Preimages of HAVAL and MD5

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Definition

"Given y, find x such that h(x) = y"

Given what?

- 1. random range element
- 2. random image
- 3. any fixed value ("weak" images)

Attacks here work for 1, 2, 3

Previous preimage attacks

```
Google(collision attack) \approx 500\,000
Google (preimage attack) \approx 15\,000
```

- ► MD2 (2004)
- ► Parallel FFT-Hashing (2007)
- ► Syndrome-based hash (2007)
- ► HAS-V (2007)
- ► Tiger (2007)
- ► MD4 (2008)
- ► GOST (2008)
- ► Snefru (2008)
- ► SHA-0/1 (2008)

Content of this talk

MD5: how to invert. . .

- ▶ 22 steps?
- ▶ 31 steps?
- ▶ 47 steps?

What about **HAVAL**?

From the **compression to hash** function

MD5

- ▶ 1991: publication (Rivest)
- ▶ 1993: **collision** attack (compression function)
- ▶ 2005: **collision** attack (hash function)
- ▶ 2005+: faster, chosen-prefix, meaningful collisions

No preimage attack.



MD5 compression function

Input

- ► chain value $H_0H_1H_2H_3$
- ► message $M_0 M_1 \dots M_{15}$

Algorithm

- ► copy $H_0H_1H_2H_3$ into $A_0B_0C_0D_0$
- ▶ for i = 1 ... 64

$$A_i = D_{i-1}$$

 $B_i = f_i(A_{i-1}, B_{i-1}, C_{i-1}, D_{i-1}, M_{\sigma(i)})$
 $C_i = B_{i-1}$
 $D_i = C_{i-1}$

► return $(A_0 + A_{64}) \| (B_0 + B_{64}) \| (C_0 + C_{64}) \| (D_0 + D_{64}) \|$

Unrolling (index, function, message word)

```
1 f_1 (A_0, B_0, C_0, D_0, 0)
                                                    17
                                                            f_{17}(A_{16}, B_{16}, C_{16}, D_{16}, 1)
 2 f_2(A_1, B_1, C_1, D_1, 1)
                                                    18
                                                            f_{18}(A_{17}, B_{17}, C_{17}, D_{17}, 6)
      f_3 (A_2, B_2, C_2, D_2, 2)
                                                     19
                                                            f_{19}(A_{18}, B_{18}, C_{18}, D_{18}, 11)
      f_4 (A_3, B_3, C_3, D_3, 3)
                                                    20
                                                            f_{20}(A_{19}, B_{19}, C_{19}, D_{19}, 0)
      f_5 (A_4, B_4, C_4, D_4, 4)
                                                    21
                                                            f_{21}(A_{20}, B_{20}, C_{20}, D_{20}, 5)
      f_6 (A_5, B_5, C_5, D_5, 5)
                                                    22
                                                            f_{22}(A_{21}, B_{21}, C_{21}, D_{21}, 10)
      f_7 (A_6, B_6, C_6, D_6, 6)
                                                     23
                                                            f_{23}(A_{22}, B_{22}, C_{22}, D_{22}, 15)
      f_8 (A_7, B_7, C_7, D_7, 7)
                                                     24
                                                            f_{24}(A_{23}, B_{23}, C_{23}, D_{23}, 4)
      f_9 (A_8, B_8, C_8, D_8, 8)
                                                    25
                                                            f_{25}(A_{24}, B_{24}, C_{24}, D_{24}, 9)
10
      f_{10}(A_9, B_9, C_9, D_9, 9)
                                                     26
                                                            f_{26}(A_{25}, B_{25}, C_{25}, D_{25}, 14)
11
       f_{11}(A_{10}, B_{10}, C_{10}, D_{10}, 10)
                                                    27
                                                            f_{27}(A_{26}, B_{26}, C_{26}, D_{26}, 3)
12
       f_{12}(A_{11}, B_{11}, C_{11}, D_{11}, 11)
                                                            f_{28}(A_{27}, B_{27}, C_{27}, D_{27}, 8)
                                                     28
13
       f_{13}(A_{12}, B_{12}, C_{12}, D_{12}, 12)
                                                     29
                                                            f_{29}(A_{28}, B_{28}, C_{28}, D_{28}, 13)
14
       f_{14}(A_{13}, B_{13}, C_{13}, D_{13}, 13)
                                                     30
                                                            f_{30}(A_{29}, B_{29}, C_{29}, D_{29}, 2)
15
       f_{15}(A_{14}, B_{14}, C_{14}, D_{14}, 14)
                                                     31
                                                            f_{31}(A_{30}, B_{30}, C_{30}, D_{30}, 7)
16
       f_{16}(A_{15}, B_{15}, C_{15}, D_{15}, 15)
                                                     32
                                                            f_{32}(A_{31}, B_{31}, C_{31}, D_{31}, 12)
```

