$$\boxed{X \sim B(n,p) \Rightarrow P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}}$$

$$\boxed{X \perp Y \Rightarrow P(X=x, Y=y) = P(X=x) \cdot P(Y=y)}$$

$$X \sim B(n_1,p) \qquad Y \sim B(n_2,p) \qquad X \perp Y$$

$$Z = X + Y$$

$$P(Z=k) = \sum_{j=0}^{k} P(X=j) \cdot P(Y=k-j) = \sum_{j=0}^{k} \binom{n_1}{j} p^j p^{n_1-j} \binom{n_2}{k-j} p^{k-j} (1-p)^{n_2-(k-j)}$$

$$= \sum_{j=0}^{k} \binom{n_1}{j}\binom{n_2}{k-j} p^k p^{n_1+n_2-k} = p^k p^{n_1+n_2-k} \underbrace{\sum_{j=0}^{k} \binom{n_1}{j}\binom{n_2}{k-j}}_{\text{tożsamość Cauchy'ego}}$$

$$= \binom{n_1+n_2}{k} p^k p^{n_1+n_2-k} = B(n_1+n_2, p)$$