
The Ancient
Secrets
Computer Vision

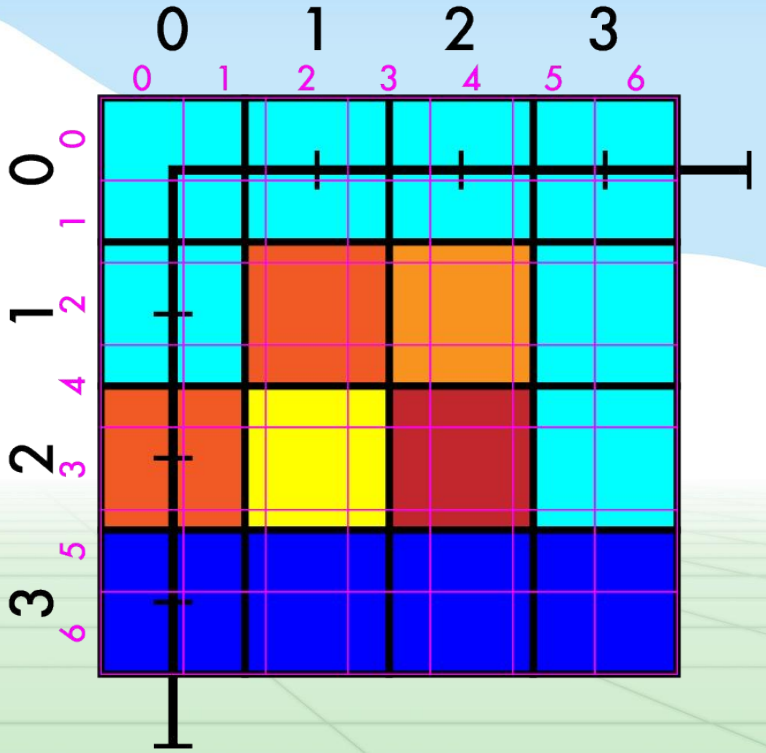
# Logistics:

- Homework 1 is out!
  - Due Thursday
-


Previously
On
the
Ancient Secrets
of Computer Vision

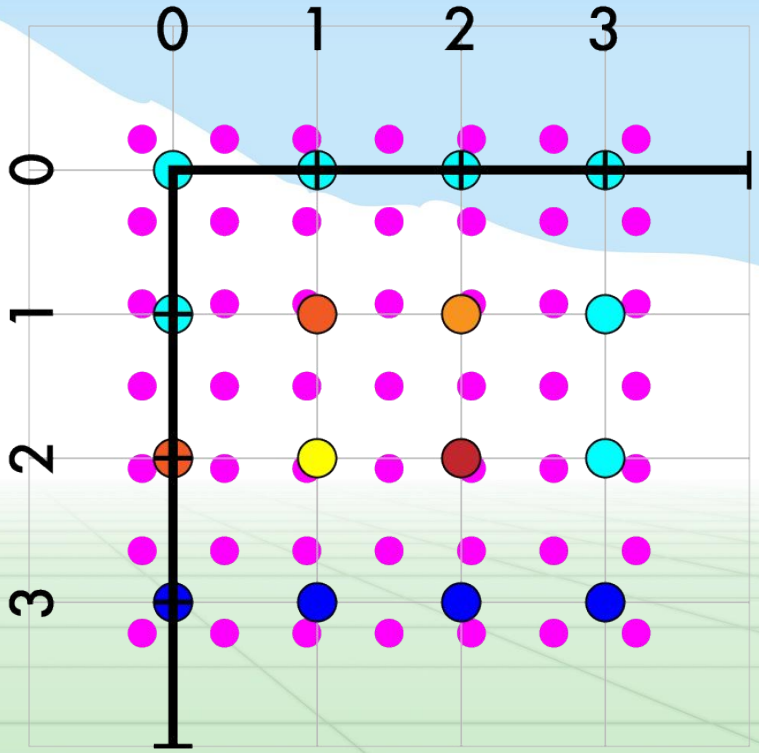
# Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates



# Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates
  - 4/7 X - 3/14 = Y
- Iterate over new pts
  - Map to old coords



# Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates
  - 4/7 X - 3/14 = Y
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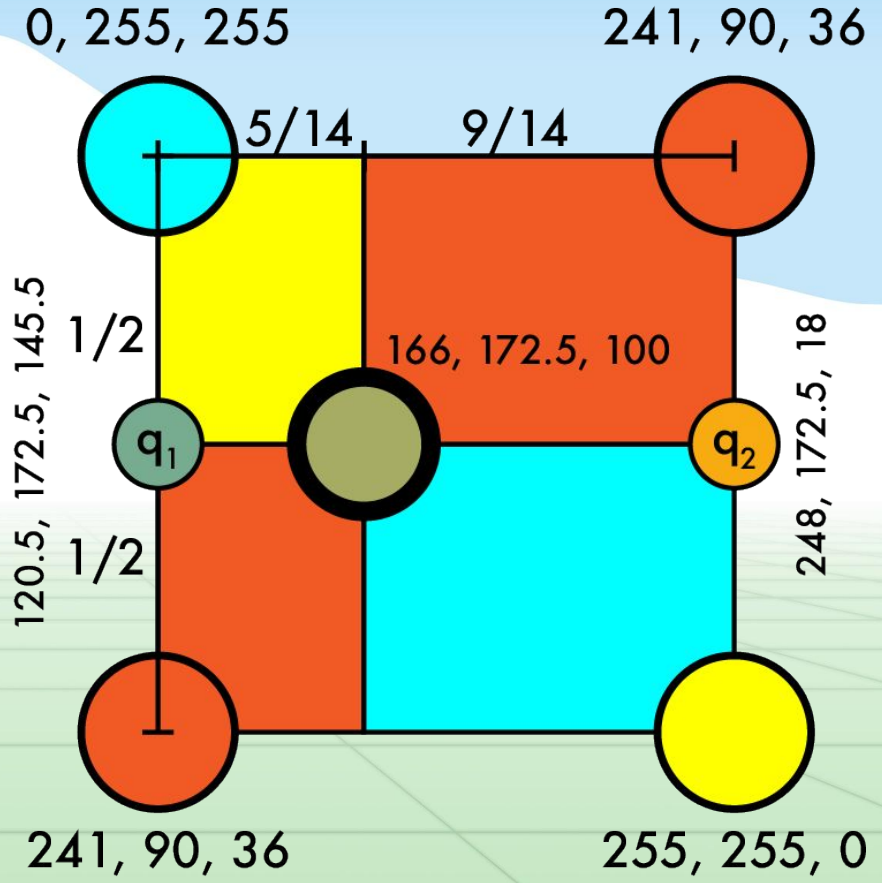
  - (1, 3) -> (5/14, 7/14)
  - Interpolate old values
    - q = (166, 172.5, 100)



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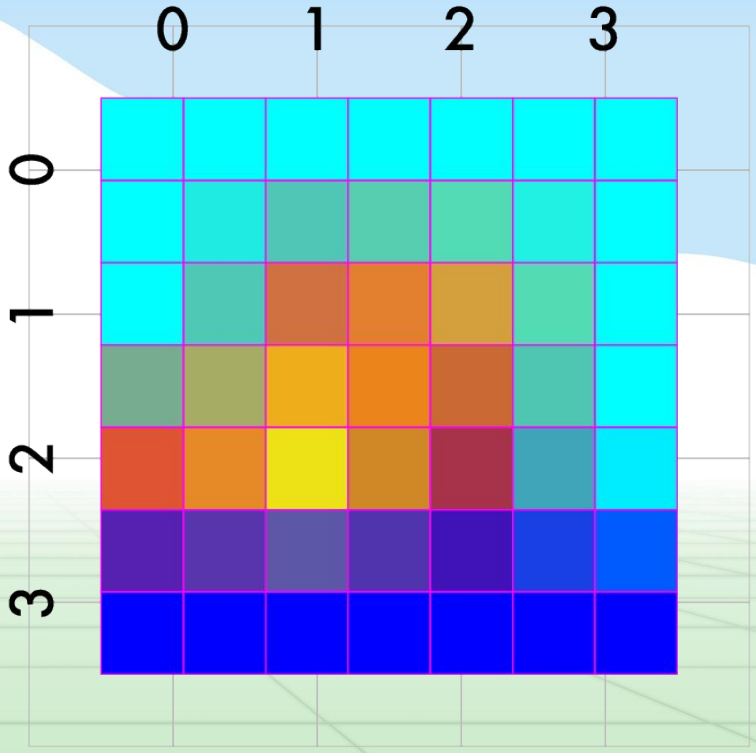
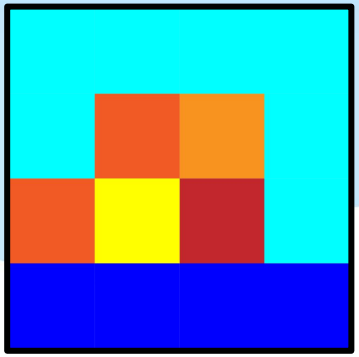
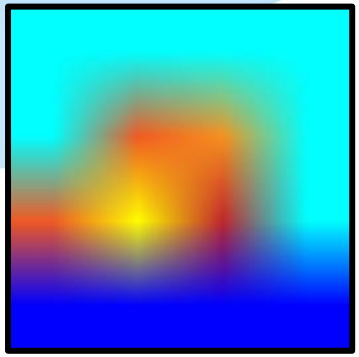
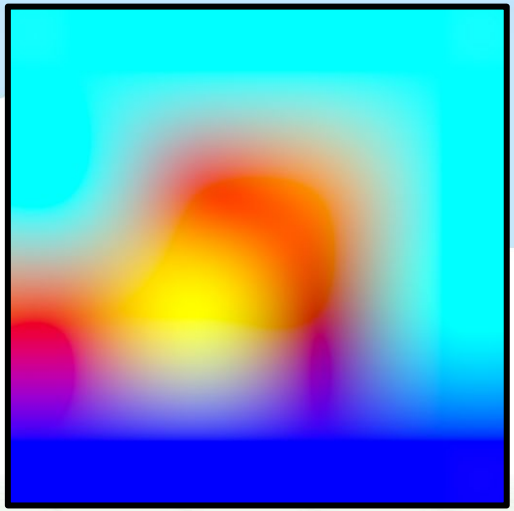
- Create our new image
- Match up coordinates
  - 4/7 X - 3/14 = Y
- Iterate over new pts
  - Map to old coords
  - (1, 3) -> (5/14, 7/14)
  - Interpolate old values
    - q = (166, 172.5, 100)
- Fill in the rest

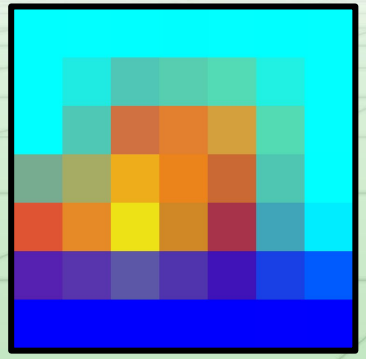


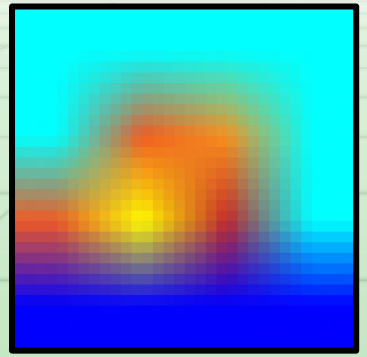
# Different methods

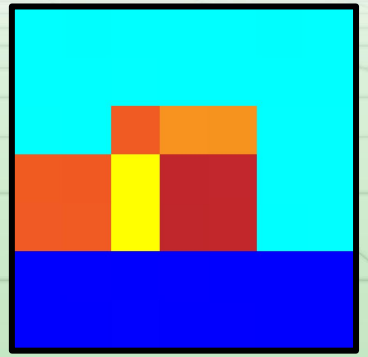




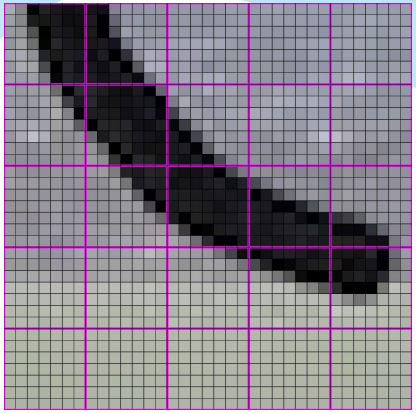




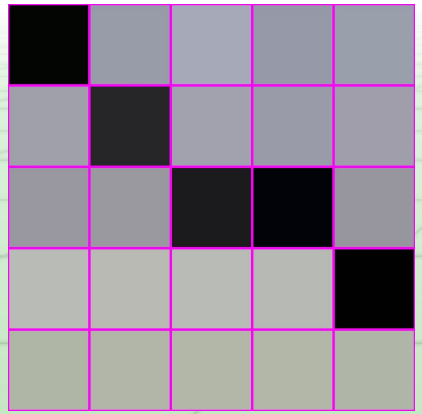




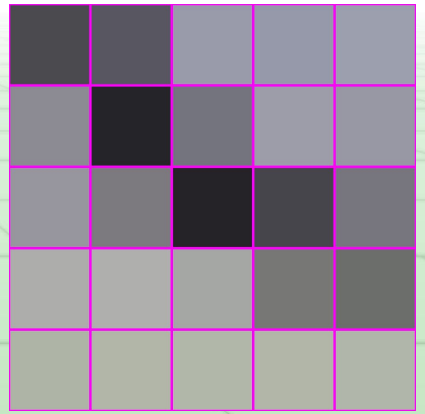
# For shrinking interpolation bad, averaging good



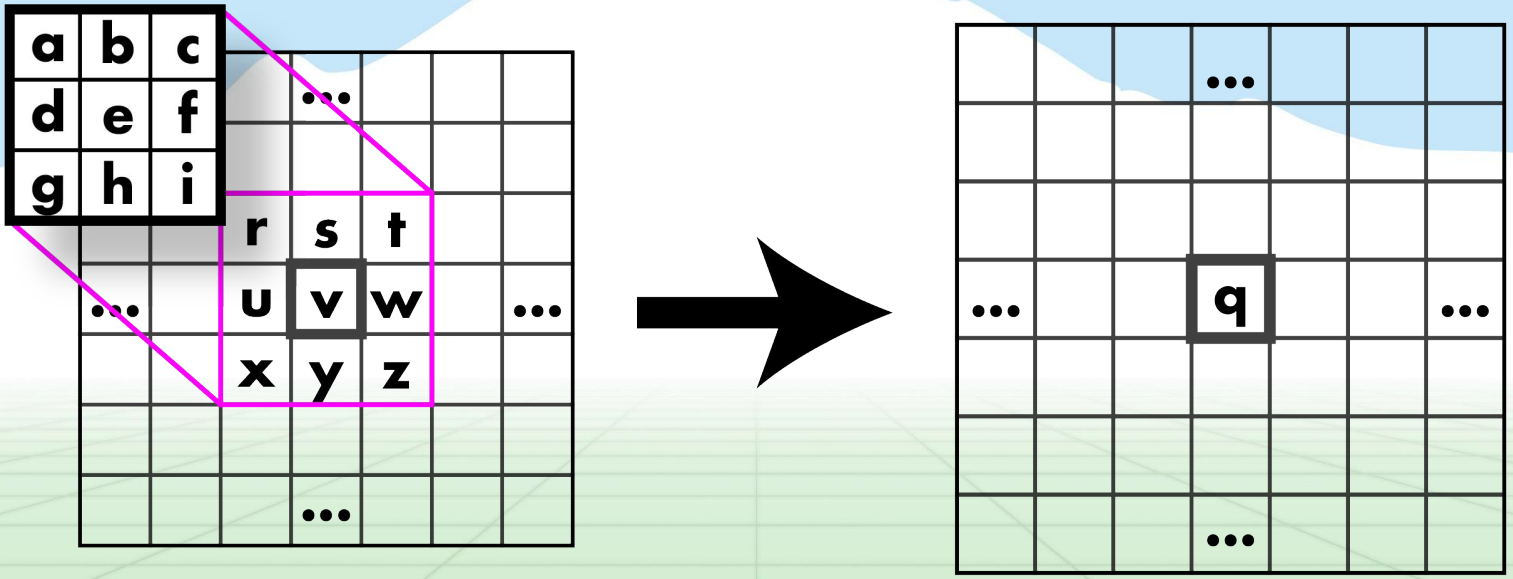
“interpolation”



averaging

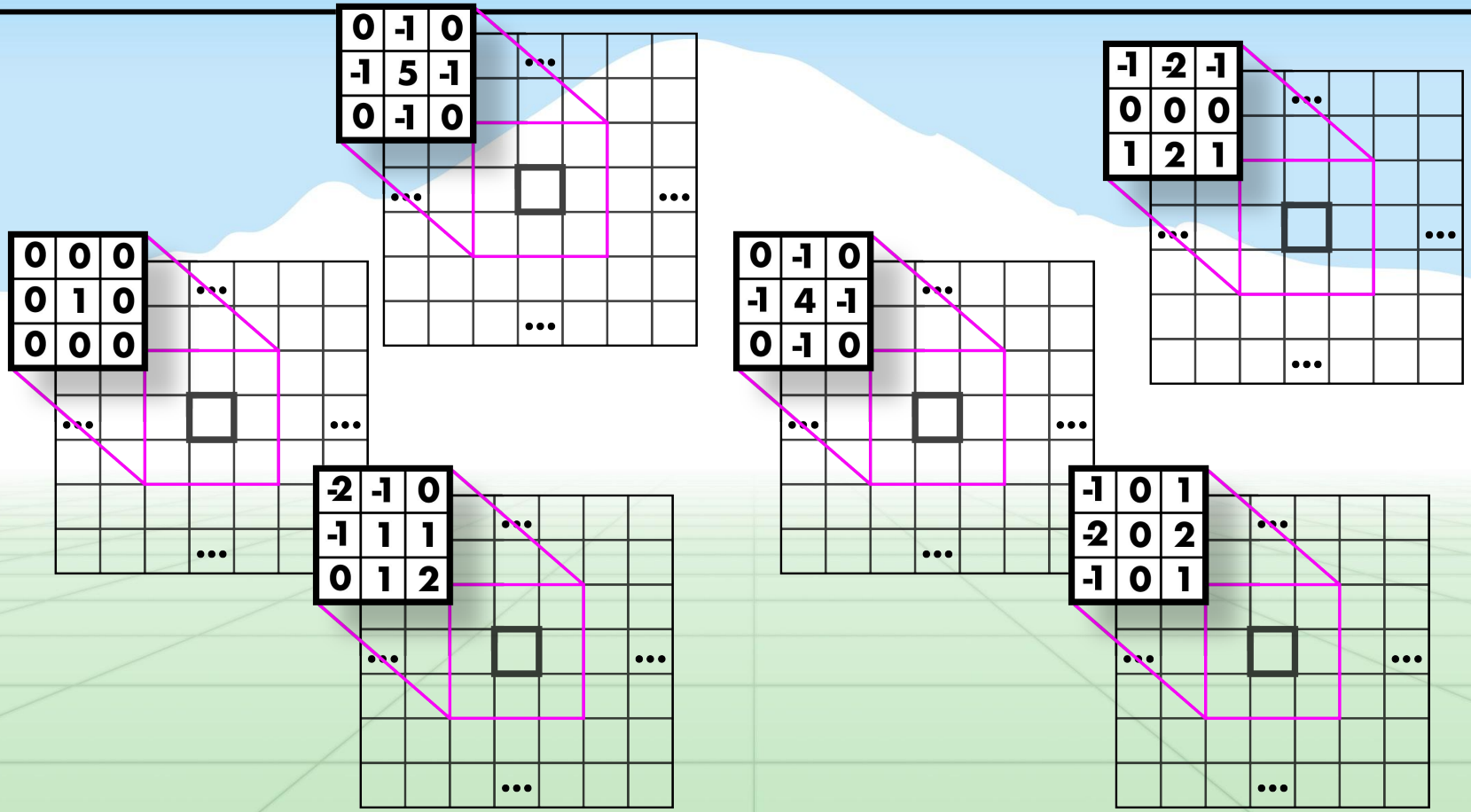


# Convolution: Weighted sum over pixels




$$q = a \times r + b \times s + c \times t + d \times u + e \times v + f \times w + g \times x + h \times y + i \times z$$

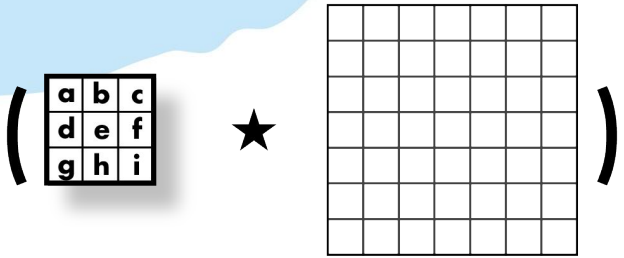
# Filters

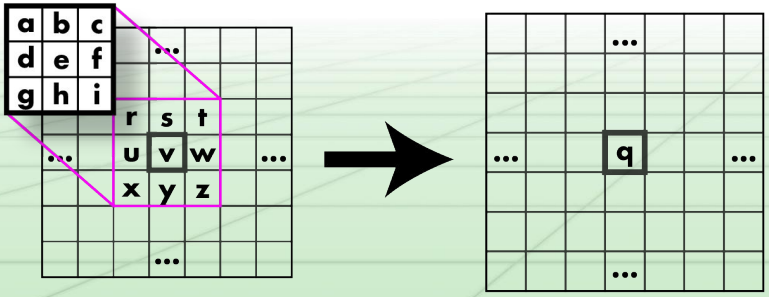

0 -1 0
-1 5 -1
0 -1 0
0 0 0
0 1 0
0 0 0
-2 -1 0
-1 1 1
0 1 2
0 -1 0
-1 4 -1
0 -1 0
-1 -2 -1
0 0 0
1 2 1
-1 0 1
-2 0 2
-1 0 1


Chapter
Five
Edges and Features

# Cross-Correlation vs Convolution

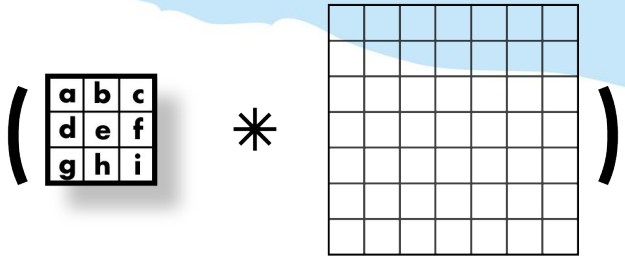
## Cross-Correlation

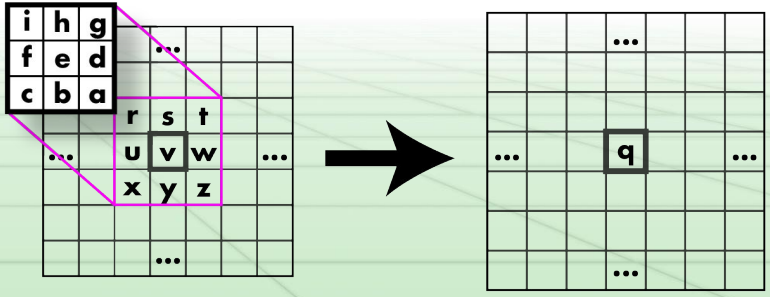



$$q = a \times r + b \times s + c \times t + d \times u + e \times v + f \times w + g \times x + h \times y + i \times z$$

## Convolution



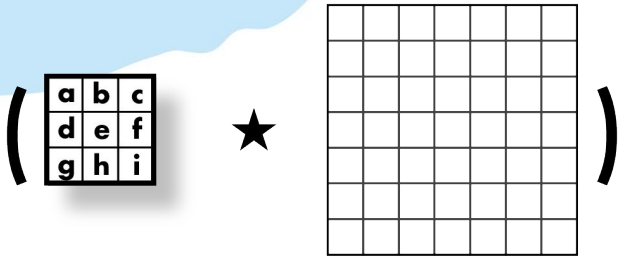



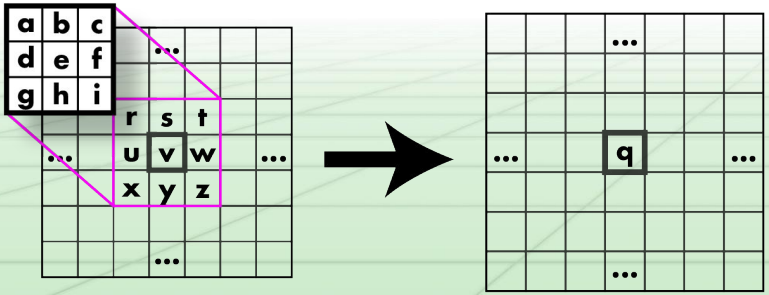

$$q = i \times r + h \times s + g \times t + f \times u + e \times v + d \times w + c \times x + b \times y + a \times z$$

# Cross-Correlation vs Convolution

Do this in HW!

