Chcemy pokazać, że $Z_1 = u^2 + v^2$, gdzie $Z_1 = (Y-\mu)^T \Sigma^{-1} (Y-\mu)$.

$$u^2 + v^2 = \frac{1}{4\cdot 15}\left(-3Y_1 + 2Y_2\right)^2 + \frac{1}{4\cdot 21}\left(3Y_1 + 2Y_2 - 12\right)^2 =$$

$$= \frac{1}{60}\left(9Y_1^2 + 4Y_2^2 - 12Y_1Y_2\right) + \frac{1}{84}\left(9Y_1^2 + 4Y_2^2 + 144 + 12Y_1Y_2 - 72Y_1 - 48Y_2\right)$$

$$= \frac{3}{20}Y_1^2 + \frac{1}{15}Y_2^2 - \frac{1}{5}Y_1Y_2 + \frac{3}{28}Y_1^2 + \frac{1}{21}Y_2^2 + \frac{\cancel{144}^{12}}{\cancel{84}_{7}} + \frac{1}{7}Y_1Y_2 - \frac{6}{7}Y_1 - \frac{4}{7}Y_2$$

$$= \frac{9}{35}Y_1^2 + \frac{7+5}{3\cdot 5\cdot 7}Y_2^2 + \frac{5-7}{5\cdot 7}Y_1Y_2 - \frac{6}{7}Y_1 - \frac{4}{7}Y_2 + \frac{12}{7}$$

$$= \frac{9}{35}Y_1^2 + \frac{\cancel{12}^{4}}{\cancel{105}_{35}}Y_2^2 - \frac{2}{35}Y_1Y_2 - \frac{6}{7}Y_1 - \frac{4}{7}Y_2 + \frac{12}{7}$$

$$\Sigma = \begin{bmatrix} 4 & 1 \\ 1 & 9 \end{bmatrix} \qquad \Sigma^{-1} = \frac{1}{35}\begin{bmatrix} 9 & -1 \\ -1 & 4 \end{bmatrix}$$

$$Z_1 = (Y-\mu)^T \Sigma^{-1} (Y-\mu) = \left(Y_1 - 2,\ Y_2 - 3\right)\frac{1}{35}\begin{bmatrix} 9 & -1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} Y_1 - 2 \\ Y_2 - 3 \end{bmatrix}$$

$$= \frac{1}{35}\left(Y_1 - 2,\ Y_2 - 3\right)\begin{bmatrix} 9(Y_1-2) - (Y_2-3) \\ -(Y_1-2) + 4(Y_2-3) \end{bmatrix}$$

$$= \frac{1}{35}\left[9(Y_1-2)^2 - (Y_1-2)(Y_2-3) - (Y_1-2)(Y_2-3) + 4(Y_2-3)^2\right]$$

$$= \frac{1}{35}\left[9\left(Y_1^2 + 4 - 4Y_1\right) - 2\left(Y_1Y_2 - 3Y_1 - 2Y_2 + 6\right) + 4\left(Y_2^2 + 9 - 6Y_2\right)\right]$$

$$= \frac{1}{35}\left[9Y_1^2 + 36 - 36Y_1 - 2Y_1Y_2 + 6Y_1 + 4Y_2 - 12 + 4Y_2^2 + 36 - 24Y_2\right]$$

$$= \frac{1}{35}\left[9Y_1^2 + 60 - 30Y_1 - 2Y_1Y_2 - 20Y_2 + 4Y_2^2\right]$$

$$= \frac{9}{35}Y_1^2 + \frac{4}{35}Y_2^2 - \frac{2}{35}Y_1Y_2 - \frac{6}{7}Y_1 - \frac{4}{7}Y_2 + \frac{12}{7} = u^2 + v^2$$