

# The Ancient Secrets



# Computer Vision

# Logistics:

- Homework 2 out! Due a week from Thursday
  - Little longer than past ones, start early
- Google group
-

Previously  
On



Ancient Secrets  
of Computer Vision

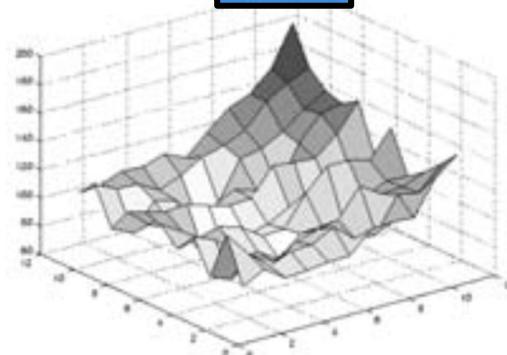
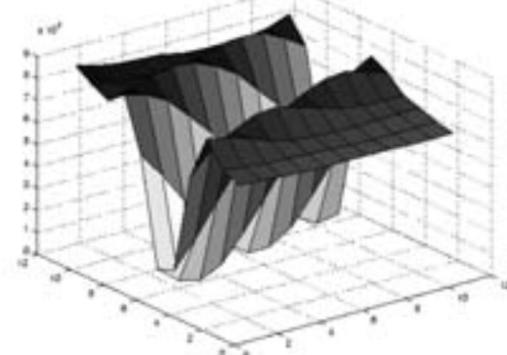
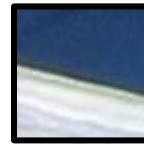
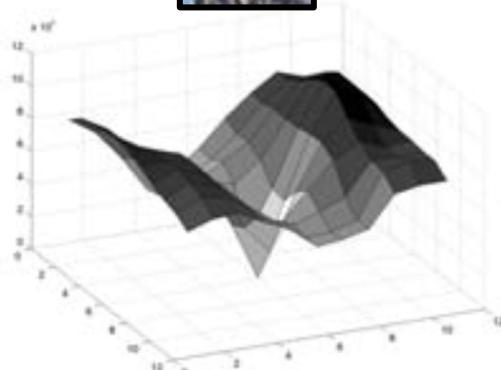
# Self-difference

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Sky: low everywhere

Edge: low along edge

Corner: mostly high



# Self-difference is still expensive

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- $\sum_d \sum_{x,y} (I(x,y) - I(x+d_x, y+d_y))^2$
- Lots of summing
- Need an approximation
  - If you want the mathy details, Szeliski pg 212

# Structure matrix

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- Weighted sum of gradient information
  - $\begin{vmatrix} \sum_i w_i I_x(i) I_x(i) & \sum_i w_i I_x(i) I_y(i) \\ \sum_i w_i I_y(i) I_x(i) & \sum_i w_i I_y(i) I_y(i) \end{vmatrix}$
- Can use Gaussian weighting (so many gaussians)
- Eigen vectors/values of this matrix summarize the distribution of the gradients nearby
- $\lambda_1$  and  $\lambda_2$  are eigenvalues
  - $\lambda_1$  and  $\lambda_2$  both small: no gradient
  - $\lambda_1 \gg \lambda_2$ : gradient in one direction
  - $\lambda_1$  and  $\lambda_2$  similar: multiple gradient directions, corner

# Estimating smallest eigen value

- A few methods:
  - $\det(S) = \lambda_1 * \lambda_2$
  - $\text{trace}(S) = \lambda_1 + \lambda_2$
- $\det(S) - \alpha \text{trace}(S)^2 = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$
- $\det(S) / \text{trace}(S) = \lambda_1 \lambda_2 / (\lambda_1 + \lambda_2)$
- If these estimates are large,  $\lambda_2$  is large

# Harris Corner Detector

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- Calculate derivatives  $I_x$  and  $I_y$
- Calculate 3 measures  $I_x I_x$ ,  $I_y I_y$ ,  $I_x I_y$
- Calculate weighted sums
  - Want a weighted sum of nearby pixels, guess what this is?
  - Gaussian!
- Estimate response based on smallest eigen value
- Non-max suppression (just like canny)

# Matching patches: descriptors!

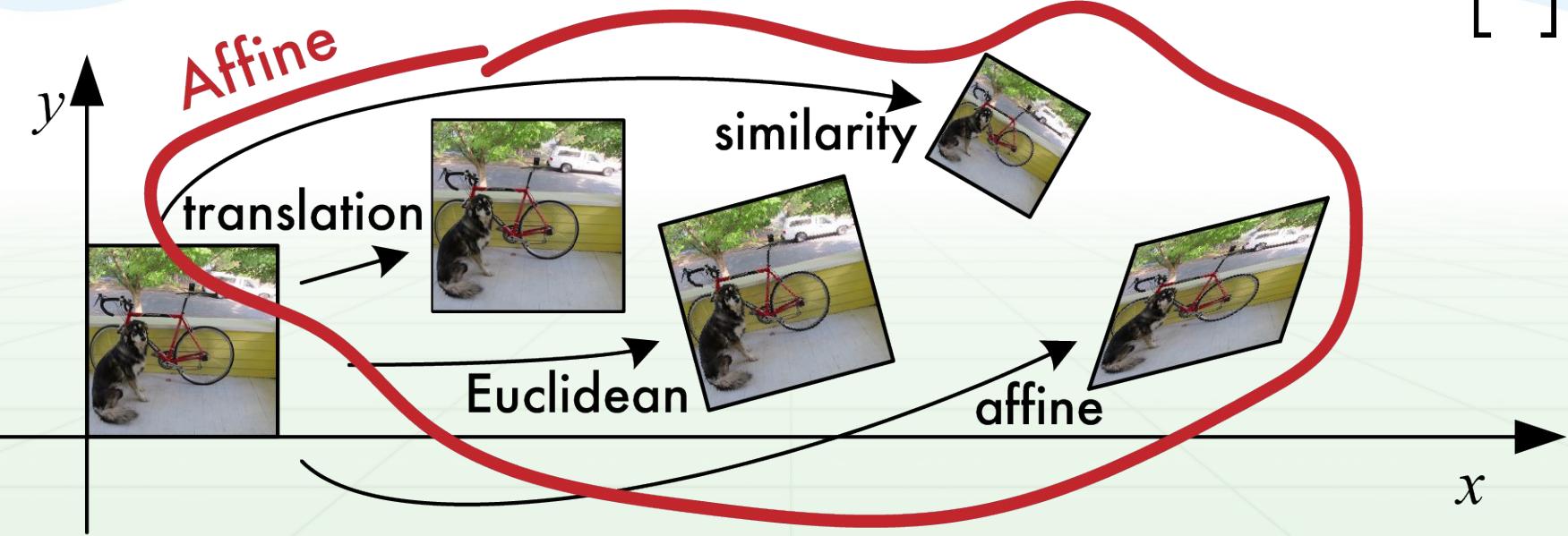
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- We want a way to represent an image patch
- Can be very simple, just pixels!
- Finding matching patch is easy, distance metric:
  - $\sum_{x,y} (I(x,y) - J(x,y))^2$
  - What problems are there with just using pixels?
- Descriptors can be more complex
  - Gradient information
  - How much context?
  - Edges, etc. we'll talk more about descriptors later!

# Affine: scale, rotate, translate, shear

General case of  $2 \times 3$  matrix

$$\mathbf{x}' = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



# Combinations are still affine

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Say you want to translate, then rotate, then translate back, then scale.

$$\mathbf{x}' = \mathbf{S} \ t \ \mathbf{R} \ t \ \bar{\mathbf{x}} = \mathbf{M} \ \bar{\mathbf{x}},$$

If  $\mathbf{M} = (\mathbf{S} \ t \ \mathbf{R} \ t)$

$\mathbf{M}$  is still affine transformation

Wait, but these are all  $2 \times 3$ , how do we multiply them together?

# Added row to transforms

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$$\bar{\mathbf{x}}' = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

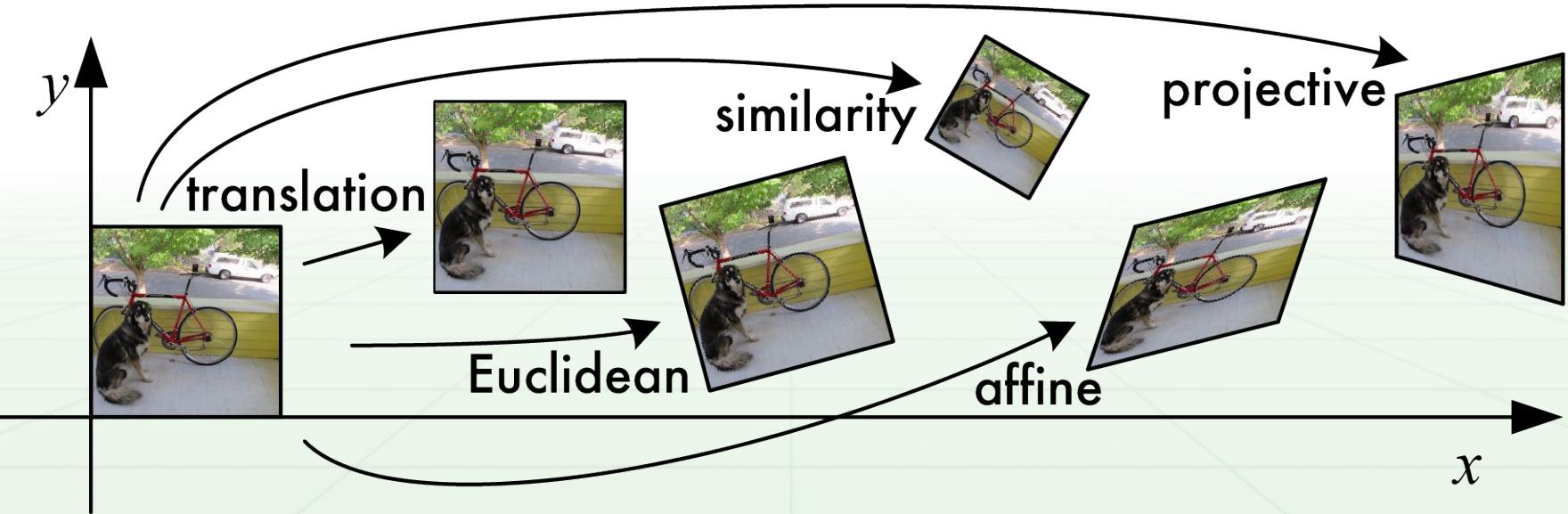
$$\bar{\mathbf{x}}' = \begin{bmatrix} \cos\theta & -\sin\theta & dx \\ \sin\theta & \cos\theta & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\bar{\mathbf{x}}' = \begin{bmatrix} a & -b & dx \\ b & a & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\bar{\mathbf{x}}' = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

# Projective transform

- AKA perspective transform or homography
- Wait but affine was any  $2 \times 3$  matrix...



# Projective transform

- AKA perspective transform or homography
- Wait but affine was any  $2 \times 3$  matrix...
- Homography is general  $3 \times 3$  matrix
- Multiplication by scalar: equivalent projection
  - $3^*H \sim H$

$$\tilde{\mathbf{x}}' = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{w} \end{bmatrix} \quad \tilde{\mathbf{x}}' = \tilde{\mathbf{H}} \tilde{\mathbf{x}}$$

# Using homography to project point

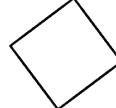
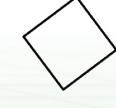
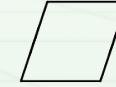
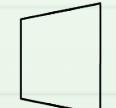
- Multiply  $\tilde{\mathbf{x}}\sim$  by  $\tilde{\mathbf{H}}$  to get  $\tilde{\mathbf{x}}'\sim$
- Convert to  $\tilde{\mathbf{x}}'$  by dividing by  $\tilde{w}'\sim$

$$\tilde{\mathbf{x}}' = \tilde{\mathbf{H}} \tilde{\mathbf{x}}$$

$$\begin{bmatrix} \tilde{x}' \\ \tilde{y}' \\ \tilde{w}' \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{w} \end{bmatrix}$$

$$\bar{\mathbf{x}} = \tilde{\mathbf{x}} / \tilde{w}$$

# Lots to choose from

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix}_{2 \times 3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$	3	lengths	
similarity	$\begin{bmatrix} s\mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$	4	angles	
affine	$\begin{bmatrix} \mathbf{A} \end{bmatrix}_{2 \times 3}$	6	parallelism	
projective	$\begin{bmatrix} \tilde{\mathbf{H}} \end{bmatrix}_{3 \times 3}$	8	straight lines	

# Say we want affine transformation

- Have our matched points
- Want to estimate  $A$  that maps from  $x$  to  $x'$
- $xA = x'$
- How many degrees of freedom?
  - 6
- How many knowns do we get with one match?  $mA = n$ 
  - 2
  - $n_x = a_{00} * m_x + a_{01} * m_y + a_{02} * 1$
  - $n_y = a_{10} * m_x + a_{11} * m_y + a_{12} * 1$

$$\mathbf{x}' = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

# Chapter Seven



**Stitching, RANSAC  
SIFT, and HOG**

# Say we want affine transformation

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  - $n_x = a_{00} * m_x + a_{01} * m_y + a_{02} * 1$
  - $n_y = a_{10} * m_x + a_{11} * m_y + a_{12} * 1$
  - Solve  $\mathbf{M} \mathbf{a} = \mathbf{b}$ 
    - $\mathbf{M}^T \mathbf{M} \mathbf{a} = \mathbf{M}^T \mathbf{b}$
    - $\mathbf{a} = (\mathbf{M}^T \mathbf{M})^{-1} \mathbf{M}^T \mathbf{b}$
  - Still works if overdetermined
    - WHY???

$$\begin{matrix} \mathbf{M} & \mathbf{a} & \mathbf{b} \\ \left[ \begin{array}{cccccc} m_{x1} & m_{y1} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{x1} & m_{y1} & 1 \\ m_{x2} & m_{y2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{x2} & m_{y2} & 1 \\ m_{x3} & m_{y3} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{x3} & m_{y3} & 1 \\ \dots & & & & & \end{array} \right] & \left[ \begin{array}{c} a_{00} \\ a_{01} \\ a_{02} \\ a_{10} \\ a_{11} \\ a_{12} \end{array} \right] & = \left[ \begin{array}{c} n_{x1} \\ n_{y1} \\ n_{x2} \\ n_{y2} \\ n_{x3} \\ n_{y3} \end{array} \right] \end{matrix}$$

# Linear least squares

Want to solve overdetermined linear system:

$$- M a = b$$

Want to minimize squared error:

$$|| b - M a ||^2 =$$

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$$b^T b - a^T M^T b - b^T M a + a^T M^T M a =$$

$$b^T b - 2a^T M^T b + a^T M^T M a$$

# Linear least squares

Want to minimize squared error:  $\| b - Ma \| ^2 =$

$$b^T b - 2a^T M^T b + a^T M^T M a$$

This is convex and minimized when gradient = 0. So we take the derivative wrt a and set = 0.

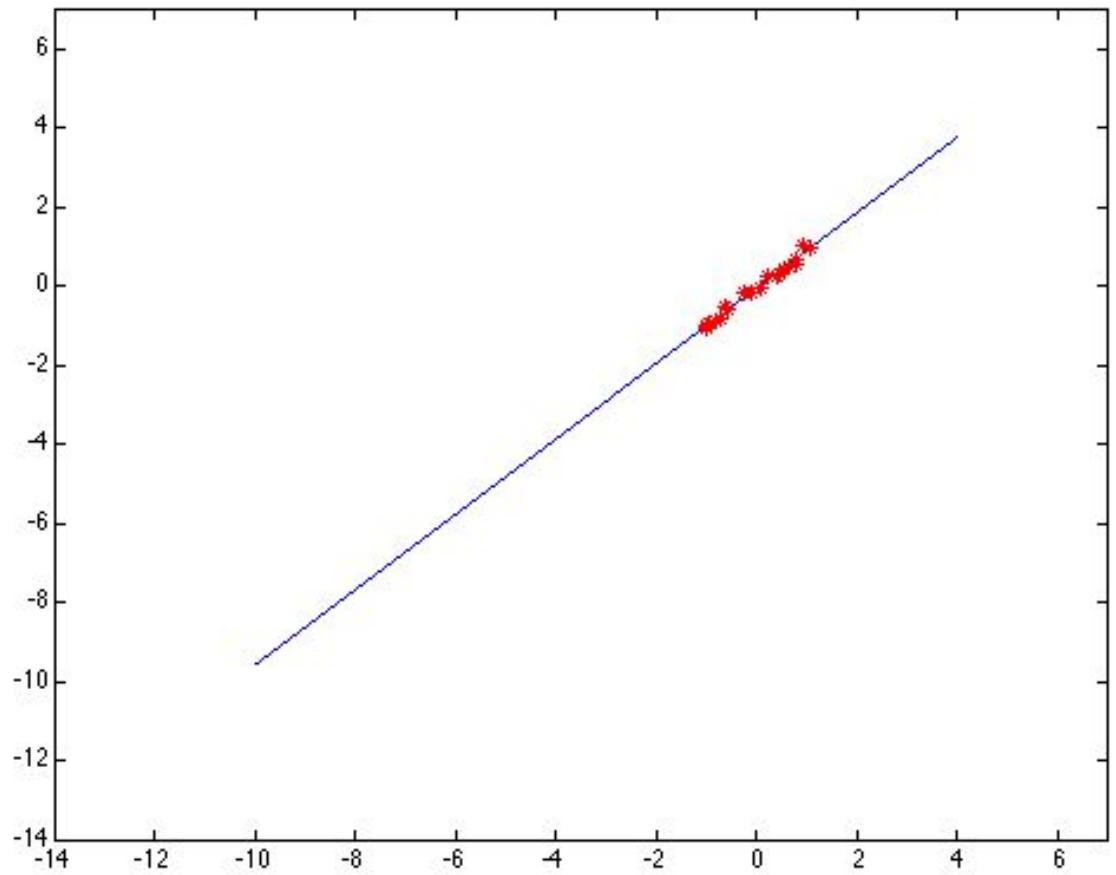
$$-M^T b + (M^T M) a = 0$$

$$(M^T M) a = M^T b$$

$$a = (M^T M)^{-1} M^T b \quad \text{yay!!!!}$$

# So how does least squares do?

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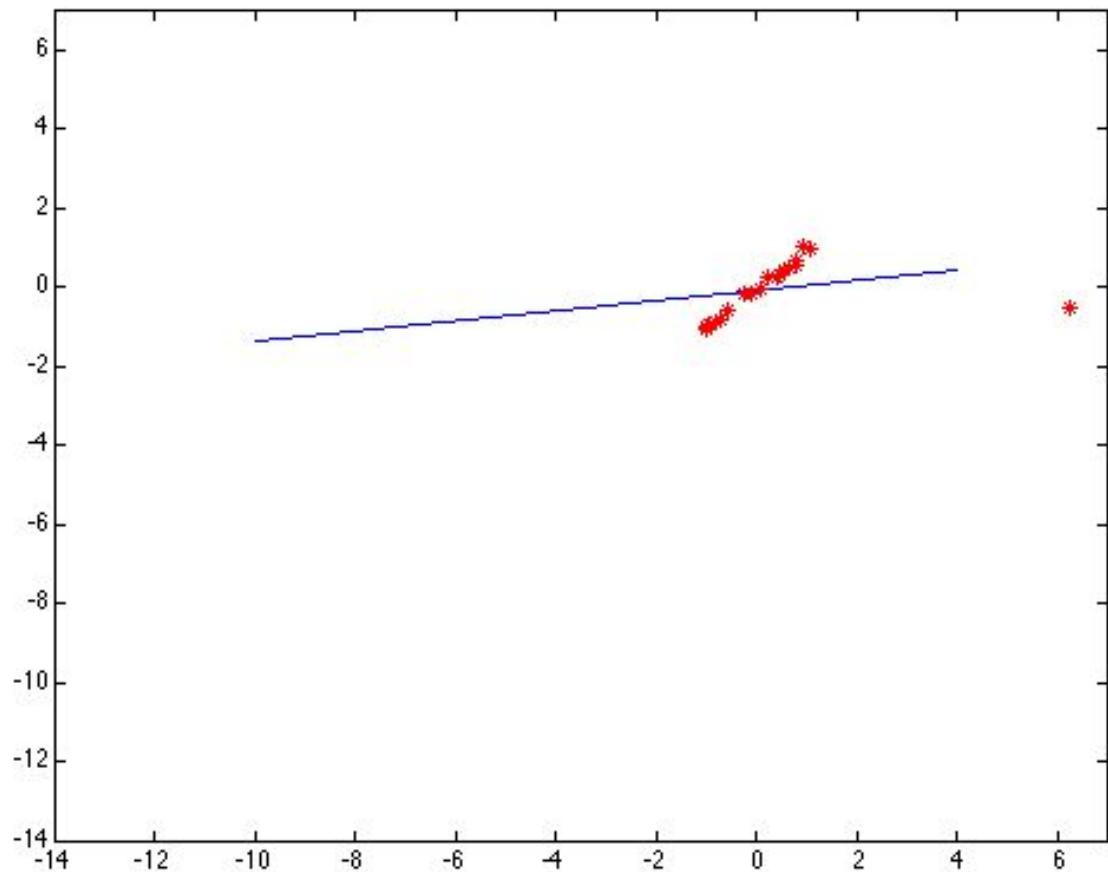