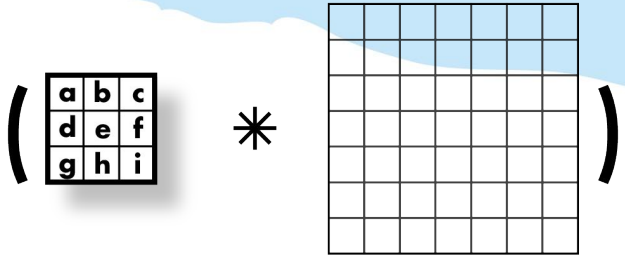
## Cross-Correlation

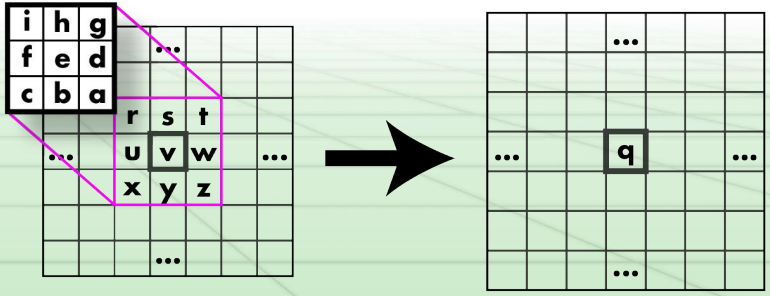



$q = a \times r + b \times s + c \times t + d \times u + e \times v + f \times w + g \times x + h \times y + i \times z$

## Convolution



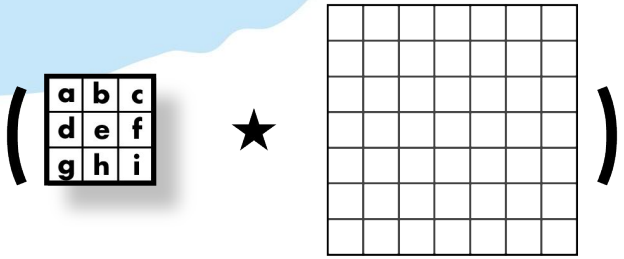



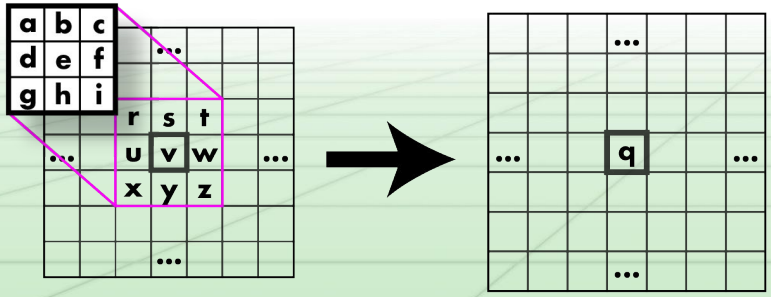

$q = i \times r + h \times s + g \times t + f \times u + e \times v + d \times w + c \times x + b \times y + a \times z$

# Cross-Correlation vs Convolution

These are nicer mathematically

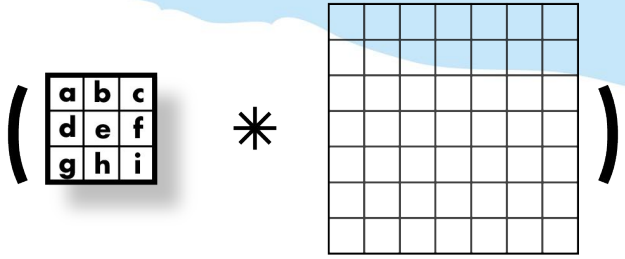
## Cross-Correlation

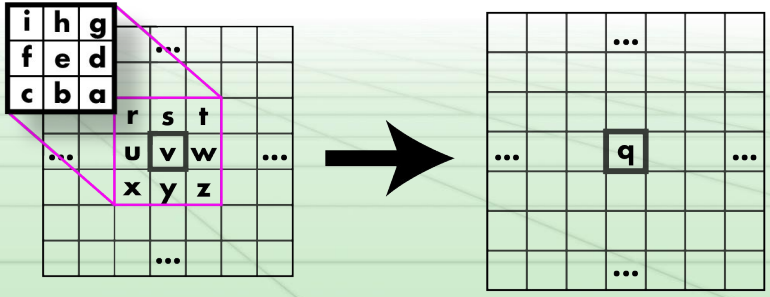



$$q = a \times r + b \times s + c \times t + d \times u + e \times v + f \times w + g \times x + h \times y + i \times z$$

## Convolution




$$q = i \times r + h \times s + g \times t + f \times u + e \times v + d \times w + c \times x + b \times y + a \times z$$

# So what can we do with these convolutions anyway?

Mathematically: all the nice things

- Commutative
  - A*B = B*A
- Associative
  - A*(B*C) = (A*B)*C
- Distributes over addition
  - A*(B+C) = A*B + A*C
- Plays well with scalars
  - x(A*B) = (xA)*B = A*(xB)

# So what can we do with these convolutions anyway?

This means some convolutions decompose:

- 2d gaussian is just composition of 1d gaussians
  - Faster to run 2 1d convolutions

# So what can we do with these convolutions anyway?

- Blurring
- Sharpening
- Edges
- Features
- Derivatives
- Super-resolution
- Classification
- Detection
- Image captioning
- ...

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All of computer vision
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(basically)

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All of computer vision
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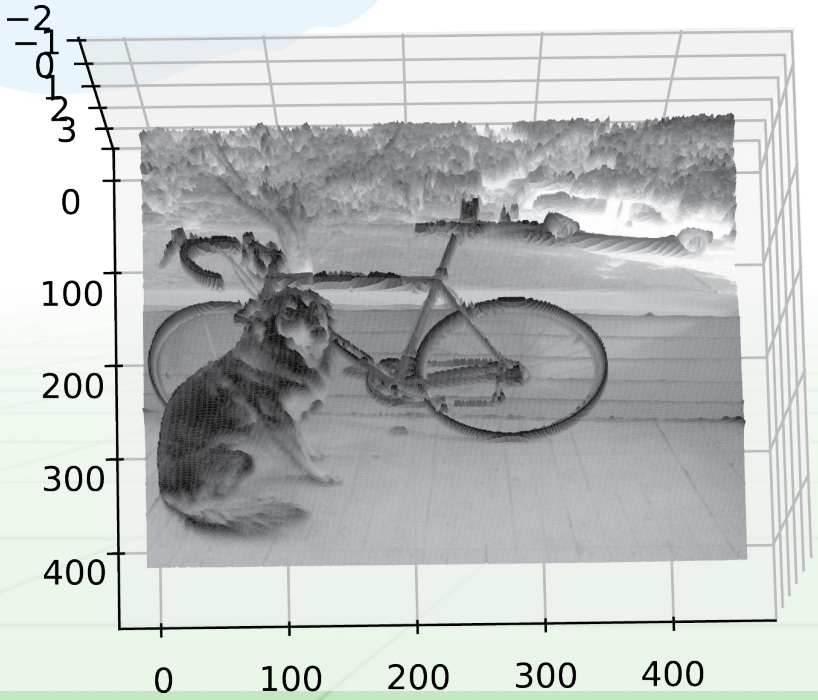
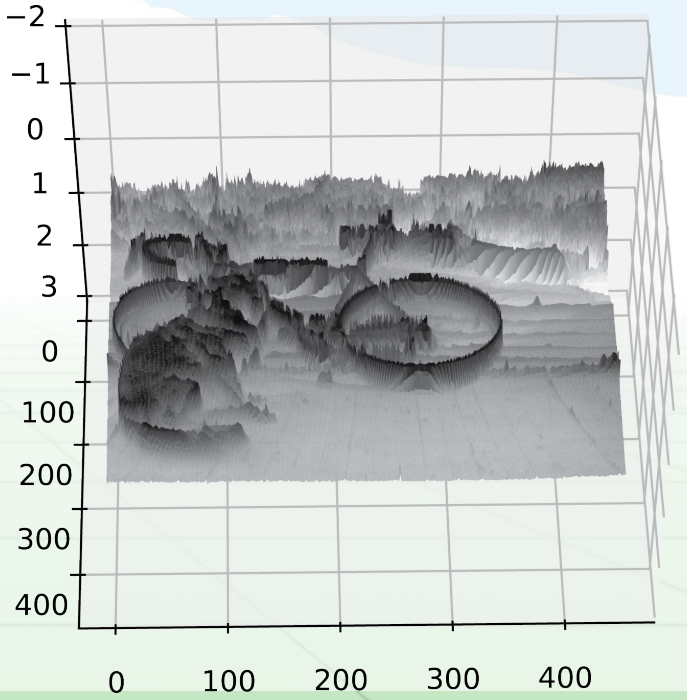
# What’s an edge?

- Image is a function
- Edges are rapid changes in this function



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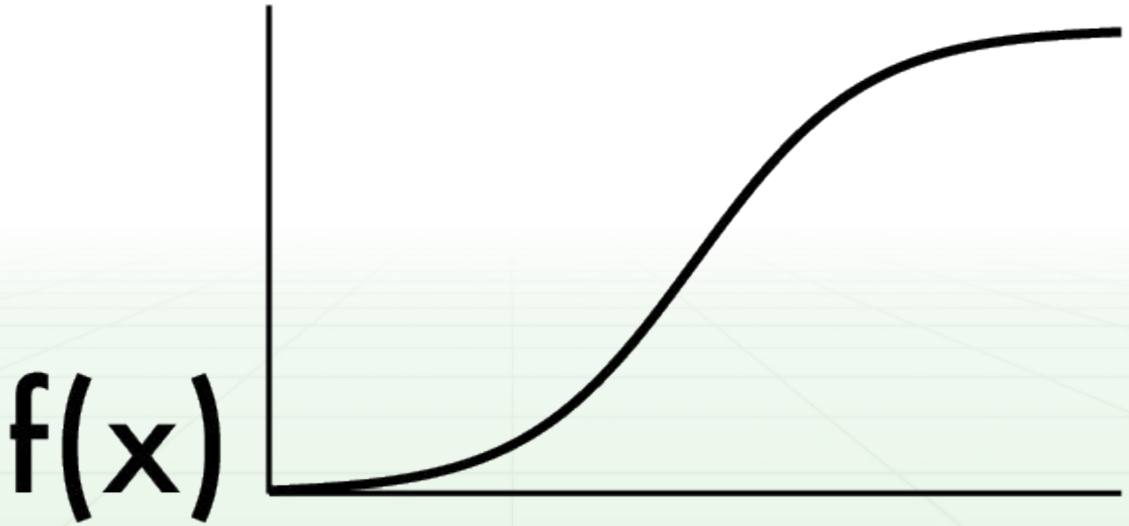
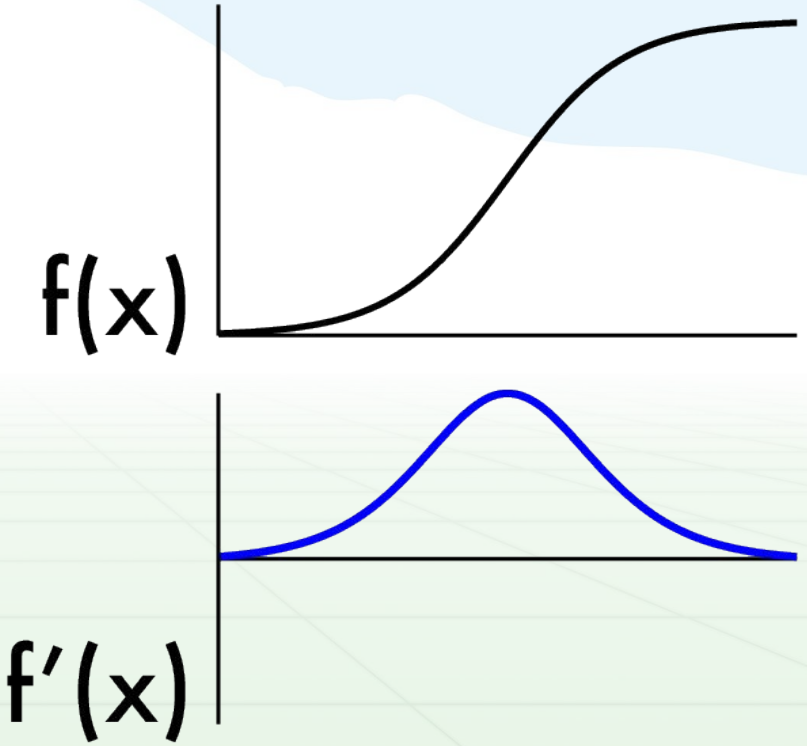


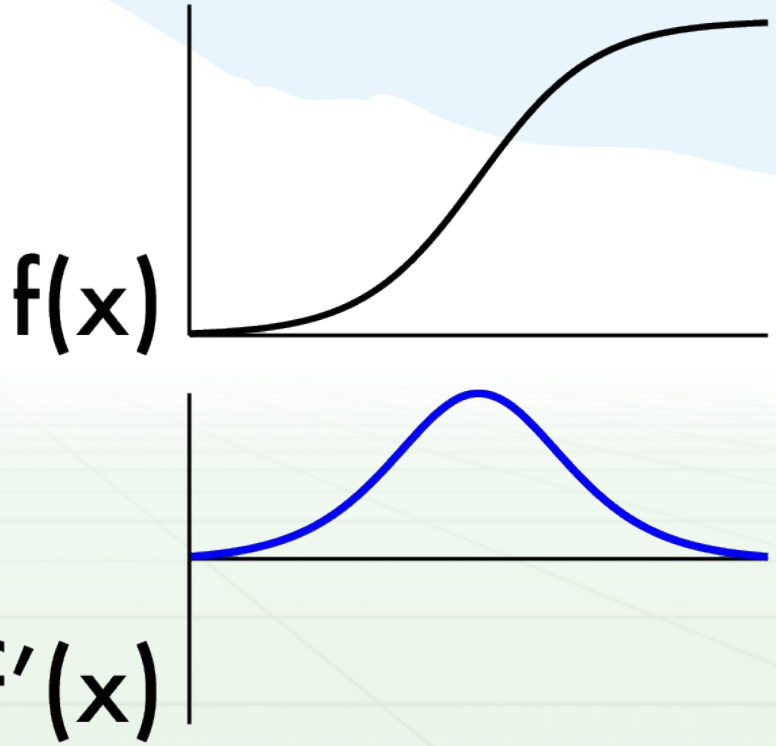
# Finding edges

- Could take derivative
- Edges = high response



# Image derivatives

- Recall:
  - $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$
- We don’t have an “actual” Function, must estimate
- Possibility: set h = 1
- What will that look like?



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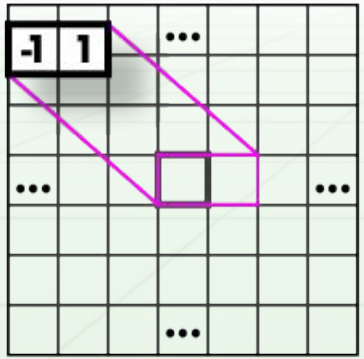


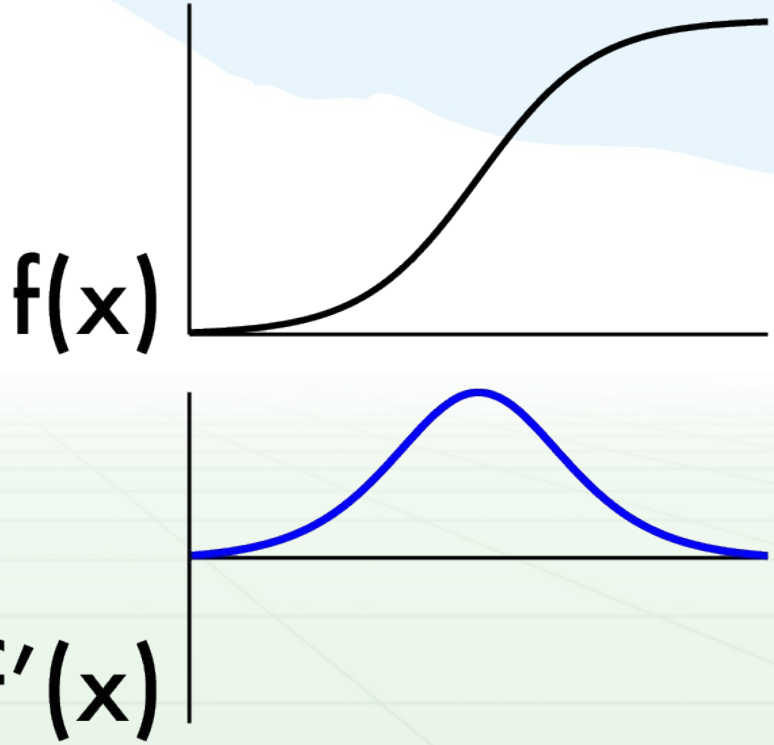

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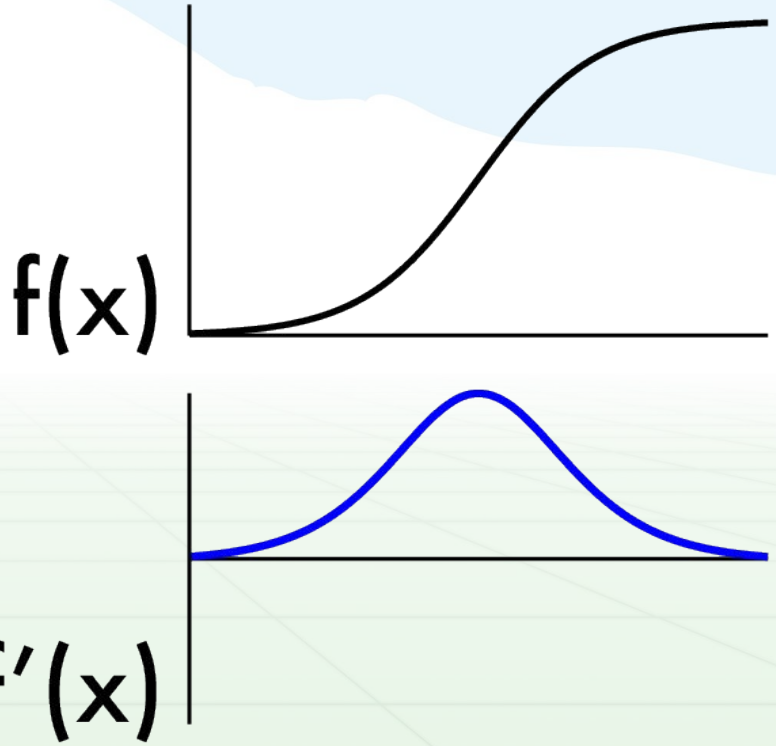
- Recall:
  - $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$
- We don’t have an “actual” Function, must estimate
- Possibility: set h = 2
- What will that look like?



