Pokażemy, że $\sum_{k=0}^{m} a_k Q_k(x) = B_0$

$$\boxed{\begin{aligned} &Q_0(x) = 1, \quad Q_1(x) = x - c_1 \\ &Q_k(x) = (x - c_k) Q_{k-1}(x) - d_k Q_{k-2}(x) \\ \\ &B_{m+1} = B_{m+2} = 0 \\ &B_k = a_k + (x - c_{k+1}) B_{k+1} - d_{k+2} B_{k+2} \\ &a_k = B_k - (x - c_{k+1}) B_{k+1} + d_{k+2} B_{k+2} \end{aligned}}$$

$$\sum_{k=0}^{m} a_k Q_k(x) = \sum_{k=0}^{m} \left(B_k - (x - c_{k+1}) B_{k+1} + d_{k+2} B_{k+2}\right) Q_k(x) =$$

$$\sum_{k=0}^{m} B_k Q_k(x) - \sum_{k=0}^{m-1} (x - c_{k+1}) B_{k+1} Q_k(x) + \sum_{k=0}^{m-2} d_{k+2} B_{k+2} Q_k(x) =$$

$$B_0 \underbrace{Q_0(x)}_{=1} + B_1 Q_1(x) + \sum_{k=2}^{m} B_k Q_k(x) - (x - c_1) B_1 \underbrace{Q_0(x)}_{=1} - \sum_{k=1}^{m-1} (x - c_{k+1}) B_{k+1} Q_k(x) + \sum_{k=0}^{m-2} d_{k+2} B_{k+2} Q_k(x) =$$

$$B_0 + B_1 \underbrace{\left(Q_1(x) - (x + c_1)\right)}_{=0} + \sum_{k=2}^{m} B_k Q_k(x) - \sum_{k=2}^{m} (x - c_k) B_k Q_{k-1}(x) + \sum_{k=2}^{m} d_k B_k Q_{k-2}(x) =$$

$$B_0 + \sum_{k=2}^{m} \left(B_k Q_k(x) - (x - c_k) B_k Q_{k-1}(x) + d_k B_k Q_{k-2}(x)\right) =$$

$$B_0 + \sum_{k=2}^{m} B_k \Big(\underbrace{Q_k(x) - (x - c_k) Q_{k-1}(x) + d_k Q_{k-2}(x)}_{=0}\Big) = B_0$$

$Q_k(x) = (x - c_k) Q_{k-1}(x) - d_k Q_{k-2}(x)$

$$\begin{cases} a_0 = a_1 = \dots = a_{m-1} \\ a_m = 1 \end{cases} \Rightarrow \sum_{k=0}^{m} a_k Q_k(x) = Q_m(x)$$

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