First 2 rounds

```
f(A_0, B_0, C_0, D_0, 0)
                                              17
                                                    g(A_{16}, B_{16}, C_{16}, D_{16}, 1)
 2 f(A_1, B_1, C_1, D_1, 1)
                                              18
                                                    g(A_{17}, B_{17}, C_{17}, D_{17}, 6)
 3 f(A_2, B_2, C_2, D_2, 2)
                                              19
                                                    q(A_{18}, B_{18}, C_{18}, D_{18}, 11)
 4 f(A_3, B_3, C_3, D_3, 3)
                                              20
                                                    g(A_{19}, B_{19}, C_{19}, D_{19}, 0)
 5 f(A_4, B_4, C_4, D_4, 4)
                                                     g(A_{20}, B_{20}, C_{20}, D_{20}, 5)
                                              21
 6 f(A_5, B_5, C_5, D_5, 5)
                                              22
                                                    q(A_{21}, B_{21}, C_{21}, D_{21}, 10)
 7 f(A_6, B_6, C_6, D_6, 6)
                                              23
                                                    a(A_{22}, B_{22}, C_{22}, D_{22}, 15)
 8 f(A_7, B_7, C_7, D_7, 7)
                                              24
                                                    q(A_{23}, B_{23}, C_{23}, D_{23}, 4)
 9 f(A_8, B_8, C_8, D_8, 8)
                                              25
                                                    g(A_{24}, B_{24}, C_{24}, D_{24}, 9)
10 f(A_9, B_9, C_9, D_9, 9)
                                              26
                                                     g(A_{25}, B_{25}, C_{25}, D_{25}, 14)
11
    f(A_{10}, B_{10}, C_{10}, D_{10}, 10)
                                              27
                                                     g(A_{26}, B_{26}, C_{26}, D_{26}, 3)
12 f(A_{11}, B_{11}, C_{11}, D_{11}, 11)
                                              28
                                                     a(A_{27}, B_{27}, C_{27}, D_{27}, 8)
13
      f(A_{12}, B_{12}, C_{12}, D_{12}, 12)
                                                     g(A_{28}, B_{28}, C_{28}, D_{28}, 13)
                                              29
14
    f(A_{13}, B_{13}, C_{13}, D_{13}, 13)
                                              30
                                                     g(A_{29}, B_{29}, C_{29}, D_{29}, 2)
15
    f(A_{14}, B_{14}, C_{14}, D_{14}, 14)
                                              31
                                                     g(A_{30}, B_{30}, C_{30}, D_{30}, 7)
16 f(A_{15}, B_{15}, C_{15}, D_{15}, 15)
                                                    q(A_{31}, B_{31}, C_{31}, D_{31}, 12)
                                              32
```

