

TAURUS



The Quantum Computing Risk & Post-Quantum Cryptography

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AMERICAN
UNIVERSITY
OF BEIRUT



Background

Co-founder & chief security officer of **Taurus SA**

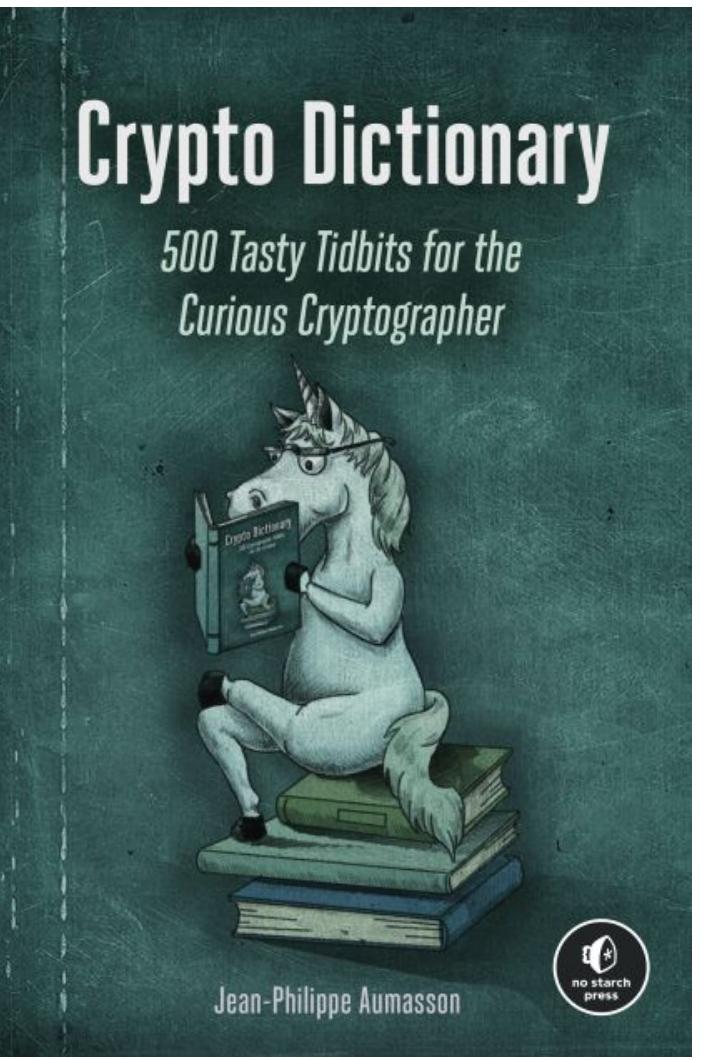
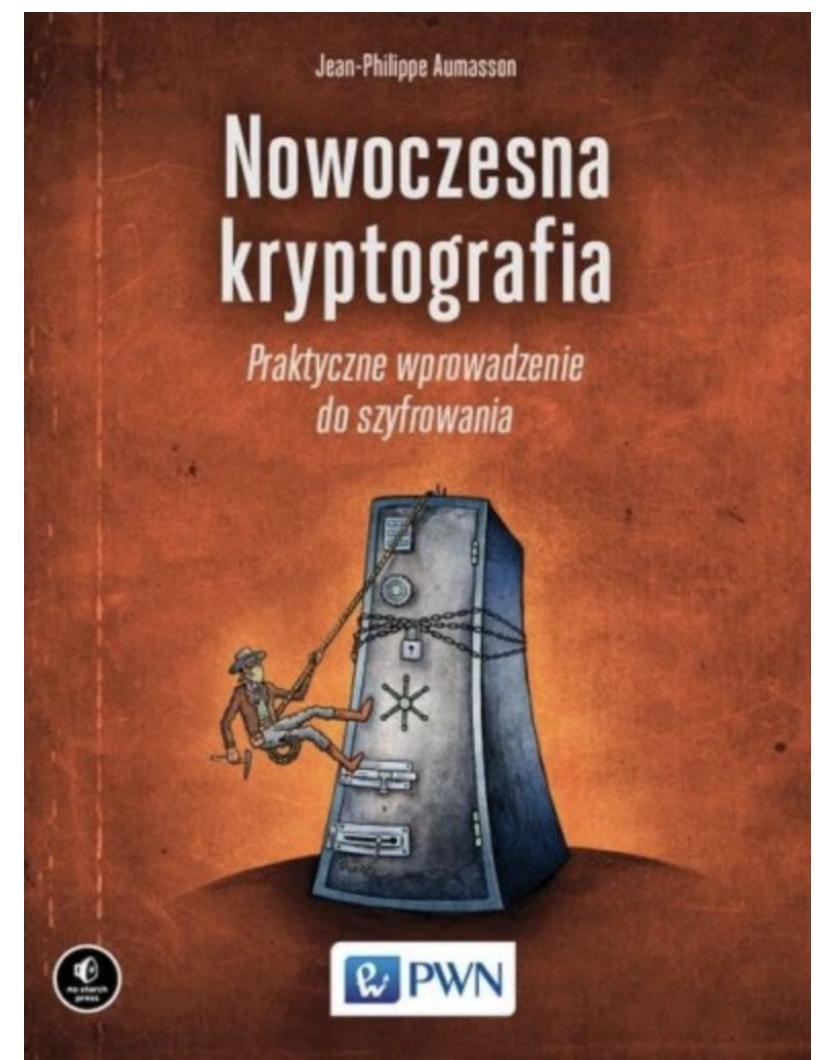
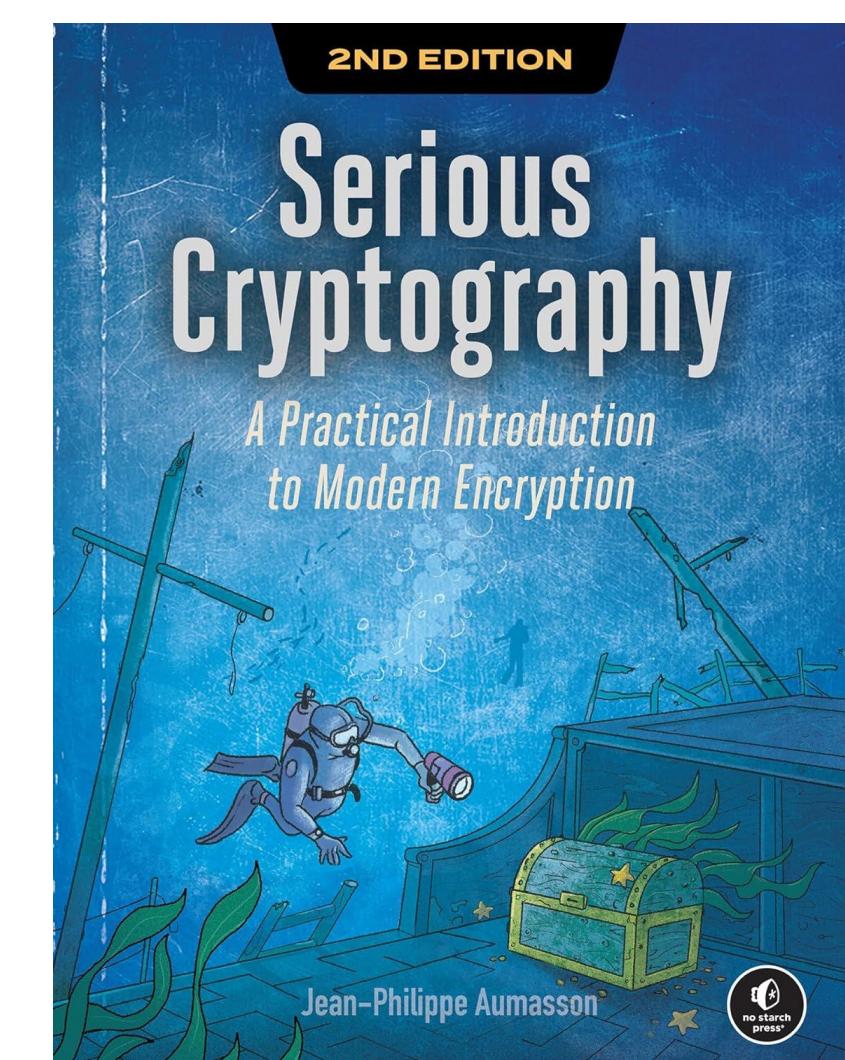


