$$\begin{cases} T_{2n}(f) = \frac{1}{2}\left(T_n(f) + M_n(f)\right) \\ M_n = h \sum_{i=1}^{n} f\left(a + \frac{1}{2}(2i-1)h\right) \end{cases}$$

$$T_n = h \sum_{i=0}^{n}{}'' (t_i) = h\left(\frac{1}{2} f(a) + f(a+h) + f(a+2h) + \dots + \frac{1}{2} f(a+nh)\right)$$

$$M_n = h \sum_{i=1}^{n} f\left(a + \frac{1}{2}(2i-1)h\right) = h\left(f\left(a + \frac{h}{2}\right) + f\left(a + \frac{3h}{2}\right) + f\left(a + \frac{5h}{2}\right) + \dots + f\left(a + \frac{2n-1}{2}h\right)\right)$$

$$T_n + M_n = h\left(\frac{1}{2} f(a) + f\left(a + \frac{h}{2}\right) + f(a+h) + f\left(a + \frac{3h}{2}\right) + \dots + f\left(a + \frac{2n-1}{2}h\right) + \frac{1}{2} f(a+nh)\right)$$

$$T_n + M_n = h \sum_{i=0}^{2n}{}'' f\left(a + \frac{ih}{2}\right) = \sum_{i=0}^{2n}{}'' f(a + i h_{2n})$$

$$\boxed{h_{2n} = \frac{h}{2}}$$

$$\frac{1}{2}(T_n + M_n) = \frac{h}{2} \sum_{i=0}^{2n}{}'' f(a + i h_{2n}) = h_{2n} \sum_{i=0}^{2n}{}'' f(a + i h_{2n}) = T_{2n}(f)$$

$$\boxed{T_{0,k} = T_{2^k}} = \frac{1}{2} T_{2^{k-1}} + \frac{1}{2} M = \frac{1}{2}\left(\frac{1}{2} T_{2^{k-2}} + \frac{1}{2} M\right) + \frac{1}{2} M = \dots$$

Created with IDroo.com