$$\text{rząd}\left(Q_n(f)\right) \geq n+1 \implies Q_n(f) = \sum_{k=0}^{n} \underbrace{\left(\int_a^b \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{x-x_i}{x_k-x_i} \, dx \right)}_{A_k} f(x_k)$$

$$\int_{-3}^{2} f(x)\,dx = \begin{bmatrix} t = \frac{2}{5}x + \frac{1}{5} & dx = \frac{5}{2}dt \\ x = \frac{5t-1}{2} & \end{bmatrix} = \frac{5}{2} \int_{-1}^{1} g(t)\,dt$$

$$g(t) = f\left(\tfrac{5t-1}{2}\right)$$

$$Q_2(f) = \frac{5}{2}\left(\sum_{k=0}^{2} \left(\int_{-1}^{1} \prod_{\substack{i=0 \\ i \neq k}}^{2} \frac{t-t_i}{t_k-t_i} \, dt \right) g(t_k) \right)$$

Z wykładu wiemy, że x_0, x_1, x_2 to miejsca zerowe P_3

$P_0(x) = 1$

$P_1(x) = x$

$P_2(x) = \frac{3}{2}x^2 - \frac{1}{2}$

$P_3(x) = \frac{5}{3}x \cdot \left(\frac{3}{2}x^2 - \frac{1}{2}\right) - \frac{2}{3}x = \frac{5}{2}x^3 - \frac{3}{2}x = \frac{1}{2}x\left(5x^2-3\right)$

Miejsca zerowe: $-\sqrt{\frac{3}{5}}, 0, \sqrt{\frac{3}{5}}$

$$Q_2(f) = \frac{5}{2}\left(\sum_{k=0}^{2} \overbrace{\left(\int_{-1}^{1} \prod_{\substack{i=0 \\ i \neq k}}^{2} \frac{t-t_i}{t_k-t_i} \, dt \right)}^{\tilde{A}_k} g(t_k) \right)$$

$$\tilde{A}_0 = \frac{5}{6} \int_{-1}^{1} t^2 - \sqrt{\tfrac{3}{5}}\, t \, dt = \frac{10}{18}$$

$$\tilde{A}_1 = \tilde{A}_2 = -\frac{5}{3} \int_{-1}^{1} t^2 \pm \frac{3}{5} \, dt = \frac{8}{9}$$

$x_0 = \dfrac{-5\sqrt{\frac{3}{5}} - 1}{2}$ $\qquad A_0 = A_2 = \frac{5}{2}\tilde{A}_0 = \frac{50}{36}$

$x_1 = -\frac{1}{2}$ $\qquad A_1 = \frac{5}{\cancel{2}} \cdot \frac{\cancel{8}^{4}}{9} = \frac{20}{9}$

$x_2 = \dfrac{5\sqrt{\frac{3}{5}} - 1}{2}$

$Q_2(f) = A_0 f(x_0) + A_1 f(x_1) + A_2 f(x_2)$