- Fintech founded in 2018, team of 90+
- **Digital asset custody** infrastructure
- Cool tech: HSMs, HA infra, smart contracts, etc.

<https://taurushq.com> <https://t-dx.com>

- 20 years in cryptography & security
- Designed industry standards
- Cryptography books

<https://aumasson.jp>



Quantum physics

Explains how Nature behaves at the smallest scales (atoms, electrons, photons)

It defies common sense:

- Particles can behave like waves (**wave-particle duality**)
- A particle is in an uncertain state until it's observed (**superposition**)
- Particles at large distances appear to influence one another (**non-locality**)

Simulating quantum physics involves complex equations of complexity growing exponentially, practically impossible even with supercomputers

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981



Not to break crypto..

5. CAN QUANTUM SYSTEMS BE PROBABILISTICALLY SIMULATED BY A CLASSICAL COMPUTER?

Now the next question that I would like to bring up is, of course, the interesting one, i.e., Can a quantum system be probabilistically simulated by a classical (probabilistic, I'd assume) universal computer? In other words, a computer which will give the same probabilities as the quantum system does. If you take the computer to be the classical kind I've described so far, (not the quantum kind described in the last section) and there're no changes in any laws, and there's no hocus-pocus, the answer is certainly, No! This is called the hidden-variable problem: it is impossible to represent the results of quantum mechanics with a classical universal device. To learn a little bit about it, I say let us try to put the quantum equations in a form as close as

... but simulate quantum physics

4. QUANTUM COMPUTERS—UNIVERSAL QUANTUM SIMULATORS

The first branch, one you might call a side-remark, is, Can you do it with a new kind of computer—a quantum computer? (I'll come back to the other branch in a moment.) Now it turns out, as far as I can tell, that you can simulate this with a quantum system, with quantum computer elements. It's not a Turing machine, but a machine of a different kind. If we disregard the continuity of space and make it discrete, and so on, as an approximation (the same way as we allowed ourselves in the classical case), it does seem to



Quantum bits (qubits)

Superposition state

$$a |0\rangle + \beta |1\rangle$$

*a, β are "probabilities" called **amplitudes**
(can be complex, and negative numbers)*

0 with probability $|a|^2$

Observation, random outcome

1 with probability $|\beta|^2$



Once observed, a qubit stays 0 or 1 forever

Different math, different computing

ON THE POWER OF QUANTUM COMPUTATION

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Abstract. The quantum model of computation is a model, analogous to the probabilistic Turing Machine, in which the normal laws of chance are replaced by those obeyed by particles on a quantum mechanical scale, rather than the rules familiar to us from the macroscopic world. We present here a problem of distinguishing between two fairly natural classes of function, which can provably be solved exponentially faster in the quantum model than in the classical probabilistic one, when the function is given as an oracle drawn equiprobably from the uniform distribution on either class. We thus offer compelling evidence that the quantum model may have significantly more complexity theoretic power than the probabilistic Turing Machine. In fact, drawing on this work, Shor has recently developed remarkable new quantum polynomial-time algorithms for the discrete logarithm and integer factoring problems.

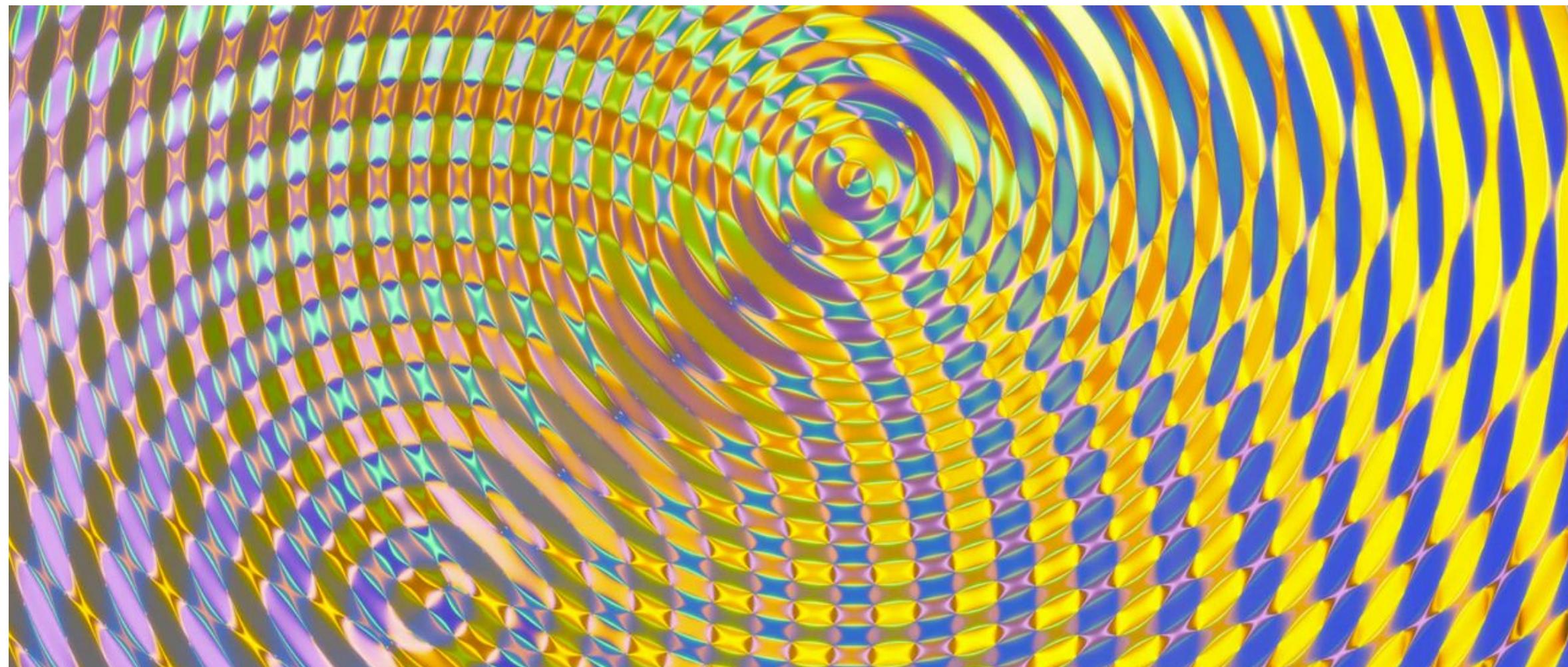
1. Introduction. *You have nothing to do but mention the quantum theory, and people will take your voice for the voice of science, and believe anything.*

