# Reinforcement Learning
# Intelligent Systems Series
# Lecture 3

Georg Martius
Slides adapted from David Silver, Deepmind

MPI for Intelligent Systems, Tübingen, Germany

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EBERHARD KARLS
UNIVERSITÄT
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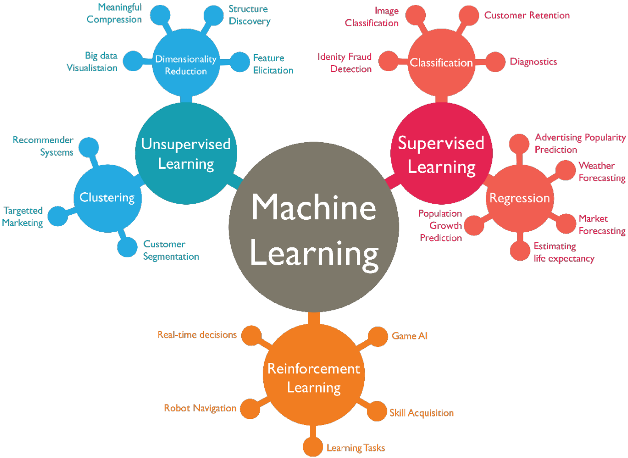
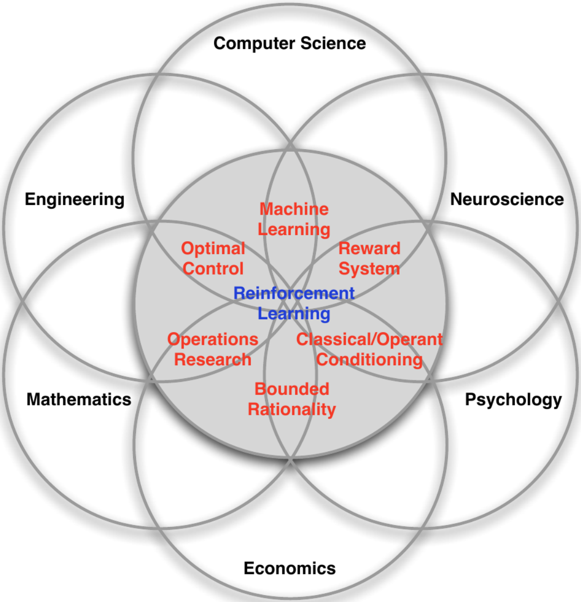
# Reminder: Branches of Machhine Learning



# Many Types and Areas of Reinforcement Learning



# Characteristics of Reinforcement Learning

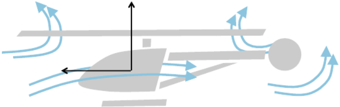
What makes reinforcement learning different from other machine learning paradigms?

- There is no supervisor, only a reward signal
- Feedback is delayed, not instantaneous
- Time really matters (sequential, non i.i.d data)
- Agent's actions affect the subsequent data it receives

# Examples of Reinforcement Learning

- Fly stunt manoeuvres in a helicopter
- Defeat the world champion at Backgammon
- Manage an investment portfolio
- Control a power station
- Make a humanoid robot walk
- Play many different Atari games better than humans
- Beat the best human player in Go

# Examples – Helicopter Manoeuvres



https://www.youtube.com/watch?v=0JL04JJjocc

# Examples – Bipedal Robots

https://www.youtube.com/watch?v=No-JwwPbSLA

# Examples – Atari Games


https://www.youtube.com/watch?v=Vr5MR5lKOc8

# Rewards

- A reward $R_t$ is a scalar feedback signal
- Indicates how well agent is doing at step $t$
- The agent's job is to maximize cumulative reward

Reinforcement learning is based on the reward hypothesis

**Definition (Reward Hypothesis)**

All goals can be described by the maximization of expected cumulative reward

Do you agree with this statement?