Inverting 22 steps

```
17 g(A_{16}, B_{16}, C_{16}, D_{16}, 1)
     f(A_0, B_0, C_0, D_0, 0)
 2 f(A_1, B_1, C_1, D_1, 1)
                                           18
                                                 g(A_{17}, B_{17}, C_{17}, D_{17}, 6)
 3 f(A_2, B_2, C_2, D_2, 2)
                                           19
                                                 g(A_{18}, B_{18}, C_{18}, D_{18}, 11)
 4 f(A_3, B_3, C_3, D_3, 3)
                                                 g(A_{19}, B_{19}, C_{19}, D_{19}, 0)
                                           20
 5 f(A_4, B_4, C_4, D_4, 4)
                                           21
                                                  g(A_{20}, B_{20}, C_{20}, D_{20}, 5)
 6 f(A_5, B_5, C_5, D_5, 5)
                                           22
                                                 q(A_{21}, B_{21}, C_{21}, D_{21}, 10)
 7 f(A_6, B_6, C_6, D_6, 6)
 8 f(A_7, B_7, C_7, D_7, 7)
 9 f(A_8, B_8, C_8, D_8, 8)
10 f(A_9, B_9, C_9, D_9, 9)
11
    f(A_{10}, B_{10}, C_{10}, D_{10}, 10)
12 f(A_{11}, B_{11}, C_{11}, D_{11}, 11)
13 f(A_{12}, B_{12}, C_{12}, D_{12}, 12)
14 f(A_{13}, B_{13}, C_{13}, D_{13}, 13)
15 f(A_{14}, B_{14}, C_{14}, D_{14}, 14)
16 f(A_{15}, B_{15}, C_{15}, D_{15}, 15)
```

Inverting 22 steps

```
Pick M_0 \dots M_{11}
```

From **1** to **12**, compute $A_{12}B_{12}C_{12}D_{12}$

From **22** to **17**, compute $A_{16}B_{16}C_{16}D_{16}$

Choose M_{12} such that $B_{13} = A_{16}$ Choose M_{13} such that $B_{14} = D_{16}$ Choose M_{14} such that $B_{15} = C_{16}$ Choose M_{15} such that $B_{16} = B_{16}$

Cost: 22 steps

31 steps: same idea...

```
f(A_0, B_0, C_0, D_0, 0)
                                              17 g(A_{16}, B_{16}, C_{16}, D_{16}, 1)
 2 f(A_1, B_1, C_1, D_1, 1)
                                              18
                                                     a(A_{17}, B_{17}, C_{17}, D_{17}, 6)
 3 f(A_2, B_2, C_2, D_2, 2)
                                              19
                                                     g(A_{18}, B_{18}, C_{18}, D_{18}, 11)
 4
     f(A_3, B_3, C_3, D_3, 3)
                                                     g(A_{19}, B_{19}, C_{19}, D_{19}, 0)
                                              20
 5 f(A_4, B_4, C_4, D_4, 4)
                                              21
                                                     g(A_{20}, B_{20}, C_{20}, D_{20}, 5)
     f(A_5, B_5, C_5, D_5, 5)
                                                     q(A_{21}, B_{21}, C_{21}, D_{21}, 10)
                                              22
 7 f(A_6, B_6, C_6, D_6, 6)
                                              23
                                                     g(A_{22}, B_{22}, C_{22}, D_{22}, 15)
    f(A_7, B_7, C_7, D_7, 7)
                                              24
                                                     g(A_{23}, B_{23}, C_{23}, D_{23}, 4)
      f(A_8, B_8, C_8, D_8, 8)
                                              25
                                                     q(A_{24}, B_{24}, C_{24}, D_{24}, 9)
10
      f(A_9, B_9, C_9, D_9, 9)
                                              26
                                                     g(A_{25}, B_{25}, C_{25}, D_{25}, 14)
11
      f(A_{10}, B_{10}, C_{10}, D_{10}, 10)
                                              27
                                                     g(A_{26}, B_{26}, C_{26}, D_{26}, 3)
12
      f(A_{11}, B_{11}, C_{11}, D_{11}, 11)
                                              28
                                                     g(A_{27}, B_{27}, C_{27}, D_{27}, 8)
13
      f(A_{12}, B_{12}, C_{12}, D_{12}, 12)
                                              29
                                                     g(A_{28}, B_{28}, C_{28}, D_{28}, 13)
14
      f(A_{13}, B_{13}, C_{13}, D_{13}, 13)
                                              30
                                                     g(A_{29}, B_{29}, C_{29}, D_{29}, 2)
15 f(A_{14}, B_{14}, C_{14}, D_{14}, 14)
                                              31
                                                     g(A_{30}, B_{30}, C_{30}, D_{30}, 7)
16
      f(A_{15}, B_{15}, C_{15}, D_{15}, 15)
```