—Bernard Shaw, *Geneva* (1938)

Quantum computing's power

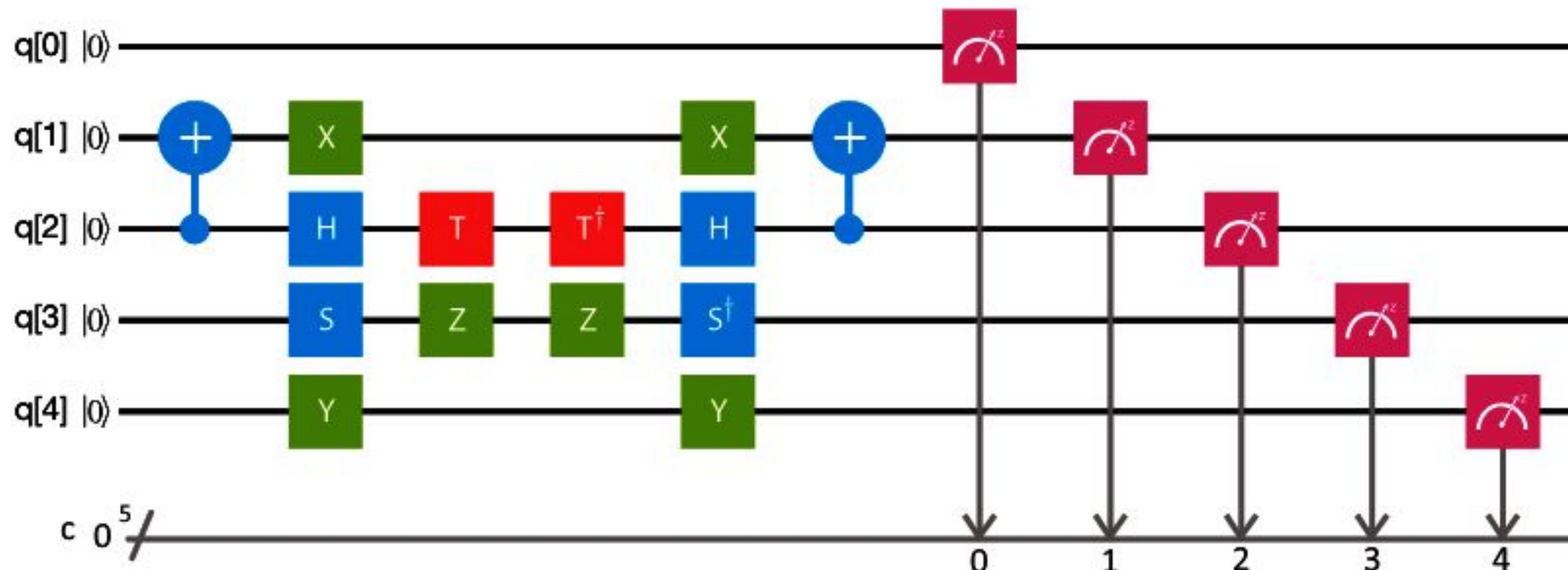
Quantum computing draws its power mainly from two phenomena:

- **Entanglement**, whereby distant particles are "correlated" and appear to influence each other even at large distances. (Cf. EPR paradox.)
- **Interference**, which lets quantum algorithms amplify correct answers and cancel out wrong ones, by arranging how their probability amplitudes combine.



