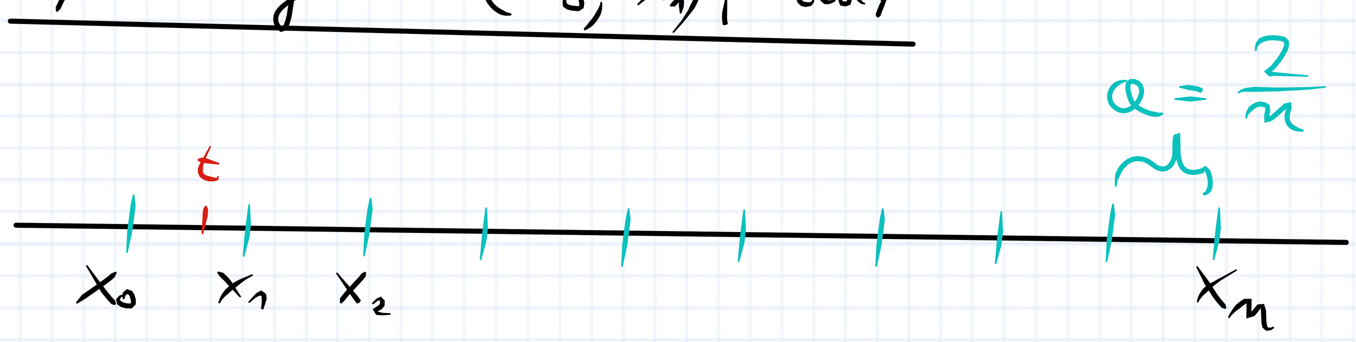
Z6

Wybieramy $t \in (x_0, x_1)$, wtedy:



$$\left|(t-x_0)(t-x_1)\right| \leq \frac{1}{4}a^2$$

$$\left|(t-x_0)(t-x_1)\prod_{j=2}^{n}(t-x_j)\right| \leq \frac{1}{4}a^2 \cdot 2a \cdot 3a \cdot \ldots \cdot na = \frac{1}{4}a^{n+1}n!$$

Sprawdźmy pozostałe przypadki (poza symetrycznym $t \in (x_{n-1}, x_n)$)

$t \in (x_{i-1}, x_i)$

$$\left|\prod_{j=0}^{n}(t-x_j)\right| \leq \frac{1}{4}a^2 \cdot 2a \cdot 3a \cdot \ldots \cdot ia \cdot 2a \cdot 3a \cdot \ldots \cdot (n-i+1)a = \frac{1}{4}a^{n+1}\, i!\,(n-i+1)!$$

$$\frac{1}{4}a^{n+1}n! \rightarrow n! = \boxed{1 \cdot 2 \cdot 3 \cdot \ldots \cdot i} \cdot \boxed{(i+1) \cdot \ldots \cdot n}$$

$$\frac{1}{4}a^{n+1}\, i!\,(n-i+1)! \rightarrow i!\,(n-i+1)! = \boxed{1 \cdot 2 \cdot 3 \cdot \ldots \cdot i} \cdot \boxed{1 \cdot 2 \cdot \ldots \cdot (n-i+1)}$$

$$\frac{1}{4}a^{n+1}n! \geq \frac{1}{4}a^{n+1}\, i!\,(n-i+1)!$$

$$\boxed{\max_{-1 \leq x \leq 1}\left|\prod_{j=0}^{n}(t-x_j)\right| \overset{*}{\leq} \frac{1}{4}a^{n+1}n!}$$

Równoodległe węzły

$$f(x) = e^{\frac{x}{3}}$$

$$f^{(n+1)}(x) = e^{\frac{x}{3}} \cdot \frac{1}{3^n}$$

$$\max_{-1 \leq \alpha \leq 1}\frac{\left|e^{\frac{\alpha}{3}} \cdot \frac{1}{3^n}\right|}{(n+1)!} \cdot \max_{-1 \leq x \leq 1}\left|\prod_{i=0}^{n}(t-x_i)\right| \underset{\alpha=1}{\overset{*}{\leq}} \frac{e^{\frac{1}{3}} \cdot \frac{1}{3^n}}{(n+1)!} \cdot \frac{1}{\cancel{4}} \cdot \frac{2^{n-1}}{n^{n+1}} \cdot n!$$

$$\frac{\sqrt[3]{e} \cdot 2^{n-1}}{(n+1)\, n^{n+1} \cdot 3^n} \leq 10^{-16} \Rightarrow \boxed{n = 11}$$

$(7.14 \cdot 10^{-17})$

Węzły Czebyszewa

$$\frac{\sqrt[3]{e}}{3^{n+1}(n+1)!\, 2^n} \leq 10^{-16} \Rightarrow \boxed{n = 11}$$

$(2.67 \cdot 10^{-18})$