$$(x_0, x_1, x_2, x_3, x_4) = (-10, -5, 0, 5, 10)$$

$$(f, g)_4 = \sum_{i=0}^{4} f(x_i)\, g(x_i)$$

Sposób I (Gram - Schmidt)

$$f_i(x) := x^i$$

$$P_k(x) = f_k(x) - \sum_{i=0}^{k-1} \frac{(f_k, P_i)_4}{(P_i, P_i)_4} P_i(x)$$

$$P_0(x) = f_0(x) = 1$$

$$P_1(x) = x - \frac{(f_1, P_0)_4}{(P_0, P_0)_4} P_0(x) = x - \frac{0}{5} = x$$

$$P_2(x) = x^2 - \frac{(f_2, P_0)_4}{(P_0, P_0)_4} P_0(x) - \frac{(f_2, P_1)_4}{(P_1, P_1)_4} P_1(x)$$

$$= x^2 - \frac{(f_2, P_0)_4}{5} - \frac{(f_2, P_1)_4}{250} x$$

$$= x^2 - \frac{250}{5} - \frac{0}{250} x$$

$$= x^2 - 50$$

Sposób II (Gram - Schmidt)

$$\begin{cases} P_0(x) = 1, \quad P_1(x) = x - c_1 \\ P_k(x) = (x - c_k)\, P_{k-1}(x) - d_k P_{k-2}(x) \end{cases}$$

$$c_k = \frac{(\tilde{x} P_{k-1}, P_{k-1})_4}{(P_{k-1}, P_{k-1})_4}, \quad d_k = \frac{(P_{k-1}, P_{k-1})_4}{(P_{k-2}, P_{k-2})_4}$$

$$P_0(x) = 1$$

$$P_1(x) = x - \frac{(\tilde{x} P_0, P_0)_4}{(P_0, P_0)_4} = x + \frac{0}{5} = x$$

$$P_2(x) = \left(x - \frac{(\tilde{x} P_1, P_1)_4}{(P_1, P_1)_4}\right) x - \frac{(P_1, P_1)_4}{(P_0, P_0)_4}$$

$$= x^2 - \frac{0}{250} x - \frac{250}{5}$$

$$= x^2 - 50$$