Quantum algorithms

Circuits of quantum gates, transforming a quantum state, ending with an observation



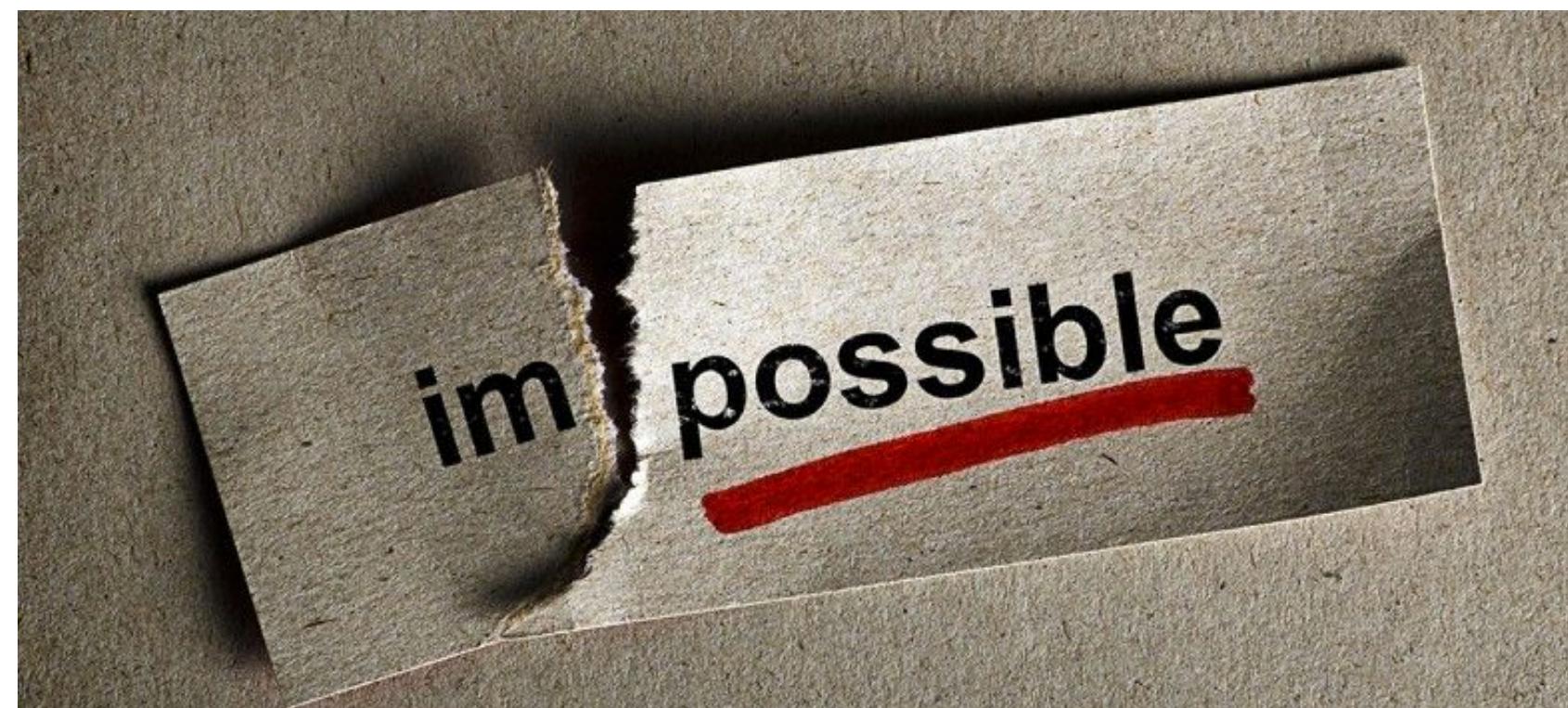
Can be simulated with basic linear algebra but does no scale, exponential cost:

- **Quantum state** = vector of 2^N amplitudes for N qubits
- **Quantum gates** = matrix multiplications, with $O(2^{3N})$ complexity

Quantum speedup

When quantum computers can solve a problem faster than classical computers

Most interesting: **Superpolynomial** quantum speedup (“exponential” boost)



List of problems on the **Quantum Zoo**: <http://math.nist.gov/quantum/zoo/>

Quantum parallelism

Quantum computers “work” on all values simultaneously, via **superposition**

But they do not “*try every answer in parallel and pick the best*”

You can only **observe one “value”** that results from the interference of all, as a projection from the Hilbert space where qubits “live” to some basis