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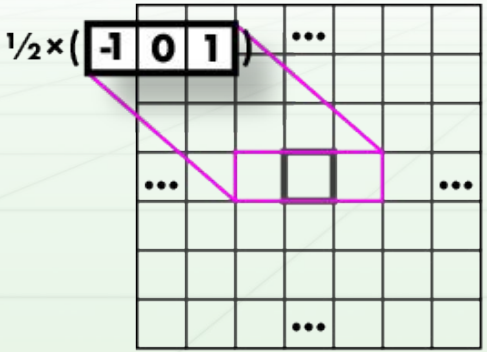
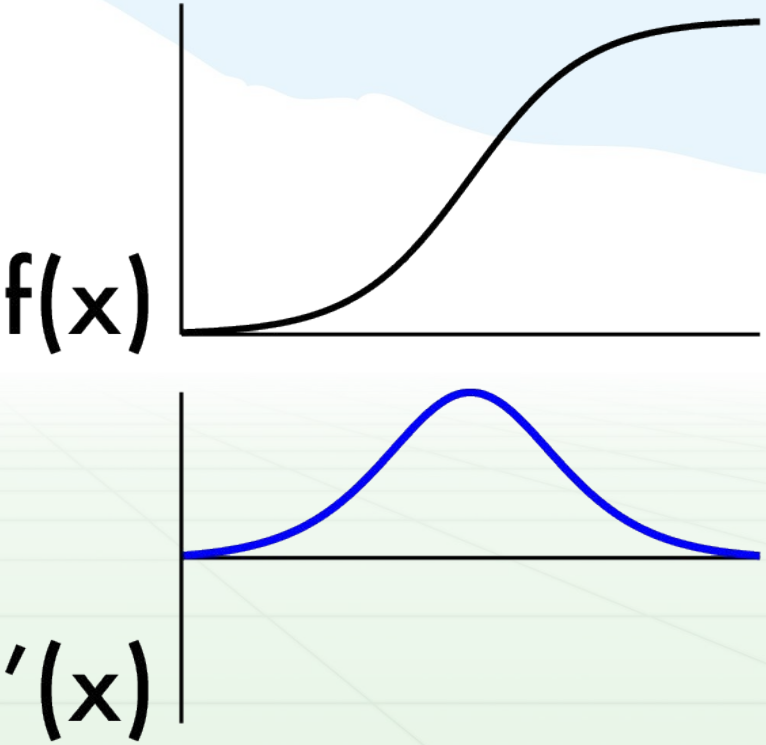
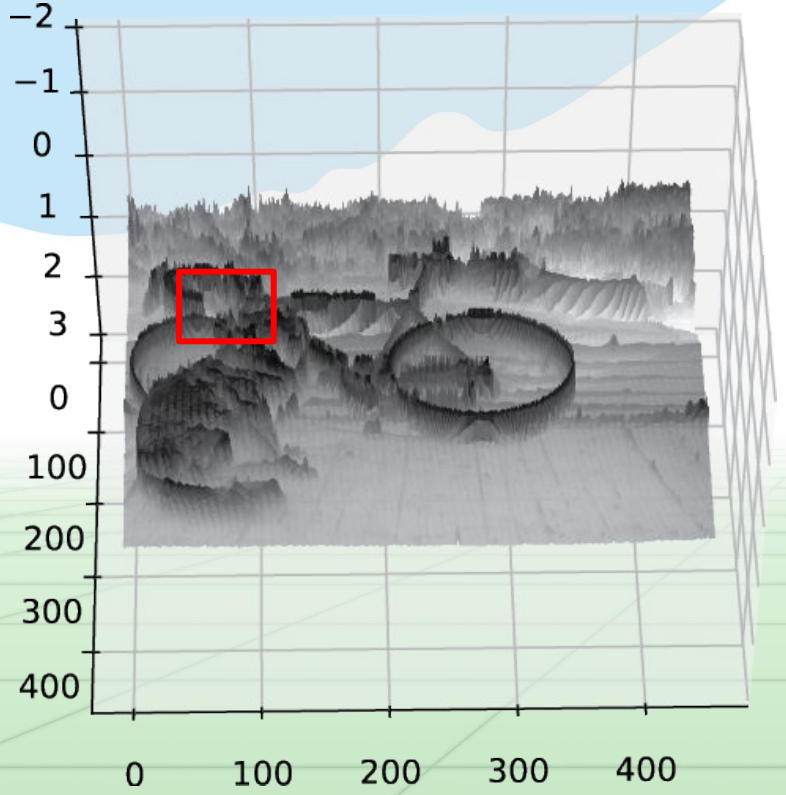
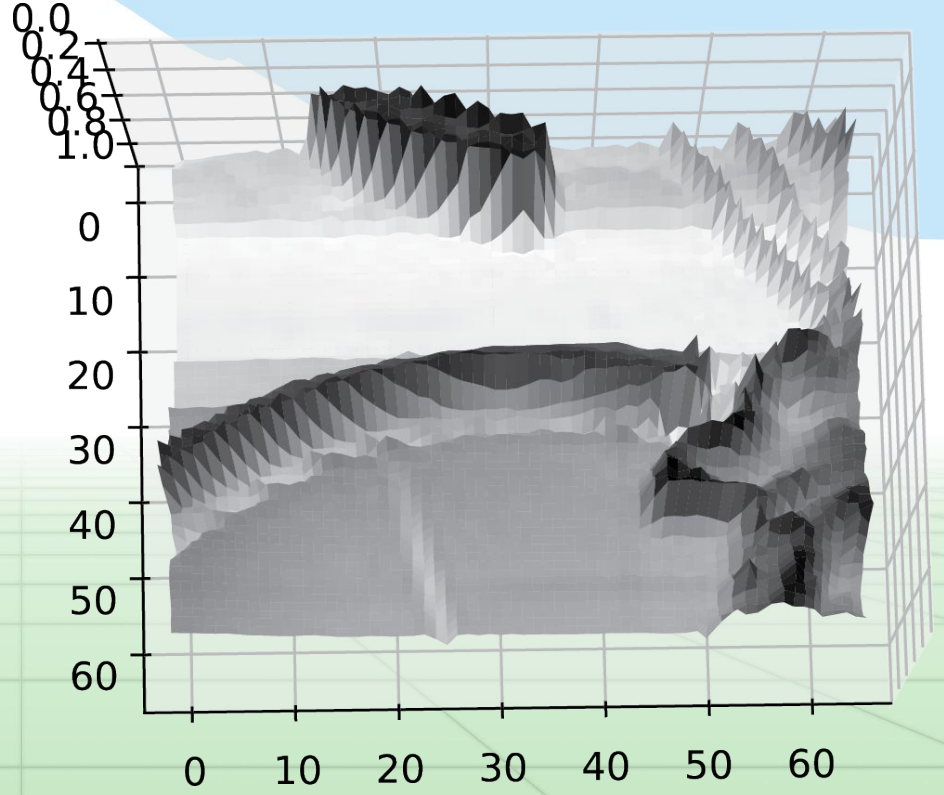
- Possibility: set h = 2
- What will that look like?



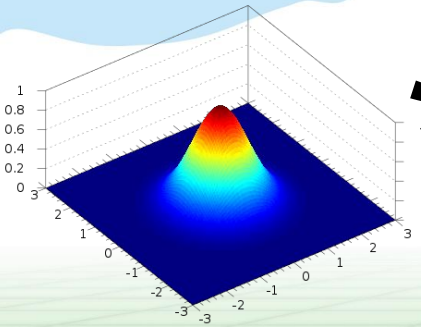



# Images are noisy!

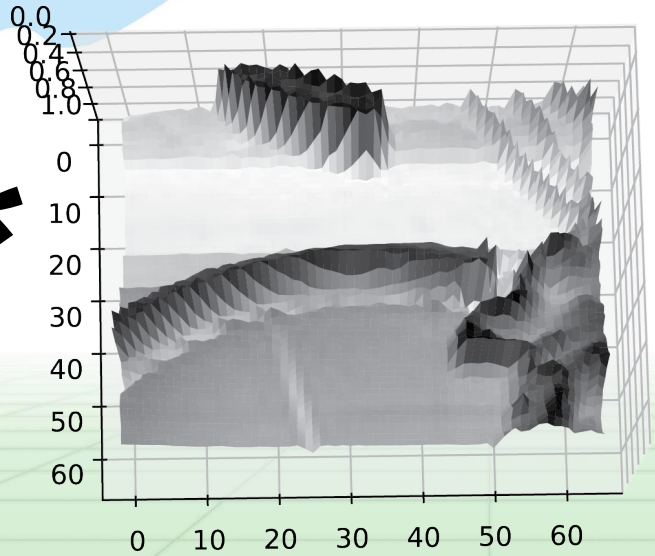


# But we already know how to smooth




*

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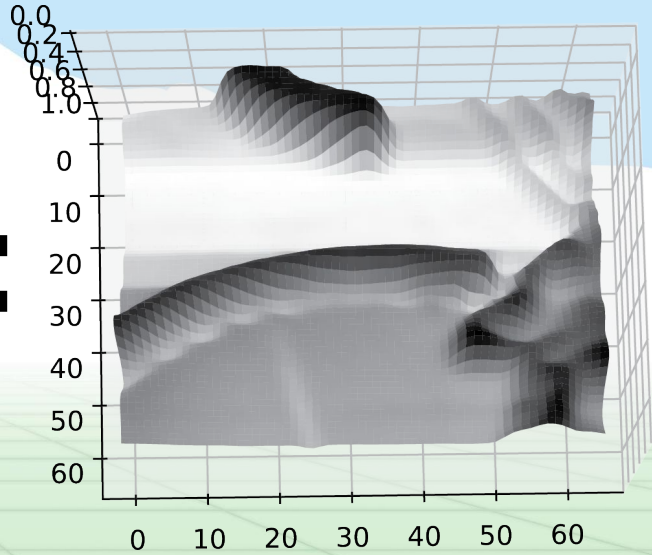
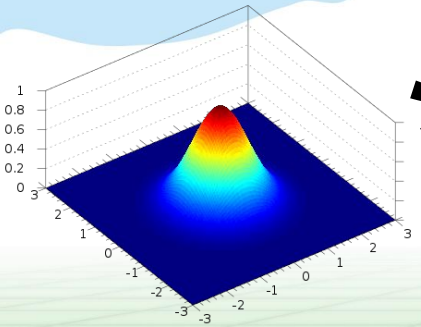


# But we already know how to smooth

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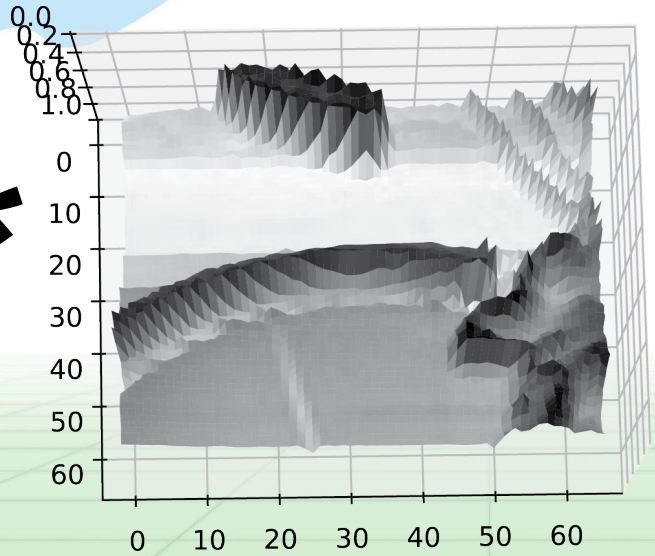


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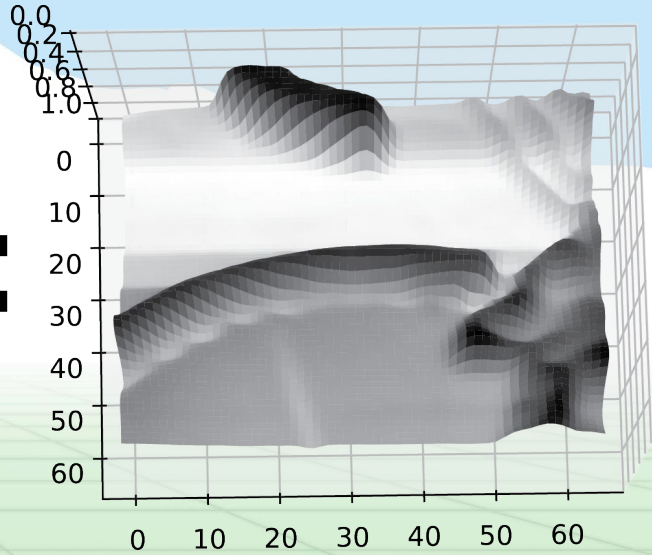
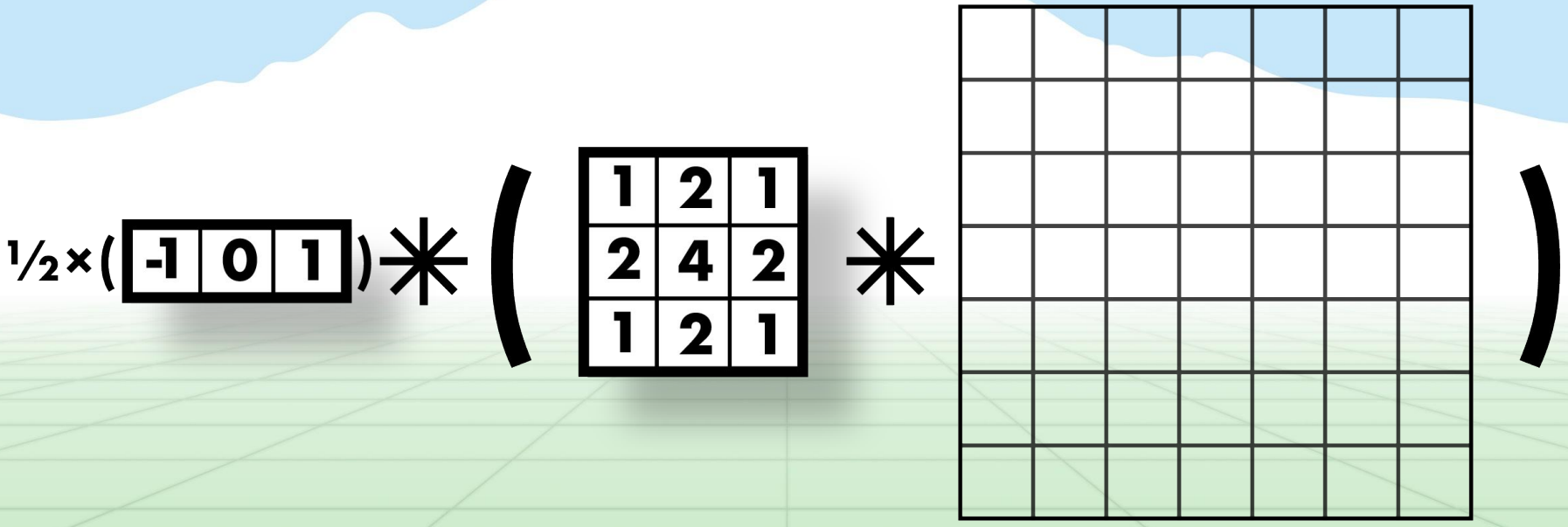
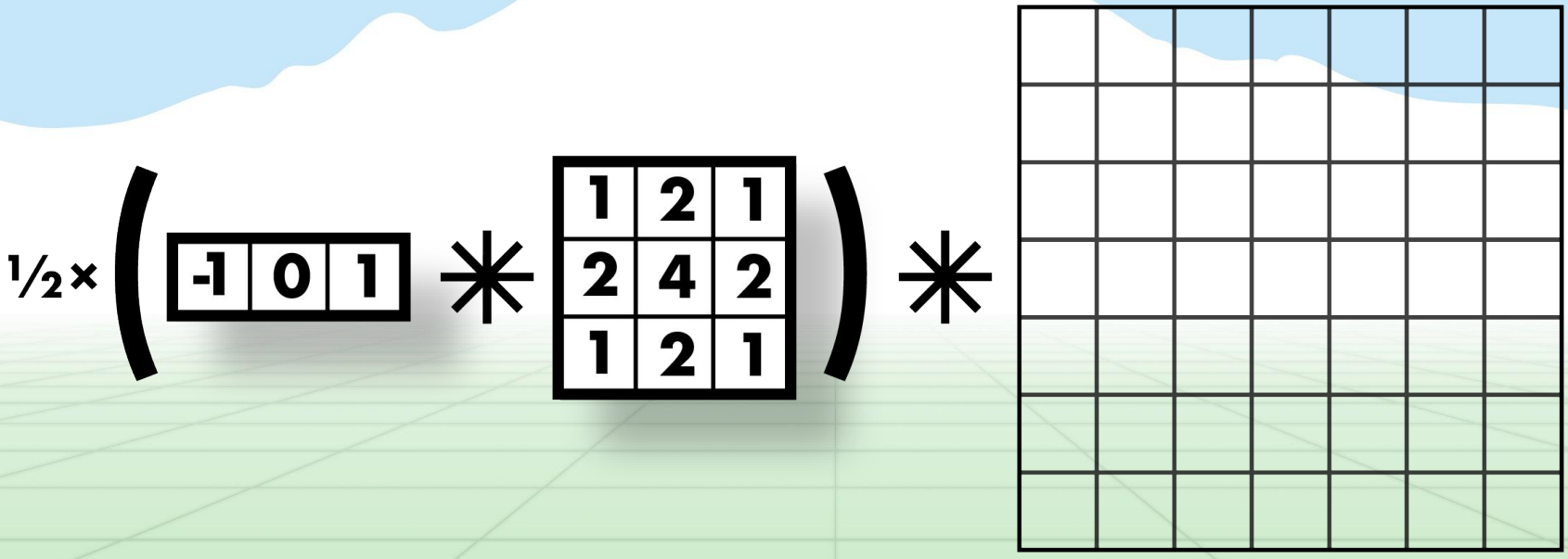


