

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma\left(\frac{1}{2}\right) = \int_0^{\infty} t^{\frac{1}{2}-1} e^{-t} dt = \int_0^{\infty} \frac{e^{-t}}{\sqrt{t}} dt = \left[\begin{array}{l} t = \frac{x^2}{2} \\ dt = x dx \end{array} \right] = \int_0^{\infty} \frac{e^{-\frac{x^2}{2}}}{\frac{x}{\sqrt{2}}} x dx$$

$$= 2\sqrt{\pi} \int_0^{\infty} \underbrace{\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}}_{\text{normalized } N(0,1) \text{ on } [0, \infty)} dx = 2\sqrt{\pi} \cdot \frac{1}{2} = \sqrt{\pi}$$