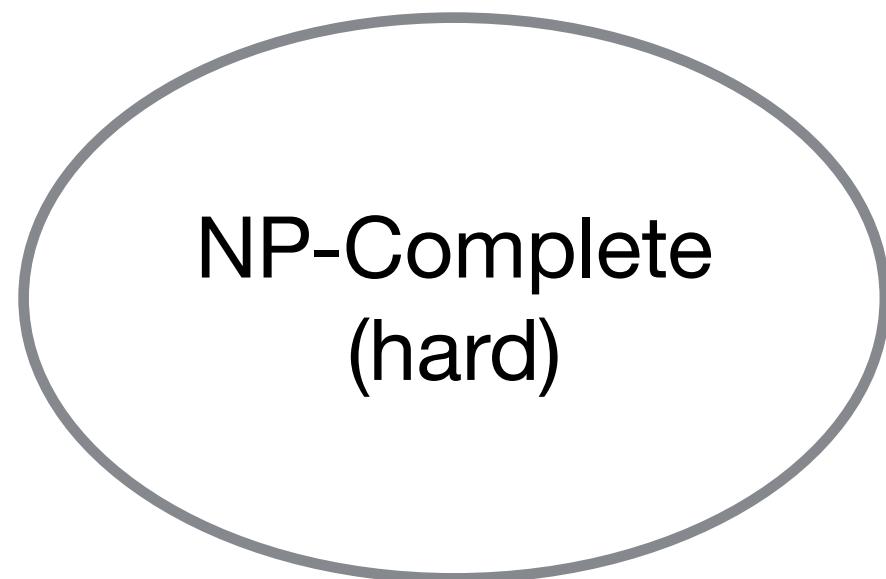


Most hard problems don't benefit from QC

NP-hard problems are very common:

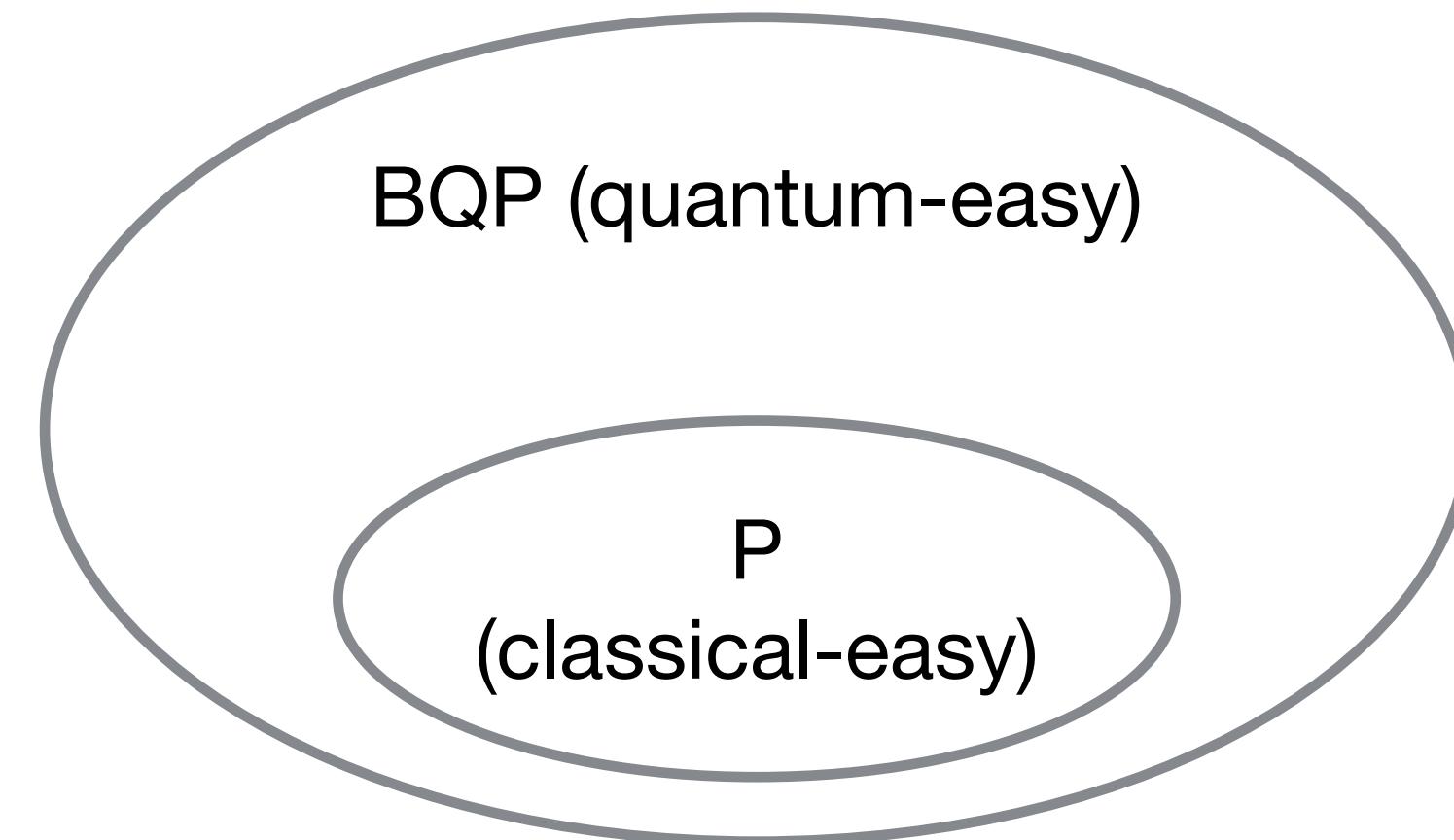
- Problems whose solution is **hard to find, but easy to verify**
- Structured like constraint satisfaction problems (scheduling, puzzle-solving, etc.)