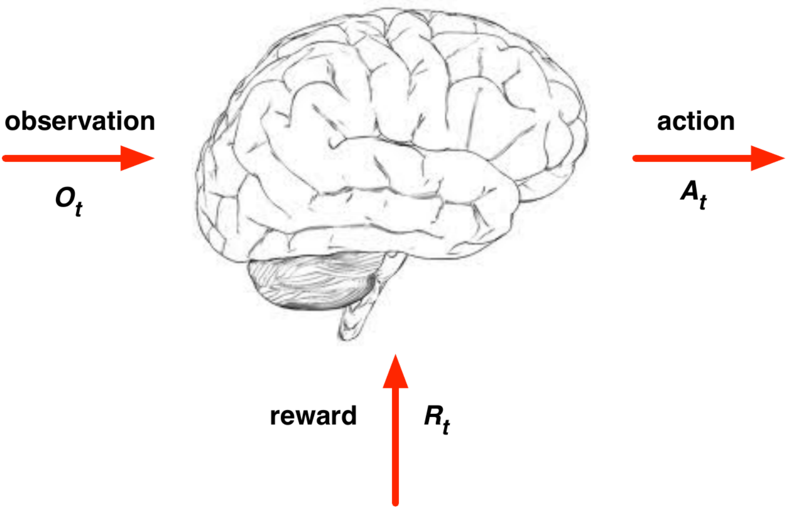
# Examples of Rewards

- Fly stunt manoeuvres in a helicopter
  - +ve reward for following desired trajectory
  - -ve reward for crashing
- Defeat the world champion at Backgammon
  - +/-ve reward for winning/losing a game
- Manage an investment portfolio
  - +ve reward for each $ in bank
- Control a power station
  - +ve reward for producing power
  - -ve reward for exceeding safety thresholds
- Make a humanoid robot walk
  - +ve reward for forward motion
  - -ve reward for falling over
- Play many different Atari games better than humans
  - +/-ve reward for increasing/decreasing score

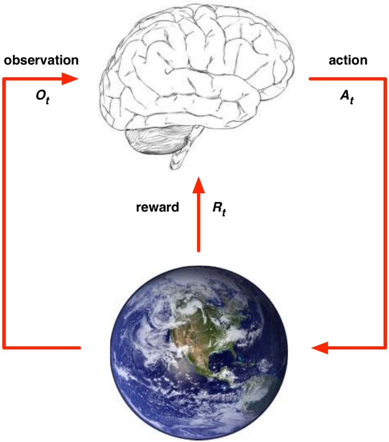
# Sequential Decision Making

- Goal: select actions to maximize total future reward
- Actions may have long term consequences
- Reward may be delayed
- It may be better to sacrifice immediate reward to gain more long-term reward
- Examples:
  - A financial investment (may take months to mature)
  - Refueling a helicopter (might prevent a crash in several hours)
  - Blocking opponent moves (might help winning chances many moves from now)

# Agent and Environment



# Agent and Environment




- At each step $t$ the agent:
    - Executes action $A_t$
    - Receives observation $O_t$
    - Receives scalar reward $R_t$
- The environment:
    - Receives action $A_t$
    - Emits observation $O_{t+1}$
    - Emits scalar reward $R_{t+1}$
- $t$ increments at env. step

# History and State

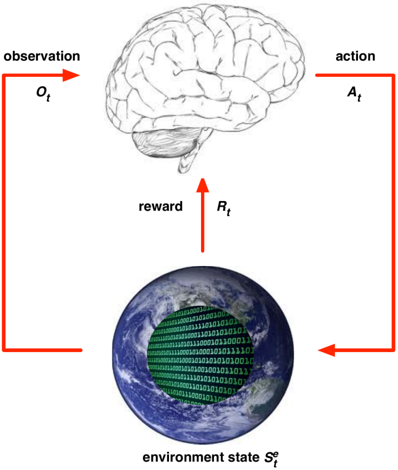
- The history is the sequence of observations, actions, rewards

$$H_t = O_1, R_1, A_1, ..., A_{t-1}, O_t, R_t$$

- i.e. all observable variables up to time $t$
- i.e. the sensorimotor stream of a robot or embodied agent
- What happens next depends on the history:
  - The agent selects actions
  - The environment selects observations/rewards
- State is the information used to determine what happens next
- Formally, state is a function of the history:

