

Z1

$$S_k(x) = \frac{1}{h_k} \left[\frac{1}{6} M_{k-1} (x_k - x)^3 + \frac{1}{6} M_k (x - x_{k-1})^3 + (y_{k-1} - \frac{1}{6} M_{k-1} h_k^2) (x_k - x) + (y_k - \frac{1}{6} M_k h_k^2) (x - x_{k-1}) \right]$$

$$h_k = x_k - x_{k-1}$$

$$\lambda_k = \frac{h_k}{h_k + h_{k+1}}$$

$$\lambda_k M_{k-1} + 2M_k + (1 - \lambda_k) M_{k+1} = 6 \cdot f[x_{k-1}, x_k, x_{k+1}]$$

a)

x_k	0	2	4
y_k	-8	8	-8

0	-8		
2	8	8	
4	-8	-8	-4 = +[x_0, x_1, x_2]

$$M_0 = M_2 = 0$$

$$\lambda_1 M_0 + 2M_1 + (1 - \lambda_1) M_2 = 6 \cdot (-4)$$

$$0 + 2M_1 + 0 = -24$$

$M_1 = -12$
$h_1 = 2$
$h_2 = 2$

$$S_1(x) = \frac{1}{2} \left[0 - 2x^3 + (-8)(2-x) + (8 + 2 \cdot 2^2)x \right]$$

$$= \frac{1}{2} [-2x^3 - 16 + 8x + (8+8)x] = -x^3 + 12x - 8$$

$$S_2(x) = \frac{1}{2} \left[-2(4-x)^3 + 0 + (8 - \frac{1}{6}(-12) \cdot 2^2)(4-x) + (-8)(x-2) \right]$$

$$= x^3 - 12x^2 + 48x - 64 + \frac{1}{2} [16 \cdot 4 - 16x - 8x + 16]$$

$$= x^3 - 12x^2 + 48x - 64 + 32 - 12x + 8 =$$

$$= x^3 - 12x^2 + 36x - 24$$

$$S(x) = \begin{cases} S_1(x), & x \in [0, 2] \\ S_2(x), & x \in [2, 4] \end{cases}$$

b)

x_k	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1
y_k	4	2	-6	-24

-1	4		
$-\frac{1}{2}$	2	-4	
$\frac{1}{2}$	-6	-8	$-\frac{8}{3} = +[x_0, x_1, x_2]$
1	-24	-36	$-\frac{56}{3} = +[x_0, x_1, x_2, x_3]$

$$M_0 = M_3 = 0$$

$$\begin{cases} 0 + 2M_1 + (1 - \lambda_1) M_2 = -\frac{8}{3} \cdot 6 = -16 \\ \lambda_2 M_1 + 2M_2 + 0 = -\frac{56}{3} \cdot 6 = -112 \\ \lambda_1 = \lambda_2 = \frac{1}{3} \end{cases}$$

$M_1 = 12$
$M_2 = -60$
$h_1 = 1/2$
$h_2 = 1$
$h_3 = 1/2$

x_k	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1
y_k	4	2	-6	-24

$$S_1(x) = 2 \left[0 + 2(x+1)^3 + 4(-\frac{1}{2} - x) + (2 - 2 \cdot (\frac{1}{2})^2)(x+1) \right]$$

$$= 4(x+1)^3 - 4 - 8x + 3(x+1)$$

$$= 4x^3 + 12x^2 + 12x + 4 - 4 - 8x + 3x + 3$$

$$= 4x^3 + 12x^2 + 7x + 3$$

$$S_2(x) = 2 \left(\frac{1}{2} - x \right)^3 - 10 \left(x + \frac{1}{2} \right)^3 + 4 \left(x + \frac{1}{2} \right)$$

$$= 2 \left(\frac{1}{8} - \frac{3}{4}x + \frac{3}{2}x^2 - x^3 \right) - 10 \left(x^3 + \frac{3}{2}x^2 + \frac{3}{4}x + \frac{1}{8} \right) - 4x + 2$$

$$= -2x^3 + 3x^2 - \frac{3}{2}x + \frac{1}{4} - 10x^3 - 15x^2 - \frac{30}{4}x - \frac{10}{8} - 4x + 2$$

$$= -12x^3 - 12x^2 - 5x + 1$$

$$S_3(x) = 2 \left[\frac{1}{6} \cdot (-60)(1-x)^3 + \frac{1}{6} \cdot 0(x - \frac{1}{2})^3 + \left(-6 - \frac{1}{6} \cdot (-60) \cdot \frac{1}{4} \right) (1-x) + \left(-24 - \frac{1}{6} \cdot 0 \cdot \frac{1}{4} \right) (x - \frac{1}{2}) \right]$$

$$= 20x^3 - 60x^2 + 19x - 3$$

$$S(x) = \begin{cases} S_1(x), & x \in [-1, -\frac{1}{2}] \\ S_2(x), & x \in [-\frac{1}{2}, \frac{1}{2}] \\ S_3(x), & x \in [\frac{1}{2}, 1] \end{cases}$$