CANNOT be solved faster with quantum computers!



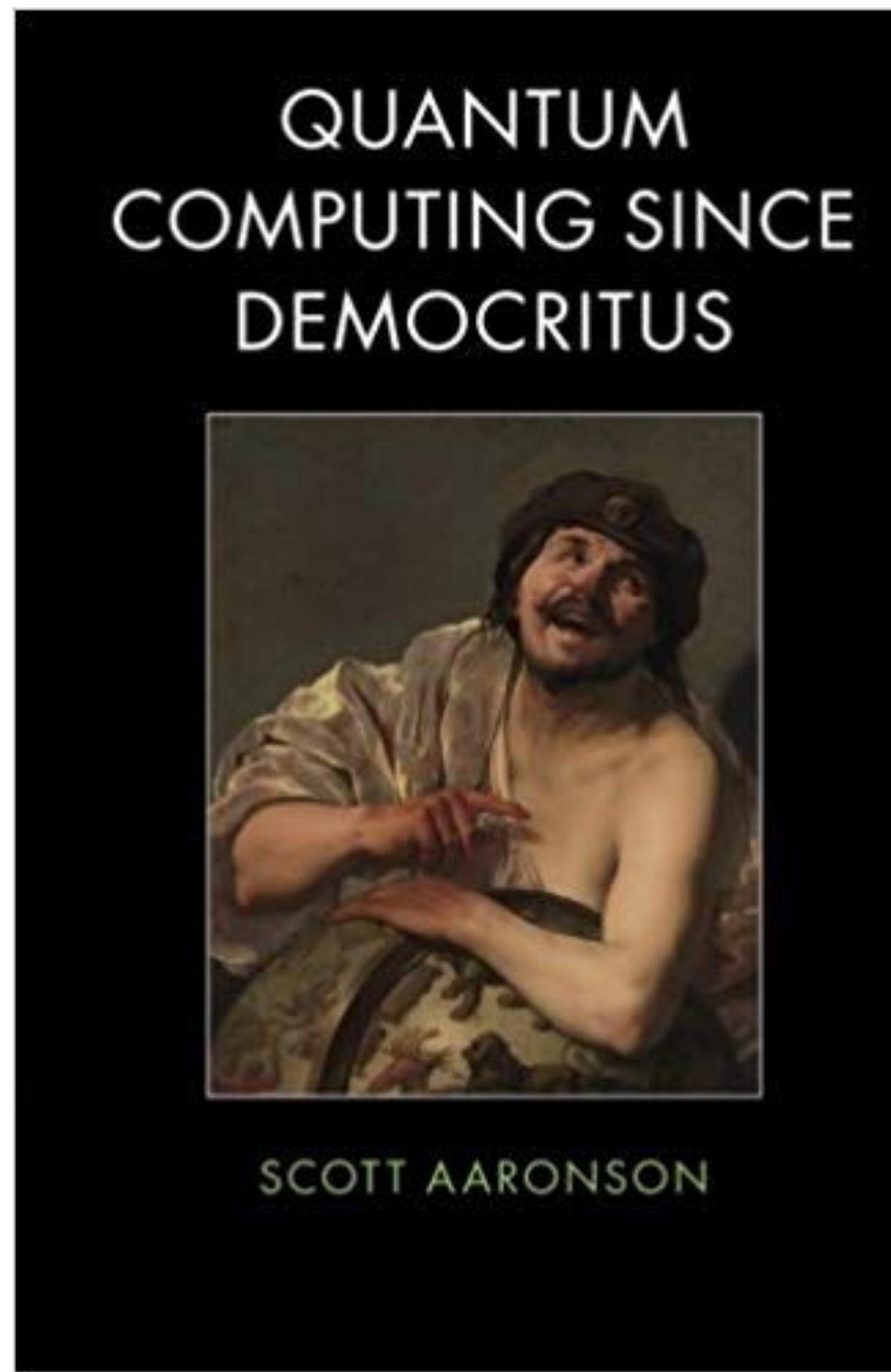
NP is not in BQP (most likely!)