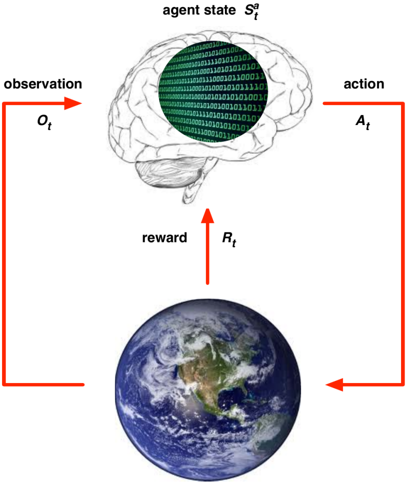
$$S_t = f(H_t)$$

# Environment/World State




- The environment state $S_t^e$ is the environment's private representation
- i.e. whatever data the environment uses to pick the next observation/reward
- The environment state is not usually visible to the agent
- Even if $S_t^e$ is visible, it may contain irrelevant information

# Agent State




- The agent state $S_t^a$ is the agent's internal representation
- i.e. whatever information the agent uses to pick the next action
- i.e. it is the information used by reinforcement learning algorithms
- It can be any function of the history:

$$S_t^a = f(H_t)$$

# Information State

An information state (a.k.a. Markov state) contains all useful information from the history.

**Definition**

A state $S_t$ is Markov if and only if

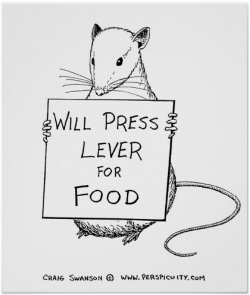
$$P(S_{t+1} \mid S_t) = P(S_{t+1} \mid S_1, ..., S_t)$$

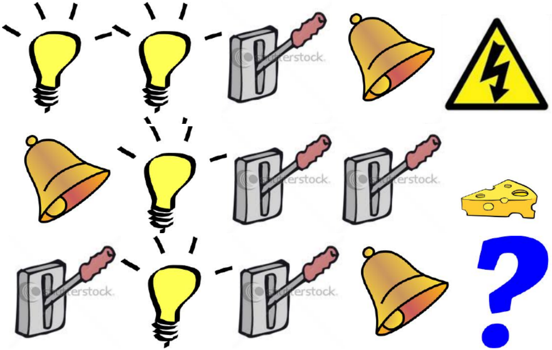
- The future is independent of the past given the present

$$H_{1:t} \rightarrow S_t \rightarrow H_{t+1:\infty}$$

- Once the state is known, the history may be thrown away
- i.e. the state is a sufficient statistic of the future
- The environment state $S_t^e$ is Markov
- The history $H_t$ is Markov

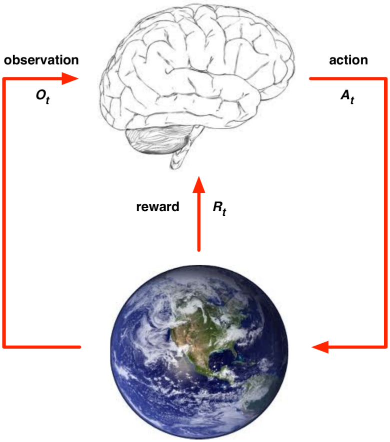
# Rat Example



- What if agent state = last 3 items in sequence?
- What if agent state = counts for lights, bells and levers?
- What if agent state = complete sequence?

# Fully Observeable Environments




Full observability: agent directly observes environment state

$$O_t = S_t^a = S_t^e$$

- Agent state = environment state = information state
- Formally, this is a Markov decision process (MDP)

# Partially Observable Environments

- Partial observability: agent indirectly observes environment:
    - A robot with camera vision isn't told its absolute location
    - A trading agent only observes current prices
    - A poker playing agent only observes public cards
- Now agent state $\neq$ environment state
- Formally this is a partially observable Markov decision process (POMDP)
- Agent must construct its own state representation $S_t^a$, e.g.
    - Complete history: $S_t^a = H_t$
    - Beliefs of environment state: $S_t^a = (P(S_t^e = s^1), ..., P(S_t^e = s^n))$
    - Recurrent neural network: $S_t^a = \sigma(S_{t-1}^a W_s + O_t W_o)$

# Major Components of an RL Agent

An RL agent may include one or more of these components:

- Policy: agent's behaviour function
- Value function: how good is each state and/or action
- Model: agent's representation of the environment

# Policy

- A policy defines the agent's behaviour
- It is a map from state to action, e.g.
- Deterministic policy: $a = \pi(s)$
- Stochastic policy: $\pi(a|s) = P(A_t = a|S_t = s)$