# So how does least squares do?

Error based on **squared** residual

Very scared of being wrong, even for just one point

Very bad at handling outliers in data

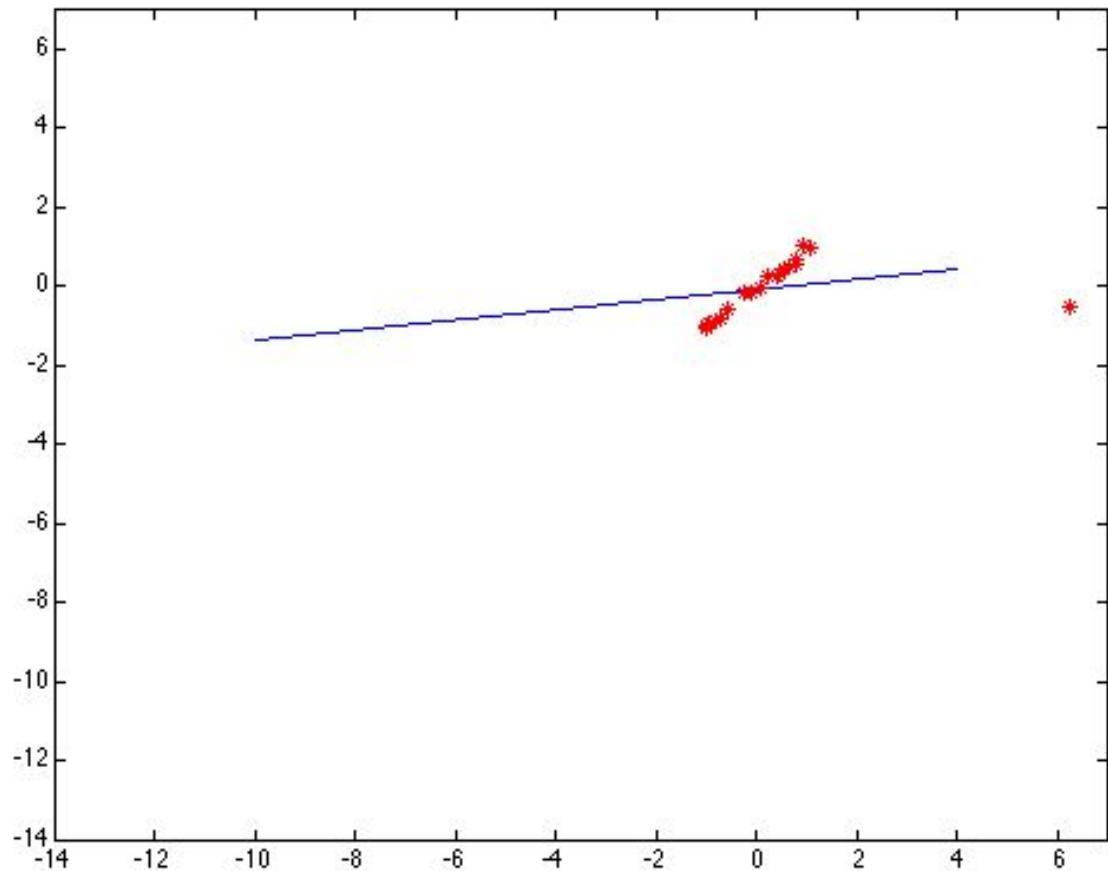


# So how does least squares do?

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Very scared of being wrong, even for just one point

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Not a problem for us, our data is perfect...



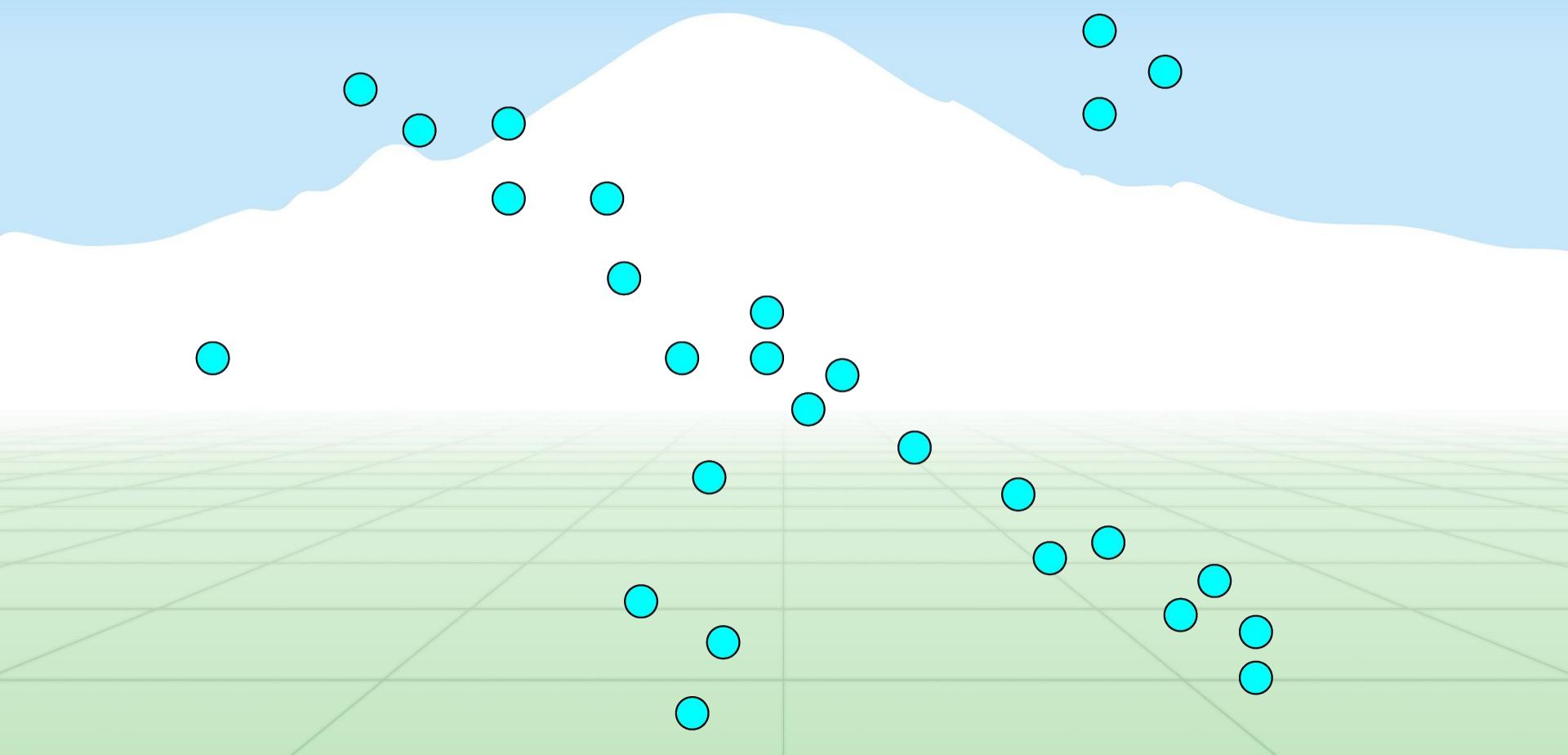
Oh wait...

## RANSAC: RANdom SAmple Consensus

- How can we fit model to inliers but ignore outliers?
- Try a bunch of models, see which ones are best!
- Inliers will all agree on a model
- Outliers are basically random, will not agree

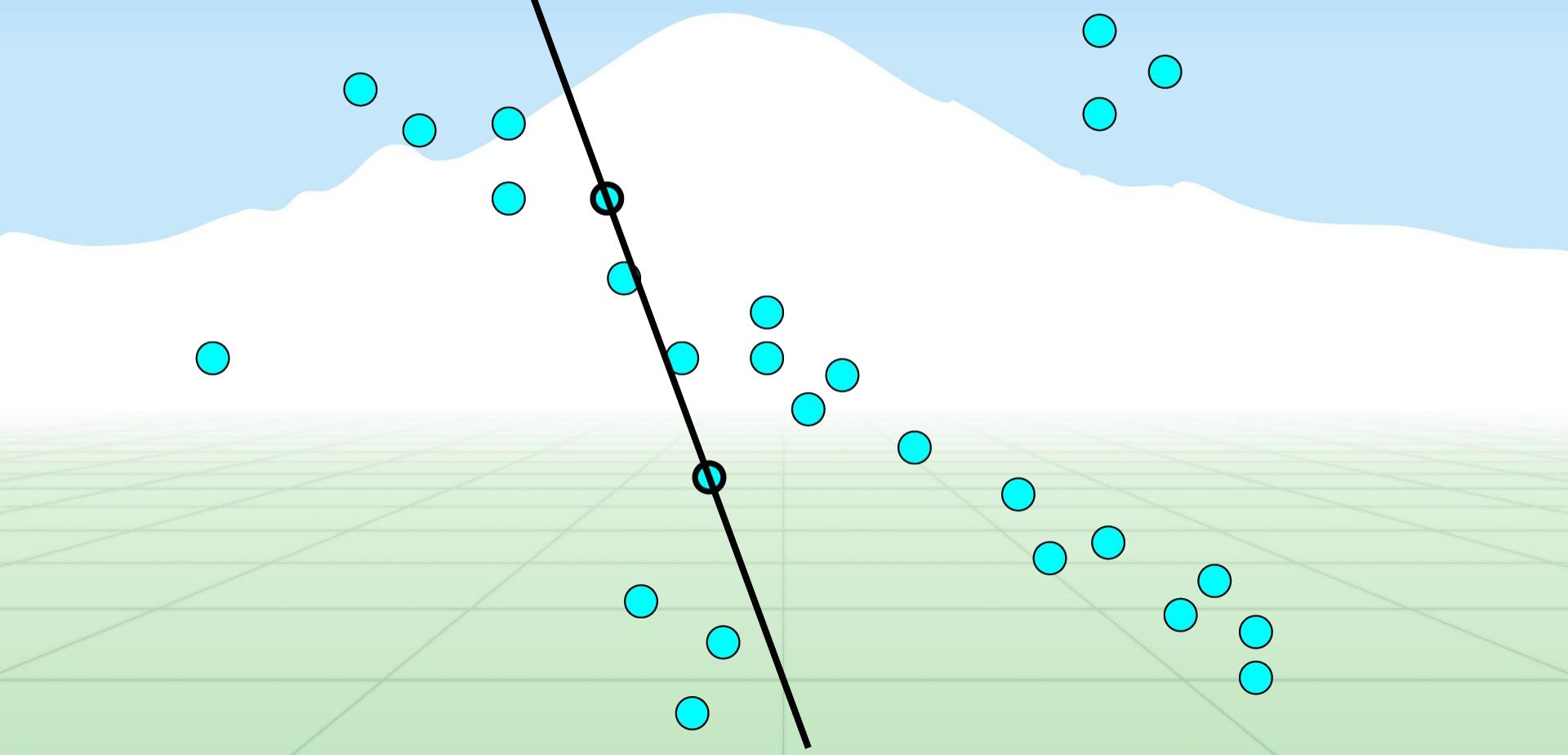
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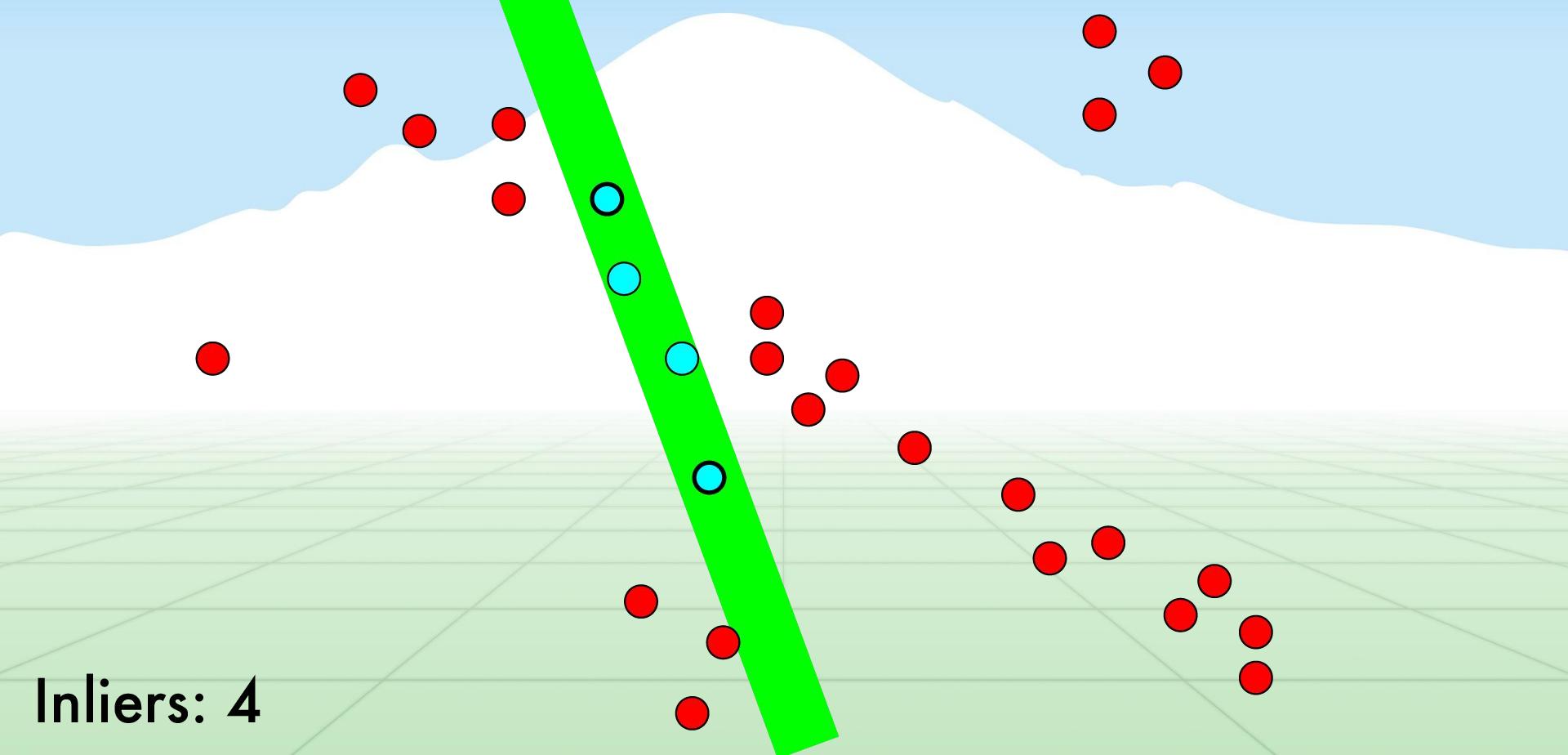
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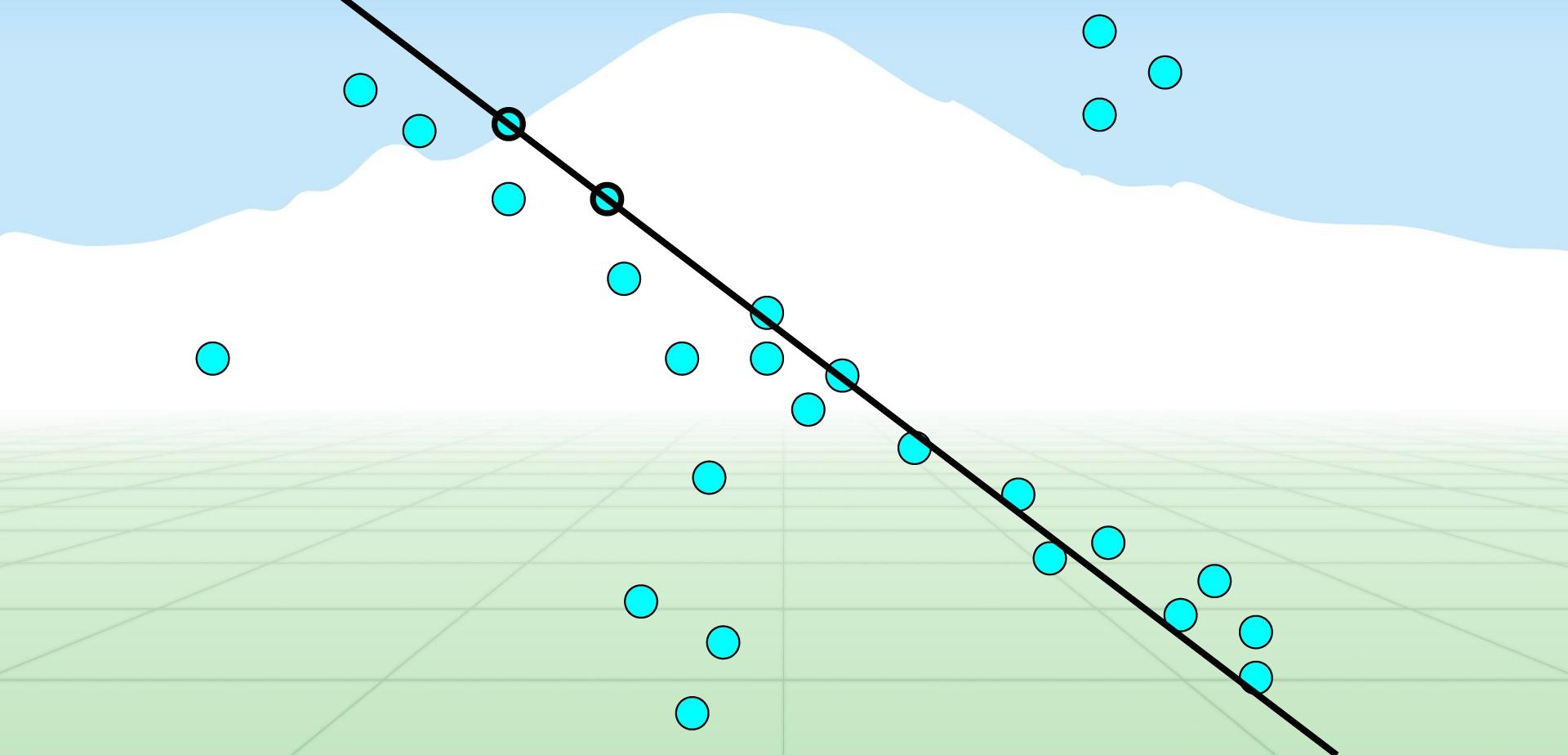
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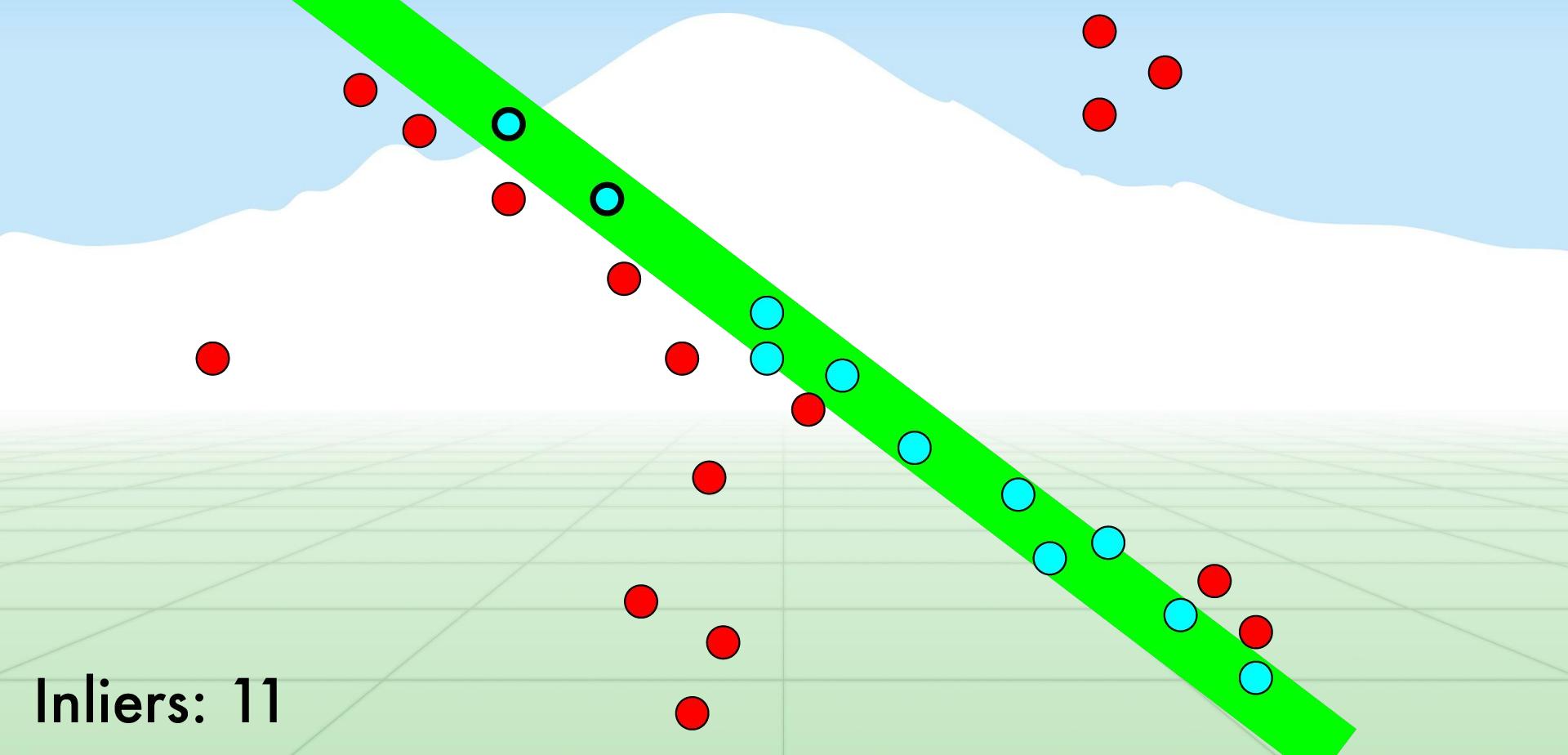
# RANSAC: RANdom SAmple Consensus