M_{12} input only once...

```
g(A_{16}, B_{16}, C_{16}, D_{16}, 1)
      f(A_0, B_0, C_0, D_0, 0)
                                               17
 2 f(A_1, B_1, C_1, D_1, 1)
                                               18
                                                      g(A_{17}, B_{17}, C_{17}, D_{17}, 6)
 3 f(A_2, B_2, C_2, D_2, 2)
                                               19
                                                      g(A_{18}, B_{18}, C_{18}, D_{18}, 11)
 4
     f(A_3, B_3, C_3, D_3, 3)
                                                      g(A_{19}, B_{19}, C_{19}, D_{19}, 0)
                                               20
 5
     f(A_4, B_4, C_4, D_4, 4)
                                               21
                                                      g(A_{20}, B_{20}, C_{20}, D_{20}, 5)
      f(A_5, B_5, C_5, D_5, 5)
                                                      q(A_{21}, B_{21}, C_{21}, D_{21}, 10)
                                               22
      f(A_6, B_6, C_6, D_6, 6)
                                               23
                                                      g(A_{22}, B_{22}, C_{22}, D_{22}, 15)
     f(A_7, B_7, C_7, D_7, 7)
                                               24
                                                      g(A_{23}, B_{23}, C_{23}, D_{23}, 4)
 9
      f(A_8, B_8, C_8, D_8, 8)
                                               25
                                                      q(A_{24}, B_{24}, C_{24}, D_{24}, 9)
10
      f(A_9, B_9, C_9, D_9, 9)
                                               26
                                                      g(A_{25}, B_{25}, C_{25}, D_{25}, 14)
11
      f(A_{10}, B_{10}, C_{10}, D_{10}, 10)
                                               27
                                                      g(A_{26}, B_{26}, C_{26}, D_{26}, 3)
12
      f(A_{11}, B_{11}, C_{11}, D_{11}, 11)
                                               28
                                                      g(A_{27}, B_{27}, C_{27}, D_{27}, 8)
13
      f(A_{12}, B_{12}, C_{12}, D_{12}, 12)
                                               29
                                                      g(A_{28}, B_{28}, C_{28}, D_{28}, 13)
14
      f(A_{13}, B_{13}, C_{13}, D_{13}, 13)
                                               30
                                                      g(A_{29}, B_{29}, C_{29}, D_{29}, 2)
15
      f(A_{14}, B_{14}, C_{14}, D_{14}, 14)
                                               31
                                                      g(A_{30}, B_{30}, C_{30}, D_{30}, 7)
16
      f(A_{15}, B_{15}, C_{15}, D_{15}, 15)
```

31 steps

Pick $M_0 \dots M_{11}, M_{13}, M_{14}, M_{15}$

From **1** to **12**, compute $A_{12}B_{12}C_{12}D_{12}$

From **22** to **14**, compute $A_{13}B_{13}C_{13}D_{13}$

If $A_{13} = D_{12}$, $C_{13} = B_{12}$, and $D_{13} = C_{12}$:

then choose M_{12} such that $B_{13} = B_{13}$

 $\textbf{Cost:} \approx 2^{96} \times 31 \text{ steps}$

47 steps: M_2 input only twice...

```
f(\ldots, 0)
                                         h(\ldots, 5)
                17 g(..., 1)
                                     33
 2  f(\ldots, 1)
                 18 g(..., 6)
                                     34
                                        h(\ldots, 8)
 3 f(\ldots, 2) 19 g(\ldots, 11)
                                         h(...,11)
                                     35
 4 f(\ldots, 3) 20 g(\ldots, 0)
                                     36
                                         h(..., 14)
 5 f(\ldots, 4) 21 g(\ldots, 5)
                                     37
                                        h(\ldots, 1)
 6 f(..., 5)
                 22 g(...,10)
                                     38
                                        h(\ldots, 4)
 7 f(..., 6)
                 23 g(..., 15)
                                     39
                                         h(\ldots, 7)
 8 f(..., 7) 24 g(..., 4)
                                     40
                                         h(...,10)
   f(..., 8) 25 g(..., 9)
                                     41
                                         h(..., 13)
10 f(..., 9)
                  26 g(..., 14)
                                        h(\ldots, 0)
                                     42
11
   f(\ldots,10)
                  27 g(..., 3)
                                     43
                                         h(\ldots, 3)
12
   f(...,11)
                 28 q(..., 8)
                                     44
                                         h(\ldots, 6)
   f(..., 12)
                  29 g(..., 13)
                                         h(\ldots, 9)
13
                                     45
14
   f(..., 13)
                 30 g(..., 2)
                                     46
                                         h(..., 12)
                                         h(..., 15)
15
   f(..., 14)
                 31 g(..., 7)
                                     47
16
   f(..., 15)
                  32 q(..., 12)
```

