z poprzedniego zadania

$$f_Y(y) \overset{\downarrow}{=} \frac{3}{2}y\left[(2-y)^2-y^2\right] = 2\cdot\frac{3}{\cancel{2}}y\left(\cancel{4}-4y\cancel{+y^2}\cancel{-y^2}\right) = 6y-6y^2\,,\ y\in(0,1)\ \text{wpp } 0$$

$$EY = \int_{-\infty}^{\infty} y\cdot f_Y(y)\,dy = \int_0^1 6y^2-6y^3\,dy = 6\left[\frac{y^3}{3}-\frac{y^4}{4}\right]_0^1 = 6\cdot\frac{1}{12} = \frac{1}{2}$$

Sprawdźmy czy $X \perp Y$:

$$(*)\quad \boxed{X\perp Y \overset{df}{\iff} \forall_{x,y\in\mathbb{R}}\ f(x,y) = f_X(x)\cdot f_Y(y)}$$

$$f(x,y) = 3xy$$

$$f_Y(y) = 6y-6y^2$$

$$f_X(x) = \begin{cases} \frac{3}{2}x^3, & x\in(0,1] \\ \frac{3}{2}x(2-x)^2, & x\in(1,2] \end{cases}$$

Dla $x=1,\ y=1$:

$$f(1,1) = 3$$

$$f_Y(1) = 0$$

$$3 \neq 0\cdot f_X(x)$$

czyli $(*)$ nie jest spełniony

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