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# RANSAC: Random SAmple Consensus

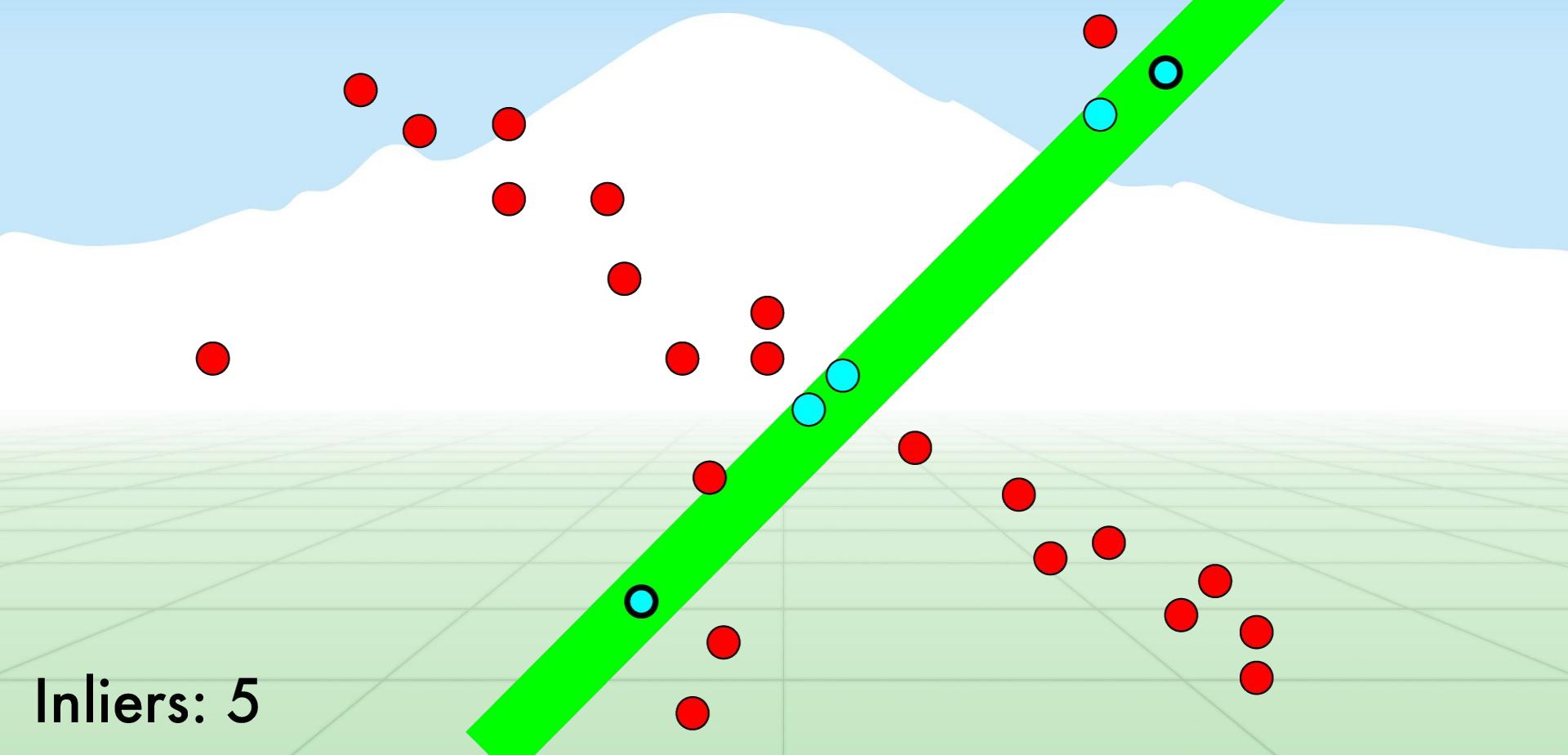
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Inliers: 11

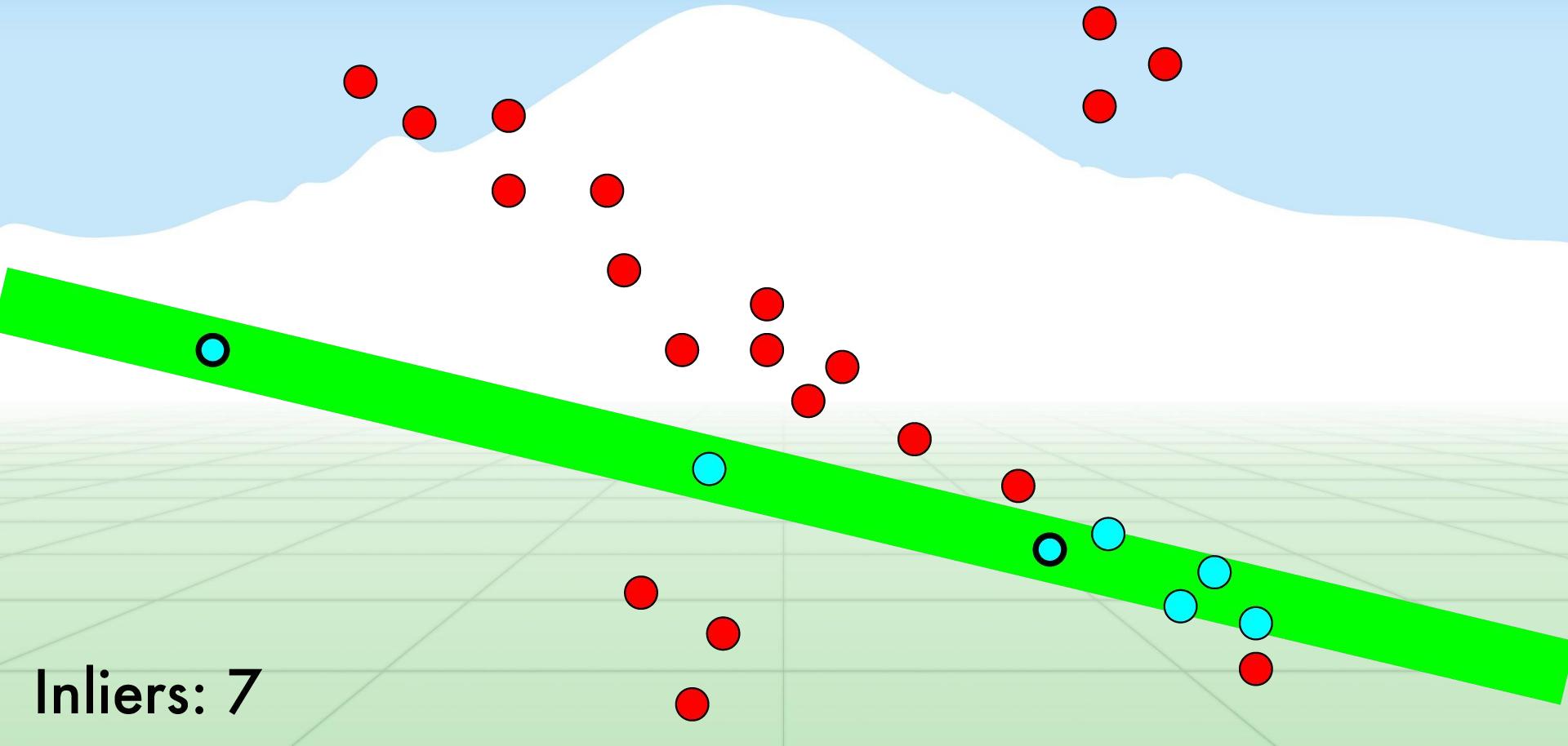
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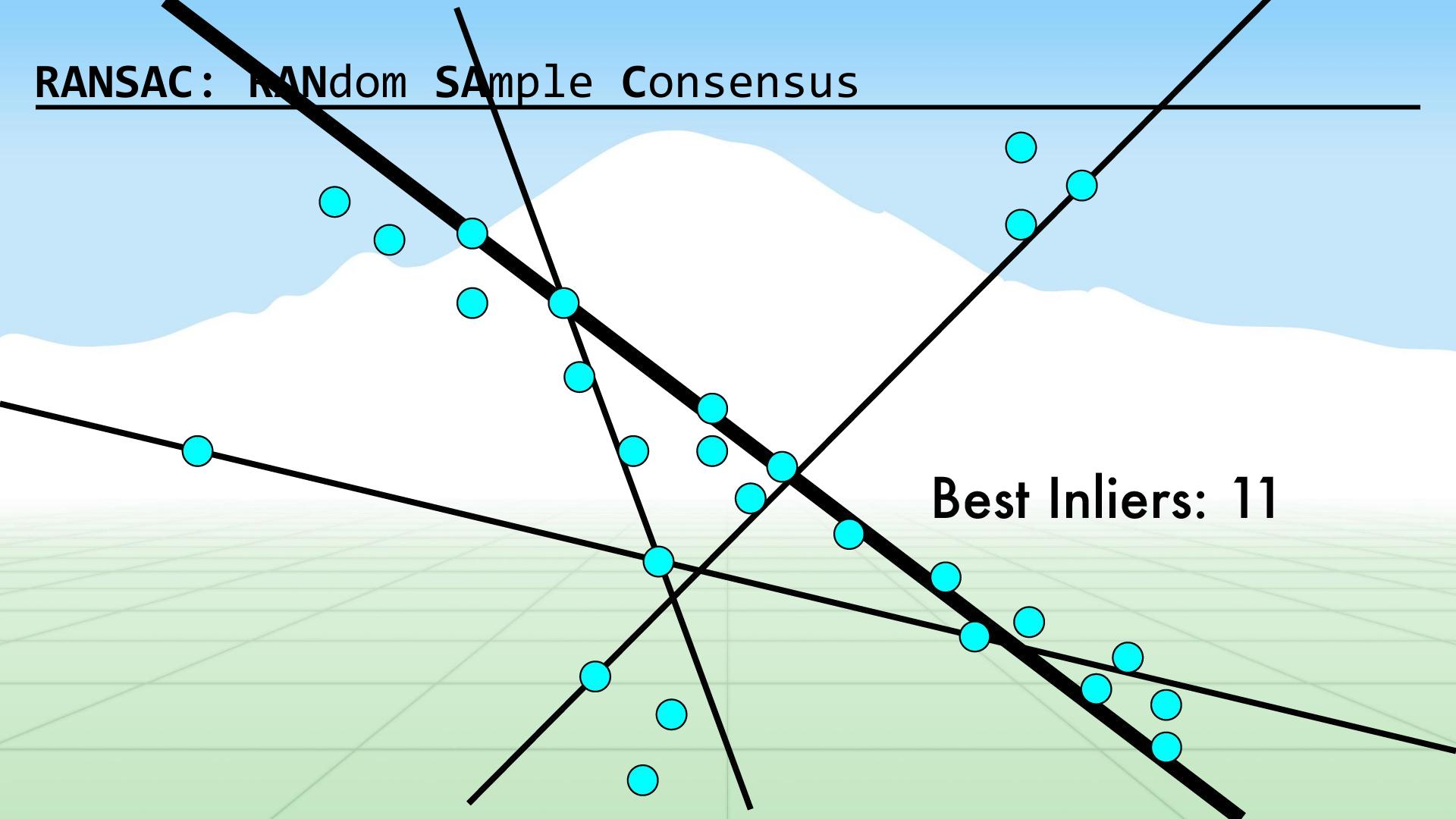


# RANSAC: RANdom SAMple Consensus

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# RANSAC: RANdom SAmple Consensus



Best Inliers: 11

# RANSAC: RANDom SAMple Consensus

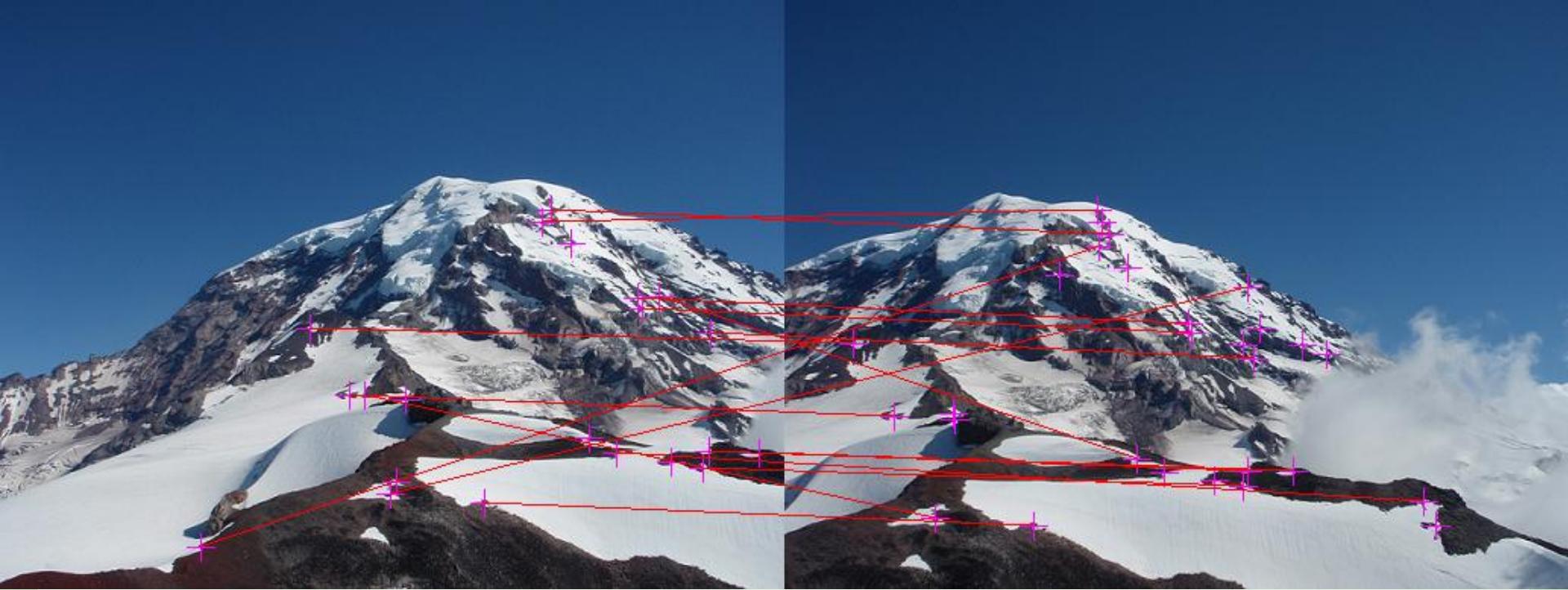
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- Parameters: data, model, n points to fit model, k iterations, t threshold, d “good” fit cutoff

```
bestmodel = None
bestfit = INF
While i < k:
    sample = draw n random points from data
    Fit model to sample
    inliers = data within t of model
    if inliers > bestfit:
        Fit model to all inliers
        bestfit = fit
        bestmodel = model
    if inliers > d:
        return model
return bestmodel
```

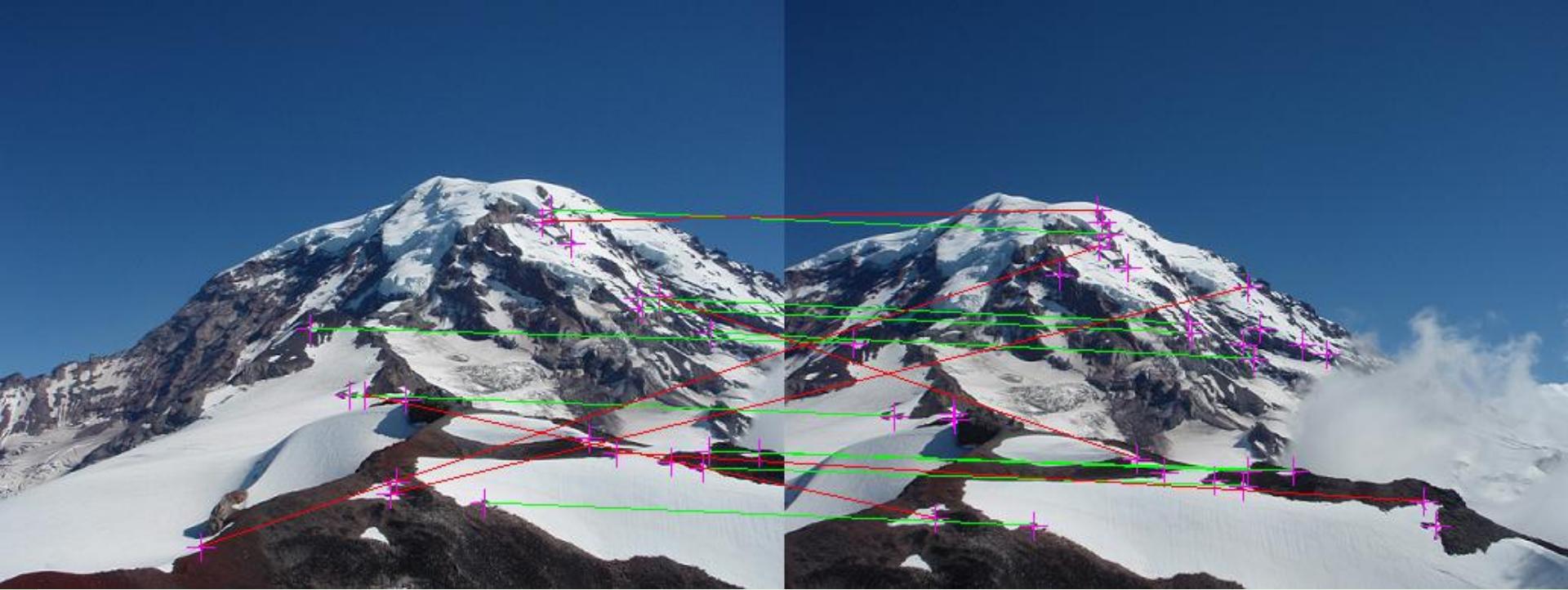
## RANSAC: RANdom SAmple Consensus

- Works well even with extreme noise.



