$$\Gamma(n) = \int_0^\infty t^{n-1} e^{-t}\, dt \;,\; n \in \mathbb{N}_+$$

Pokażemy, że $\Gamma(n) = (n-1)!$

Indukcja

- Baza $(n=1)$:

$$\Gamma(1) = \int_0^\infty e^{-t}\, dt = \left[-e^{-t}\right]_0^\infty = -\lim_{t\to\infty} \frac{1}{e^t} + e^0 = 0 + 1 = 1 = 0!$$

- Krok $(\Gamma(n) \Rightarrow \Gamma(n+1))$:

$$\int_0^\infty t^n e^{-t}\, dt = -\int_0^\infty t^n (e^{-t})'\, dt = \left[-t^n e^{-t}\right]_0^\infty + n\int_0^\infty t^{n-1} e^{-t}\, dt \overset{\text{zał}}{=} \left[-t^n e^{-t}\right]_0^\infty + n!$$

$$\left[-t^n e^{-t}\right]_0^\infty = \lim_{t\to\infty} -t^n e^{-t} = -\lim_{t\to\infty} \frac{t^n}{e^t} = -\lim_{t\to\infty} \frac{n\, t^{n-1}}{e^t} \overset{\text{dlH}}{=} \dots \overset{\text{dlH}}{=} -\lim_{t\to\infty} \frac{n!}{e^t} = 0$$