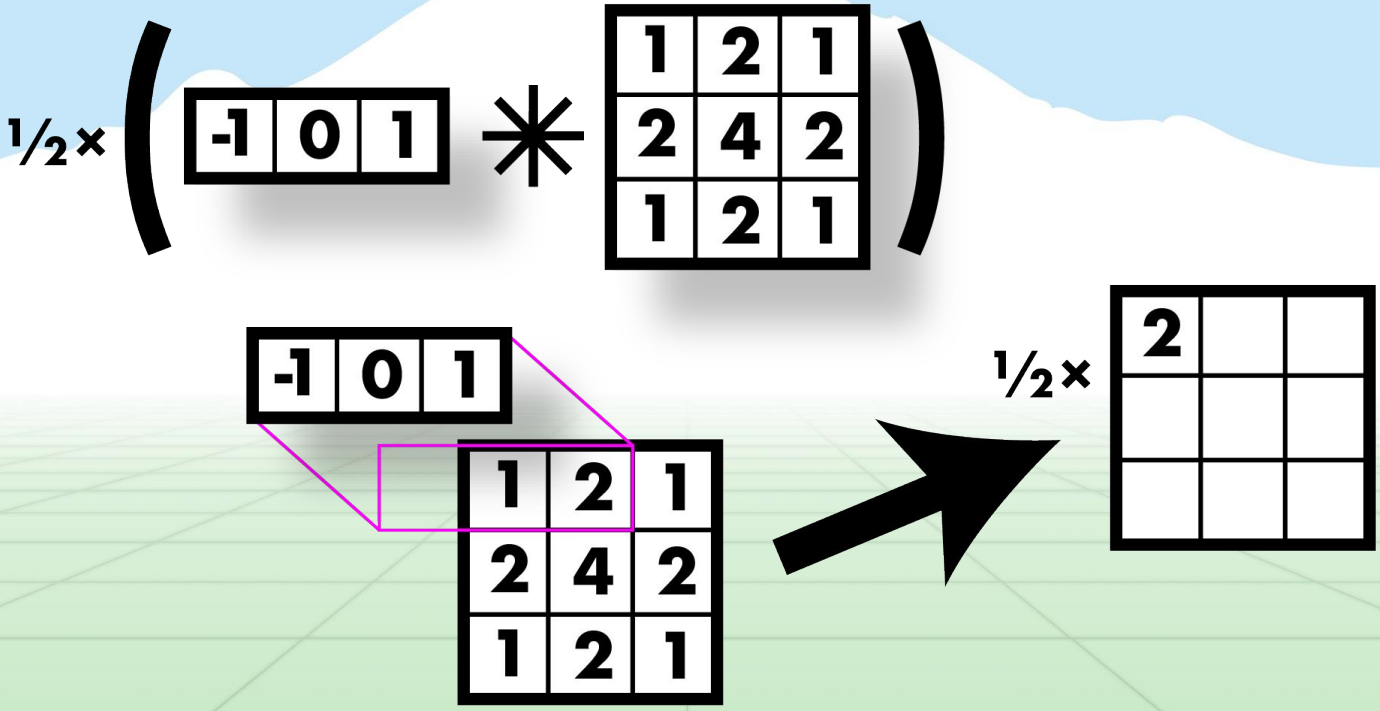
# Smooth first, then derivative



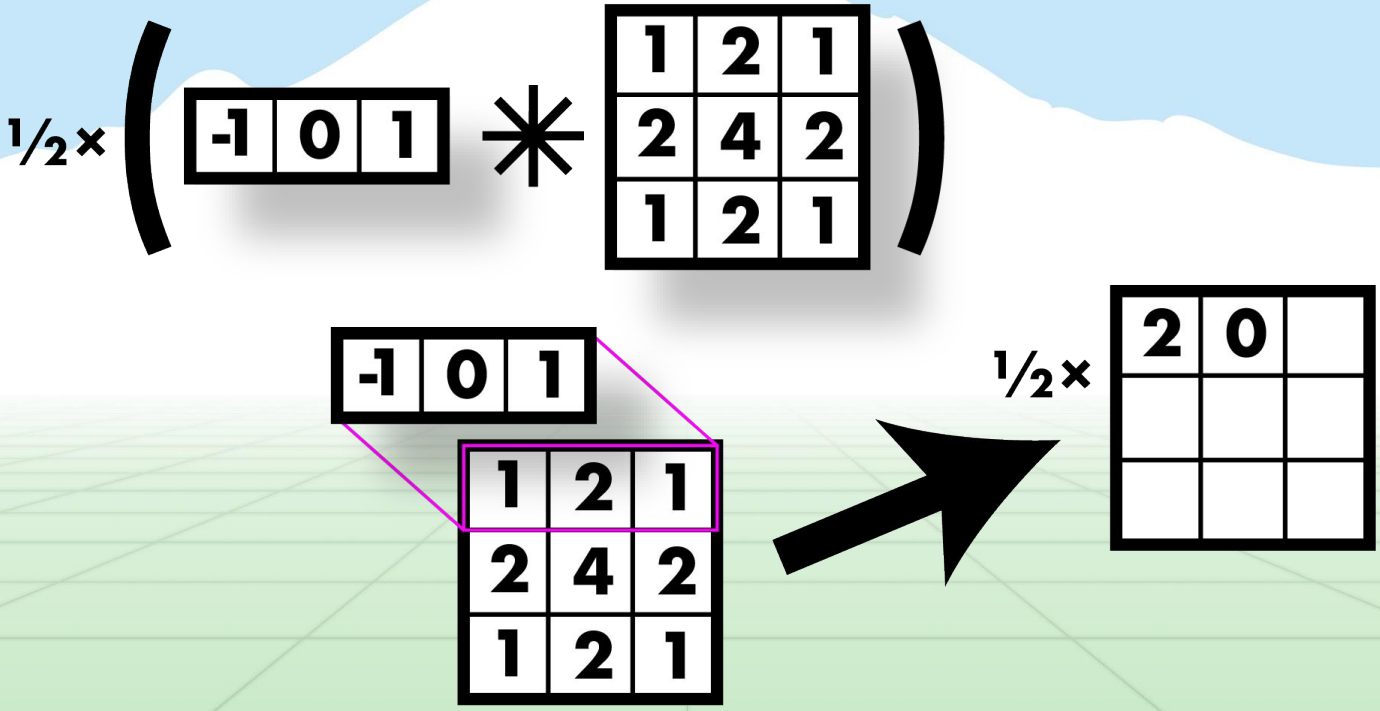
# Smooth first, then derivative



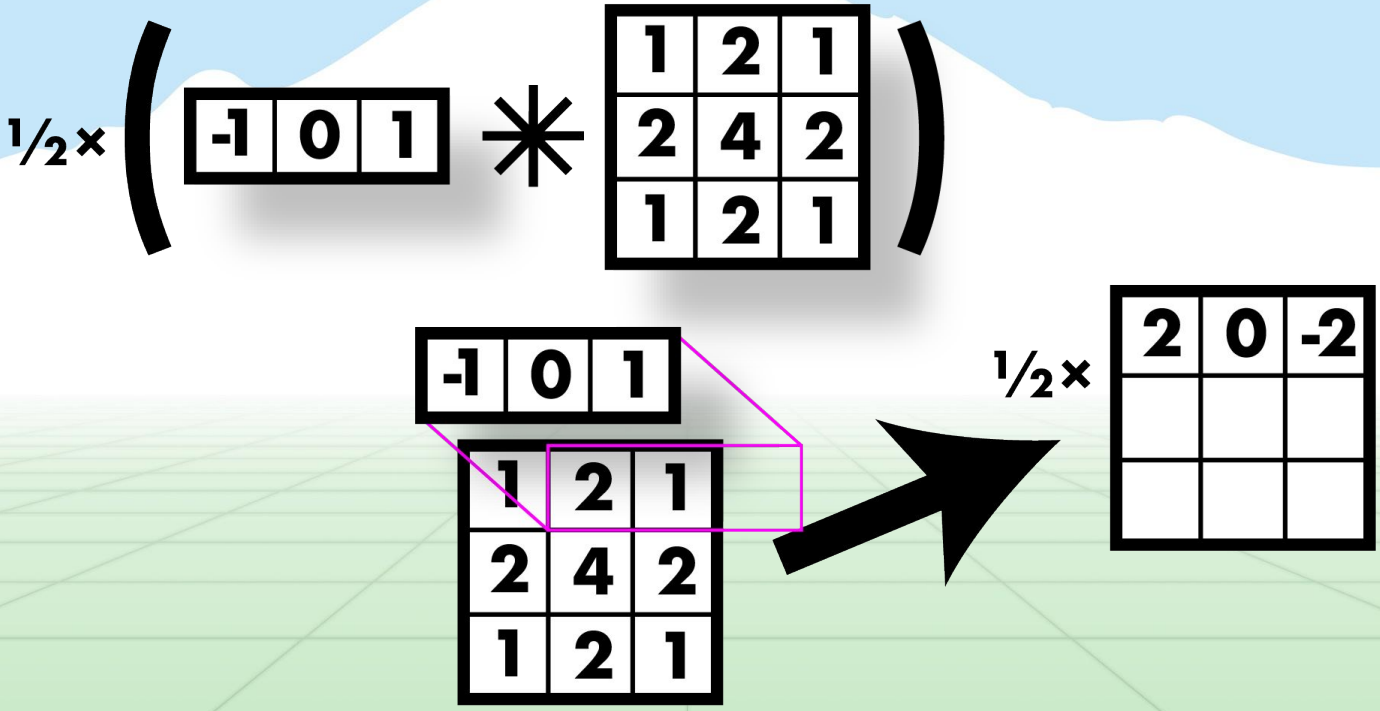
# Smooth first, then derivative



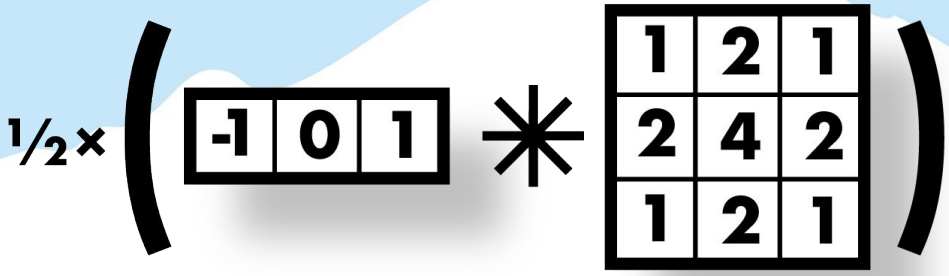
# Smooth first, then derivative

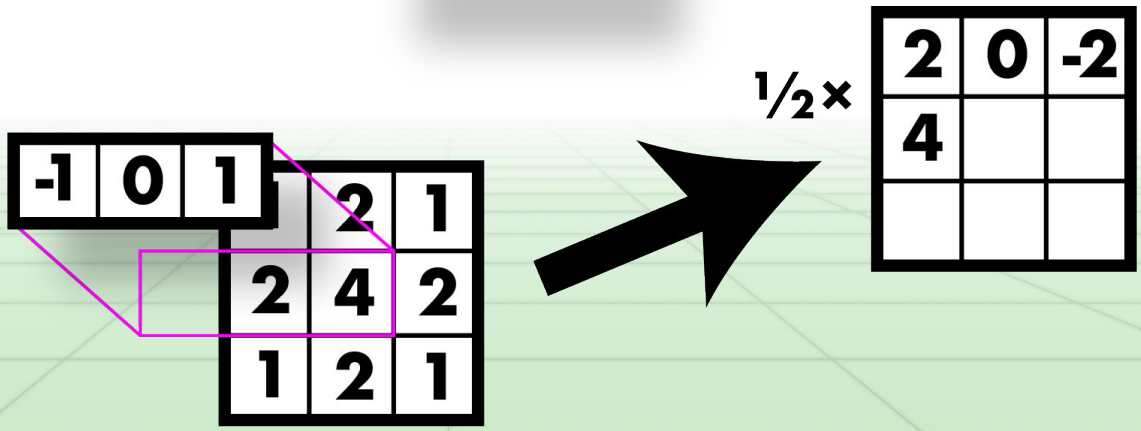


# Smooth first, then derivative



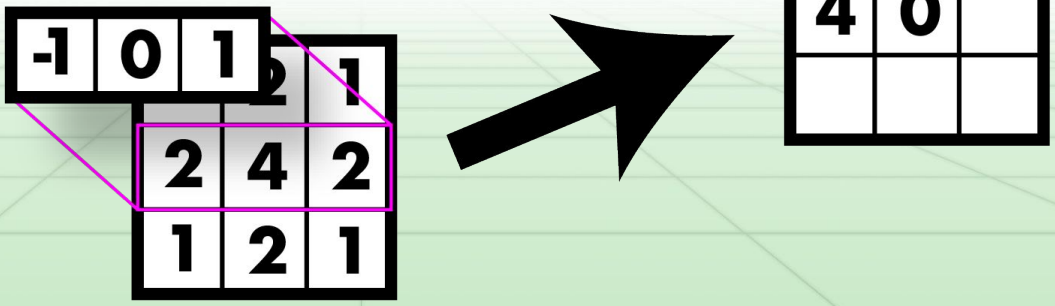
# Smooth first, then derivative



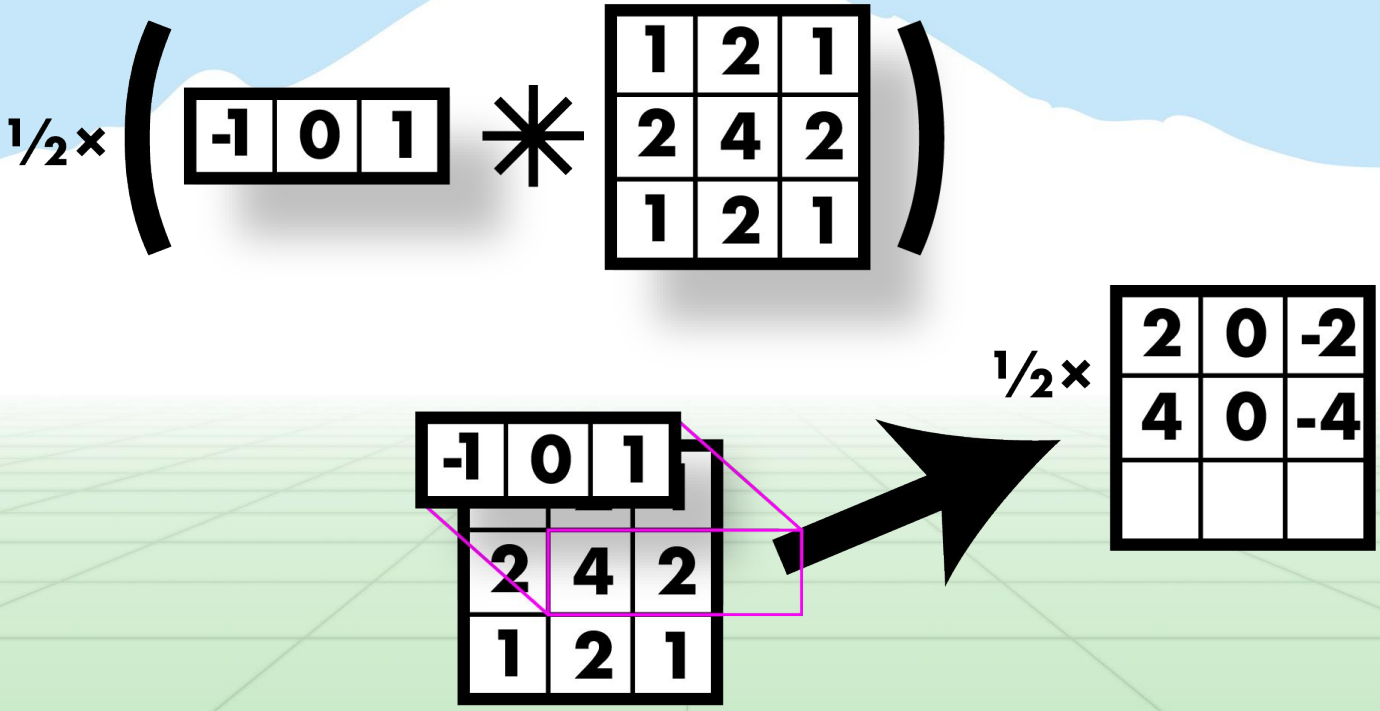



# Smooth first, then derivative

$$\tfrac{1}{2} \times \left( \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right)$$

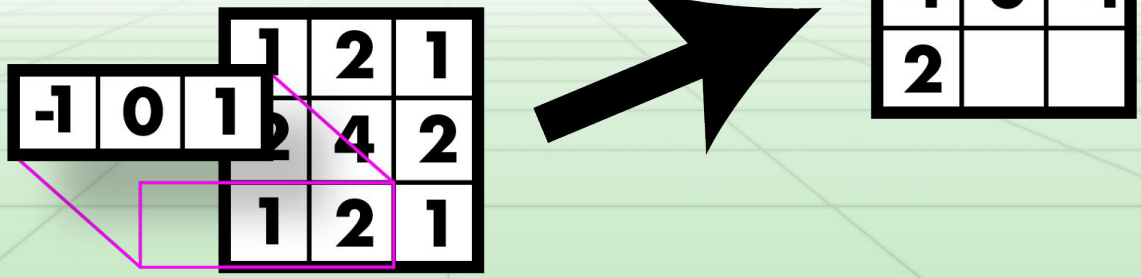


# Smooth first, then derivative



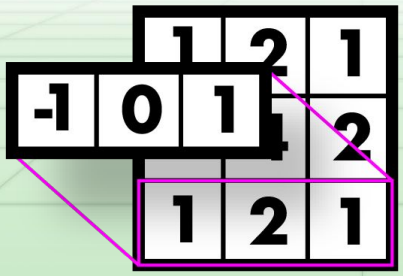
# Smooth first, then derivative

$$\tfrac{1}{2} \times \left( \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right)$$



# Smooth first, then derivative

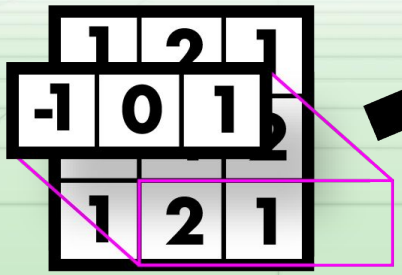
$$\tfrac{1}{2} \times \left( \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right)$$




$$\tfrac{1}{2} \times \begin{bmatrix} 2 & 0 & -2 \\ 4 & 0 & -4 \\ 2 & 0 & \end{bmatrix}$$

# Smooth first, then derivative

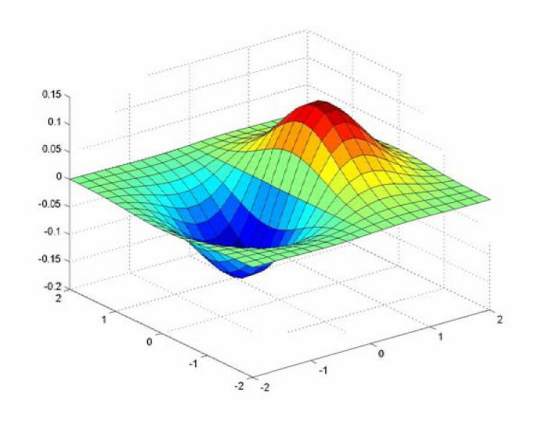
$$\tfrac{1}{2} \times \left( \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right)$$

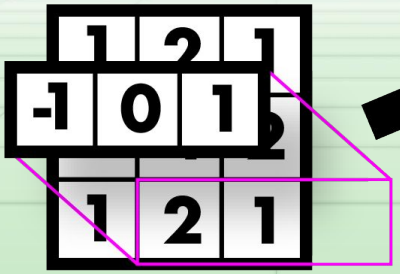



$$\tfrac{1}{2} \times \begin{bmatrix} 2 & 0 & -2 \\ 4 & 0 & -4 \\ 2 & 0 & -2 \end{bmatrix}$$

# Sobel filter! Smooth & derivative

$$\tfrac{1}{2} \times \left( \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right)$$

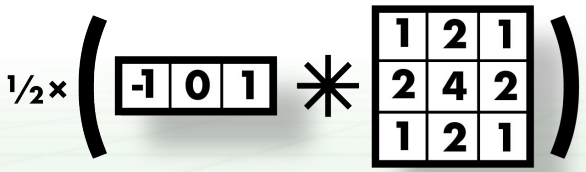


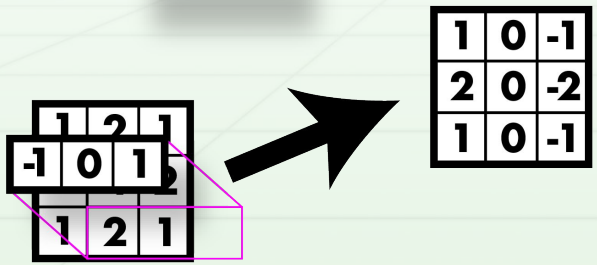


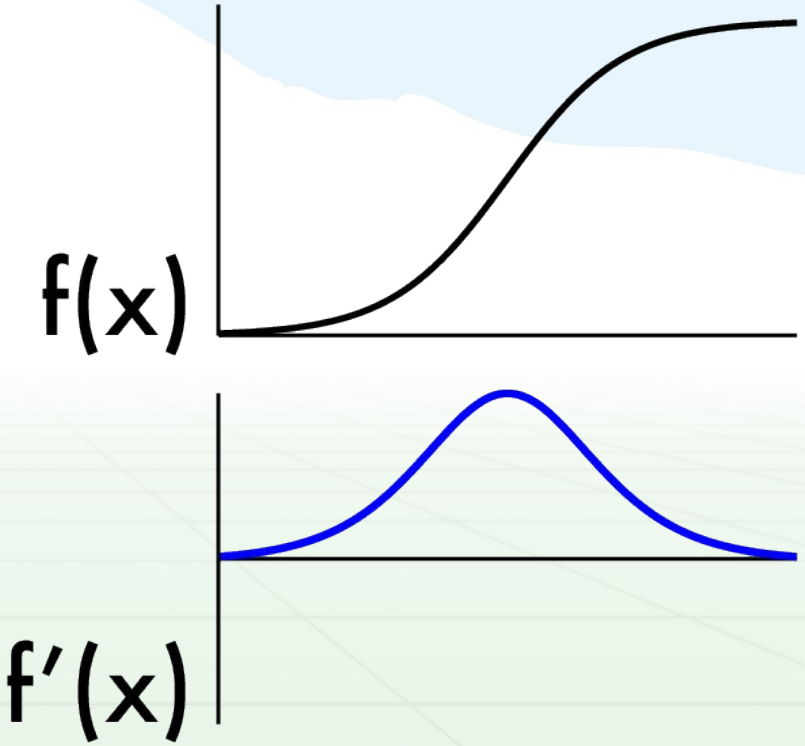

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

# Image derivatives

- Recall:
  - $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$
- Want smoothing too!

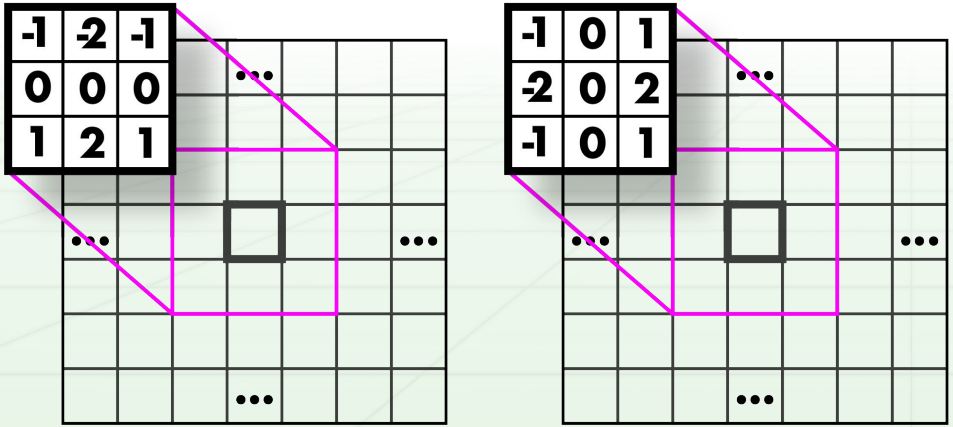


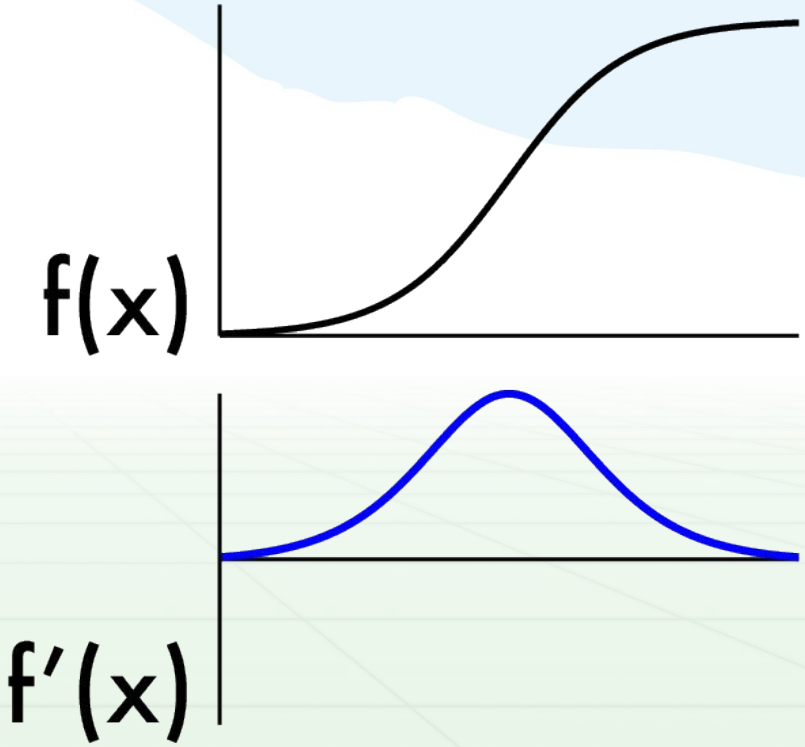



# Finding edges

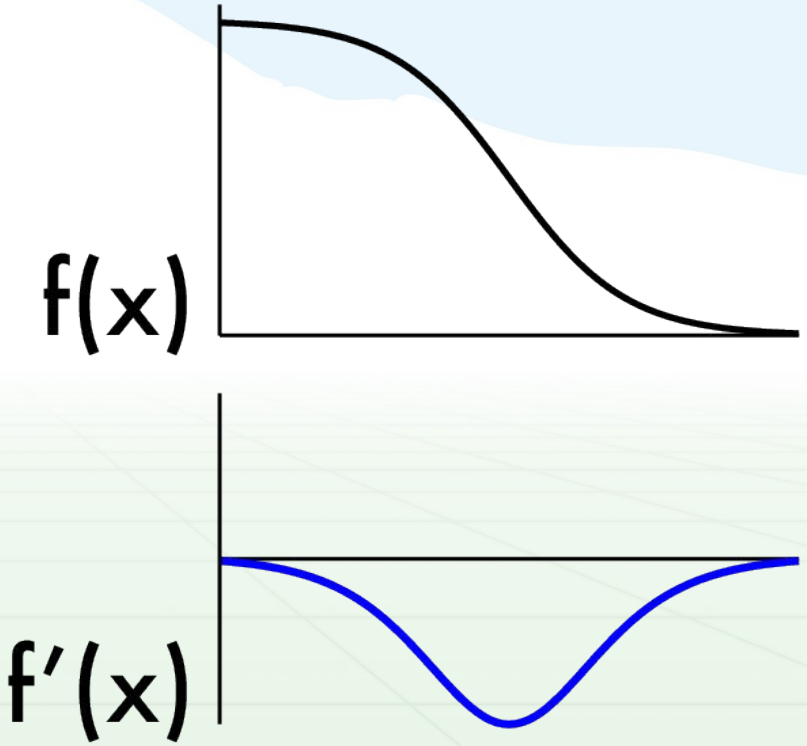
- Could take derivative
- Find high responses
- Sobel filters!
- But…



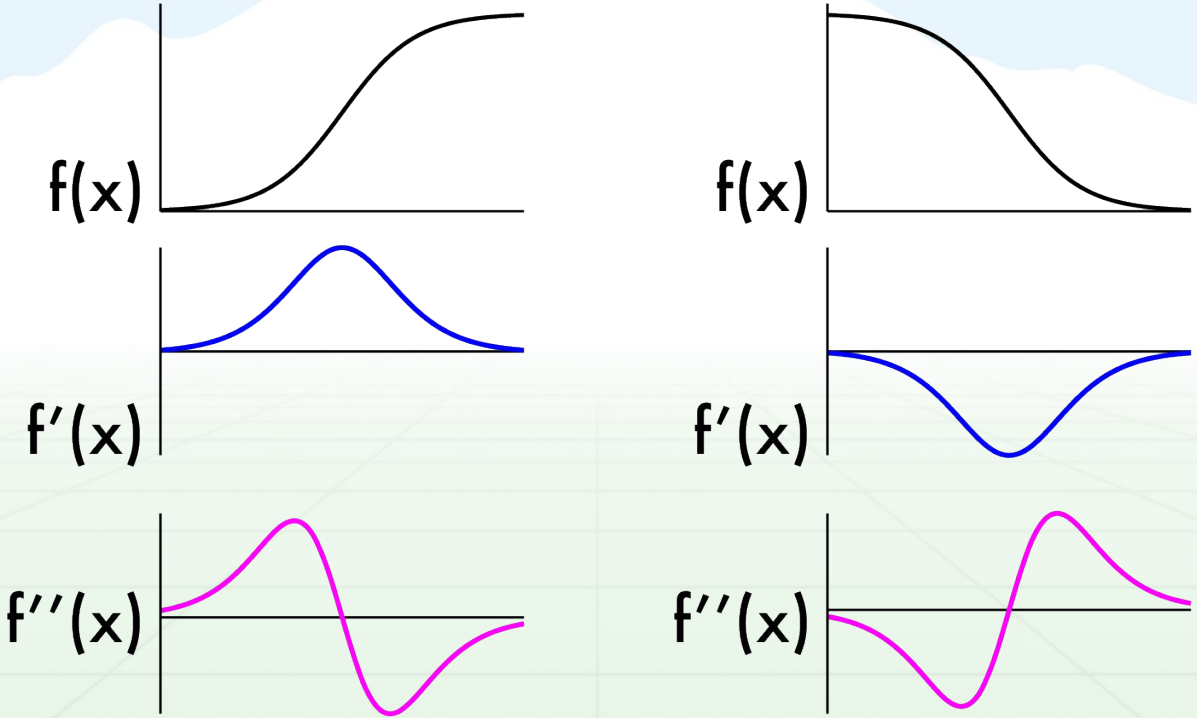
# Finding edges

- Could take derivative
- Find high responses
- Sobel filters!
- But…
- Edges go both ways
- Want to find extrema



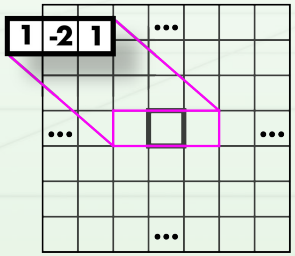
# 2nd derivative!