## RANSAC: RANdom SAMple Consensus

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## RANSAC: RANDom SAmple Consensus

- Parameters: data, model, n points to fit model, k iterations, t threshold, d “good” fit cutoff
- Lots of tunable parameters
- Want high probability of recovering “right” model
- t: often quite small, assume “good” inliers
- n: should be just enough to fit model, no extra
- k: can be very high
- d: should be  $\gg n$

# We can estimate affine...

- How many knowns do we get with one match?  $m\mathbf{A} = \mathbf{n}$ 
  - $n_x = a_{00} * m_x + a_{01} * m_y + a_{02} * 1$
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# We want projective (homography)

- What are our equations now?

- $n_x = (h_{00} * m_x + h_{01} * m_y + h_{02} * m_w) / (h_{20} * m_x + h_{21} * m_y + h_{22} * m_w)$
- $n_y = (h_{10} * m_x + h_{11} * m_y + h_{12} * m_w) / (h_{20} * m_x + h_{21} * m_y + h_{22} * m_w)$

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- Assume  $h_{22}$  and  $m_w$  are 1, now 8 DOF

- $n_x = (h_{00} * m_x + h_{01} * m_y + h_{02}) / (h_{20} * m_x + h_{21} * m_y + 1)$
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- More algebra on  $n_x$

- $n_x * (h_{20} * m_x + h_{21} * m_y + 1) = (h_{00} * m_x + h_{01} * m_y + h_{02})$
- $n_x * h_{20} * m_x + n_x * h_{21} * m_y + n_x = h_{00} * m_x + h_{01} * m_y + h_{02}$
- $n_x = h_{00} * m_x + h_{01} * m_y + h_{02} - n_x * h_{20} * m_x - n_x * h_{21} * m_y$

- Similar for  $n_y$

# We want projective (homography)

- What are our equations now?

$$\begin{aligned} - n_x &= h_{00} * m_x + h_{01} * m_y + h_{02} - n_x * h_{20} * m_x - n_x * h_{21} * m_y \\ - n_y &= h_{10} * m_x + h_{11} * m_y + h_{12} - n_x * h_{20} * m_x - n_x * h_{21} * m_y \end{aligned}$$

- New matrix equations:

$$\mathbf{M} \begin{bmatrix} m_{x1} & m_{y1} & 1 & 0 & 0 & 0 & -m_{x1}n_{x1} & -m_{y1}n_{x1} \\ 0 & 0 & 0 & m_{x1} & m_{y1} & 1 & -m_{x1}n_{y1} & -m_{y1}n_{y1} \\ m_{x2} & m_{y2} & 1 & 0 & 0 & 0 & -m_{x2}n_{x2} & -m_{y2}n_{x2} \\ 0 & 0 & 0 & m_{x2} & m_{y2} & 1 & -m_{x2}n_{y2} & -m_{y2}n_{y2} \\ m_{x3} & m_{y3} & 1 & 0 & 0 & 0 & -m_{x3}n_{x3} & -m_{y3}n_{x3} \\ 0 & 0 & 0 & m_{x3} & m_{y3} & 1 & -m_{x3}n_{y3} & -m_{y3}n_{y3} \\ m_{x4} & m_{y4} & 1 & 0 & 0 & 0 & -m_{x4}n_{x4} & -m_{y4}n_{x4} \\ 0 & 0 & 0 & m_{x4} & m_{y4} & 1 & -m_{x4}n_{y4} & -m_{y4}n_{y4} \end{bmatrix} \begin{bmatrix} \mathbf{h}_{00} \\ \mathbf{h}_{01} \\ \mathbf{h}_{02} \\ \mathbf{h}_{10} \\ \mathbf{h}_{11} \\ \mathbf{h}_{12} \\ \mathbf{h}_{20} \\ \mathbf{h}_{21} \end{bmatrix} = \begin{bmatrix} n_{x1} \\ n_{y1} \\ n_{x2} \\ n_{y2} \\ n_{x3} \\ n_{y3} \\ n_{x4} \\ n_{y4} \end{bmatrix}$$

# We want projective (homography)

- New matrix equations:

$$\begin{matrix} \mathbf{M} & & \mathbf{a} & & \mathbf{b} \\ \left[ \begin{array}{ccccccccc} m_{x1} & m_{y1} & 1 & 0 & 0 & 0 & -m_{x1}n_{x1} & -m_{y1}n_{x1} \\ 0 & 0 & 0 & m_{x1} & m_{y1} & 1 & -m_{x1}n_{y1} & -m_{y1}n_{y1} \\ m_{x2} & m_{y2} & 1 & 0 & 0 & 0 & -m_{x2}n_{x2} & -m_{y2}n_{x2} \\ 0 & 0 & 0 & m_{x2} & m_{y2} & 1 & -m_{x2}n_{y2} & -m_{y2}n_{y2} \\ m_{x3} & m_{y3} & 1 & 0 & 0 & 0 & -m_{x3}n_{x3} & -m_{y3}n_{x3} \\ 0 & 0 & 0 & m_{x3} & m_{y3} & 1 & -m_{x3}n_{y3} & -m_{y3}n_{y3} \\ m_{x4} & m_{y4} & 1 & 0 & 0 & 0 & -m_{x4}n_{x4} & -m_{y4}n_{x4} \\ 0 & 0 & 0 & m_{x4} & m_{y4} & 1 & -m_{x4}n_{y4} & -m_{y4}n_{y4} \end{array} \right] & = & \left[ \begin{array}{c} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \end{array} \right] & = & \left[ \begin{array}{c} n_{x1} \\ n_{y1} \\ n_{x2} \\ n_{y2} \\ n_{x3} \\ n_{y3} \\ n_{x4} \\ n_{y4} \end{array} \right] \end{matrix}$$

- Same procedure, Solve  $\mathbf{M} \mathbf{a} = \mathbf{b}$ 
  - Exact if #rows of  $\mathbf{M} = 8$
  - Least squares if #rows of  $\mathbf{M} > 8$

# Are there any problems with this??

- New matrix equations:

$$\mathbf{M} \quad \mathbf{a} \quad \mathbf{b}$$
$$\begin{bmatrix} m_{x1} & m_{y1} & 1 & 0 & 0 & 0 & -m_{x1}n_{x1} & -m_{y1}n_{x1} \\ 0 & 0 & 0 & m_{x1} & m_{y1} & 1 & -m_{x1}n_{y1} & -m_{y1}n_{y1} \\ m_{x2} & m_{y2} & 1 & 0 & 0 & 0 & -m_{x2}n_{x2} & -m_{y2}n_{x2} \\ 0 & 0 & 0 & m_{x2} & m_{y2} & 1 & -m_{x2}n_{y2} & -m_{y2}n_{y2} \\ m_{x3} & m_{y3} & 1 & 0 & 0 & 0 & -m_{x3}n_{x3} & -m_{y3}n_{x3} \\ 0 & 0 & 0 & m_{x3} & m_{y3} & 1 & -m_{x3}n_{y3} & -m_{y3}n_{y3} \\ m_{x4} & m_{y4} & 1 & 0 & 0 & 0 & -m_{x4}n_{x4} & -m_{y4}n_{x4} \\ 0 & 0 & 0 & m_{x4} & m_{y4} & 1 & -m_{x4}n_{y4} & -m_{y4}n_{y4} \end{bmatrix} = \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \end{bmatrix} = \begin{bmatrix} n_{x1} \\ n_{y1} \\ n_{x2} \\ n_{y2} \\ n_{x3} \\ n_{y3} \\ n_{x4} \\ n_{y4} \end{bmatrix}$$

- Same procedure, Solve  $\mathbf{M} \mathbf{a} = \mathbf{b}$ 
  - Exact if #rows of  $\mathbf{M} = 8$
  - Least squares if #rows of  $\mathbf{M} > 8$

# Panorama algorithm:

Find corners in both images

Calculate descriptors

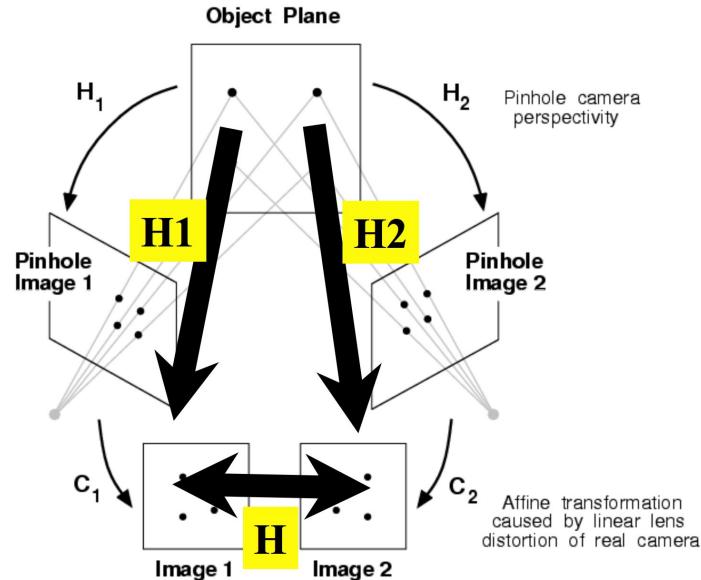
Match descriptors

RANSAC to find homography

Stitch together images with homography

# Stitching panoramas:

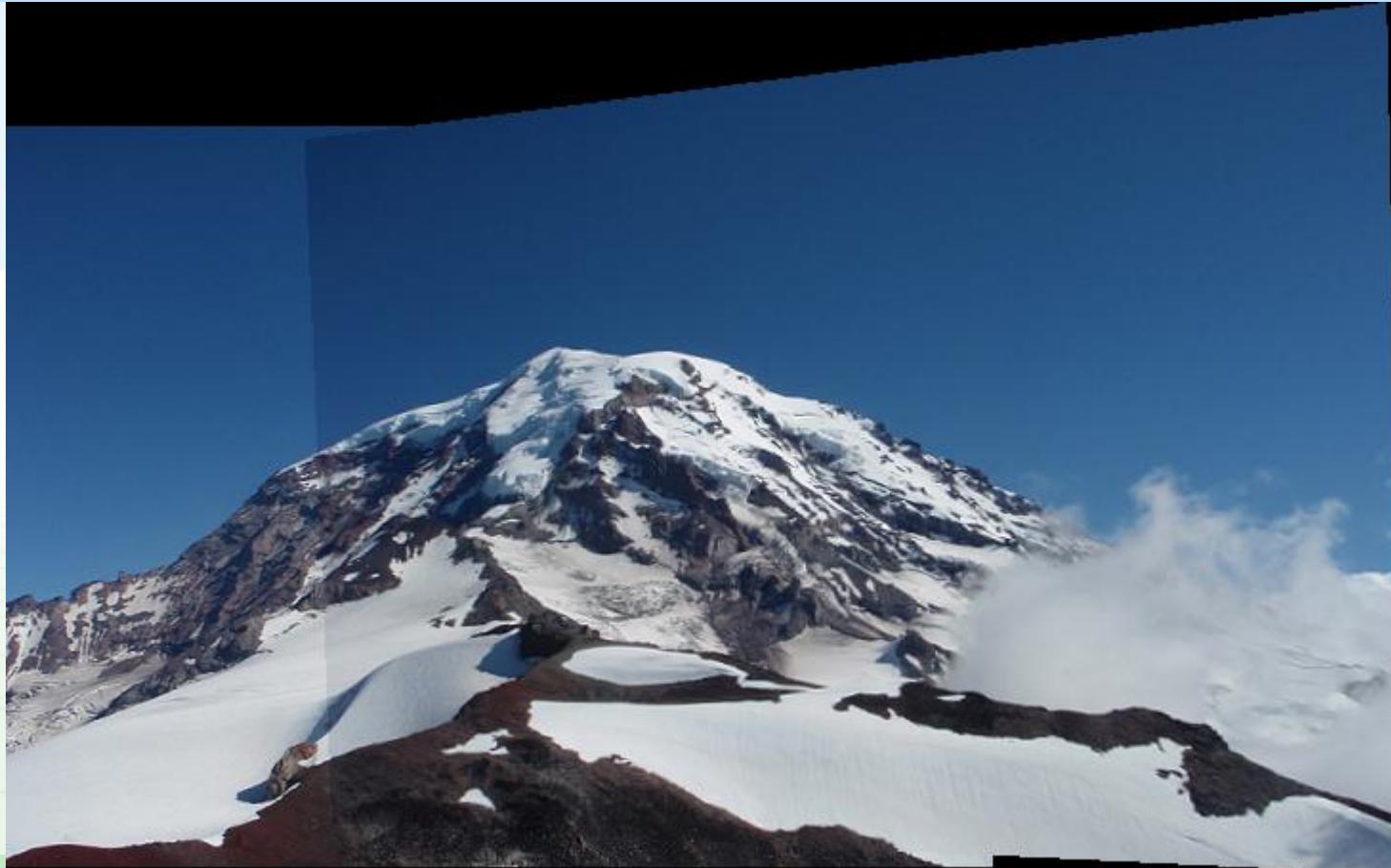
- We know homography is right choice under certain assumption:
  - Assume we are taking multiple images of planar object



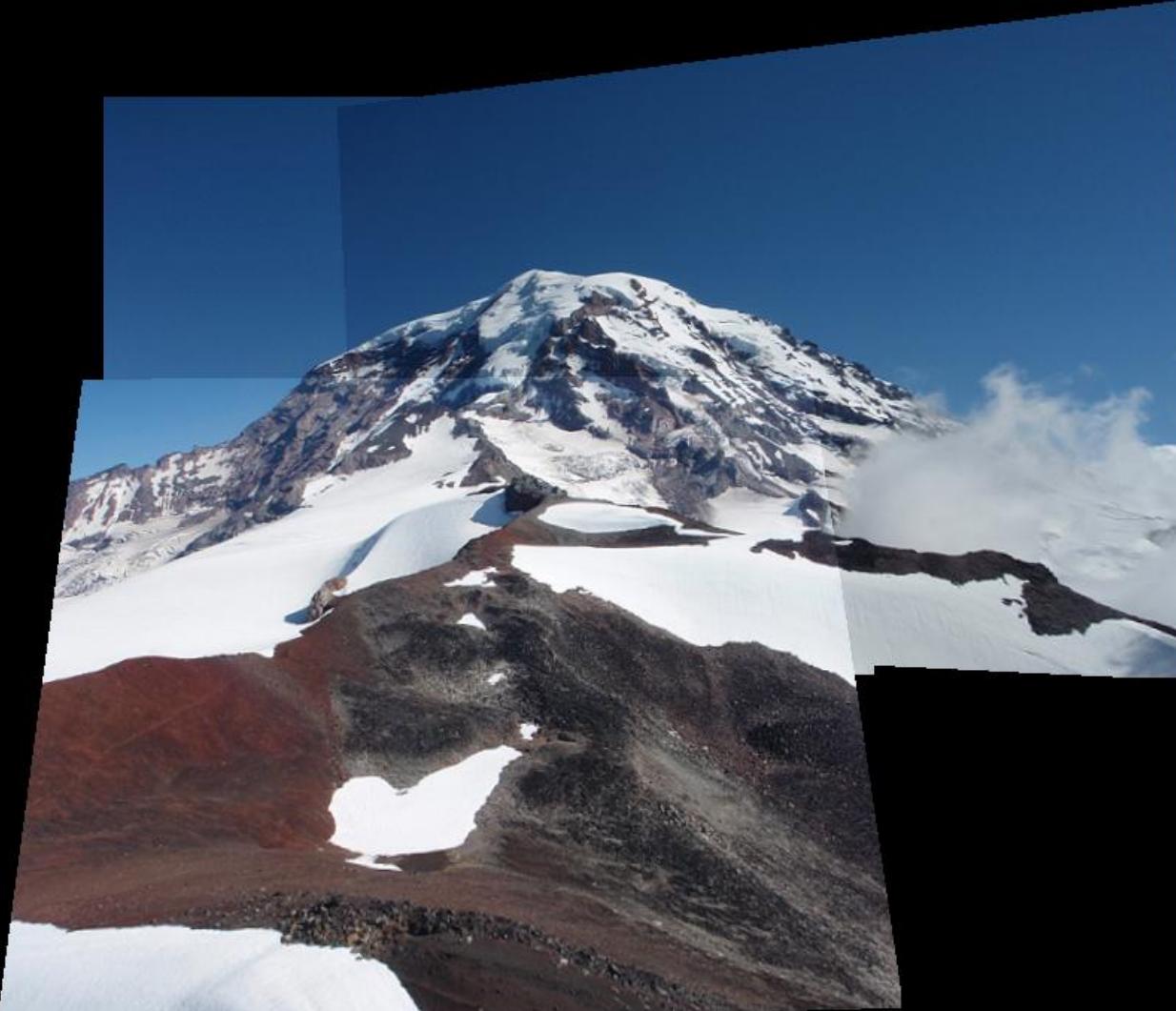
# In practice:



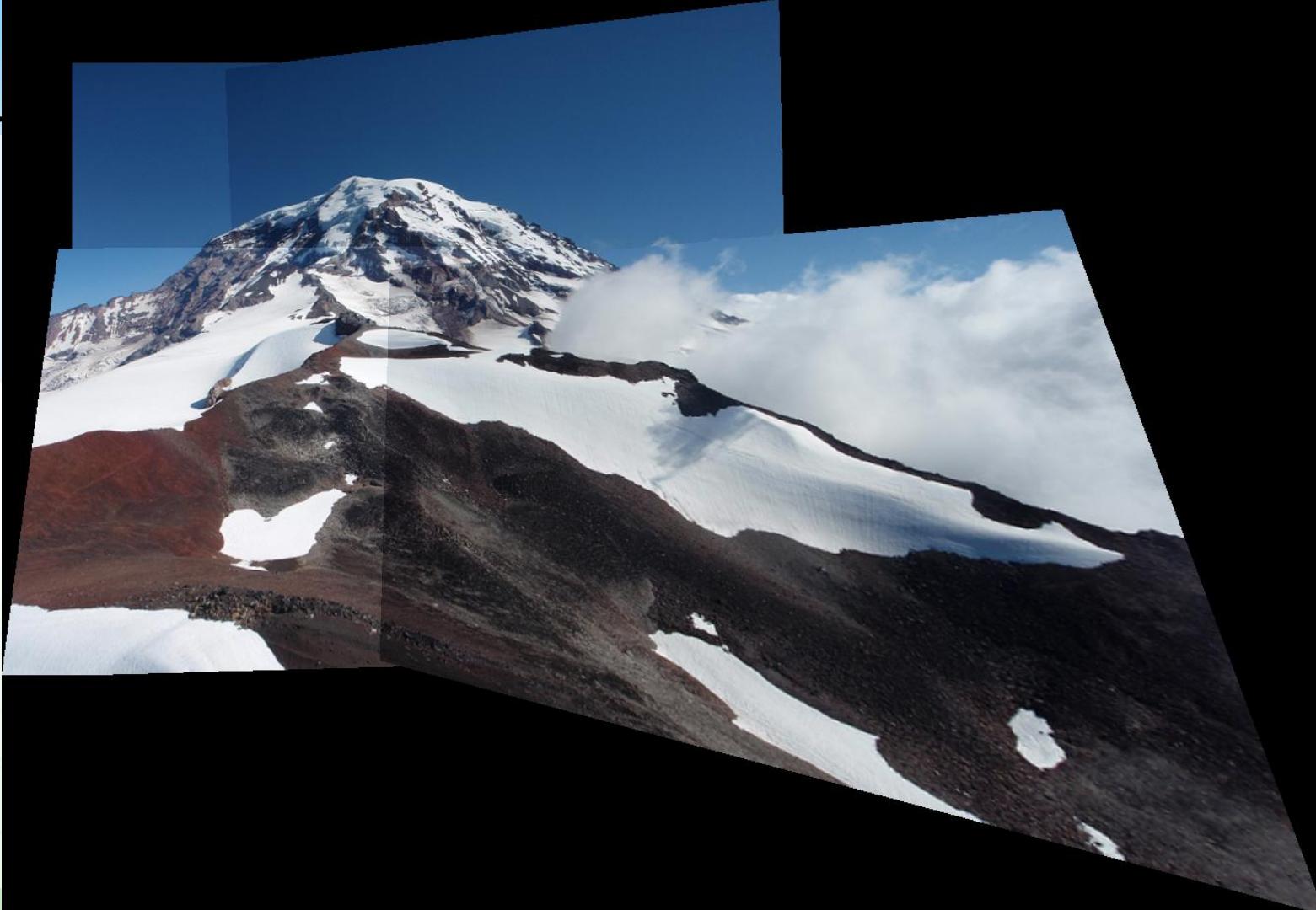
# In practice:

