Z2

a) $$A(x) = \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} n^2 x^n \overset{\text{akrobatyka}}{=} x \sum_{n=0}^{\infty} n \cdot n x^{n-1} = x \sum_{n=0}^{\infty} n \cdot (x^n)'$$

$$= x\left(\sum_{n=0}^{\infty} n x^n\right)' = x\left(x \sum_{n=0}^{\infty} n x^{n-1}\right)' = x\left(x\left(\sum_{n=0}^{\infty} x^n\right)'\right)' = x\left(x\left(\frac{1}{1-x}\right)'\right)' =$$

$$= x \cdot \frac{(x)'(1-x)^2 - x\left((1-x)^2\right)'}{\left((1-x)^2\right)^2} = x \cdot \frac{1-x+2x}{(1-x)^3} = \frac{x(x+1)}{(1-x)^3}$$

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b) $$B(x) = \sum_{n=0}^{\infty} b x^n = \sum_{n=0}^{\infty} n^3 x^n = x\left(\sum_{n=0}^{\infty} n^2 x^n\right)' = x\big(\overset{\text{z a)}}{A(x)}\big)'$$

$$= x\left(\frac{x(x+1)}{(1-x)^3}\right)' = \frac{x^3 + 4x^2 + x}{(1-x)^4}$$

c) $$C(x) = \sum_{n=0}^{\infty} \binom{n+k}{k} x^n = \binom{k}{k} + \binom{k+1}{k} x + \binom{k+2}{k} x^2 + \binom{k+3}{k} x^3 + \dots$$

$$= 1 + \frac{(k+1)\,x}{1!} + \frac{(k+1)(k+2)\,x^2}{2!} + \frac{(k+1)(k+2)(k+3)\,x^3}{3!} + \dots$$

$$\frac{1}{(1-x)^2} = \sum_{n=0}^{\infty} (n+1) x^n$$

$$\frac{1}{(1-x)^3} = \sum_{n=0}^{\infty} \frac{(n+1)(n+2)\,x^n}{2!}$$

$$\frac{1}{(1-x)^4} = \sum_{n=0}^{\infty} \frac{(n+1)(n+2)(n+3)\,x^n}{3!}$$

$$\frac{1}{(1-x)^{k+1}} = \sum_{n=0}^{\infty} \frac{(n+1)(n+2)\dots(n+k)}{k!} x^n = \sum_{n=0}^{\infty} \binom{n+k}{k} x^n$$

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