = \sum_{i=0}^{N} w(x_i) \cdot P_k(x_i) = \sum_{i=0}^{N} \left(\sum_{j=0}^{k-1} \alpha_j P_j(x_i) \right) P_k(x_i)$$

$$= \sum_{i=0}^{N} \sum_{j=0}^{k-1} \alpha_j P_j(x_i) P_k(x_i)$$

$$= \sum_{j=0}^{k-1} \alpha_j \sum_{i=0}^{N} P_j(x_i) P_k(x_i)$$

$$= \sum_{j=0}^{k-1} \alpha_j (P_j, P_k)_N \overset{*}{=} \sum_{j=0}^{k-1} \alpha_j \cdot 0 = 0$$