# Value Function

- Value function is a prediction of future reward
- Used to evaluate the goodness/badness of states
- ➡ used to select which action to take
- e.g. models the expected discounted future reward

$$v_\pi(s) = \mathbb{E}_\pi[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots \mid S_t = s]$$

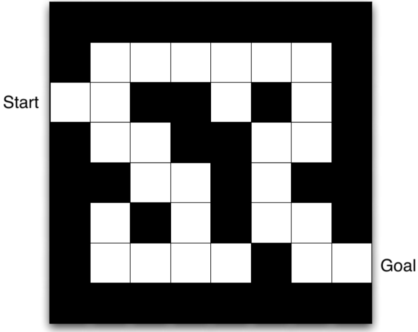
# Model

- A model predicts what the environment will do next
- $\mathcal{P}$ predicts the next state
- $\mathcal{R}$ predicts the next (immediate) reward, e.g.

$$\mathcal{P}^a_{ss'} = P(S_{t+1} = s' \mid S_t = s, A_t = a)$$
$$\mathcal{R}^a_s = \mathbb{E}[R_{t+1} \mid S_t = s, A_t = a]$$

- Can be used for planning without actually performing actions

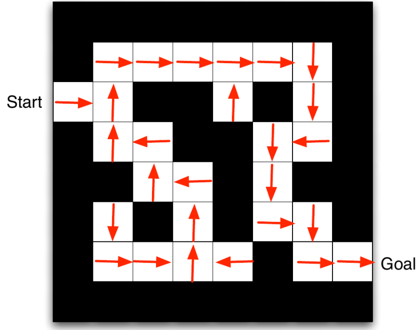
# Maze Example



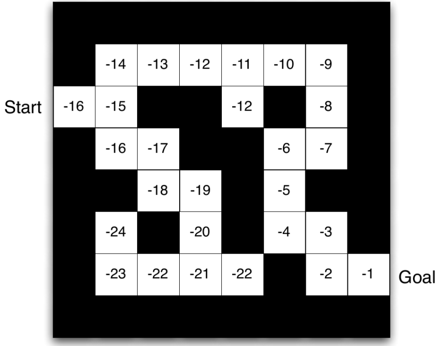

Environment

- Rewards: $-1$ per time-step
- Actions: N, E, S, W
- States: Agent's location
- End at Goal state

# Maze Example: Policy and Value Function

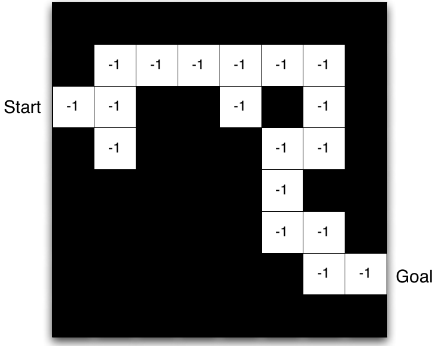



Arrows represent policy $\pi(s)$




Numbers represent value $v_\pi(s)$

# Maze Example: Model




- Agent may have an internal model of the environment
- Dynamics: how actions change the state
- Rewards: how much reward from each state
- The model may be imperfect (most likely is)

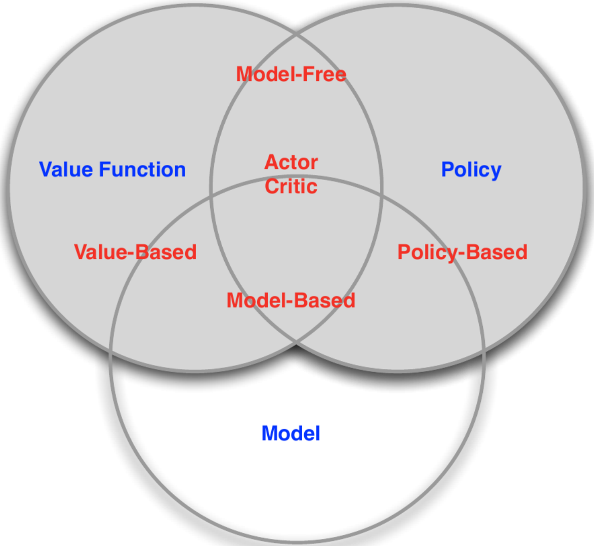
- Grid layout represents transition model $\mathcal{P}^a_{ss'}$
- Numbers represent immediate reward $\mathcal{R}^a_s$ from each state $s$ (same for all $a$)

