$\boxed{Z4}$

$$f(\alpha) \overset{*}{=} 0, \quad f'(\alpha) \neq 0$$

$$x_{n+1} = F(x_n) = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$F(\alpha) = \alpha - \frac{f(\alpha)}{f'(\alpha)} \overset{*}{=} \alpha - 0 = \alpha$$

$$F'(x_n) = 1 - \left(\frac{f(x_n)}{f'(x_n)}\right)' = 1 - \frac{\cancel{f'(x_n) f'(x_n)}}{\cancel{f'(x_n)^2}} - \frac{f''(x_n) f(x_n)}{f'(x_n)^2} = - \frac{f''(x_n) f(x_n)}{f'(x_n)^2}$$

$$F'(\alpha) = - \frac{f''(\alpha) f(\alpha)}{f'(\alpha)^2} \overset{*}{=} \frac{f''(\alpha) \cdot 0}{f'(\alpha)^2} = 0$$

$$F''(x_n) = \frac{2 f''(x_n) f'(x_n) - f'''(x_n) f(x_n) - f''(x_n) f'(x_n)}{f'(x_n)^4}$$

$$F''(\alpha) = - \frac{f''(\alpha)}{f'(\alpha)^3} + \frac{f'''(\alpha) \overbrace{f(\alpha)}^{=0}}{f'(\alpha)^4} = - \frac{f''(\alpha)}{f'(\alpha)^3}$$

Przypadki:

1) $f''(\alpha) \neq 0 \Rightarrow F''(\alpha) \neq 0$

2) $f''(\alpha) = 0 \Rightarrow F''(\alpha) = 0$

Korzystając z zadania L5.3 dostajemy, że rząd metody wynosi co najmniej 2.