

(a)

Assuming the angle between velocity v and x axis is α

So,

$$v_x = v \cos \alpha$$

$$v_y = v \sin \alpha$$

and

$$v^2 = v_x^2 + v_y^2$$

the air resistance is:

$$\vec{F} = \frac{1}{2} \pi R^2 \rho C \vec{v} |v|$$

So,

$$ma_x = -\frac{1}{2} \pi R^2 \rho C v_x |v| = -\frac{1}{2} \pi R^2 \rho C \cdot \frac{dx}{dt} \cdot \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$ma_y = -mg - \frac{1}{2} \pi R^2 \rho C v_y |v| = -mg - \frac{1}{2} \pi R^2 \rho C \cdot \frac{dy}{dt} \cdot \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

So,

$$a_x = -\frac{1}{2m} \pi R^2 \rho C v_x |v| = -\frac{1}{2m} \pi R^2 \rho C \cdot \frac{dx}{dt} \cdot \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$a_y = -g - \frac{1}{2m} \pi R^2 \rho C v_y |v| = -g - \frac{1}{2m} \pi R^2 \rho C \cdot \frac{dy}{dt} \cdot \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

(b)+(c)

The distances travelled by the projectile are as below:

Pb 2453.14 meters

Pu 2872.74 meters

Nd 2011.56 meters

Pu is the farthest.

All these cannonball are in same size, which means the one has bigger density will has bigger mass.

From the equations of motions, we can see that when the mass is bigger, the acceleration will be smaller.

Also, we know that when the ball hits the ground (when we have the maximum x as the distance), we can have this equation in y direction:

$$y = y_0 + v_{y0} \cdot t + \frac{1}{2} a_y t^2 = 0$$

When we have a smaller acceleration a_y , we have more time for the cannonball to travel in the air, that means the ball has more time to travel in x direction. That's why it will have a larger distance.