PLOTTING CIRCLES UNDER MOLLWEIDE PROJECTION

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Consider an origin-centered sphere of radius ρ. Place a circle (representing an impact crater) with radius r and center (ρ, θ, ϕ), where θ, ϕ are the usual azimuthal and polar angles in radians, with $0 \leq \theta < 2\pi$ and $0 \leq \phi \leq \pi$.

The angle subtended by a radius of the crater is given in radians by $\alpha = r/\rho$. We assume that $\alpha < \pi/2$.

Consider a rotation of the sphere that sends the north pole to the center of the crater. Evidently this rotation sends the circle given by $\phi = \alpha$ (equivalently the line of latitude $\delta = \pi/2 - \alpha$) to the rim of the crater.

This rotation is given by the composition of a rotation of angle ϕ about the x-axis and a rotation of angle θ around the z-axis. Using rotation matrices,

$$\begin{aligned} R(\theta,\phi) = R_z(\theta)R_x(\phi) &= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \\ &= \begin{bmatrix} \cos\theta & -\sin\theta\cos\phi & \sin\theta\sin\phi \\ \sin\theta & \cos\theta\cos\phi & -\cos\theta\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}. \end{aligned}$$

The image of a point $\mathbf{P} = (\rho, \lambda, \alpha)$ on the rim of the crater is then

$$R(\theta,\phi)\mathbf{P} = \begin{bmatrix} \cos\theta & -\sin\theta\cos\phi & \sin\theta\sin\phi \\ \sin\theta & \cos\theta\cos\phi & -\cos\theta\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \rho\cos\lambda\sin\alpha \\ \rho\sin\lambda\sin\alpha \\ \rho\cos\alpha \end{bmatrix}.$$