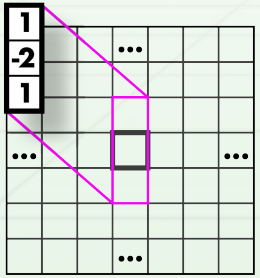
- Crosses zero at extrema

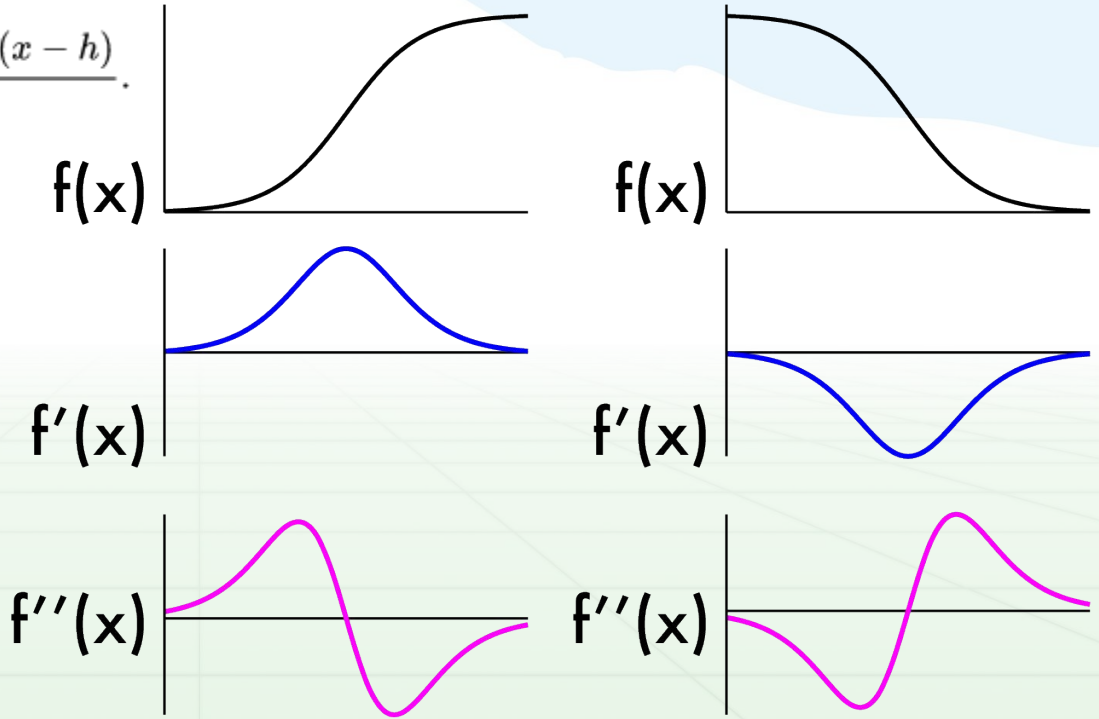


# Laplacian (2nd derivative)!

- Crosses zero at extrema
- Recall:
  - $f''(x) = \lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$
- Laplacian:
  - $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$
- Again, have to estimate f’’(x):



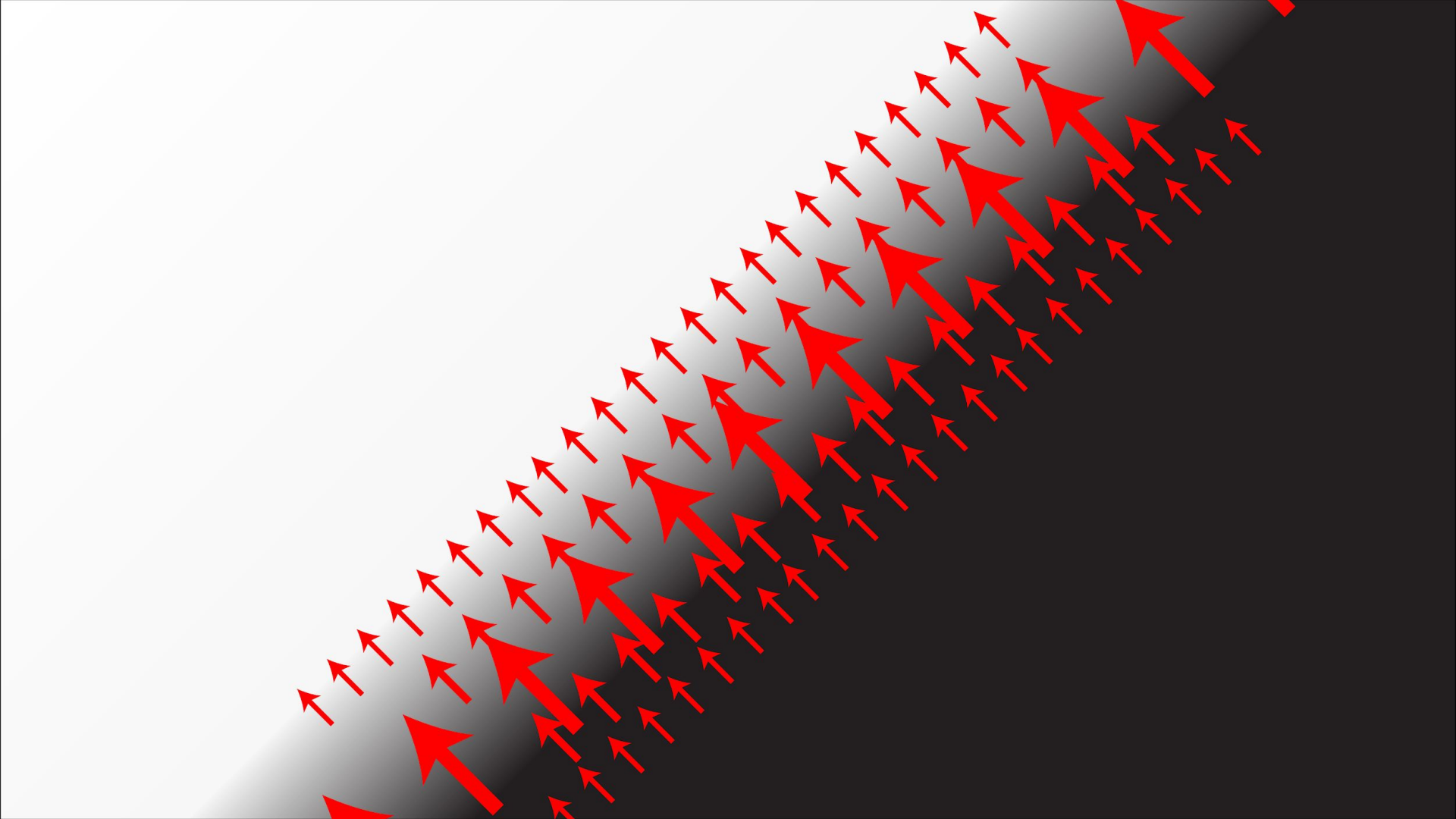
# Laplacians

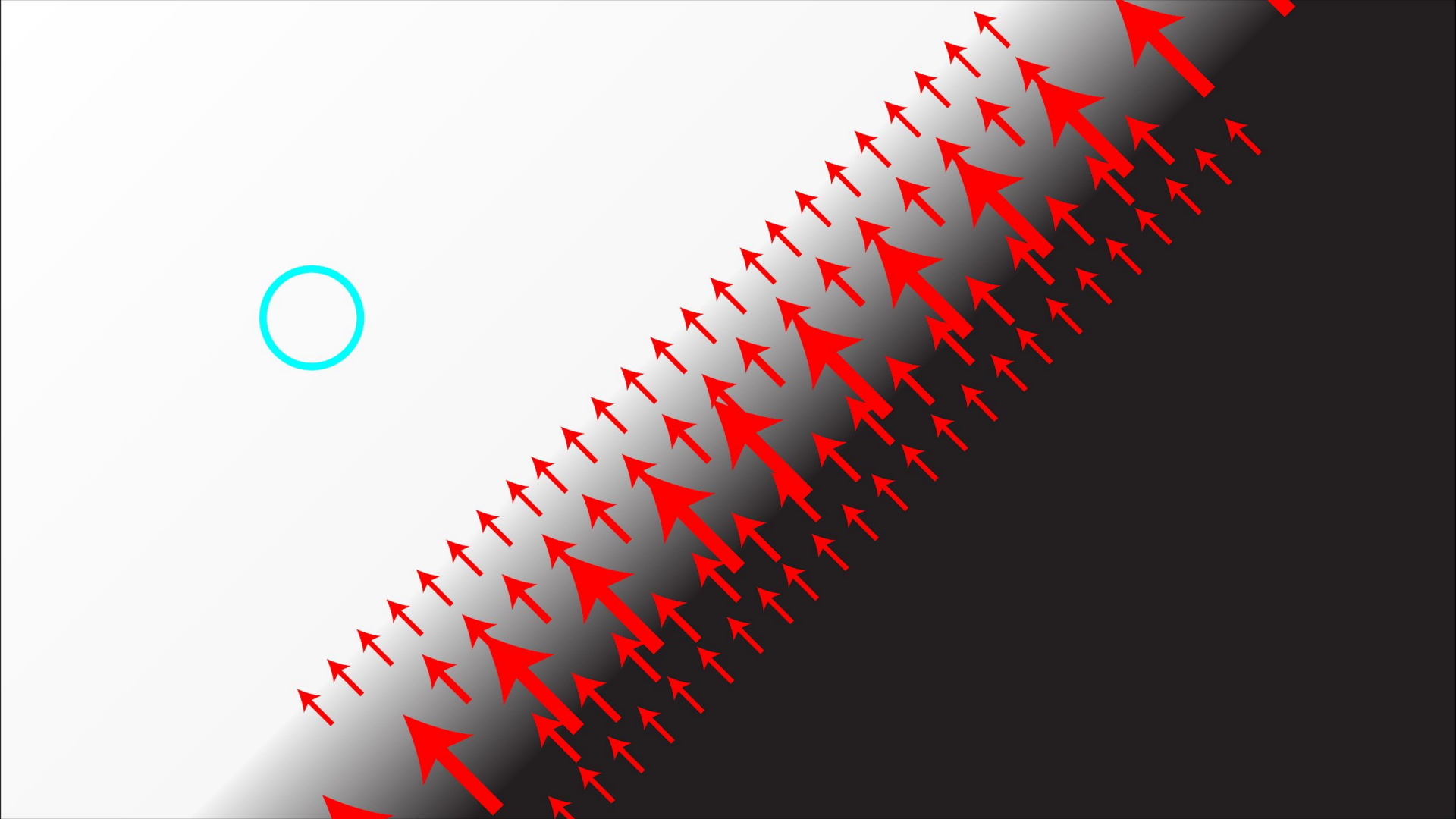
- Laplacian:
  - $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

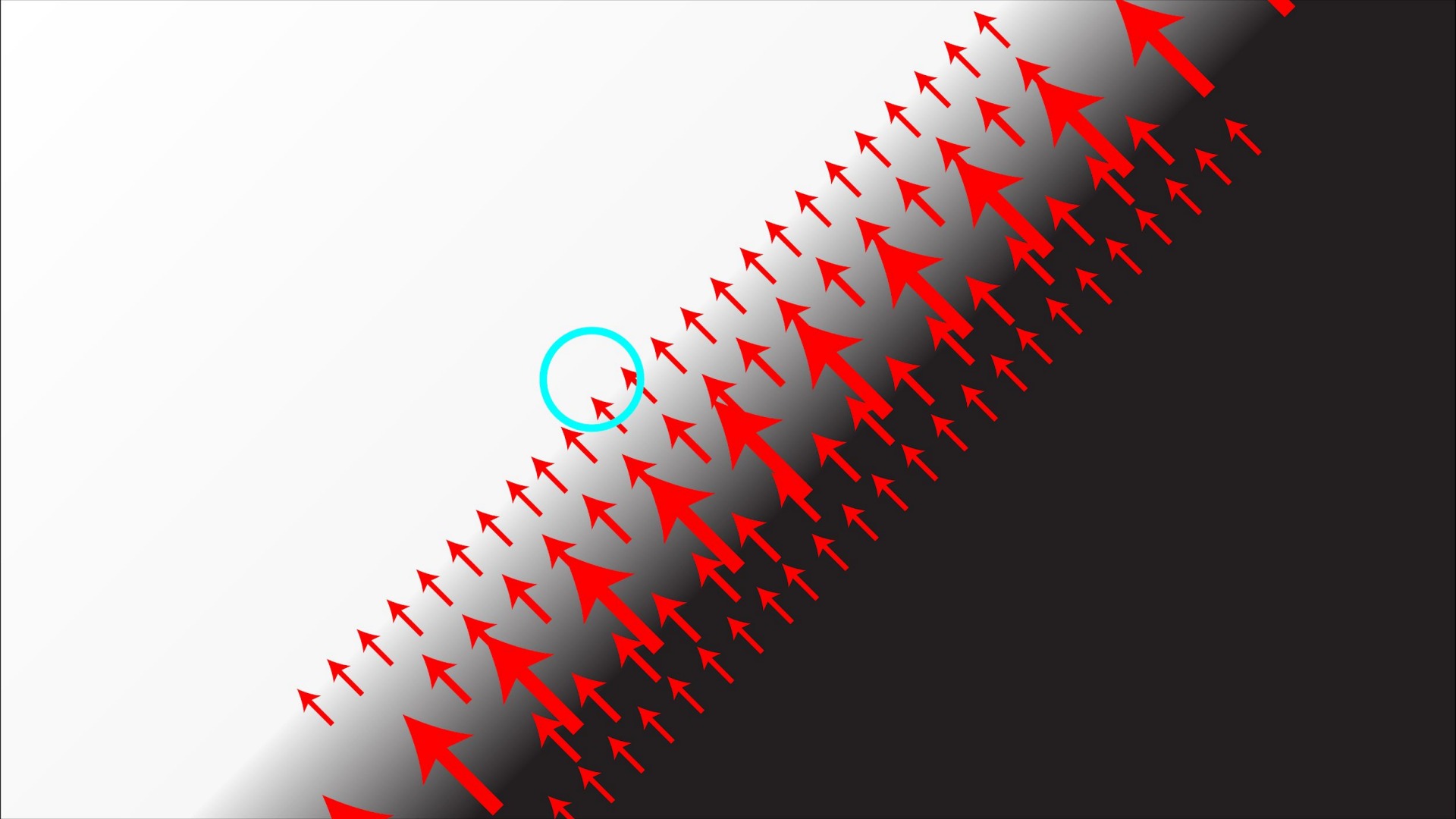
# Laplacians

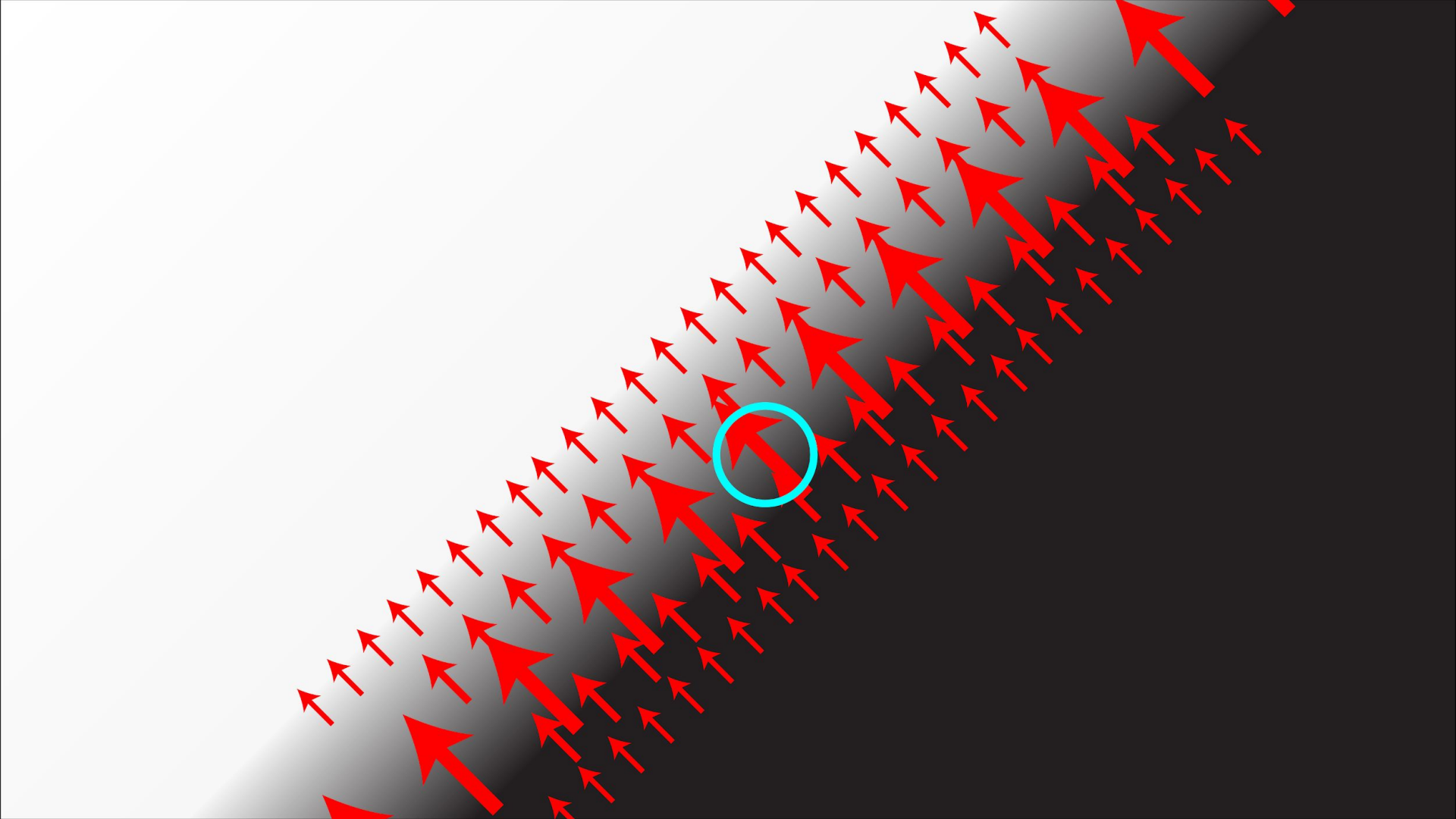
- Laplacian:
  - $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

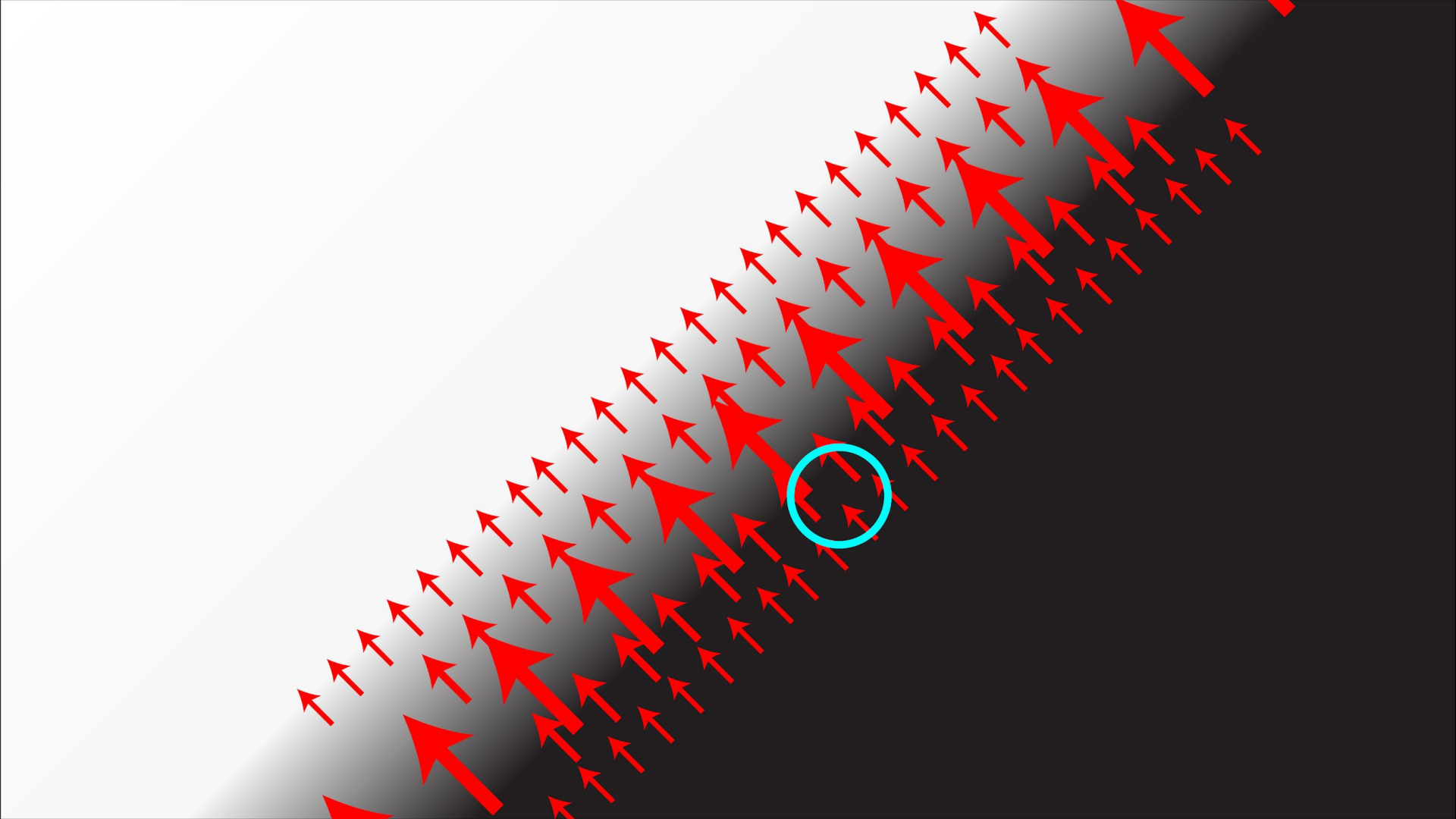
- Measures the divergence of the gradient
  - Flux of gradient vector field through small area