# Categorization of RL agents

- Value Based
  - No Policy (Implicit)
  - Value Function
- Policy Based
  - Policy
  - No Value Function
- Actor Critic
  - Policy
  - Value Function

- Model Free
  - Policy and/or Value Function
  - No Model
- Model Based
  - Policy and/or Value Function
  - Model

# RL Agent Taxonomy

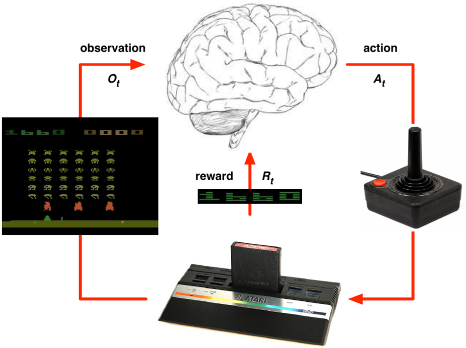


# Learning and Planning

Two fundamental problems in sequential decision making

- Reinforcement Learning:
  - The environment is initially unknown
  - The agent interacts with the environment
  - The agent improves its policy
  - a.k.a. learning by doing, trail and error learning
- Planning:
  - A model of the environment is known
  - The agent performs computations with its model (without any external interaction)
  - The agent improves its policy
  - a.k.a. deliberation, reasoning, introspection, pondering, thought, search

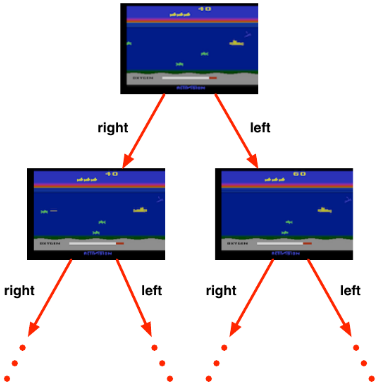
# Atari Example: Reinforcement Learning




- Rules of the game are unknown
- Learn directly from interactive game-play
- Pick actions on joystick, see pixels and scores

# Atari Example: Planning

- Rules of the game are known
- Can query emulator
  perfect model inside agent's brain
- If I take action a from state s:
  - what would the next state be?
  - what would the score be?
- Plan ahead to find optimal policy
  e.g. tree search



# Exploration and Exploitation

- Reinforcement learning is like trial-and-error learning
- The agent should discover a good policy
- From its experiences of the environment
- Without losing too much reward along the way

- Exploration finds more information about the environment
- Exploitation exploits known information to maximize reward
- It is usually important to explore as well as exploit

# Examples

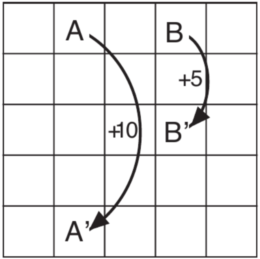
- Restaurant Selection
  Exploitation Go to your favorite restaurant
  Exploration Try a new restaurant
- Online Banner Advertisements
  Exploitation Show the most successful advert
  Exploration Show a different advert
- Game Playing
  Exploitation Play the move you believe is best
  Exploration Play an experimental move
- Robot Control
  Exploitation Do the movement you know works best
  Exploration Try a different movement

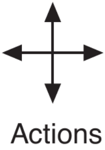
# Prediction and Control

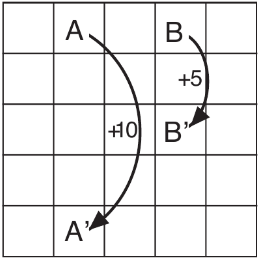
Subproblems within Reinforcement Learning:

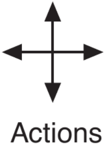
- Prediction: evaluate the future
  How do I do given a policy?
- Control: optimize the future
  Find the best policy