Differences propagation, general case

Pick random A_0 , B_0 , C_0 , D_0 and M

1
$$f(A_0, B_0, C_0, D_0, 0)$$

2 $f(A_1, B_1, C_1, D_1, 1)$

3
$$f(A_2, B_2, C_2, D_2, 2)$$

Modify C_0 to C_0^*

$$X$$
 1 $f(A_0, B_0, C_0^*, D_0, 0)$
 X 2 $f(A_1, B_1, C_1, C_0^*, 1)$
 X 3 $f(C_0^*, B_2, C_2, D_2, 2)$

⇒ all first steps affected (X=state modified)

Difference in C_0 + chosen IV

Pick random A_0 , C_0 , D_0 and M and set $B_0 = 0$

1
$$f(A_0, B_0, C_0, D_0, 0)$$

2 $f(A_1, B_1, C_1, D_1, 1)$

3
$$f(A_2, B_2, C_2, D_2, 2)$$

Modify C_0 to C_0^*

$$\sqrt{ 1 } f(A_0, 0, C_0^*, D_0, 0)
\sqrt{ 2 } f(A_1, B_1, 0, C_0^*, 1)
X 3 $f(C_0^*, B_2, C_2, 0, 2)$$$

⇒ only step 3 affected

Difference in M₂

Pick random A_0, B_0, C_0, D_0 and M

1
$$f(A_0, B_0, C_0, D_0, 0)$$

2 $f(A_1, B_1, C_1, D_1, 1)$
3 $f(A_2, B_2, C_2, D_2, 2)$

Modify M₂

⇒ only step 3 affected

Absorbing differences

Pick random A_0 , C_0 , D_0 and M and set $B_0 = 0$

1
$$f(A_0, B_0, C_0, D_0, 0)$$

2
$$f(A_1, B_1, C_1, D_1, 1)$$

3
$$f(A_2, B_2, C_2, D_2, 2)$$

Modify C_0 to C_0^* and M_2

$$\sqrt{ 1} f(A_0, 0, C_0^*, D_0, 0)
\sqrt{ 2} f(A_1, B_1, 0, C_0^*, 1)
\sqrt{ 3} f(C_0^*, B_2, C_2, 0, 2)$$

 \Rightarrow nothing changes!

Application to 47-step MD5: key steps

```
1 f(..., 0)
                                       h(\ldots, 5)
              17 g(..., 1)
                                  33
2 f(..., 1)
                18 g(..., 6)
                                  34
                                      h(\ldots, 8)
3 f(\ldots, 2) 19 g(\ldots, 11)
                                      h(...,11)
                                   35
4 f(\ldots, 3) 20 g(\ldots, 0)
                                   36
                                       h(..., 14)
5 f(\ldots, 4) 21 g(\ldots, 5)
                                  37 h(..., 1)
6 f(..., 5)
                22 g(..., 10)
                                   38 h(..., 4)
7 f(..., 6)
                23 g(..., 15)
                                   39
                                       h(\ldots, 7)
8 f(..., 7) 24 g(..., 4)
                                  40
                                      h(...,10)
   f(..., 8) 25 g(..., 9)
                                  41
                                      h(..., 13)
10 f(..., 9)
                 26 g(..., 14)
                                      h(\ldots, 0)
                                  42
11
   f(..., 10)
                 27 g(\ldots, 3)
                                  43
                                       h(\ldots, 3)
12
   f(...,11)
                28 g(..., 8)
                                  44
                                       h(\ldots, 6)
   f(..., 12)
                 29 g(..., 13)
                                       h(\ldots, 9)
13
                                  45
14
   f(..., 13)
                30 g(..., 2)
                                  46
                                       h(..., 12)
15
   f(..., 14)
                31 g(..., 7)
                                  47
                                       h(..., 15)
16
  f(..., 15)
                 32 q(..., 12)
```