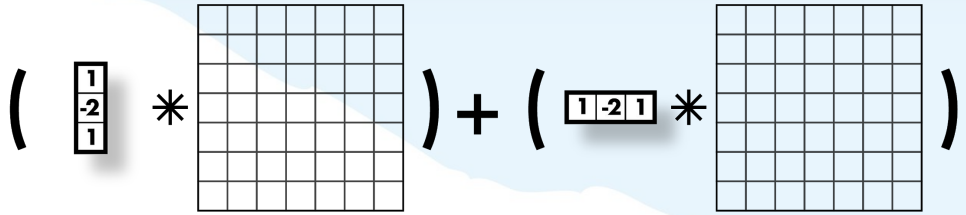






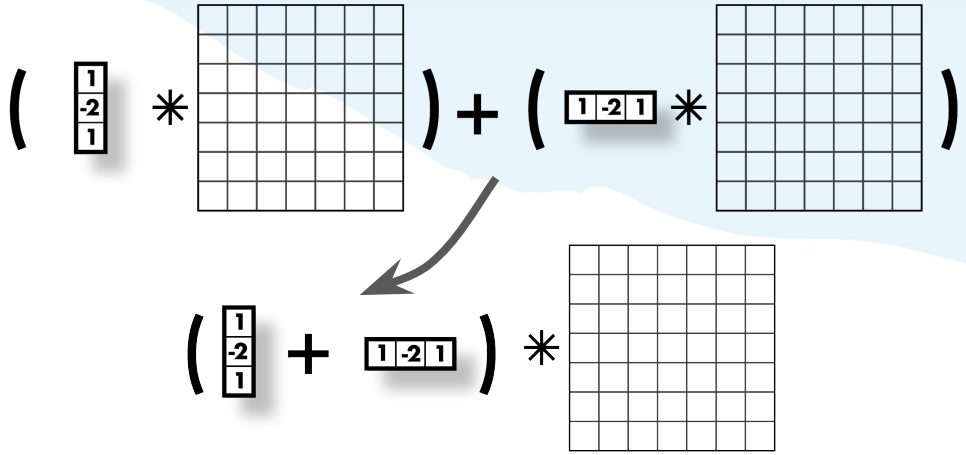
# Laplacians

- Laplacian:
  - $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$



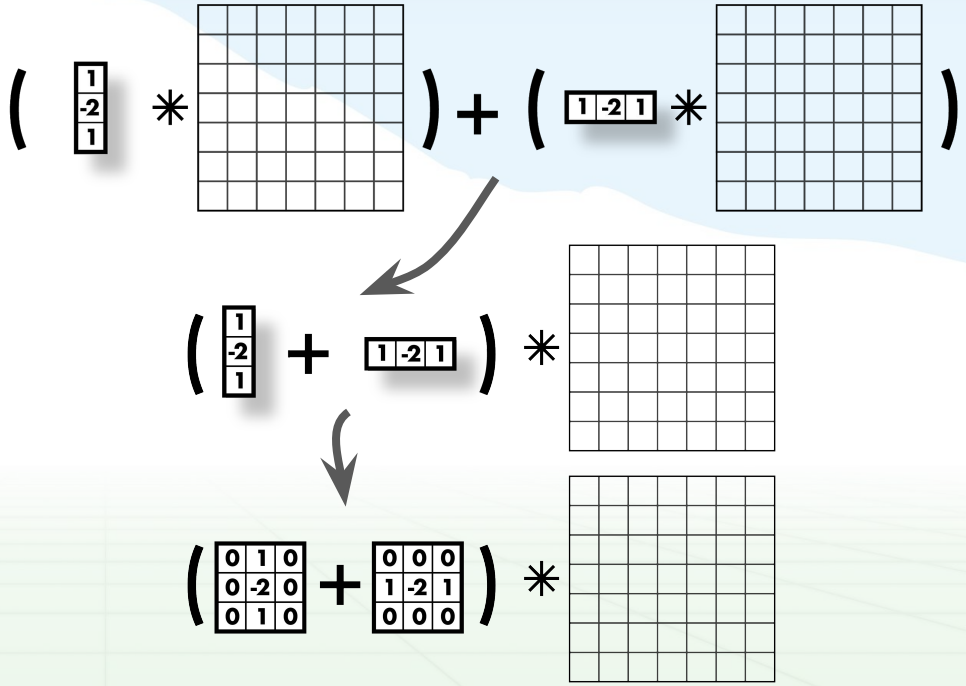
# Laplacians

- Laplacian:
  - $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$



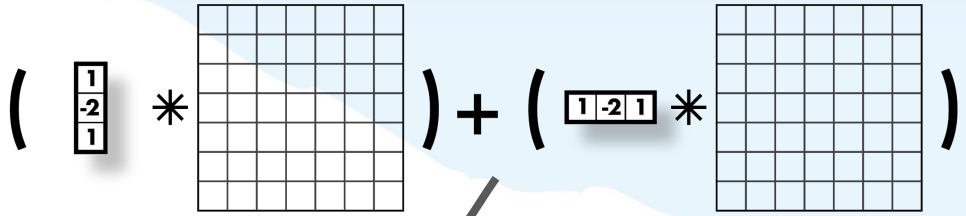
# Laplacians

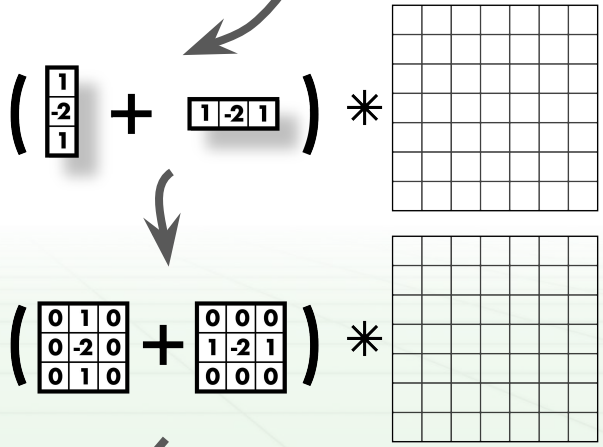
- Laplacian:
  - $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

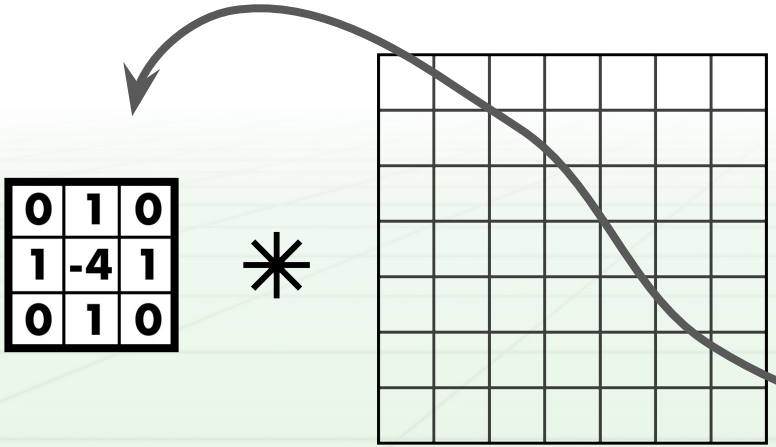


# Laplacians

- Laplacian:
  - $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$



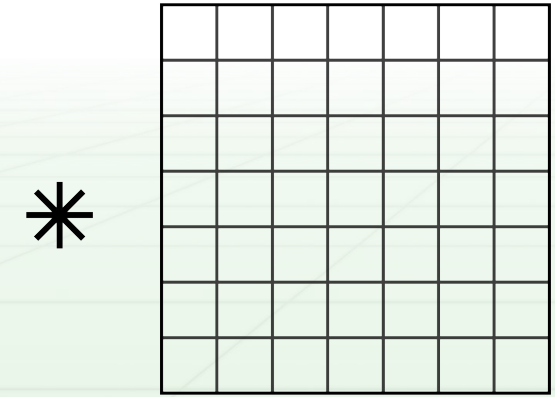



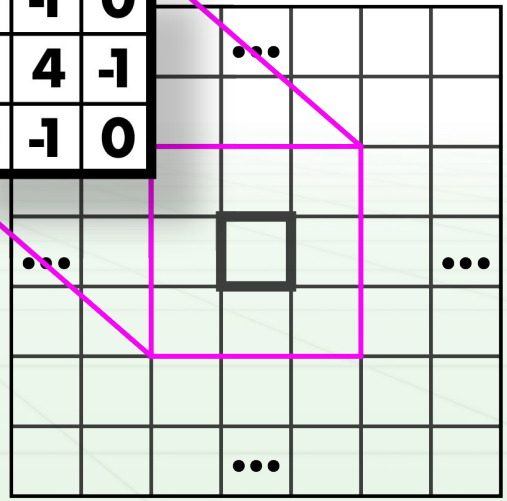
# Laplacians

- Laplacian:
  - $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$
- Negative Laplacian, -4 in middle
- Positive Laplacian  --->

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | -4 | 1 |
| 0 | 1 | 0 |

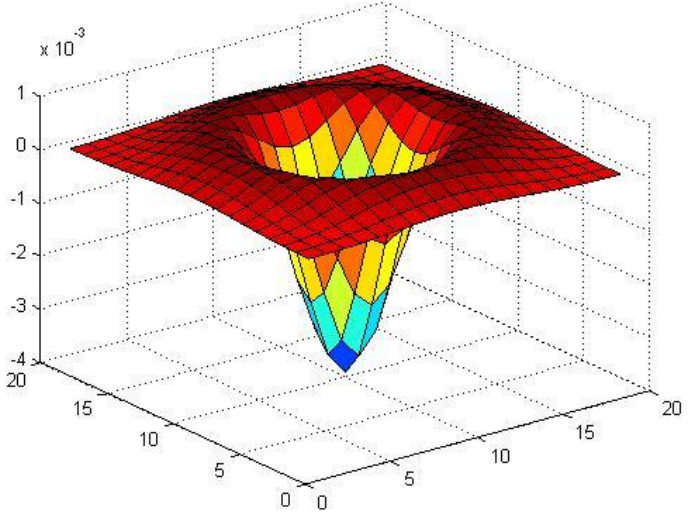


| | | |
|---|---|---|
| 0 | -1 | 0 |
| -1 | 4 | -1 |
| 0 | -1 | 0 |



# Laplacians also sensitive to noise

- Again, use gaussian smoothing
- Can just use one kernel since convs commute
- Laplacian of Gaussian, LoG
- Can get good approx. with
  5x5 - 9x9 kernels



# Another edge detector:

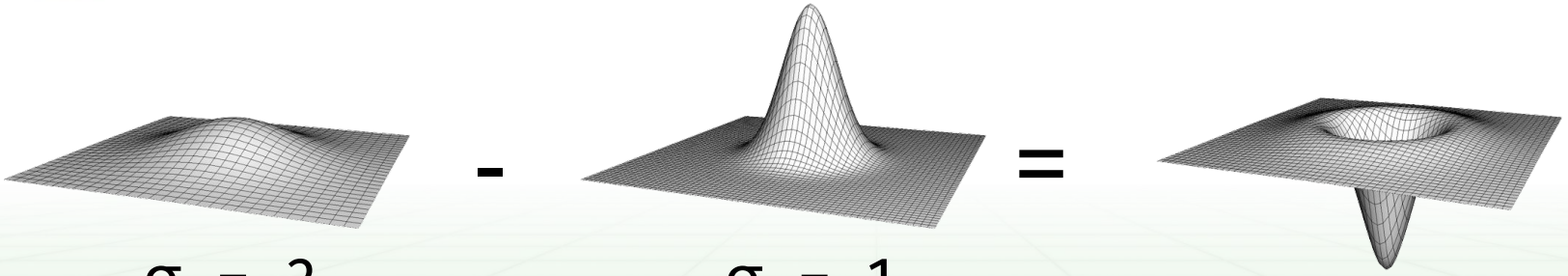
- Image is a function:
  - Has high frequency and low frequency components
  - Think in terms of fourier transform
- Edges are high frequency changes
- Maybe we want to find edges of a specific size (i.e. specific frequency)

# Difference of Gaussian (DoG)

- Gaussian is a low pass filter
- Strongly reduce components with frequency $f < \sigma$
- $(g*I)$ low frequency components
- $I - (g*I)$ high frequency components
- $g(\sigma1)*I - g(\sigma2)*I$
  - Components in between these frequencies
- $g(\sigma1)*I - g(\sigma2)*I = [g(\sigma1) - g(\sigma2)]*I$

# Difference of Gaussian (DoG)

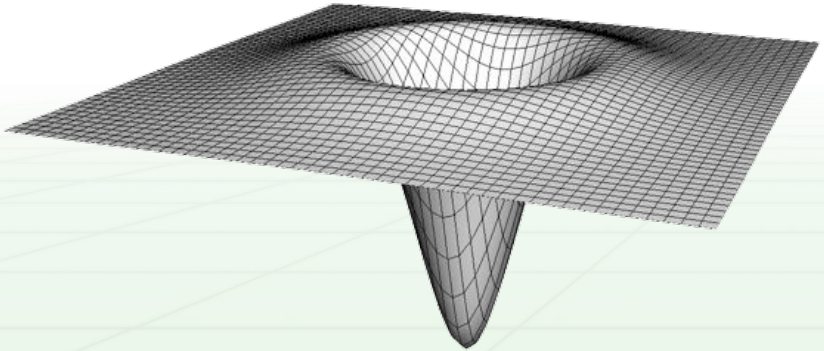
- $g(\sigma 1)*I - g(\sigma 2)*I = [g(\sigma 1) - g(\sigma 2)]*I$

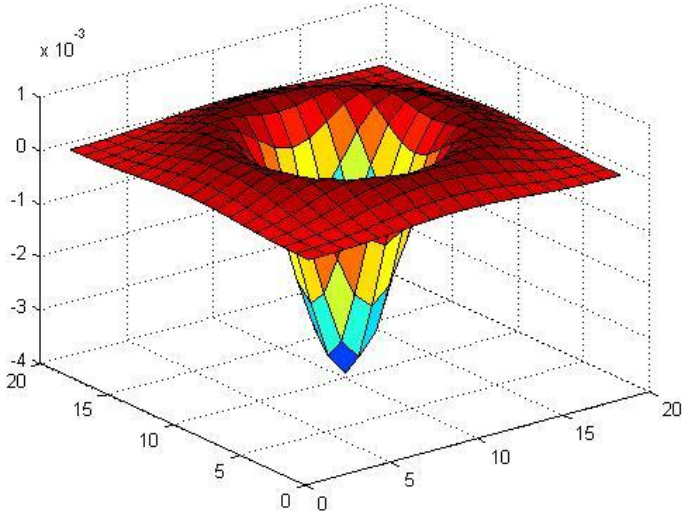


$\sigma = 2$ $\sigma = 1$

# Difference of Gaussian (DoG)

- g(σ1)*I - g(σ2)*I = [g(σ1) - g(σ2)]*I
- This looks a lot like our LoG!
- (not actually the same but similar)





# DoG (1 - 0)

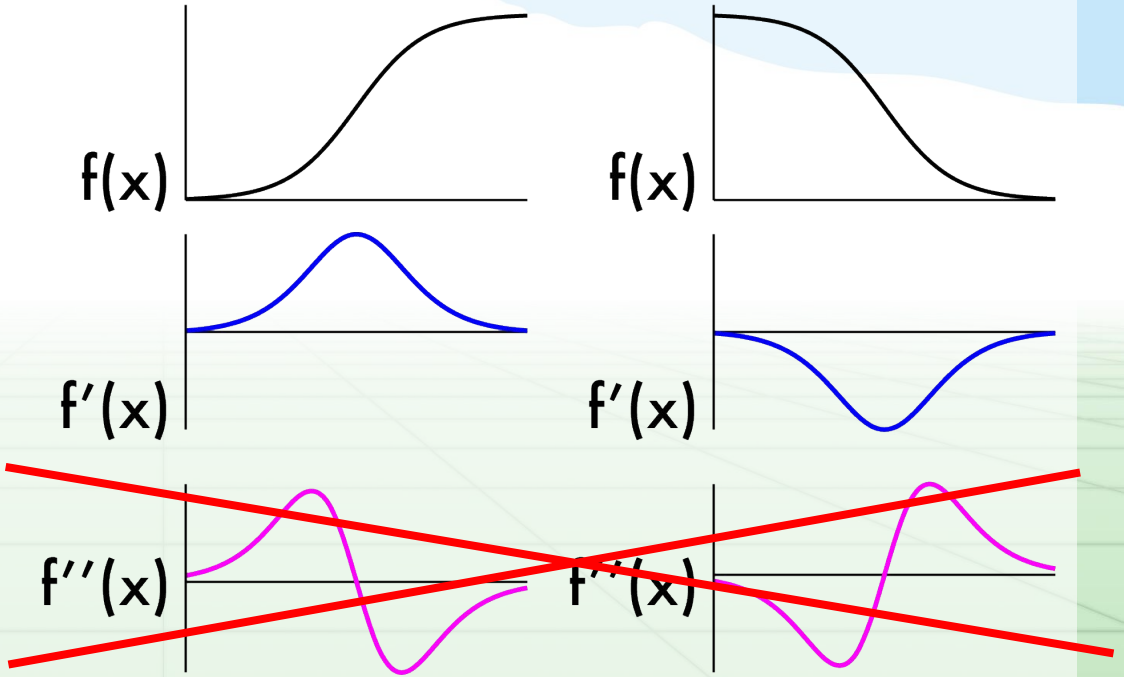
# DoG (2 - 1)

# DoG (3 - 2)

# DoG (4 - 3)

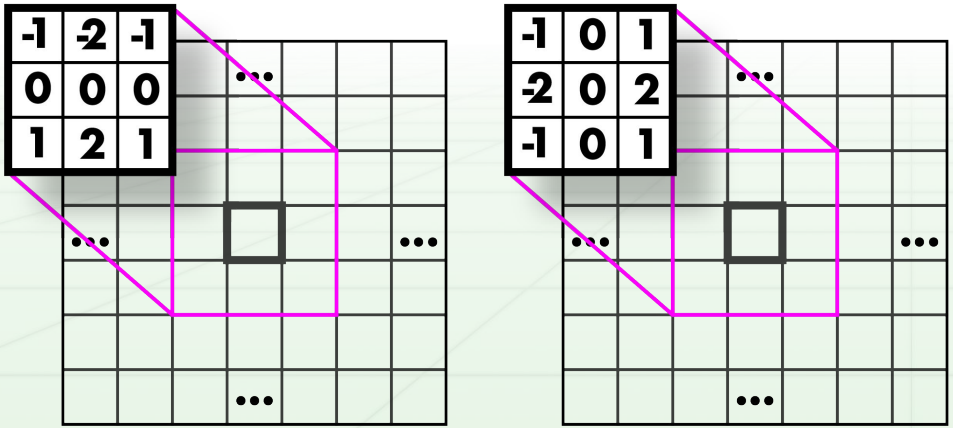
# Another approach: gradient magnitude

- Don’t need 2nd derivatives
- Just use magnitude of gradient



# Another approach: gradient magnitude

- Don’t need 2nd derivatives
- Just use magnitude of gradient
- Are we done? No!




# What we really want: line drawing

# Canny Edge Detection

- Your first image processing pipeline!
  - Old-school CV is all about pipelines

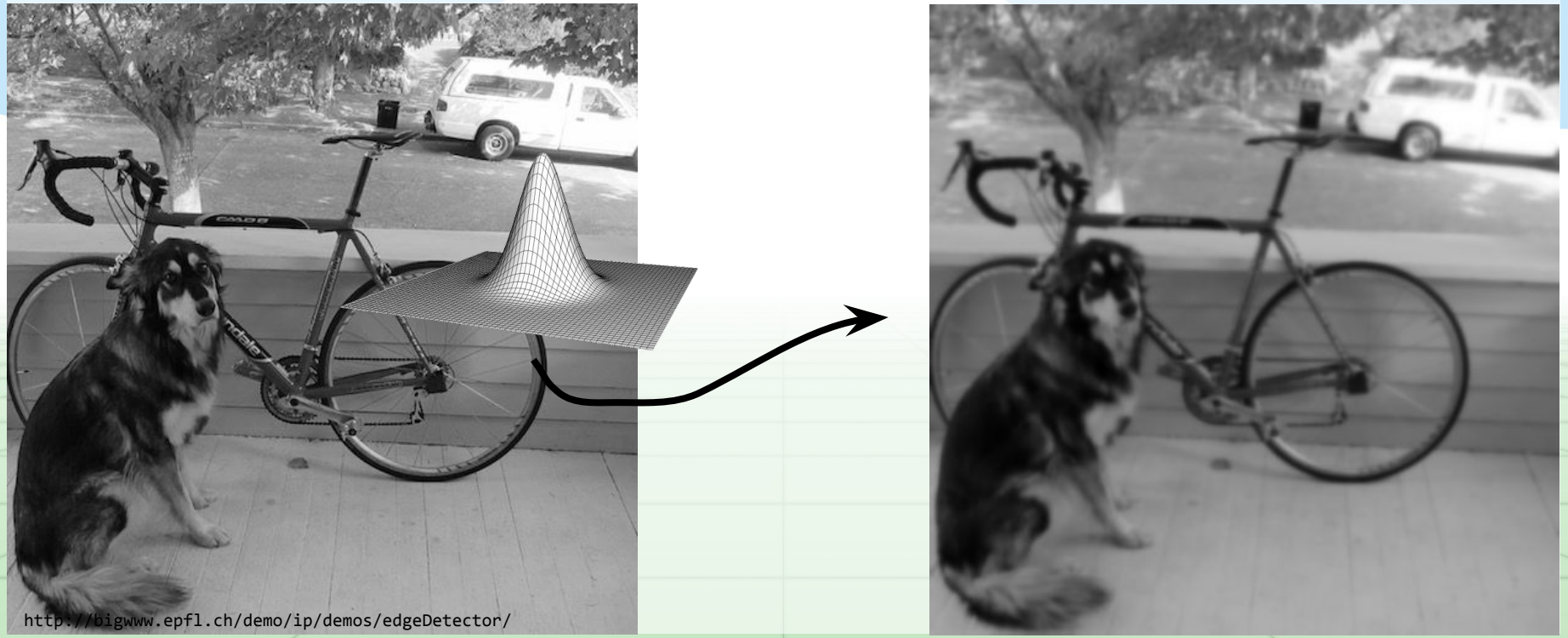
Algorithm:

- Smooth image (only want “real” edges, not noise)
- Calculate gradient direction and magnitude
- Non-maximum suppression perpendicular to edge
- Threshold into strong, weak, no edge
- Connect together components

http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/

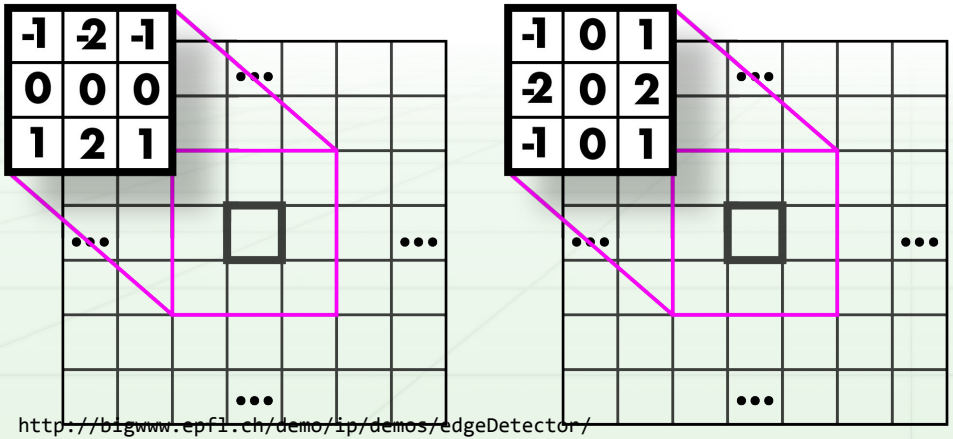
# Smooth image

- You know how to do this, gaussians!