# Gridworld Example: Prediction



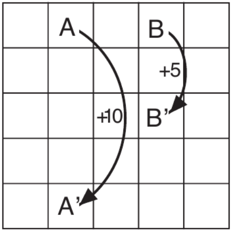




| | | | | |
|---|---|---|---|---|
| 3.3 | 8.8 | 4.4 | 5.3 | 1.5 |
| 1.5 | 3.0 | 2.3 | 1.9 | 0.5 |
| 0.1 | 0.7 | 0.7 | 0.4 | -0.4 |
| -1.0 | -0.4 | -0.4 | -0.6 | -1.2 |
| -1.9 | -1.3 | -1.2 | -1.4 | -2.0 |

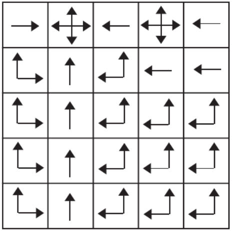
# Gridworld Example: Control




a) gridworld

| | | | | |
|---|---|---|---|---|
| 22.0 | 24.4 | 22.0 | 19.4 | 17.5 |
| 19.8 | 22.0 | 19.8 | 17.8 | 16.0 |
| 17.8 | 19.8 | 17.8 | 16.0 | 14.4 |
| 16.0 | 17.8 | 16.0 | 14.4 | 13.0 |
| 14.4 | 16.0 | 14.4 | 13.0 | 11.7 |

b) $v_*$



c) $\pi_*$

- What is the optimal value function over all possible policies?
- What is the optimal policy?

# Markov Decision Processes

# Markov Process

A Markov process is a memoryless random process, i.e. a sequence of random states $S_1, S_2, \dots$ with the Markov property.

**Reminder: Markov property**

A state $S_t$ is Markov if and only if

$$P(S_{t+1} \mid S_t) = P(S_{t+1} \mid S_1, \dots, S_t)$$

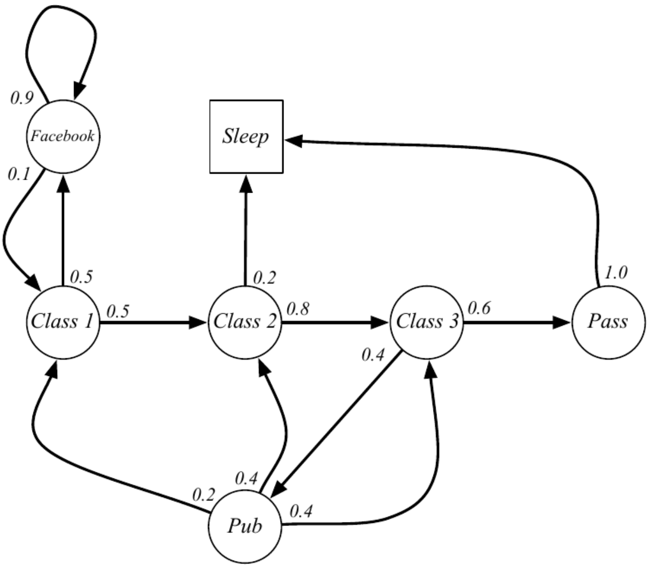
**Definition (Markov Process/ Markov Chain)**

A *Markov Process* (or *Markov Chain*) is a tuple $(\mathcal{S}, \mathcal{P})$

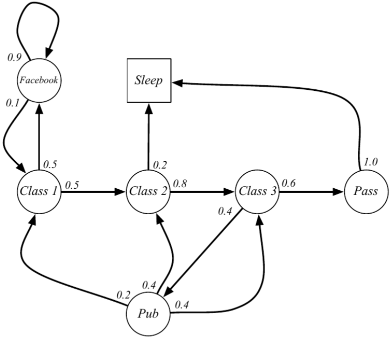
- $\mathcal{S}$ is a (finite) set of states
- $\mathcal{P}$ is a state transition 0 probability matrix,

$$P_{ss'} = P(S_{t+1} = s' \mid S_t = s)$$

# Example: Student Markov Chain



# Example: Student Markov Chain Episodes

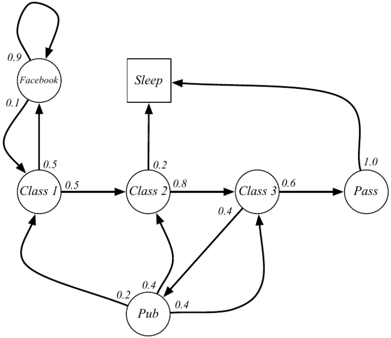



Sample episodes for starting from $S_1 =$C1

$$S_1, S_2, \ldots, S_T$$

- C1 C2 C3 Pass Sleep
- C1 FB FB C1 C2 Sleep
- C1 C2 C3 Pub C2 C3 Pass Sleep
- C1 FB FB C1 C2 C3 Pub C1 FB FB FB C1 C2 C3 Pub C2 Sleep