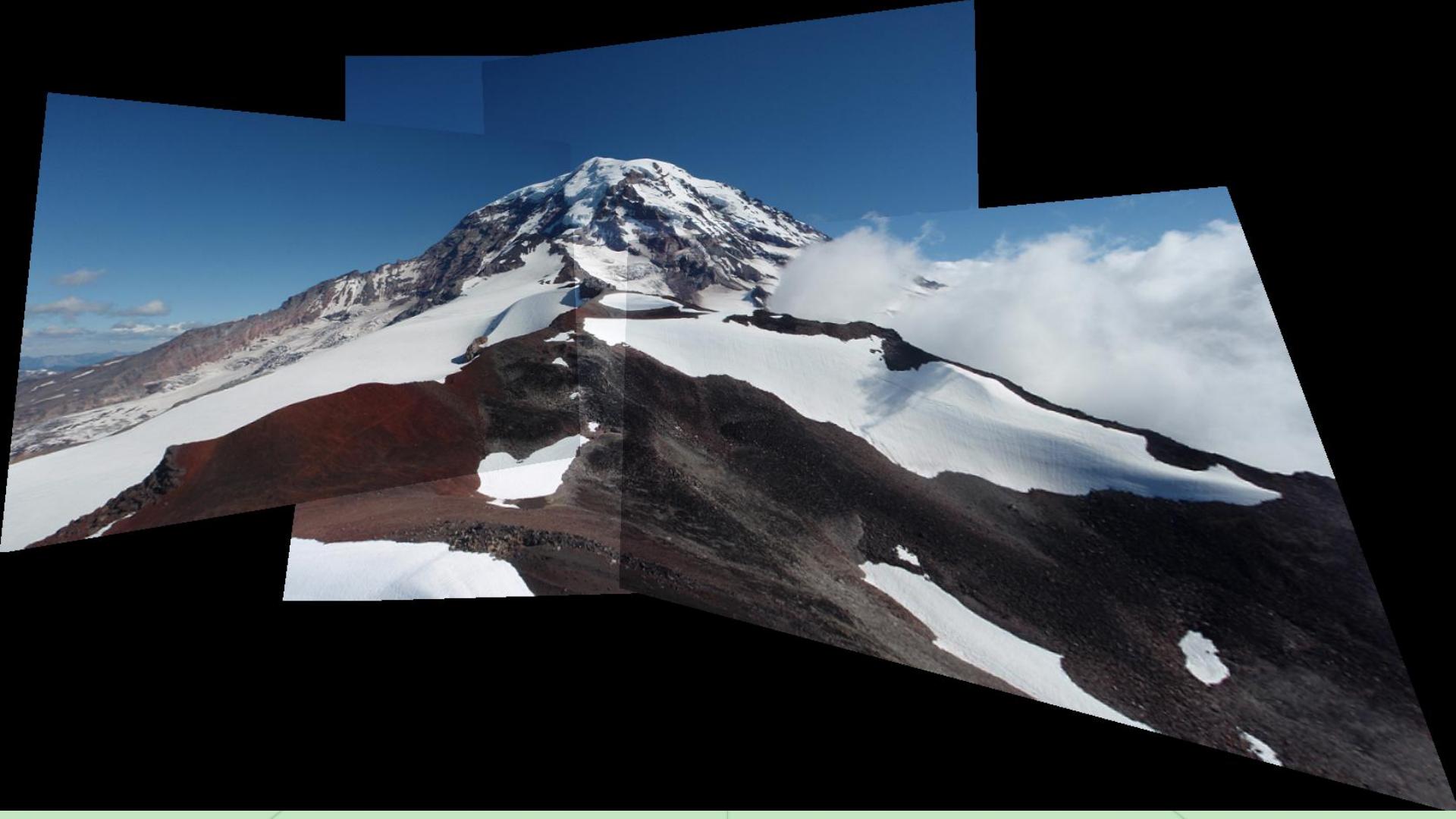


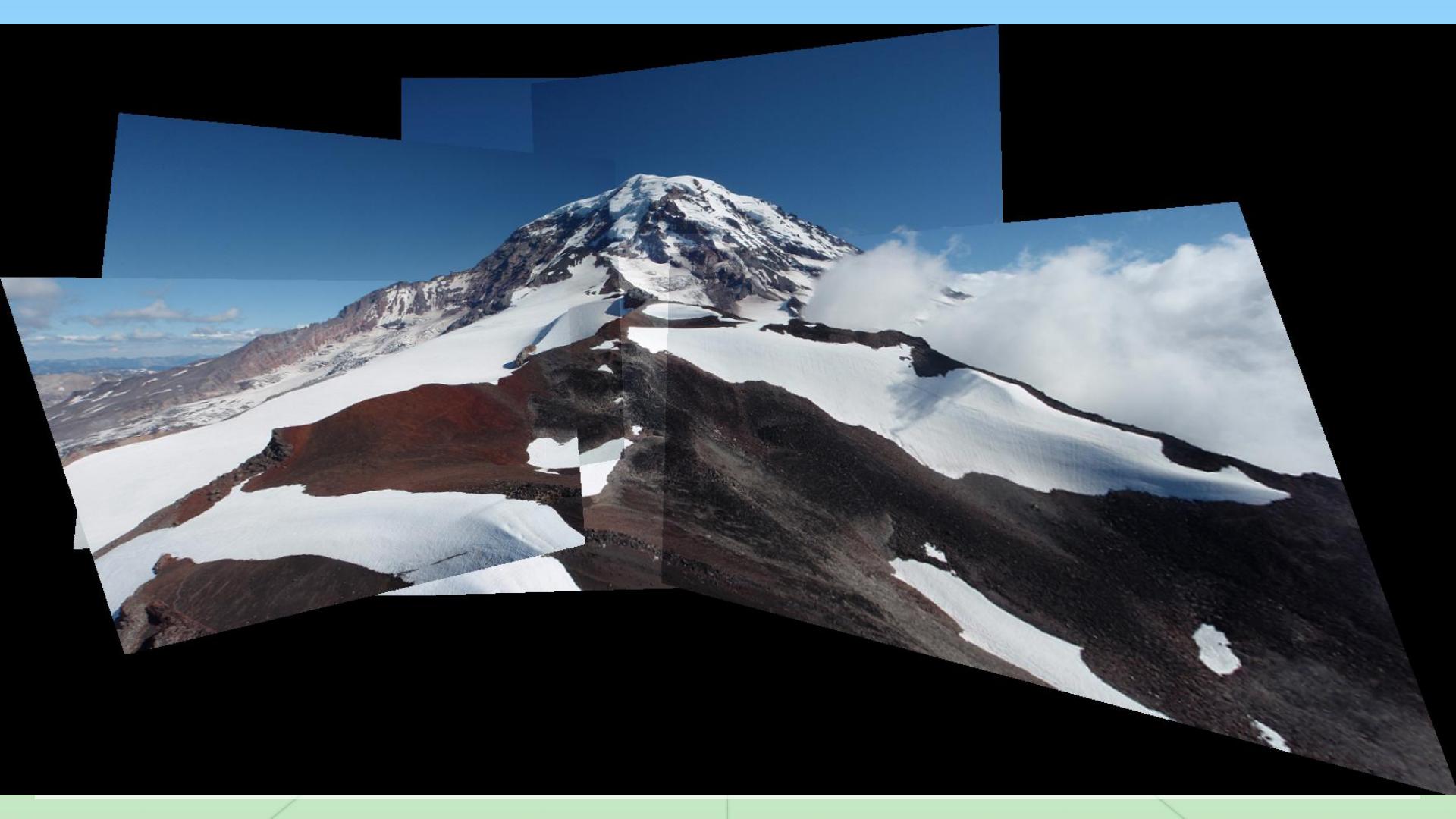
In pr



In

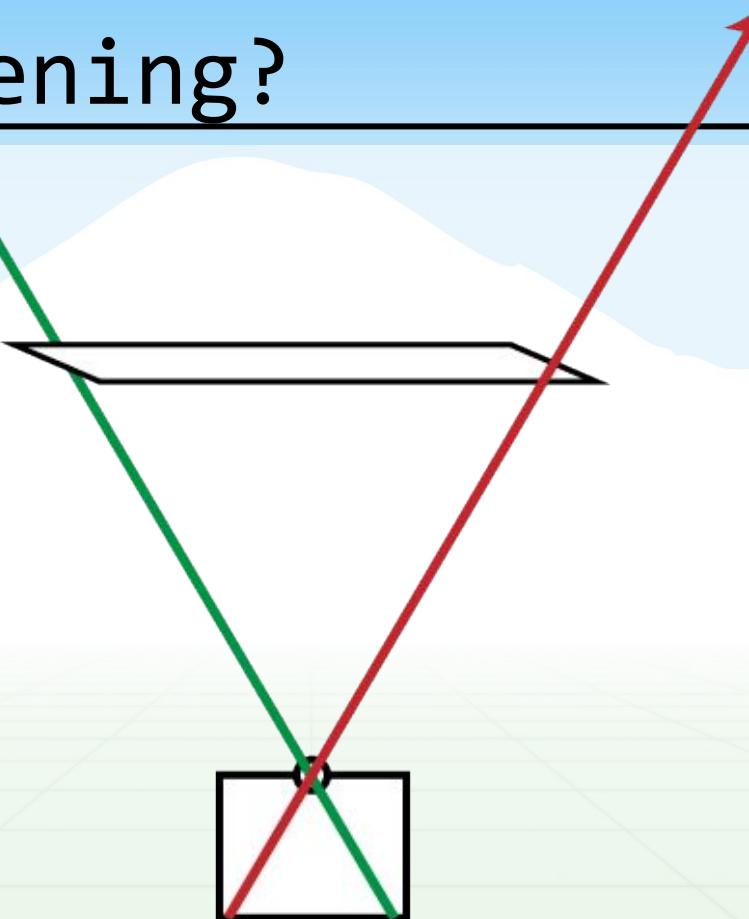






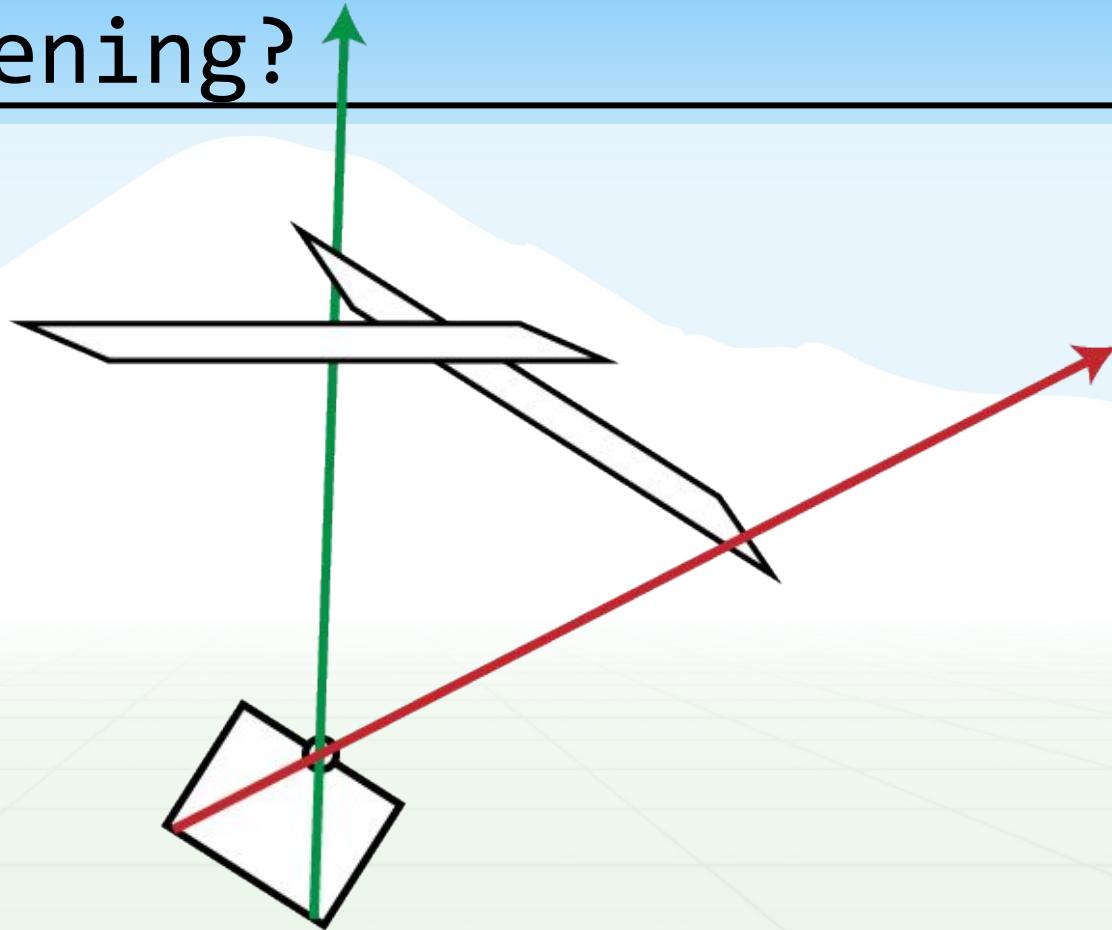
# What's happening?

---



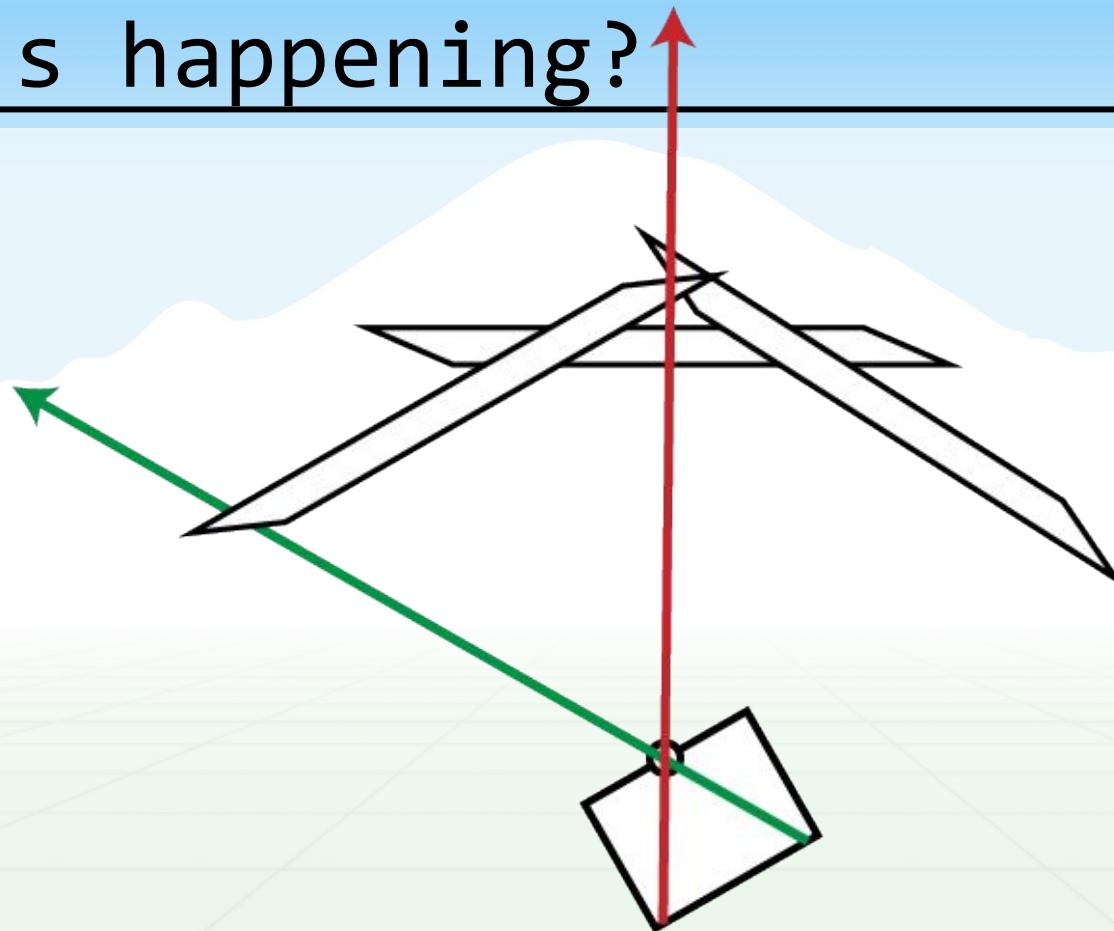
# What's happening?