# Gradient magnitude and direction

- Sobel filter



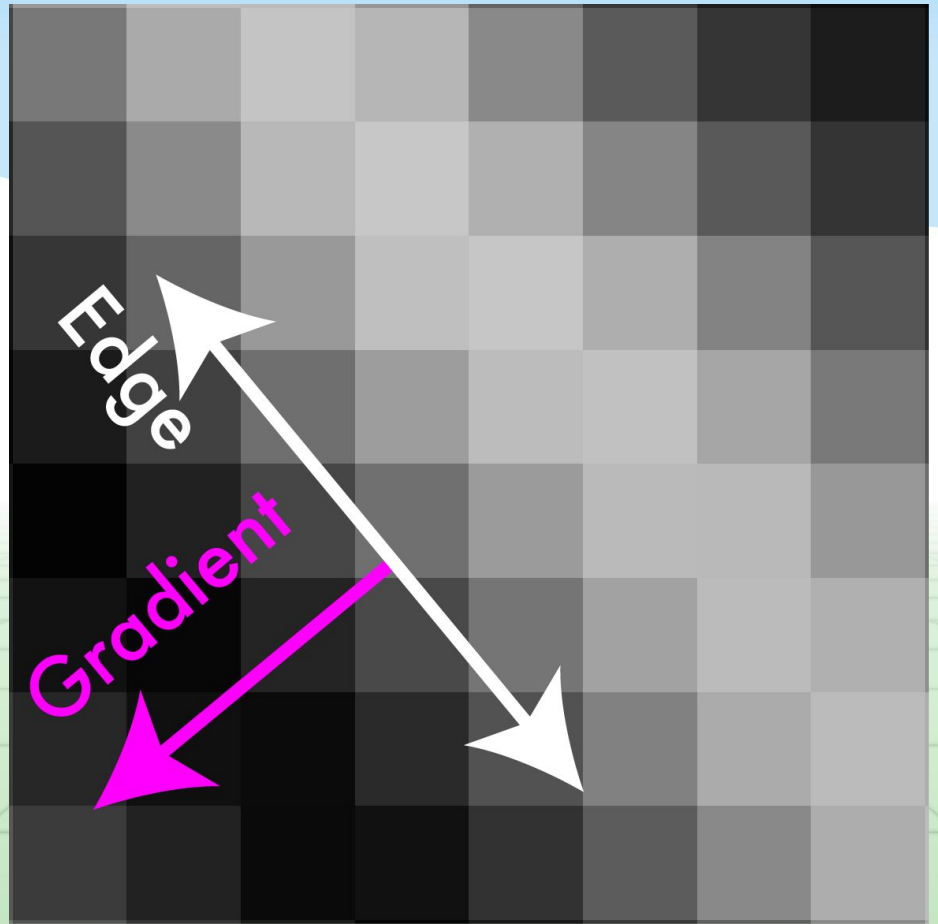

http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/

# Non-maximum suppression

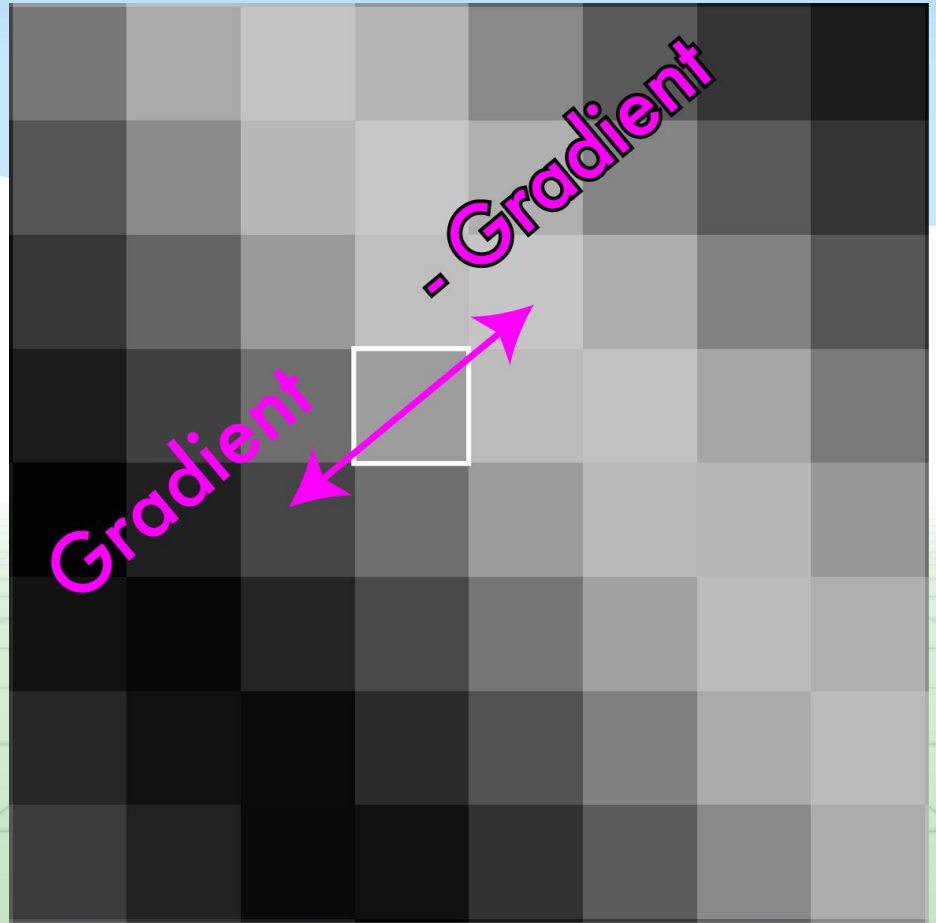
- Want single pixel edges, not thick blurry lines
- Need to check nearby pixels
- See if response is highest

http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/

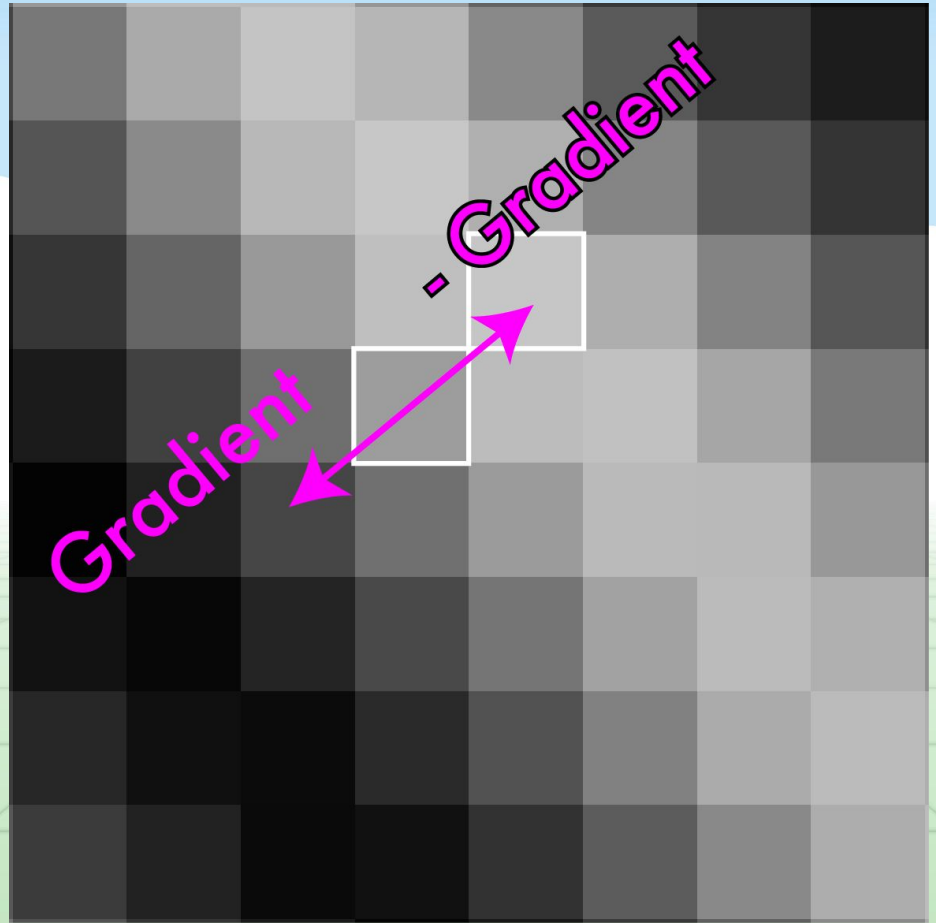
# Non-maximum suppression



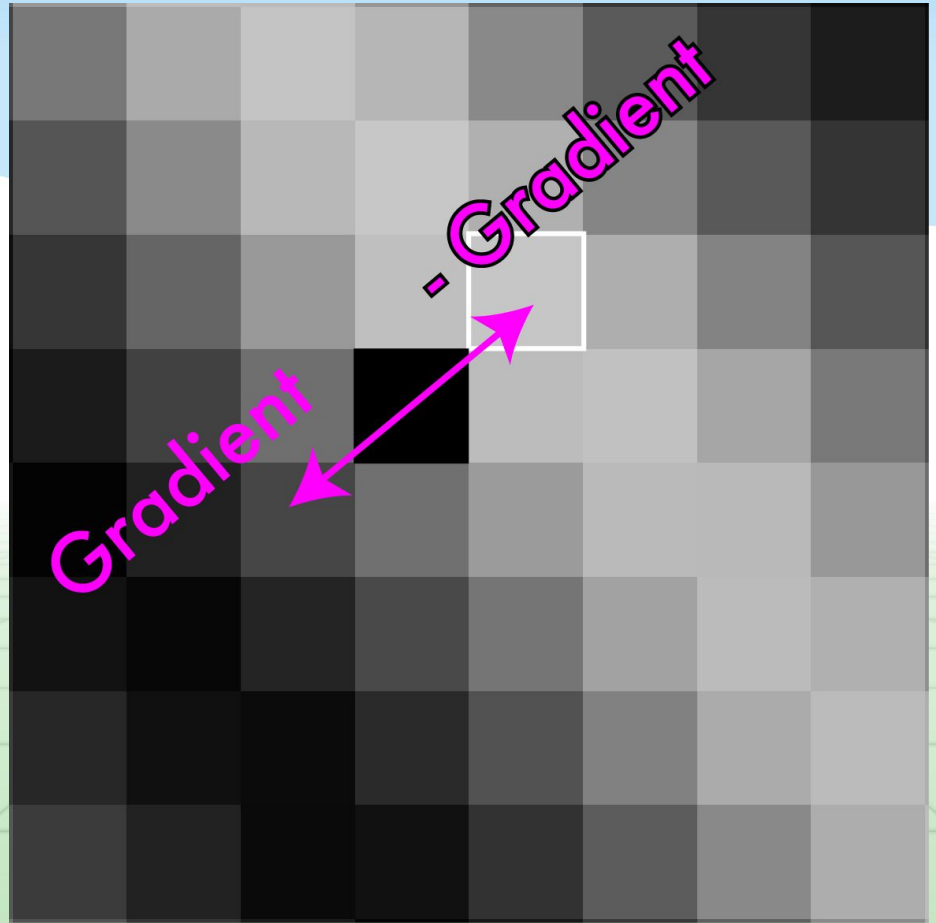
# Non-maximum suppression



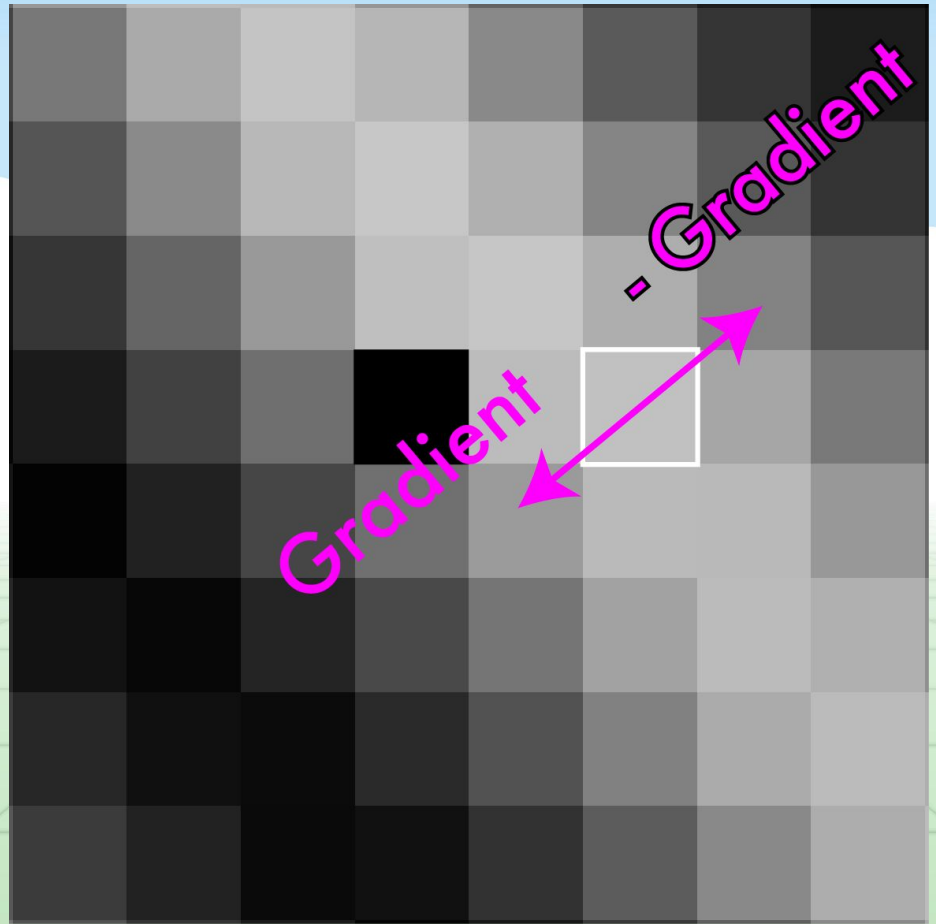
# Non-maximum suppression



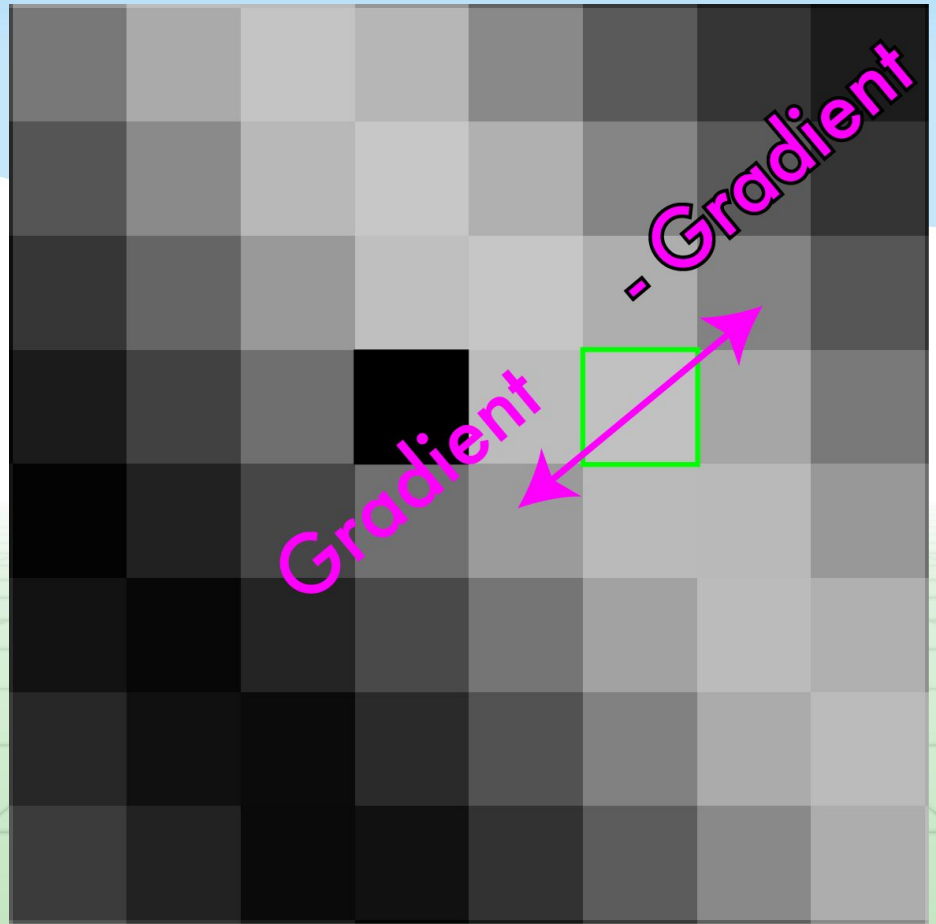
# Non-maximum suppression



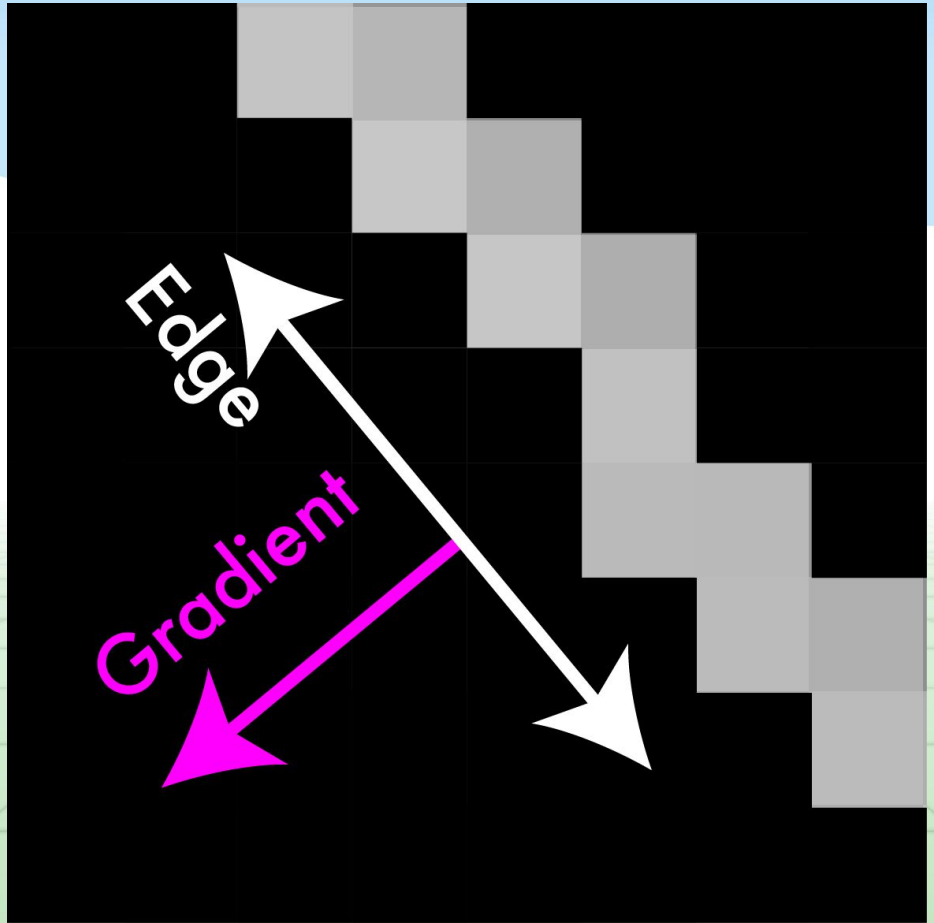
# Non-maximum suppression



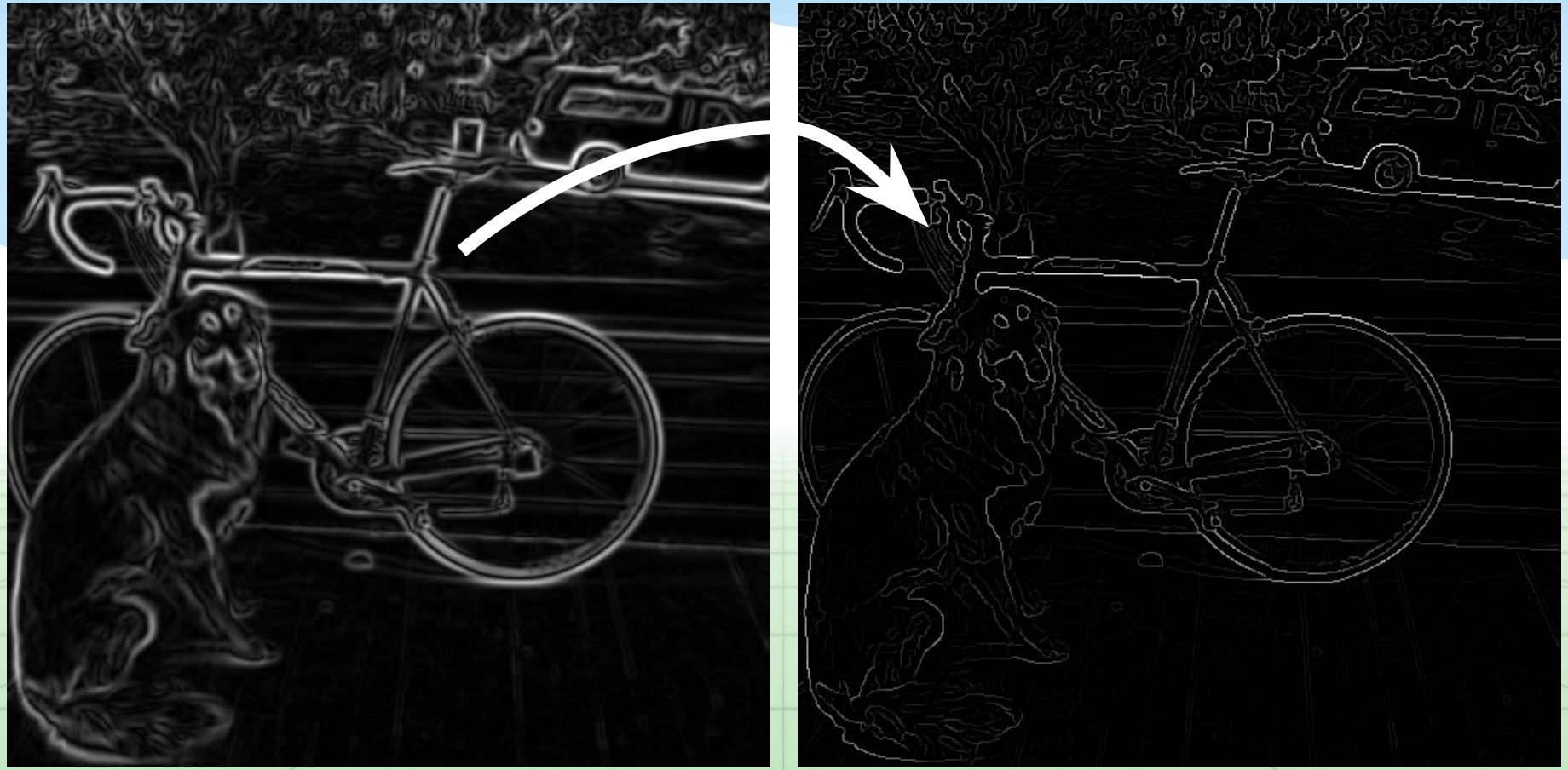
# Non-maximum suppression



# Non-maximum suppression



# Non-maximum suppression



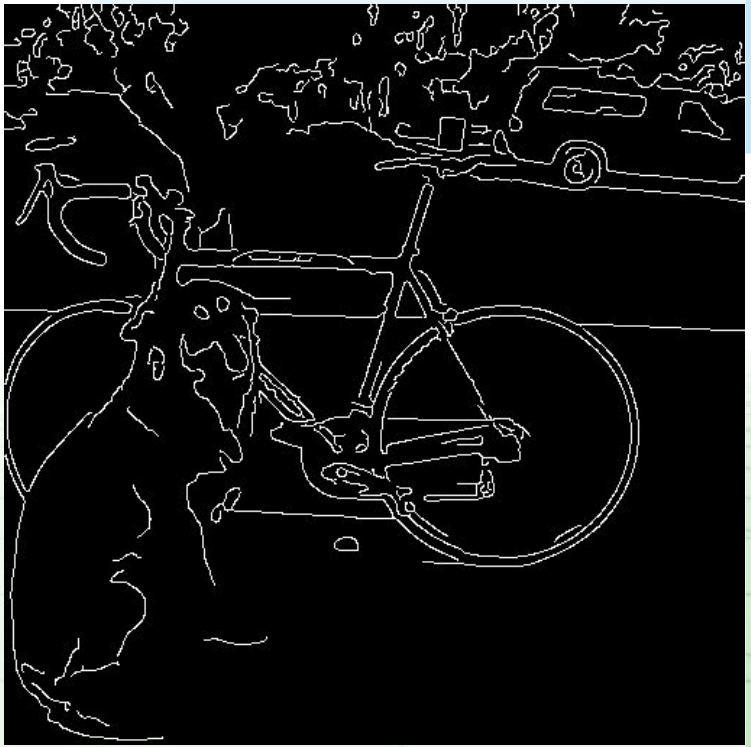
# Threshold edges

- Still some noise
- Only want strong edges
- 2 thresholds, 3 cases
  - R > T: strong edge
  - R < T but R > t: weak edge
  - R < t: no edge
- Why two thresholds?

http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/

# Connect ‘em up!

- Strong edges are edges!
- Weak edges are edges
  iff they connect to strong
- Look in some neighborhood
  (usually 8 closest)



http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/

# Canny Edge Detection

- Your first image processing pipeline!
  - Old-school CV is all about pipelines

Algorithm:

- Smooth image (only want “real” edges, not noise)
- Calculate gradient direction and magnitude
- Non-maximum suppression perpendicular to edge
- Threshold into strong, weak, no edge
- Connect together components
- Tunable: Sigma, thresholds

http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/

# Canny Edge Detection

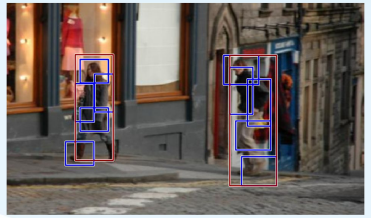
# What else is there?