# Example: Student Markov Chain Transition Matrix




$$\mathcal{P} = \begin{array}{cc} & \begin{array}{ccccccc} C1 & C2 & C3 & Pass & Pub & FB & Sleep \end{array} \\ \begin{array}{c} C1 \\ C2 \\ C3 \\ Pass \\ Pub \\ FB \\ Sleep \end{array} & \begin{bmatrix} & 0.5 & & & & 0.5 & \\ & & 0.8 & & & & 0.2 \\ & & & 0.6 & 0.4 & & \\ & & & & & & 1.0 \\ 0.2 & 0.4 & 0.4 & & & & \\ 0.1 & & & & & 0.9 & \\ & & & & & & 1 \end{bmatrix} \end{array}$$

# Markov Reward Process

A Markov reward process is a Markov chain with values.

**Definition (MRP)**

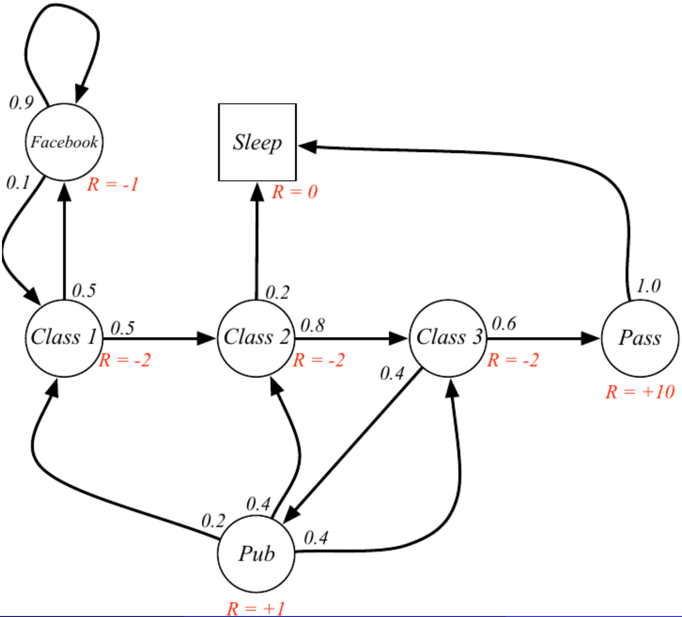
A *Markov Reward Process* is a tuple $(\mathcal{S}, \mathcal{P}, \mathcal{R}, \gamma)$

- $\mathcal{S}$ is a finite set of states
- $\mathcal{P}$ is a state transition probability matrix,

$$P(S_{t+1} \mid S_t) = P(S_{t+1} \mid S_1, \ldots, S_t)$$

- $\mathcal{R}$ is a reward function, $\mathcal{R}_s = \mathbb{E}[R_{t+1} | S_t = s]$
- $\gamma$ is a discount factor, $\gamma \in [0, 1]$

# Example: Student MRP



# Return

## Definition

The *return* $G_t$ is the total discounted reward from time-step $t$.

$$G_t = R_{t+1} + \gamma R_{t+2} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

- The discount $\gamma \in [0, 1]$ is the present value of future rewards
- The value of receiving reward $R$ after $k+1$ time-steps is $\gamma^k R$.
- This values immediate reward above delayed reward.
  - $\gamma$ close to 0 leads to "myopic" evaluation
  - $\gamma$ close to 1 leads to "far-sighted" evaluation

# Why discount?

Most Markov reward and decision processes are discounted. Why?

- Mathematically convenient to discount rewards
- Avoids infinite returns in cyclic Markov processes
- Uncertainty about the future may not be fully represented
- If the reward is financial, immediate rewards may earn more interest than delayed rewards
- Animal/human behaviour shows preference for immediate reward
- It is sometimes possible to use *undiscounted* Markov reward processes (i.e. $\gamma = 1$), e.g. if all sequences terminate.

# Next time

- Continue with MDP's and Bellmann Equation
- Dynamic Programming and Q-Learning