Pokażmy, że $A_k = A_{n-k}$

$$A_k = \frac{(-1)^{n-k}}{k!(n-k)!}\int_0^n \prod_{\substack{j\neq k\\ j=0}}^{n}(t-j)\,dt = \begin{bmatrix} t = n-s \\ dt = -ds \end{bmatrix} =$$

$$\frac{(-1)^{n-k}}{k!(n-k)!}\int_n^0 \prod_{\substack{j\neq k\\ j=0}}^{n}(n-s-j)(-ds) =$$

$$\frac{(-1)^{n-k}}{k!(n-k)!}\int_0^n (-1)^n \prod_{\substack{j\neq k\\ j=0}}^{n}(s-(n-j))\,ds =$$

$$\frac{(-1)^{2n-k}}{k!(n-k)!}\int_0^n \prod_{\substack{j\neq k\\ j=0}}^{n}(s-(n-j))\,ds = \frac{(-1)^{k}}{k!(n-k)!}\int_0^n \prod_{\substack{j\neq k\\ j=0}}^{n}(s-(n-j)) =$$

$$\boxed{\begin{aligned}(-1)^{2n-k} &= (-1)^{2n}\cdot(-1)^{-k} \\ &= 1\cdot\frac{1}{(-1)^k} = (-1)^k\end{aligned}}$$

$$\boxed{\begin{aligned} p &= n-j \\ j &= n-p\end{aligned}}$$

$$A_k = \frac{(-1)^{k}}{k!(n-k)!}\int_0^n \prod_{\substack{p=0\\ p\neq n-k}}^{n}(s-p)$$

$$A_{n-k} = \frac{(-1)^{k}}{k!(n-k)!}\int_0^n \prod_{\substack{j\neq n-k\\ j=0}}^{n}(t-j)\,dt$$

Created with IDroo.com