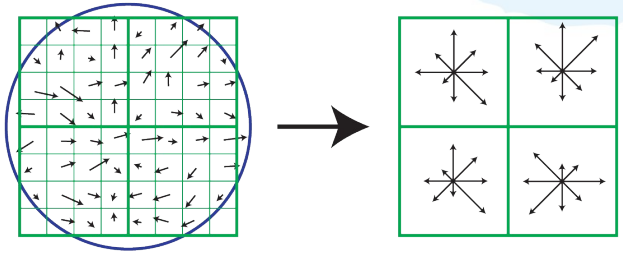
# So what can we do with these convolutions anyway?

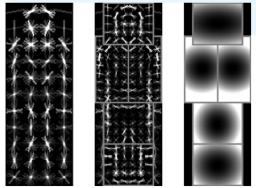
- Blurring
- Sharpening
- Edges
- Features
- Derivatives
- Super-resolution
- Classification
- Detection
- Image captioning
- ...

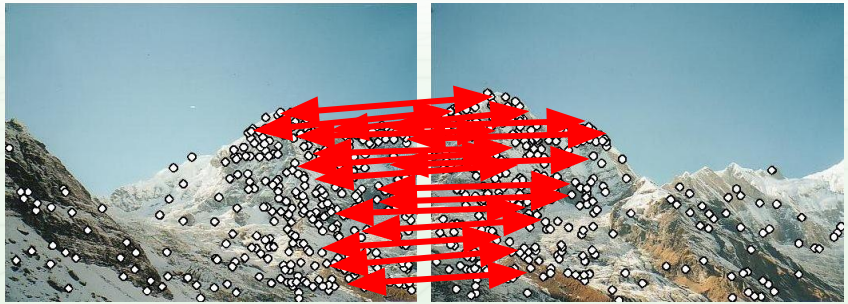
# Features!

- Highly descriptive local regions
- Ways to describe those regions
- Useful for:
  - Matching
  - Recognition
  - Detection







# What makes a good feature?

- Want to find patches in image that are useful or have some meaning
- For objects, want a patch that is common to that object but not in general
- For panorama stitching, want patches that we can find easily in another image of same place
- Good features are unique!
  - Can find the “same” feature easily
  - Not mistaken for “different” features

# How close are two patches?

- Sum squared difference
- Images I, J
- $\Sigma_{x,y} (I(x,y) - J(x,y))^2$

# How can we find unique patches?

- Say we are stitching a panorama
- Want patches in image to match to other image
- Need to only match one spot

# How can we find unique patches?

- Sky: bad
  - Very little variation
  - Could match any other sky

# How can we find unique patches?

- Sky: bad
  - Very little variation
  - Could match any other sky
- Edge: ok
  - Variation in one direction
  - Could match other patches along same edge

# How can we find unique patches?

- Sky: bad
  - Very little variation
  - Could match any other sky
- Edge: ok
  - Variation in one direction
  - Could match other patches along same edge
- Corners: good!
  - Only one alignment matches

# How can we find unique patches?

- Want a patch that is unique in the image
- Can calculate distance between patch and every other patch, lot of computation

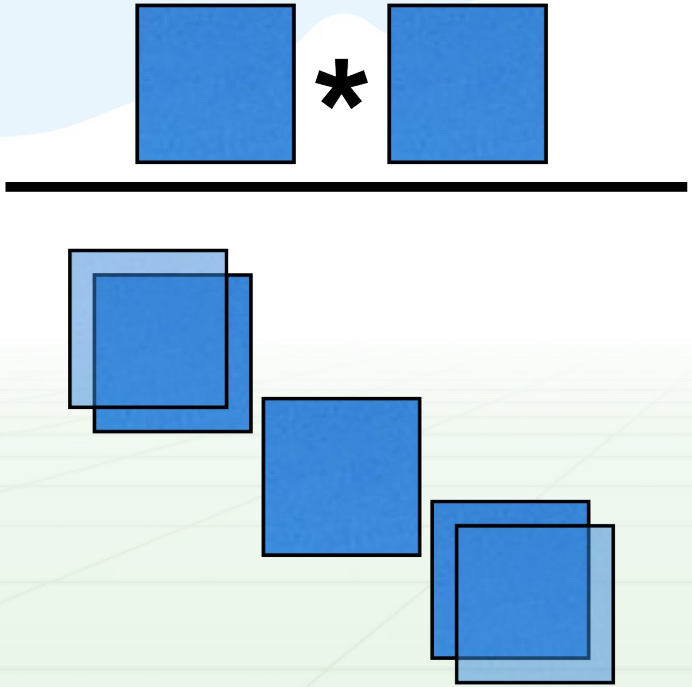
# How can we find unique patches?

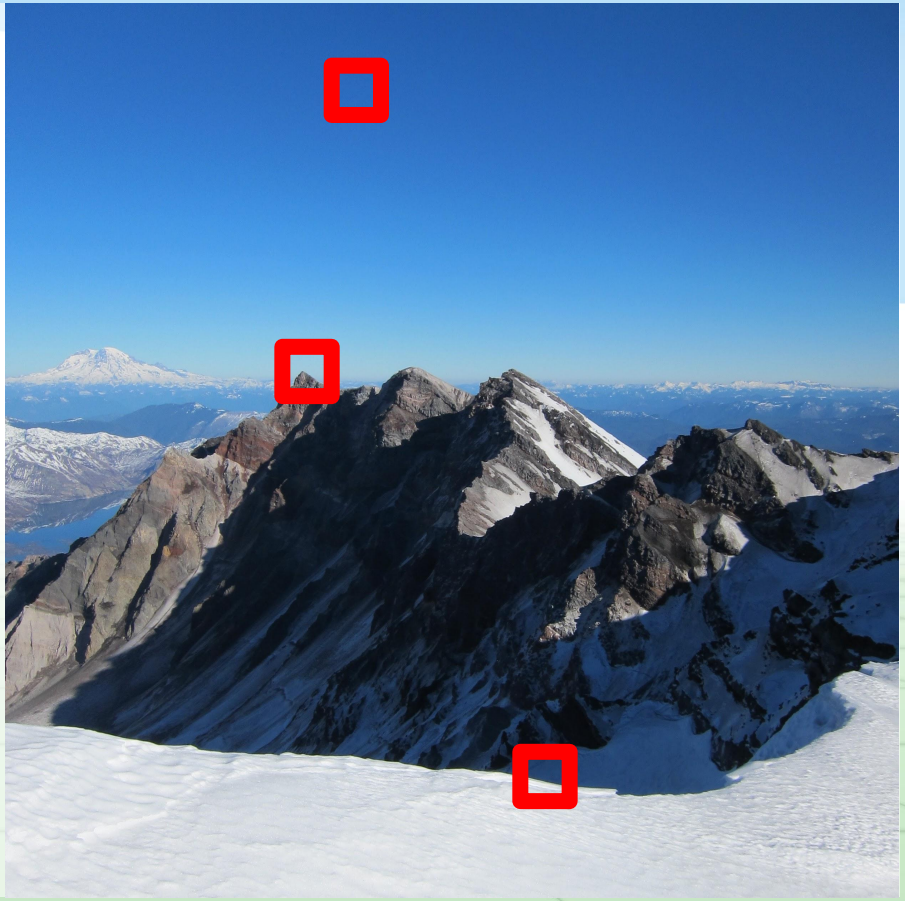
- Want a patch that is unique in the image
- Can calculate distance between patch and every other patch, lot of computation
- Instead, we could think about auto-correlation:
  - How well does image match shifted version of itself?
- $\Sigma_{\mathbf{d}}\Sigma_{x,y}\ (I(x,y) - I(x+\mathbf{d}_x, y+\mathbf{d}_y))^2$

- Measure of self-difference (how am I not myself?)

# Self-difference

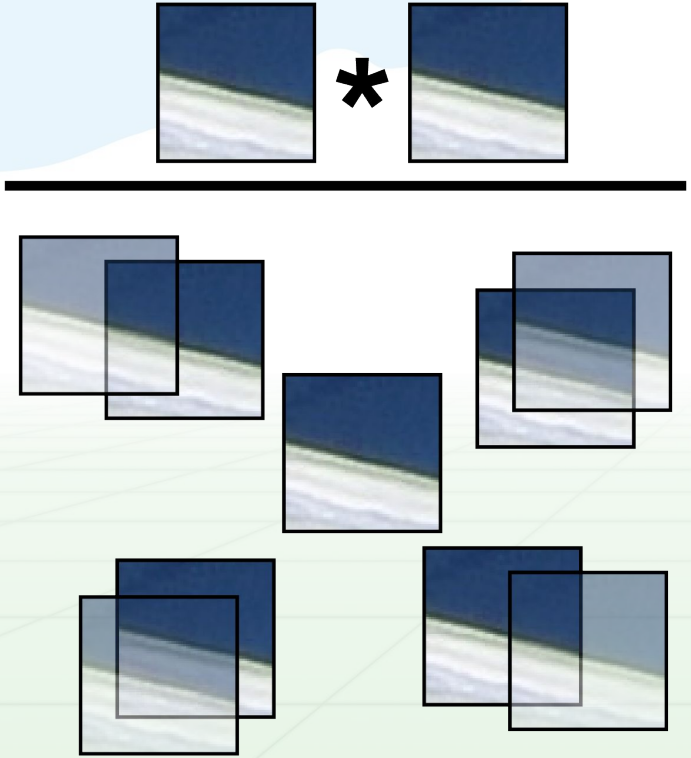
Sky: low everywhere

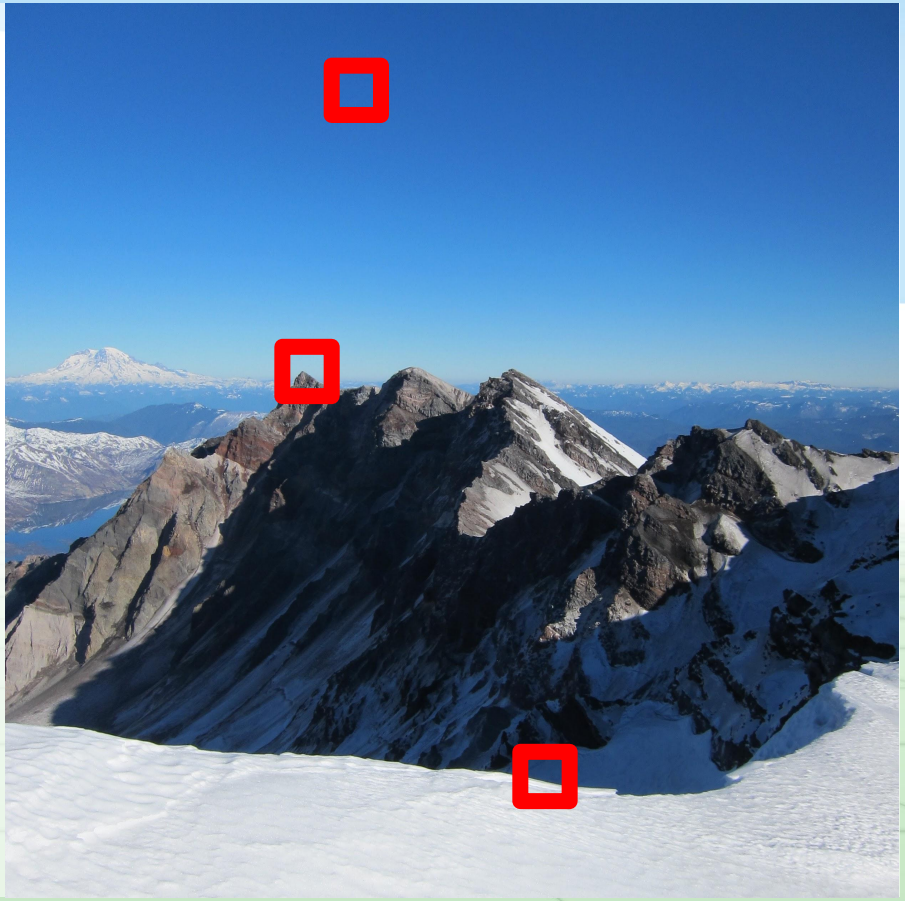




# Self-difference

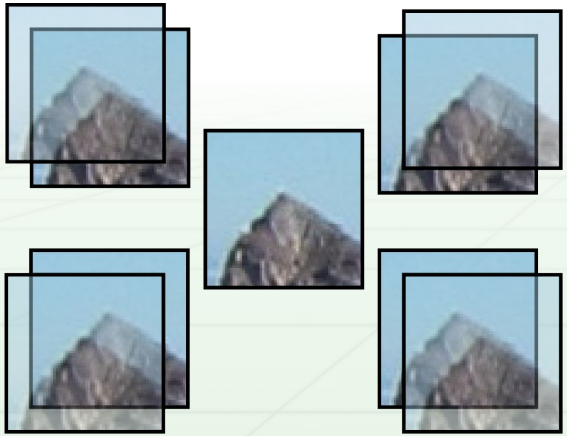
Edge: low along edge

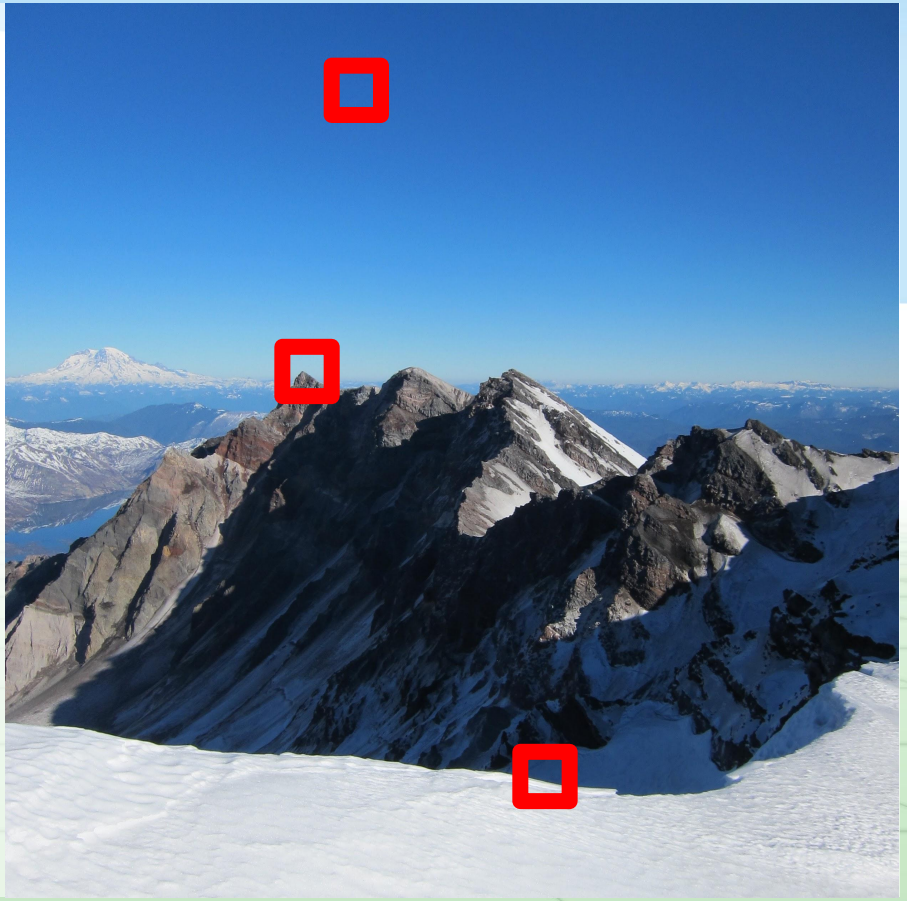




# Self-difference

Corner: mostly high



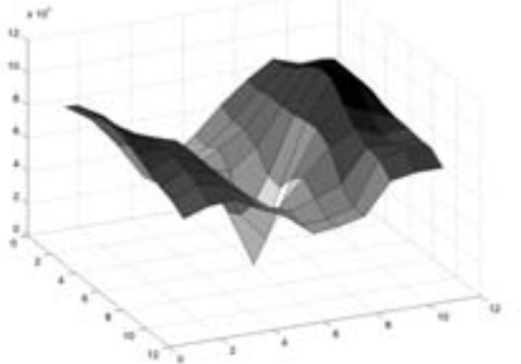


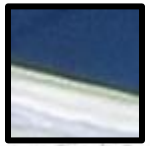
# Self-difference

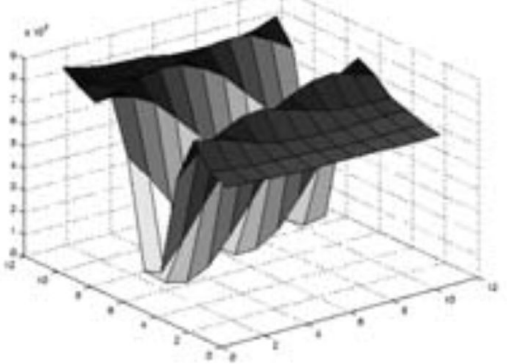
Sky: low everywhere

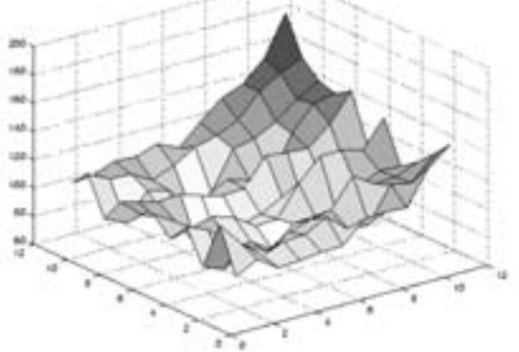
Edge: low along edge

Corner: mostly high









# Self-difference is still expensive

- $\Sigma_{\mathbf{d}}\Sigma_{x,y}\ (I(x,y) - I(x+\mathbf{d}_x, y+\mathbf{d}_y))^2$
- Lots of summing
- Need an approximation
  - If you want the mathy details, Szeliski pg 212

# Approximate self-difference

- Look at nearby gradients $I_x$ and $I_y$
- If gradients are mostly zero, not a lot going on
  - Low self-difference
- If gradients are mostly in one direction, edge
  - Still low self-difference
- If gradients are in twoish directions, corner!
  - High self-difference, good patch!

# Approximate self-difference

- How do we tell what's going on with gradients?
- Eigen vectors/values!
- Need structure matrix for patch, just a weighted sum of nearby gradient information

$$S_w[p] = \begin{bmatrix} \sum_r w[r](I_x[p-r])^2 & \sum_r w[r] I_x[p-r] I_y[p-r] \\ \sum_r w[r] I_x[p-r] I_y[p-r] & \sum_r w[r](I_y[p-r])^2 \end{bmatrix}$$

- Not as complex as it looks, weighted sum of gradients near pixel

# Structure matrix

- Weighted sum of gradient information
  - $| \Sigma_i w_i I_x(i) I_x(i) \quad \Sigma_i w_i I_x(i) I_y(i) |$
  - $| \Sigma_i w_i I_x(i) I_y(i) \quad \Sigma_i w_i I_y(i) I_y(i) |$
- Can use Gaussian weighting (so many gaussians)
- Eigen vectors/values of this matrix summarize the distribution of the gradients nearby
- $\lambda_1$ and $\lambda_2$ are eigenvalues
  - $\lambda_1$ and $\lambda_2$ both small: no gradient
  - $\lambda_1 >> \lambda_2$: gradient in one direction
  - $\lambda_1$ and $\lambda_2$ similar: multiple gradient directions, corner

# Estimating smallest eigen value

- A few methods:
  - $\det(S) = \lambda_1 * \lambda_2$
  - $\text{trace}(S) = \lambda_1 + \lambda_2$
- $\det(S) - \alpha\ \text{trace}(S)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1+\lambda_2)^2$
- $\det(S) / \text{trace}(S) = \lambda_1\lambda_2/(\lambda_1+\lambda_2)$
- If these estimates are large, $\lambda_2$ is large

# Harris Corner Detector

- Calculate derivatives $I_x$ and $I_y$
- Calculate 3 measures $I_xI_x$, $I_yI_y$, $I_xI_y$
- Calculate weighted sums
  - Want a weighted sum of nearby pixels, guess what this is?
  - Gaussian!
- Estimate response based on smallest eigen value
- Non-max suppression (just like canny)