---

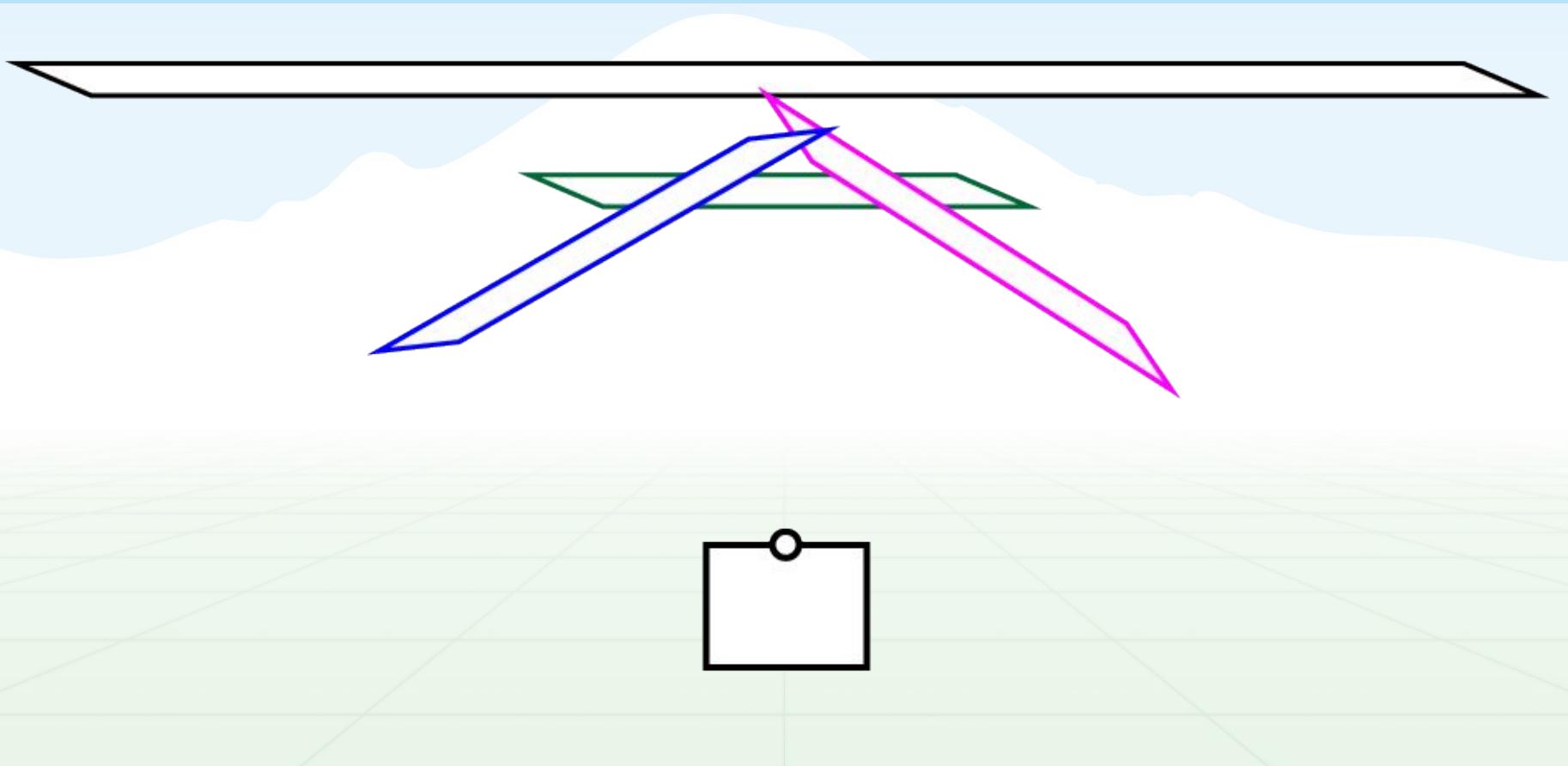


# What's happening?

---

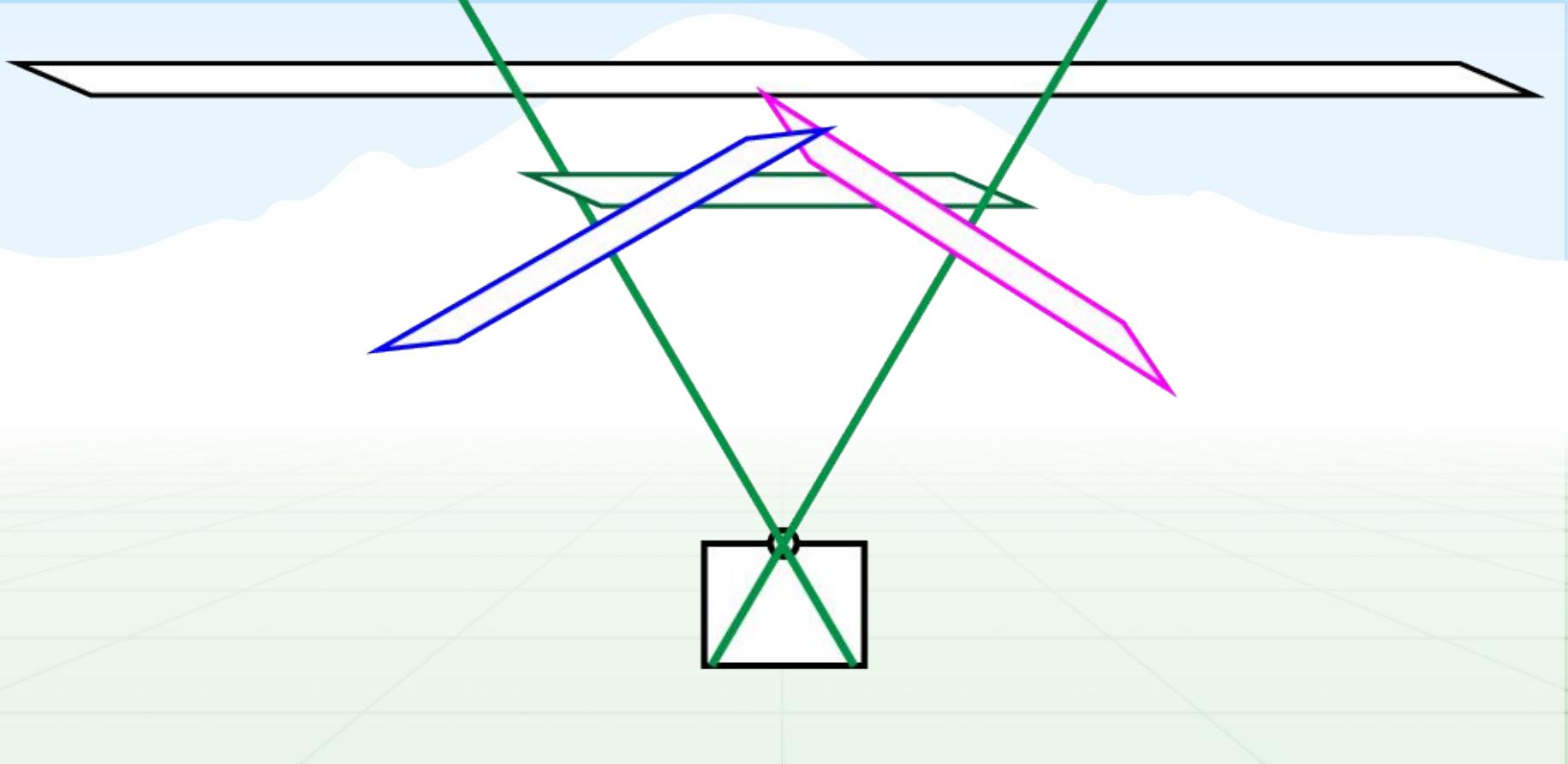


# What's happening?

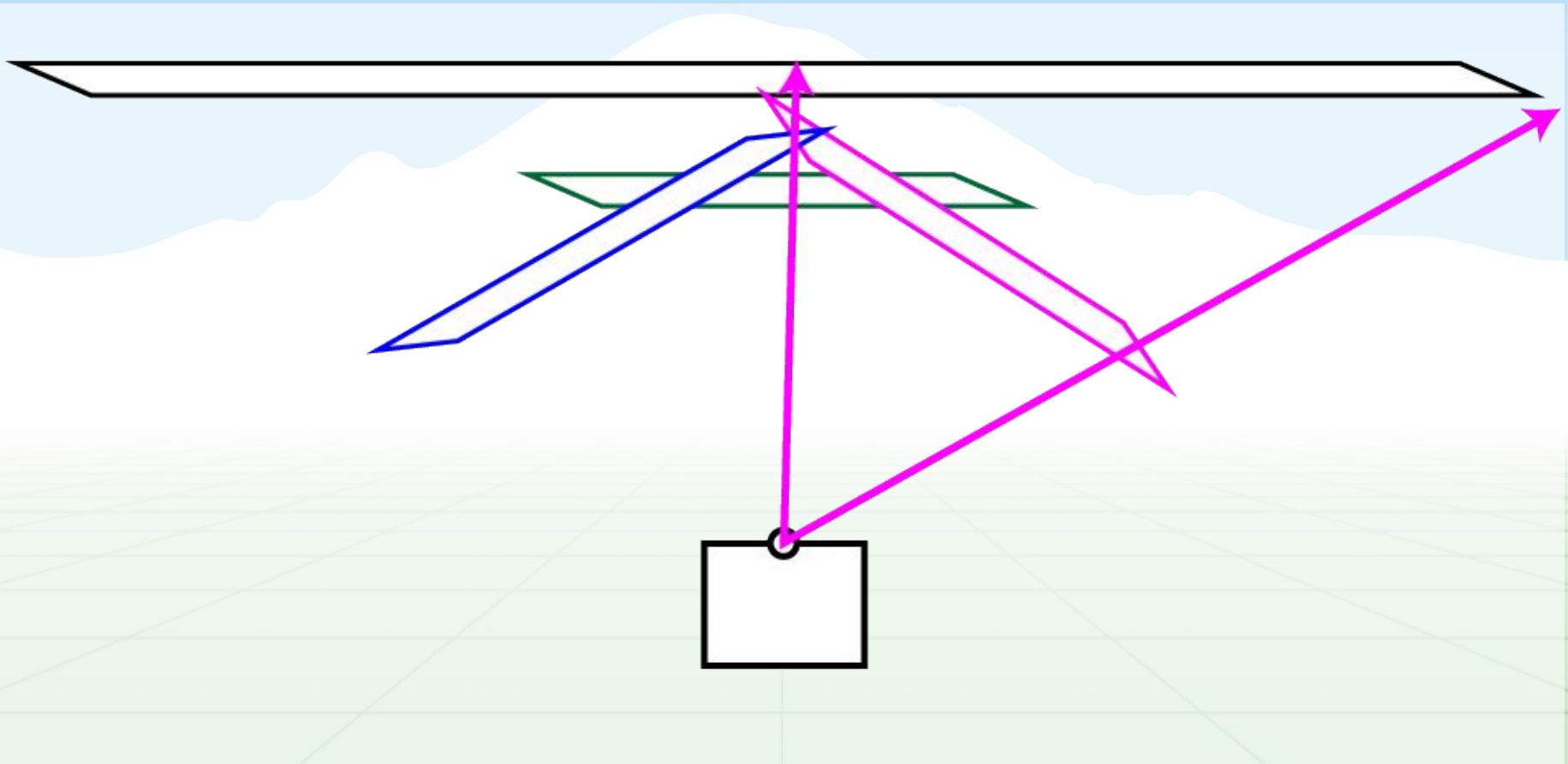


# What's happening?

---

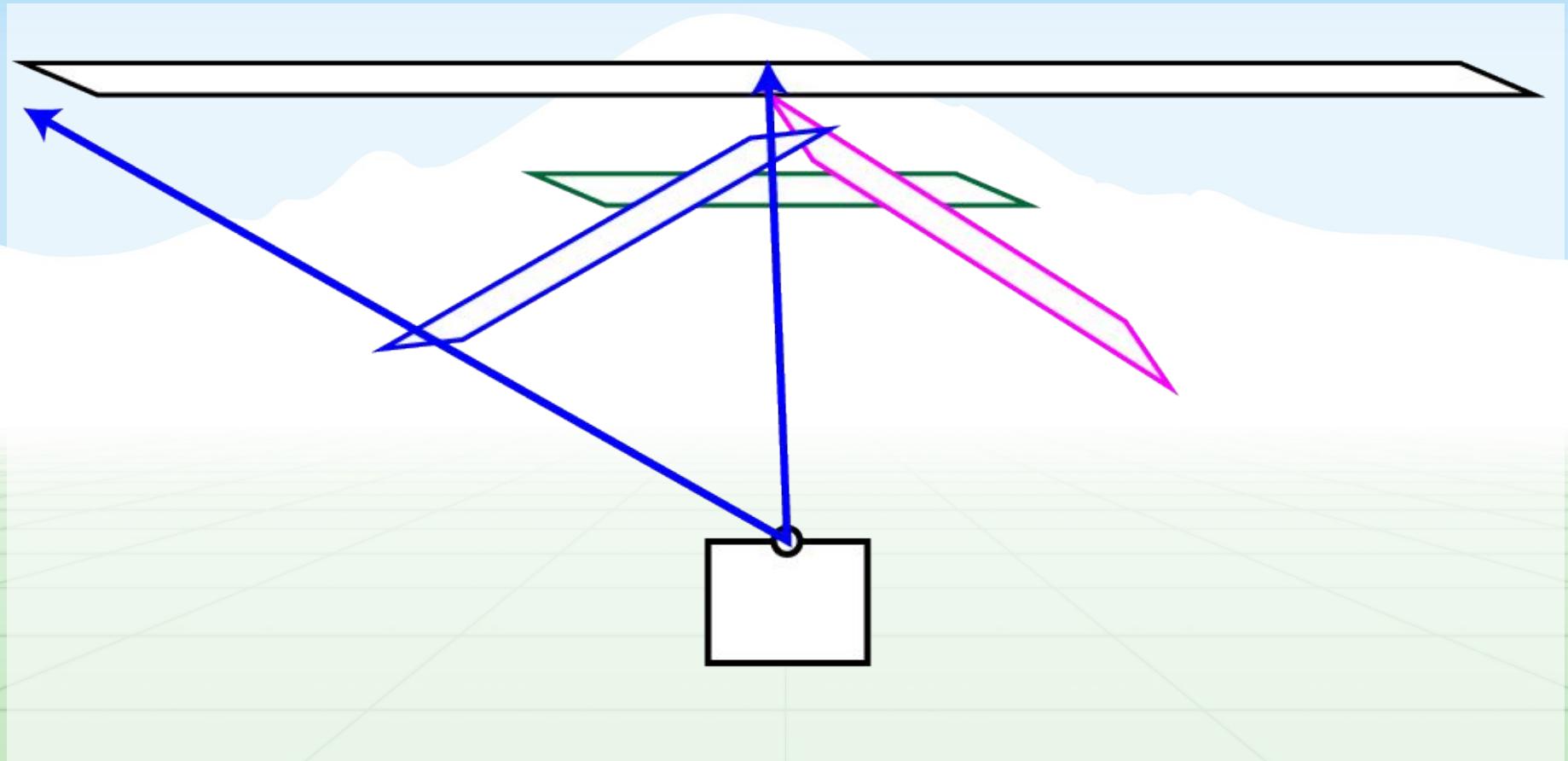


# What's happening?



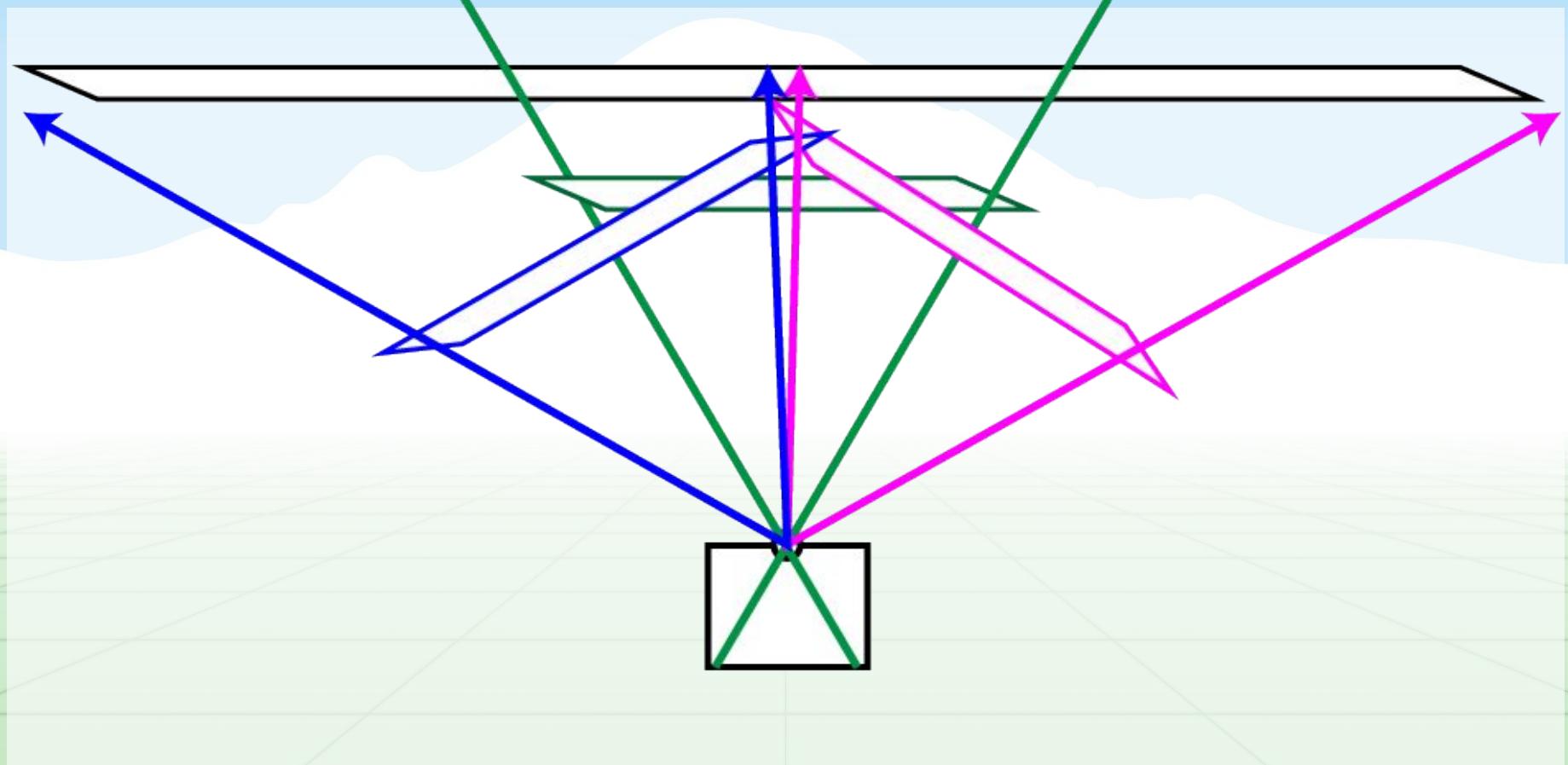
# What's happening?

