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$$f[x_0] = y_0$$

$$f[x_i, \dots, x_j] = \frac{f[x_{i+1}, \dots, x_j] - f[x_i, \dots, x_{j-1}]}{x_j - x_i}$$

a)

x_k	-3	-1	0	1
y_k	-16	0	-16	32

-3	-16	$= b_0$		
-1	0	8	$= b_1$	
0	-16	-16	-8	$= b_2$
1	32	48	32	10 $= b_3$

$$L(x) = -16 + 8(x+3) - 8(x+3)(x+1) + 10(x+3)(x+1)x$$

$$= \boxed{10x^3 + 32x^2 + 6x - 16}$$

b)

x_k	-3	-1	0	1
y_k	-16	0	-16	-8

-3	-16	$= b_0$		
-1	0	8	$= b_1$	
0	-16	-16	-8	$= b_2$
1	-8	8	12	5 $= b_3$

$$L(x) = -16 + 8(x+3) - 8(x+3)(x+1) + 5(x+3)(x+1)x$$

$$= \boxed{5x^3 + 12x^2 - 9x - 16}$$

c)

x_k	-3	-1	0	1
y_k	-16	0	-16	32

-3	-16	$= b_0$			
-1	0	8	$= b_1$		
0	-16	-16	-8	$= b_2$	
1	32	48	32	10	$= b_3$
3	560	264	72	10	0 $= b_4$

$$L(x) = -16 + 8(x+3) - 8(x+3)(x+1) + 10(x+3)(x+1)x$$

$$= \boxed{10x^3 + 32x^2 + 6x - 16}$$