

a)

$$\sum_{k=0}^n \binom{n}{k} p^k (1-p)^{n-k} \stackrel{\substack{\text{distributiv} \\ \text{Newton's}}}{=} (p + (1-p))^n = 1$$

b)

$$\begin{aligned} \sum_{k=0}^n k \binom{n}{k} p^k (1-p)^{n-k} &= \sum_{k=0}^n \cancel{k} \frac{n!}{(n-k)! \cancel{k!}} p^k (1-p)^{n-k} = \sum_{k=-1}^{n-1} \left(\frac{n!}{(n-1-k)! k!} p^{k+1} (1-p)^{n-1-k} \right) = np \cdot \sum_{k=-1}^{n-1} \binom{n-1}{k} p^k (1-p)^{(n-1)-k} = np \cdot \boxed{\sum_{k=0}^{n-1} \binom{n-1}{k} p^k (1-p)^{(n-1)-k}} = np \end{aligned}$$

\nearrow
 $\binom{n-1}{k}$

\nwarrow
 $\sum \text{ alle } k=-1 \text{ to zero}$

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