---

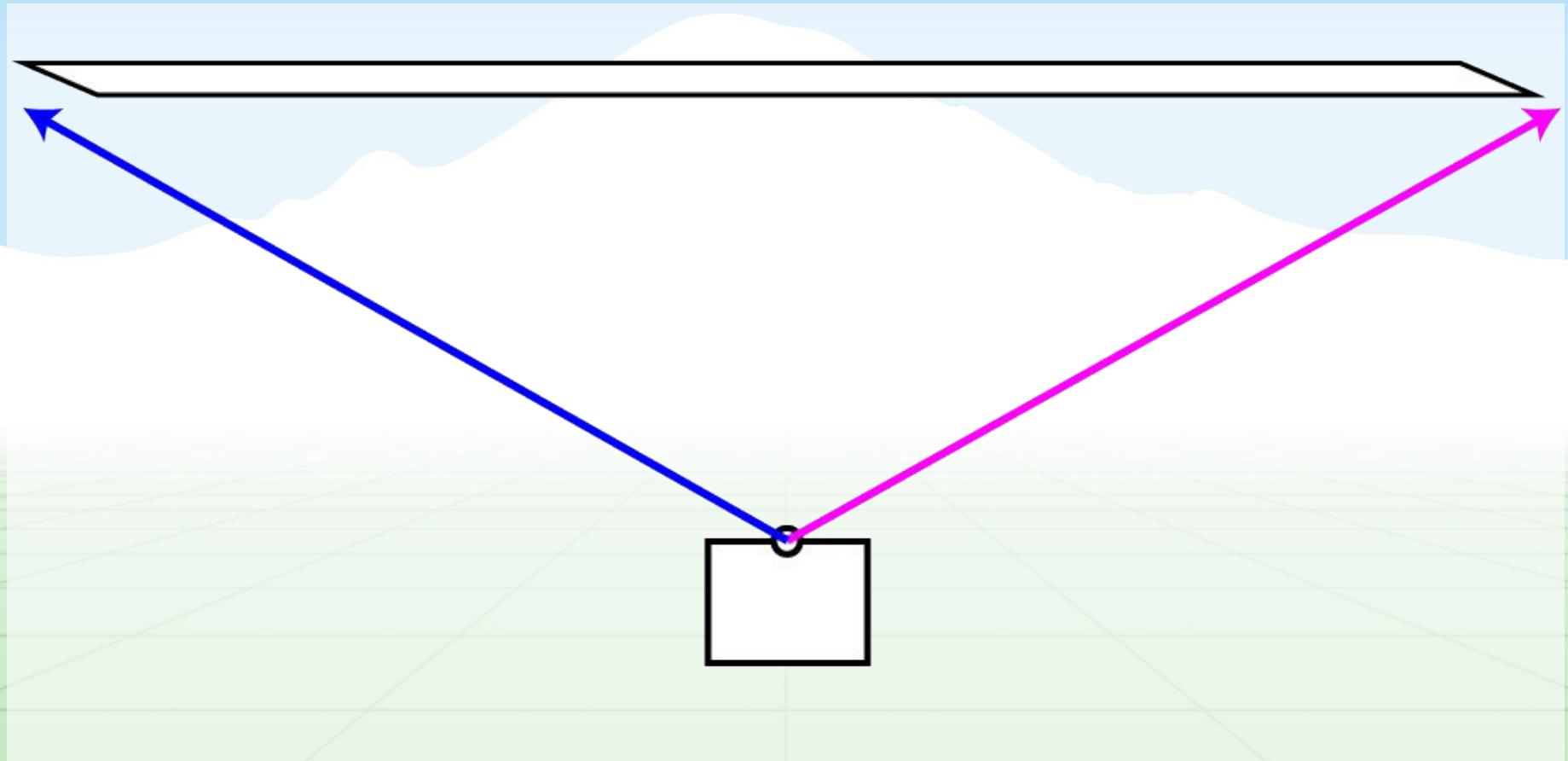


# What's happening?

---



# What's happening?



# Very bad for big panoramas!



# Very bad for big panoramas!

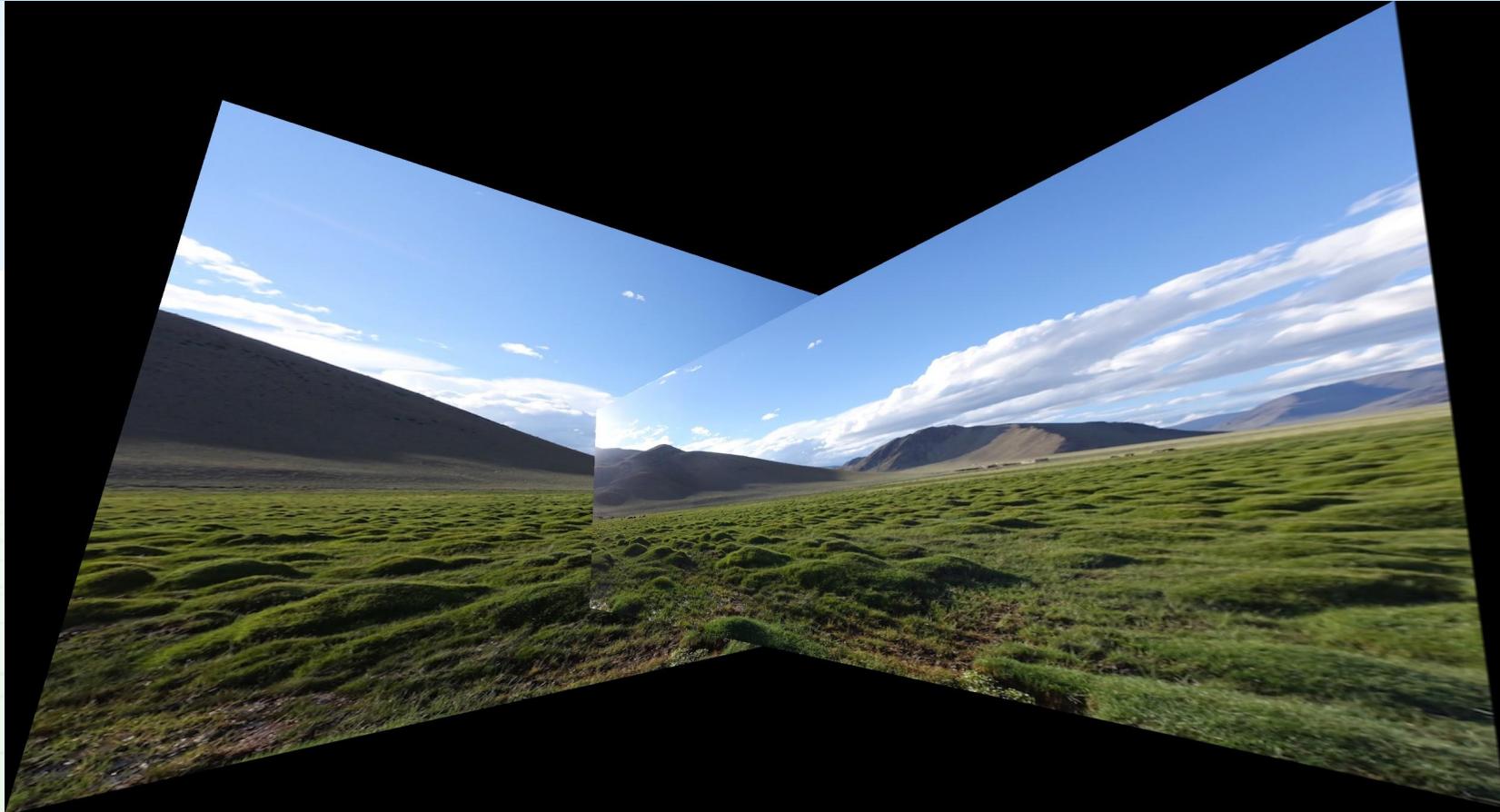


# Very bad for big panoramas!



# Fails :- (

---



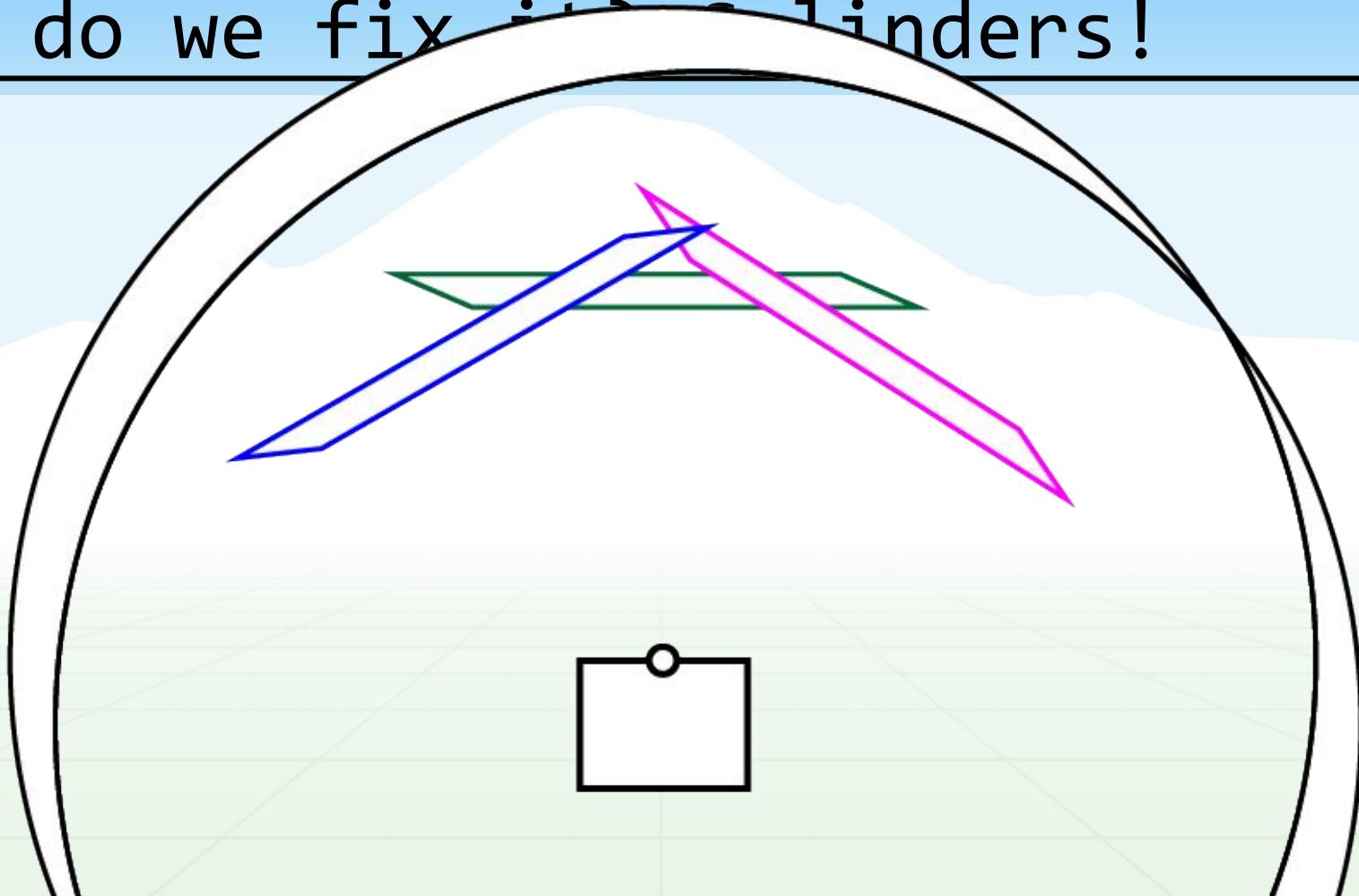
# How do we fix it? Cylinders!

---

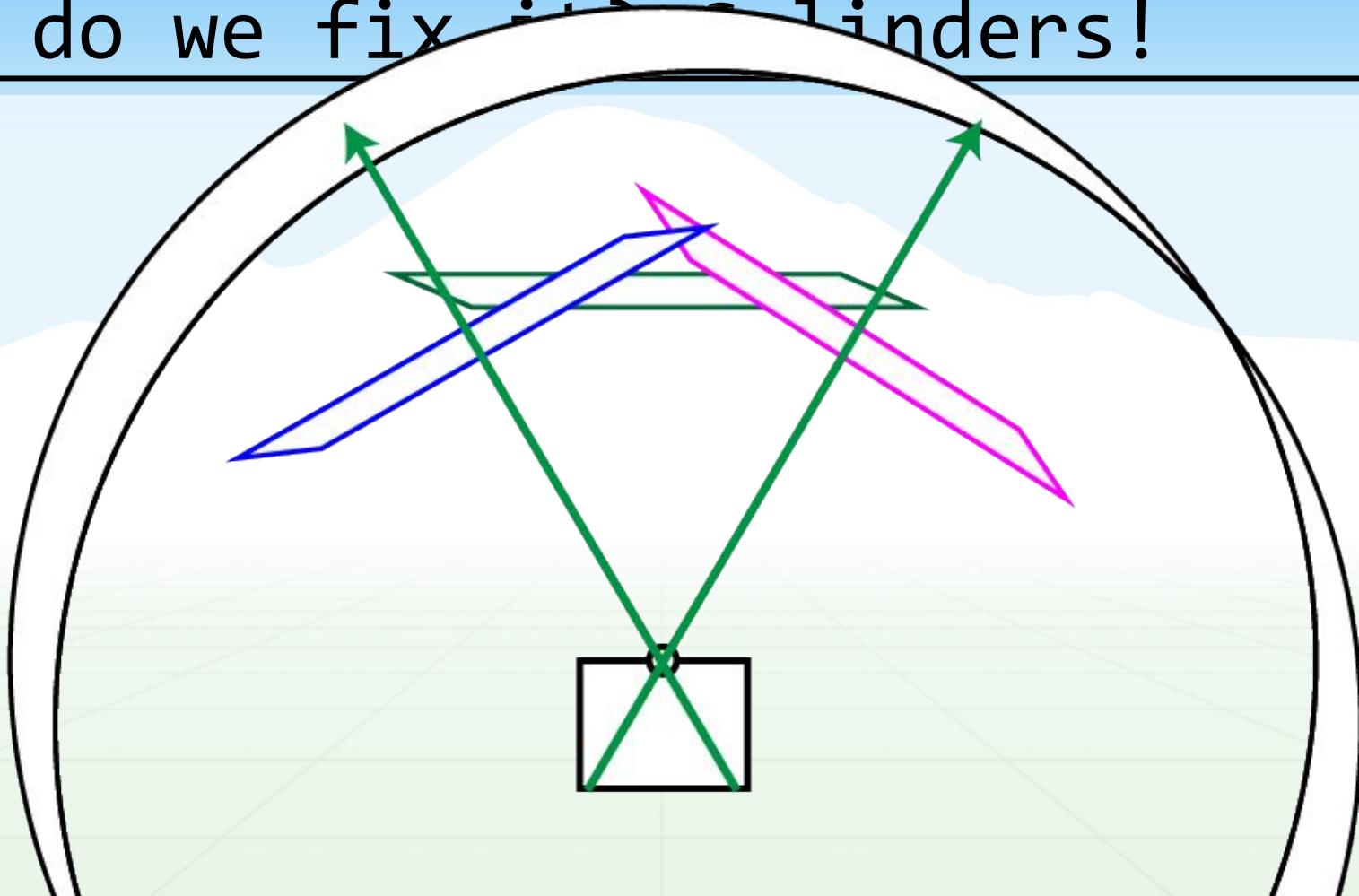


# How do we fix it - cylinders!

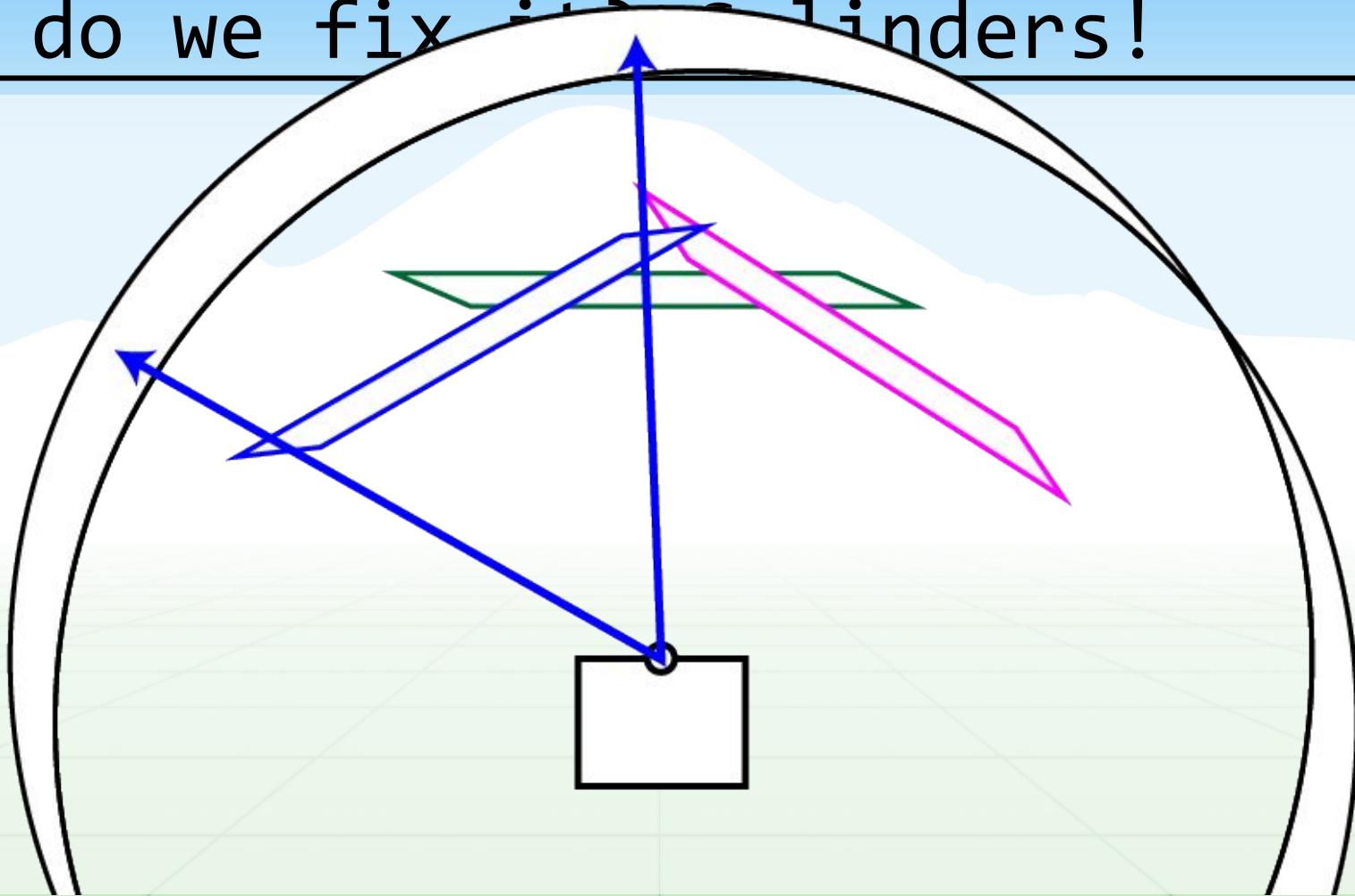
---



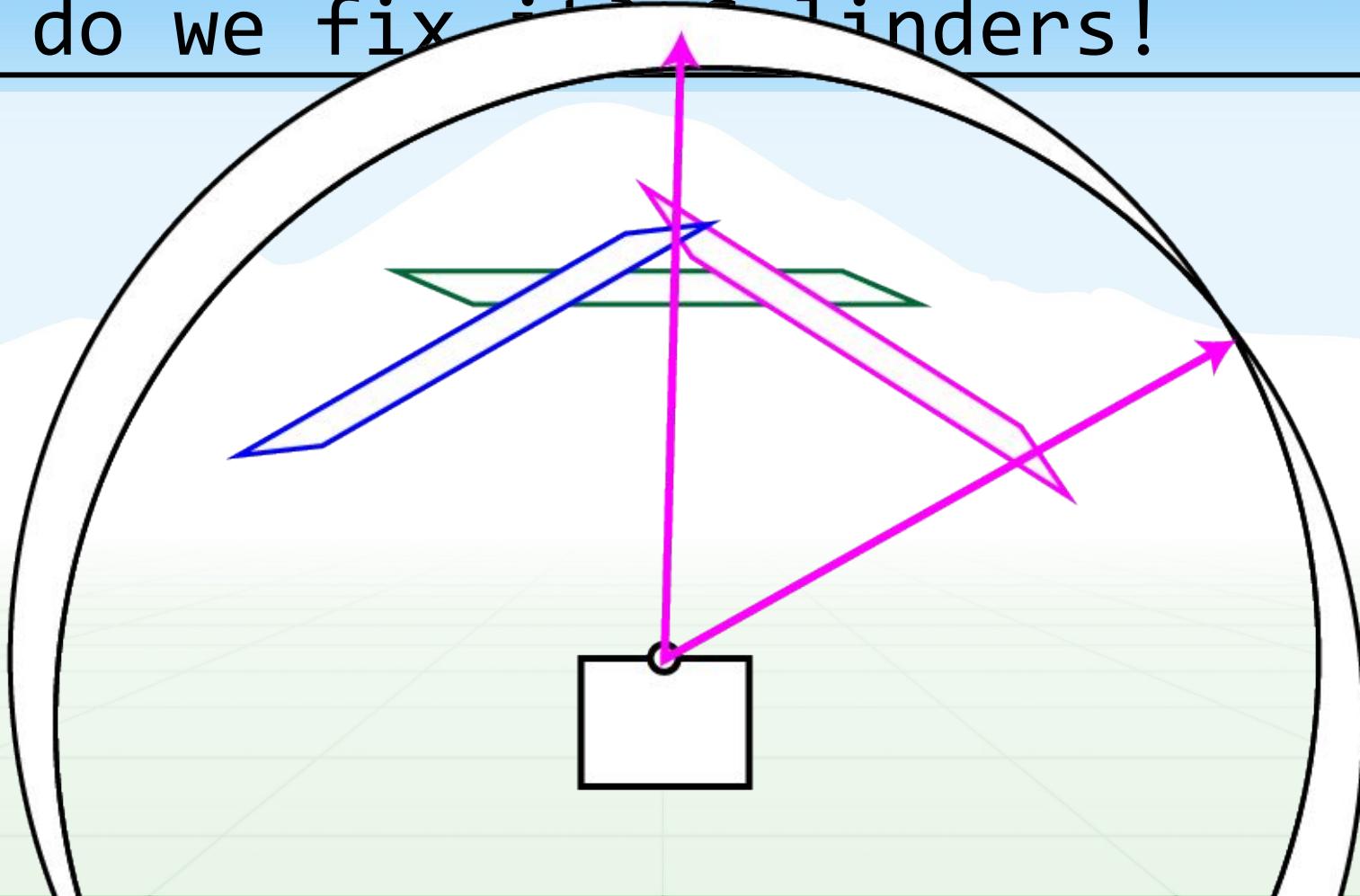
# How do we fix it - cylinders!



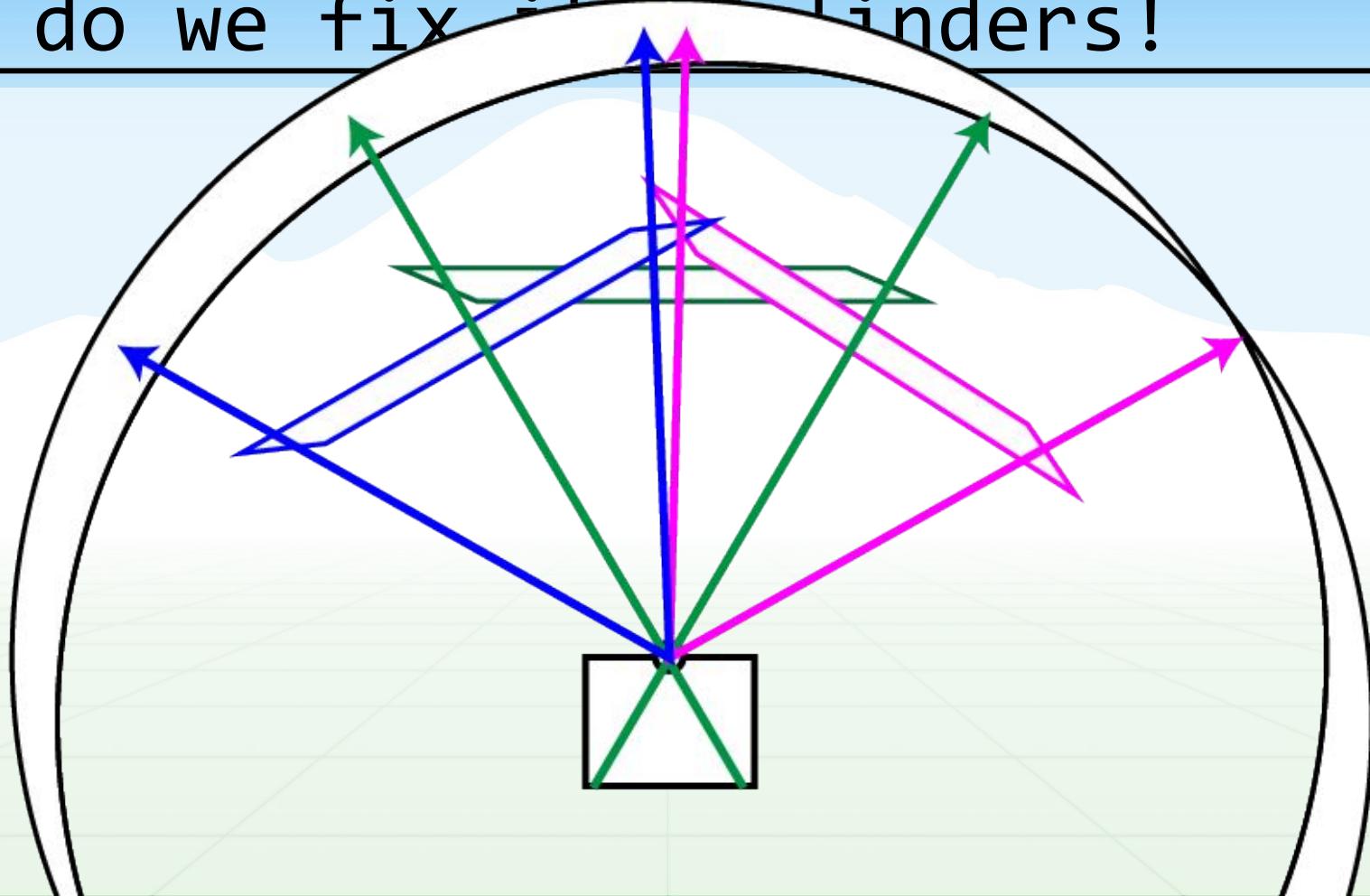
# How do we fix it - cylinders!



