

x_i	-10	-5	0	5	10
$h(x_i)$	3	-5	-1	-5	3

$$P_0(x) = 1, P_1(x) = x, P_2(x) = x^2 - 50$$

$$w_2^*(x) = \sum_{k=0}^2 a_k P_k(x), \quad a_k = \frac{(h, P_k)_4}{(P_k, P_k)_4}$$

$$a_0 = \frac{-5}{5} = -1$$

$$a_1 = \frac{-30 + 25 + 0 - 25 + 30}{250} = 0$$

$$\begin{aligned} a_2 &= \frac{2(3 \cdot 50 + 125) - 50}{\sum_{k=0}^4 (x_k^2 - 50)^2} \\ &= \frac{600}{2 \cdot 50^2 + 2 \cdot (-25)^2 + (-50)^2} \\ &= \frac{600}{3 \cdot 50^2 + 2 \cdot 25^2} = \frac{60}{875} \end{aligned}$$

$$\begin{aligned} P_2(x_0) &= P_2(x_4) = 50 \\ P_2(x_1) &= P_2(x_3) = -25 \\ P_2(x_2) &= -50 \end{aligned}$$

$$w_2^* = 1 + \frac{60}{875} (x^2 - 50)$$