The attack

Stage 1: MITM

Pick M and IV with $B_0 = 0$,

- 1. store $(A_{29}, B_{29}, C_{29}, D_{29})$ for all 2^{32} C_0 's (forward)
- 2. store $(A_{30}, B_{30}, C_{30}, D_{30})$ for all 2^{32} c C_{47} 's (backward)

Find entries such that

$$A_{30} = D_{29}$$

 $D_{30} = C_{29}$
 $C_{30} = B_{29}$

 \equiv 96-bit equality; 2⁶⁴ choices \Rightarrow repeat 2³² times

The attack

Stage 2: correction

Modify M2 such that

$$B_{30} = g(A_{29}, B_{29}, C_{29}, D_{29}, 2)$$

Modify C_0 accordingly

 \Rightarrow 96-bit preimage ($C_0 + C_{47}$ is random) with prob. 2^{-32}

Total cost: 2⁹⁶ trials for a 128-bit preimage

Summary

Preimages for MD5's compression function with

- ▶ chosen message except M_1 and M_2
- ▶ IV with $B_0 = 0$ and random C_0
- ► storage for 2³⁶ bytes (64 Gb)
- ▶ 2⁹⁶ compressions

By bruteforce:

- ▶ random message
- chosen IV
- negligible memory
- ▶ 2¹²⁸ compressions

HAVAL

- ▶ 1992: publication (Zheng, Pieprzyk, Seberry)
- ▶ 2003: collision attack (3-pass)
- ▶ 2006: **collision** attack (4- and 5-pass)
- ► 2008: (partial) **second-preimage** attack (3-pass)

Function similar to MD5 with

- ▶ 256-bit chain values
- ▶ 1024-bit blocks
- ▶ 3, 4, or 5 rounds

Preimages for 3-pass HAVAL

Same strategy as for MD5

- identify absorption properties in the initial steps
- ► MITM
- ▶ modify a M_i to complete the MITM
- correct initial steps
- \Rightarrow 2 attacks in 2^{224} and storage of 2^{69} bytes (vs. 2^{256} and negligible memory)

Extension to the iterated hash

Attacks presented for the compression function (with IV partially random, no padding)

Restrictions for the hash function:

- ▶ padding: not a problem, because message chosen
- ▶ fixed IV: makes direct application impossible

Iterated hash: basic MITM

Given image H:

compute a list of images from the fixed IV

$$(M_i, compress(IV, M_i))_i$$

2. compute a list of preimages

$$(H_i, M_i')_i$$
, compress $(H_i, M_i') = \mathbf{H}$

Find entries such that

$$compress(IV, M_i)) = H_i$$

Cost: 2^{113} trials + 2^{36} bytes for MD5, 2^{241} + 2^{69} for HAVAL

Iterated hash: tree technique

Mendel & Rijmen (ICISC'07), Leurent (FSE'08)

Build a tree using multi-target preimages

Cost: 2^{102} trials + 2^{39} bytes for MD5, 2^{230} + 2^{71} for HAVAL

Conclusion

	compressions	bytes
47-step MD5	2 ¹⁰²	2 ³⁹
3-pass HAVAL	2 ²³⁰	2 ⁷¹

Independent work by Sasaki & Aoki @ACISP 2008: 44-step MD5 (starting from step 6) in 2⁹⁶

Questions?

Is it effectively faster than bruteforce?

 \rightarrow Arguably yes (but not 2²⁶ times faster)

Same strategy applies to MD4?

 \rightarrow No (because no M_i at very start and very end)

Same strategy applies to SHA-0/1/2?

 \rightarrow No (nontrivial message expansion), cf. next talk