# How do we fix it? - cylinders!



# How do we fix it - cylinders!



# How do we fix it? Cylinders!

Calculate angle and height:

$$\theta = (x - xc) / f$$

$$h = (y - yc) / f$$

Find unit cylindrical coords:

$$X' = \sin(\theta)$$

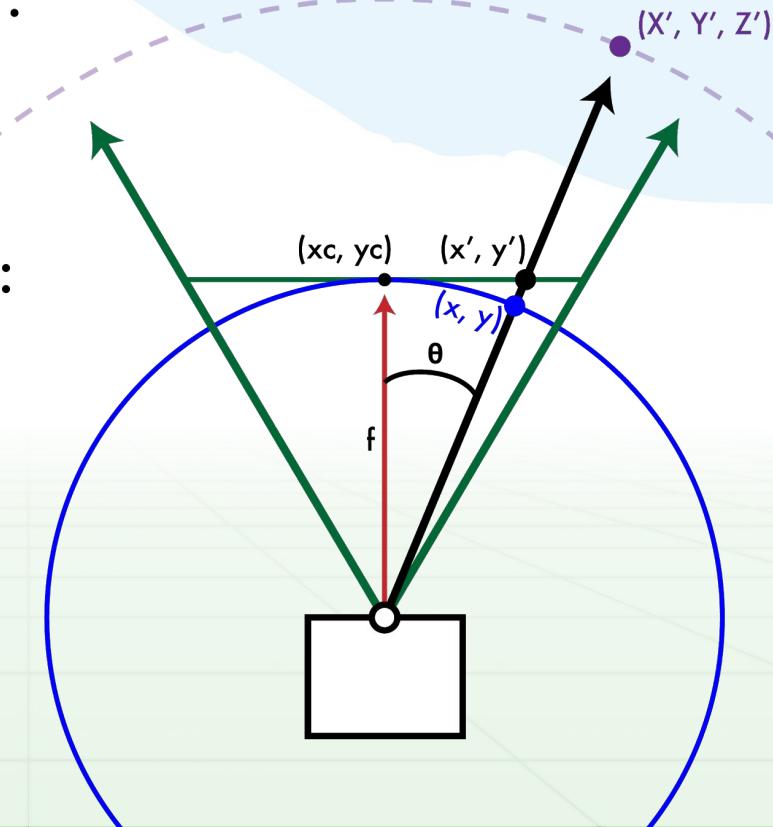
$$Y' = h$$

$$Z' = \cos(\theta)$$

Project to image plane:

$$x' = f X' / Z' + xc$$

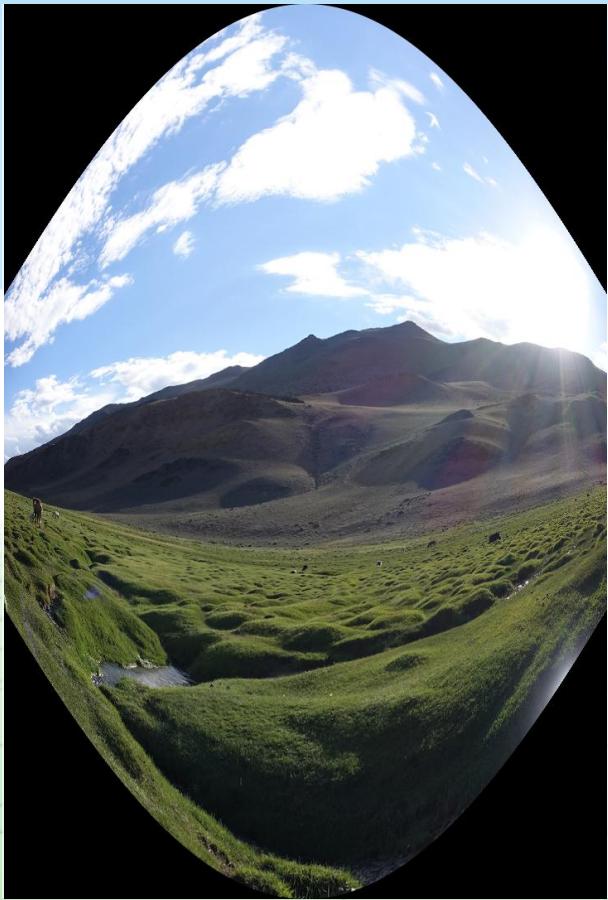
$$y' = f Y' / Z' + yc$$



# Dependant on focal length!



$f = 300$



$f = 500$



$f = 1000$

---



**f = 1400**

---



$f = 10,000$

---



$f = 10,000$

---



# Does it work?



# Does it work?

---



# Does it work?

---



# Does it work?

---



# Does it work?

---



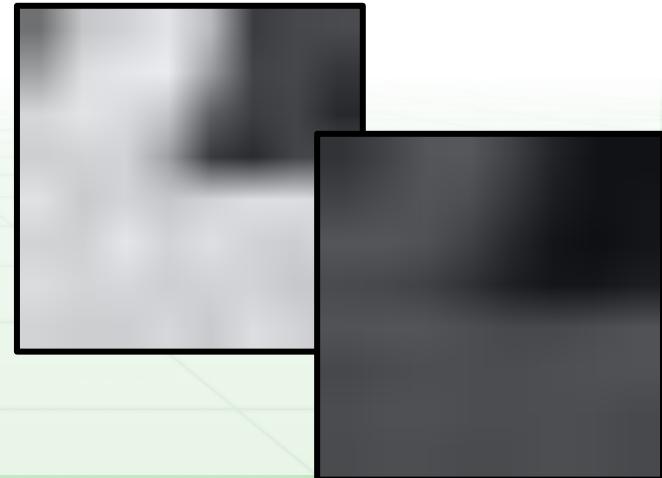
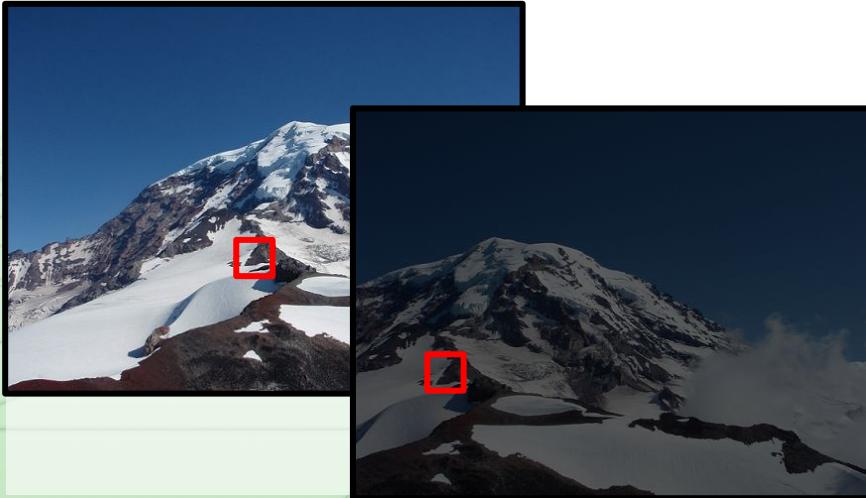
# Does it work? Yay!



# THIS IS AS GOOD AS IT GETS!

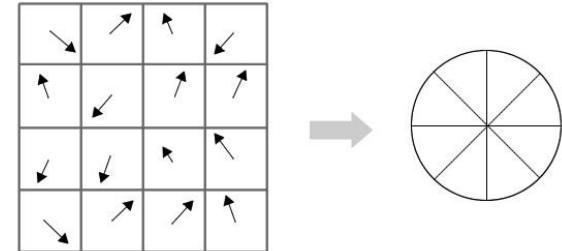
---

- Not so fast...
- Our descriptor has some issues:
  - Not invariant to lighting changes!
- Want features invariant to lighting

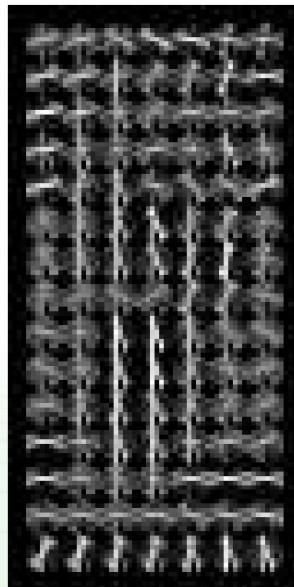


# Histogram of Oriented Gradients (HOG)

- Dalal and Triggs 2005
- Better image descriptor
  - Compute gradients
  - Bin gradients
  - Aggregate blocks (4x4, 16x16 cells)
  - Normalize gradient magnitudes
- Not reliant on magnitude, just direction
  - Invariant to some lighting changes
- Train SVM to recognize people



# Histogram of Oriented Gradients (HOG)

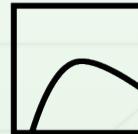


# THIS IS AS GOOD AS IT GETS!

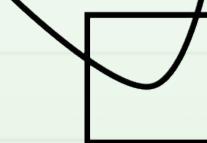
---

- Not so fast...
- Harris and has some issues:
  - Corners detection is rotation invariant
  - Harris not invariant to scale
- Descriptors are also hard
  - Just looking at pixels is not rotation invariant!
  - HOG also not rotation invariant

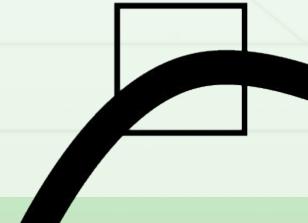
Corner



Still Corner



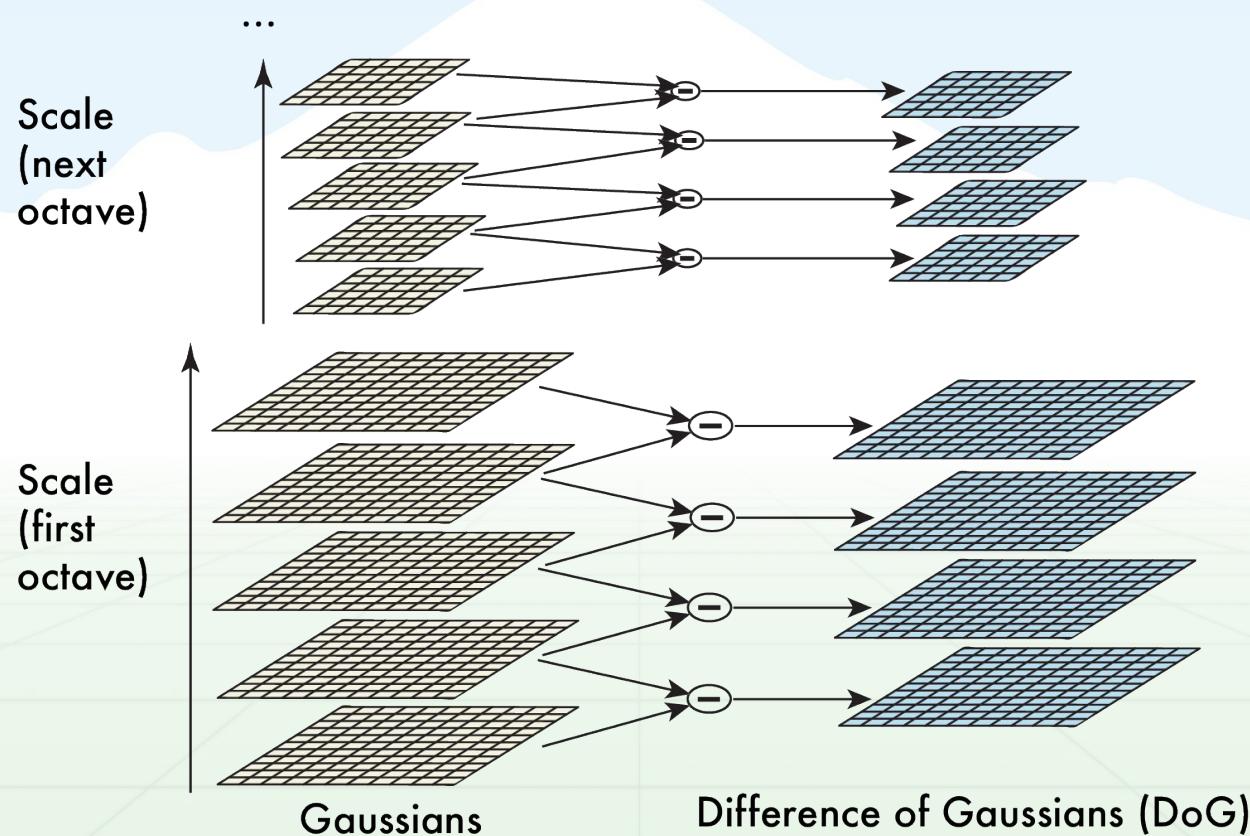
Not Corner



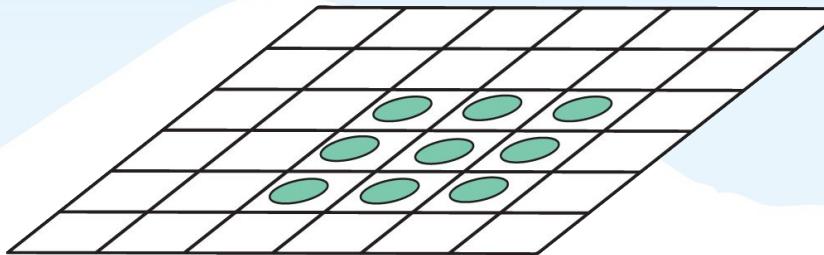
Want features invariant to scaling, rotation, etc.

- Scale Invariant Feature Transform (SIFT)
  - Lowe et al. 2004, many images from that paper
- Get scale-invariant response map
- Find keypoints
- Extract rotation-invariant descriptors
  - Normalize based on orientation
  - Normalize based on lighting

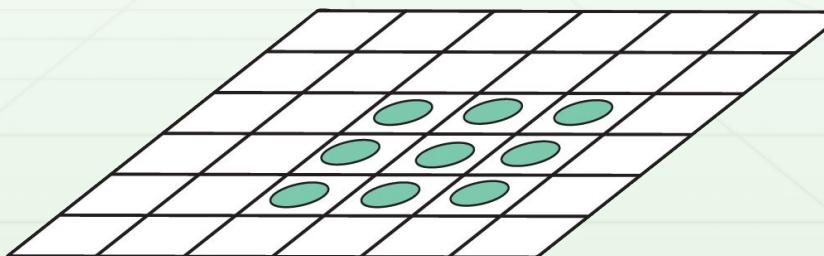
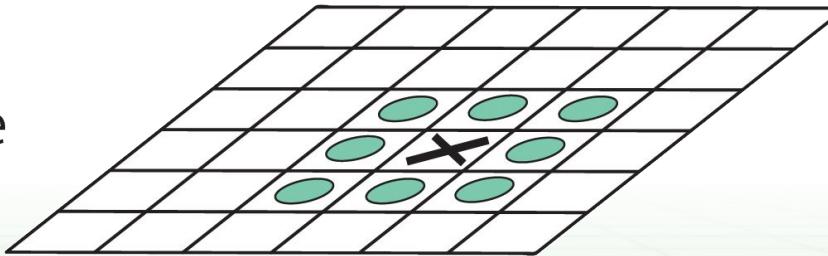
# Extract DoG features at multiple scales



# Find local-maxima in location and scale



Scale



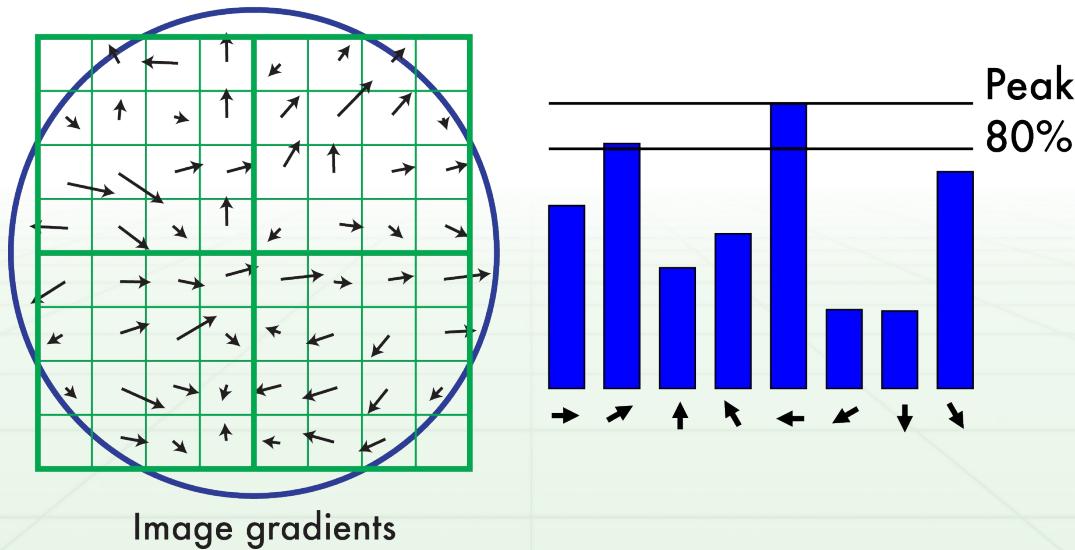
# Throw out weak responses and edges

- Estimate gradients
  - Similar to before, look at nearby responses
  - Not whole image, only a few points! Faster!
  - Throw out weak responses
- Find cornery things
  - Same deal, structure matrix, use det and trace information

$$\frac{\text{Tr}(\mathbf{H})^2}{\text{Det}(\mathbf{H})} < \frac{(r+1)^2}{r}$$

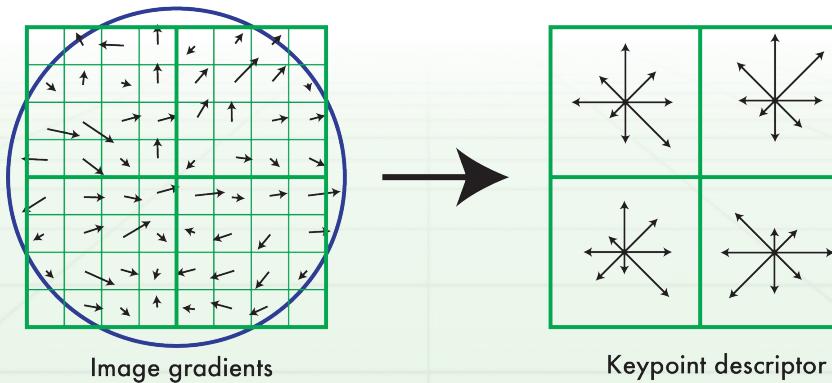
# Find main orientation of patches

- Look at weighted histogram of nearby gradients
  - Any gradient within 80% of peak gets its own descriptor
    - Multiple keypoints per pixel
  - Descriptors are normalized based on main orientation



## Keypoints are normalized gradient histograms

- Divide into subwindows ( $2 \times 2$ ,  $4 \times 4$ )
- Bin gradients within subwindow, get histogram
  - Normalize to unit length
  - Clamp at maximum .2
  - Normalize again
  - Helps with lighting changes!



# SIFT is great!

- Find good keypoints, describe them
- Finding objects, recognition, panoramas, etc.



# SIFT is great!