.....
Therefore quantum computers
can't solve NP-hard problems



BQP = bounded-error quantum polynomial time, what QC can solve efficiently

Recommended reading

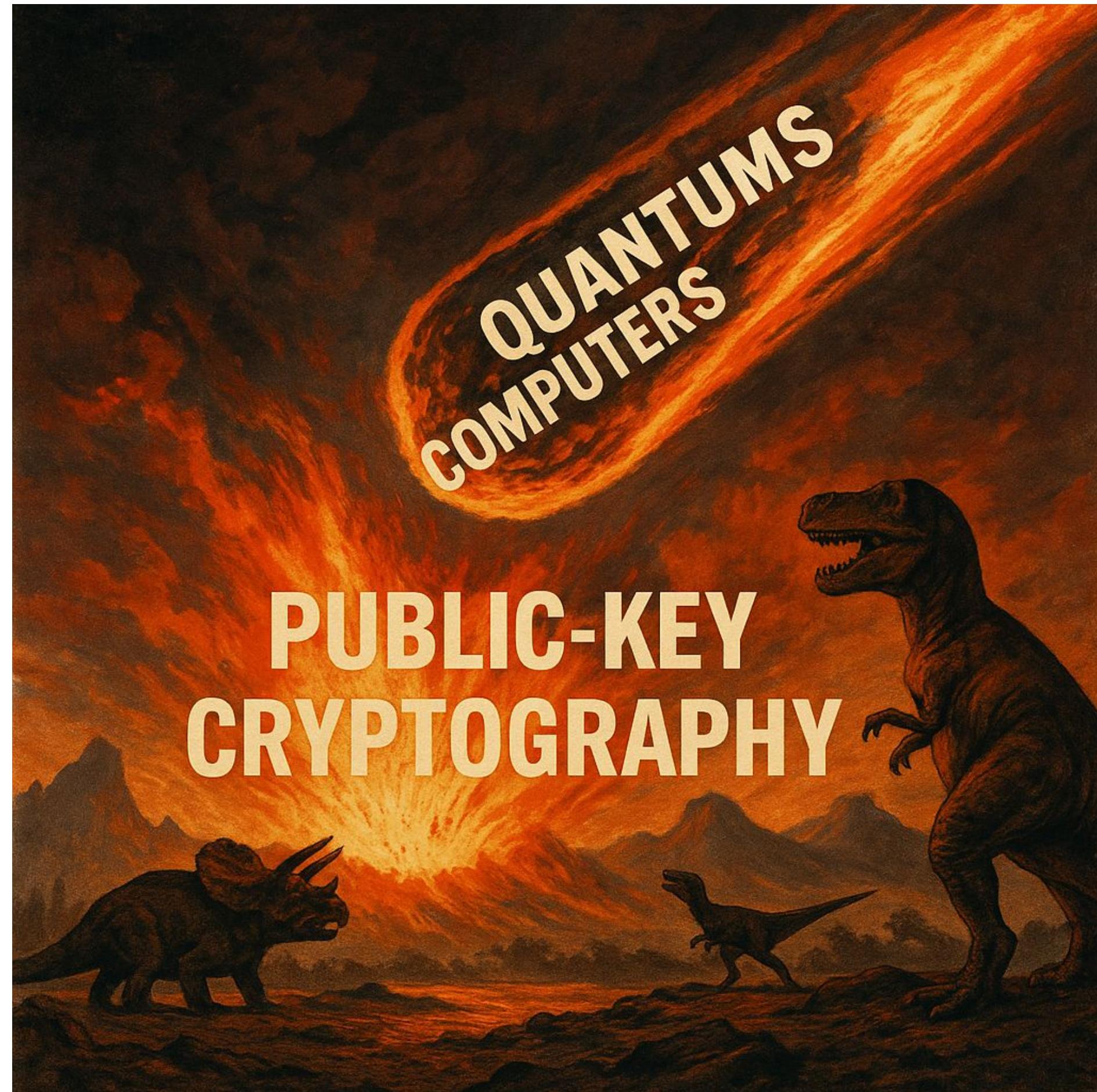


Contents	
Preface	page ix
Acknowledgments	xxix
1. Atoms and the void	1
2. Sets	8
3. Gödel, Turing, and friends	18
4. Minds and machines	29
5. Paleocomplexity	44
6. P, NP, and friends	54
7. Randomness	71
8. Crypto	93
9. Quantum	109
10. Quantum computing	132
11. Penrose	150
12. Decoherence and hidden variables	160
13. Proofs	186

viii CONTENTS	
14. How big are quantum states?	200
15. Skepticism of quantum computing	217
16. Learning	228
17. Interactive proofs, circuit lower bounds, and more	243
18. Fun with the Anthropic Principle	266
19. Free will	290
20. Time travel	307
21. Cosmology and complexity	325
22. Ask me anything	343
Index	363



Impact on cryptography



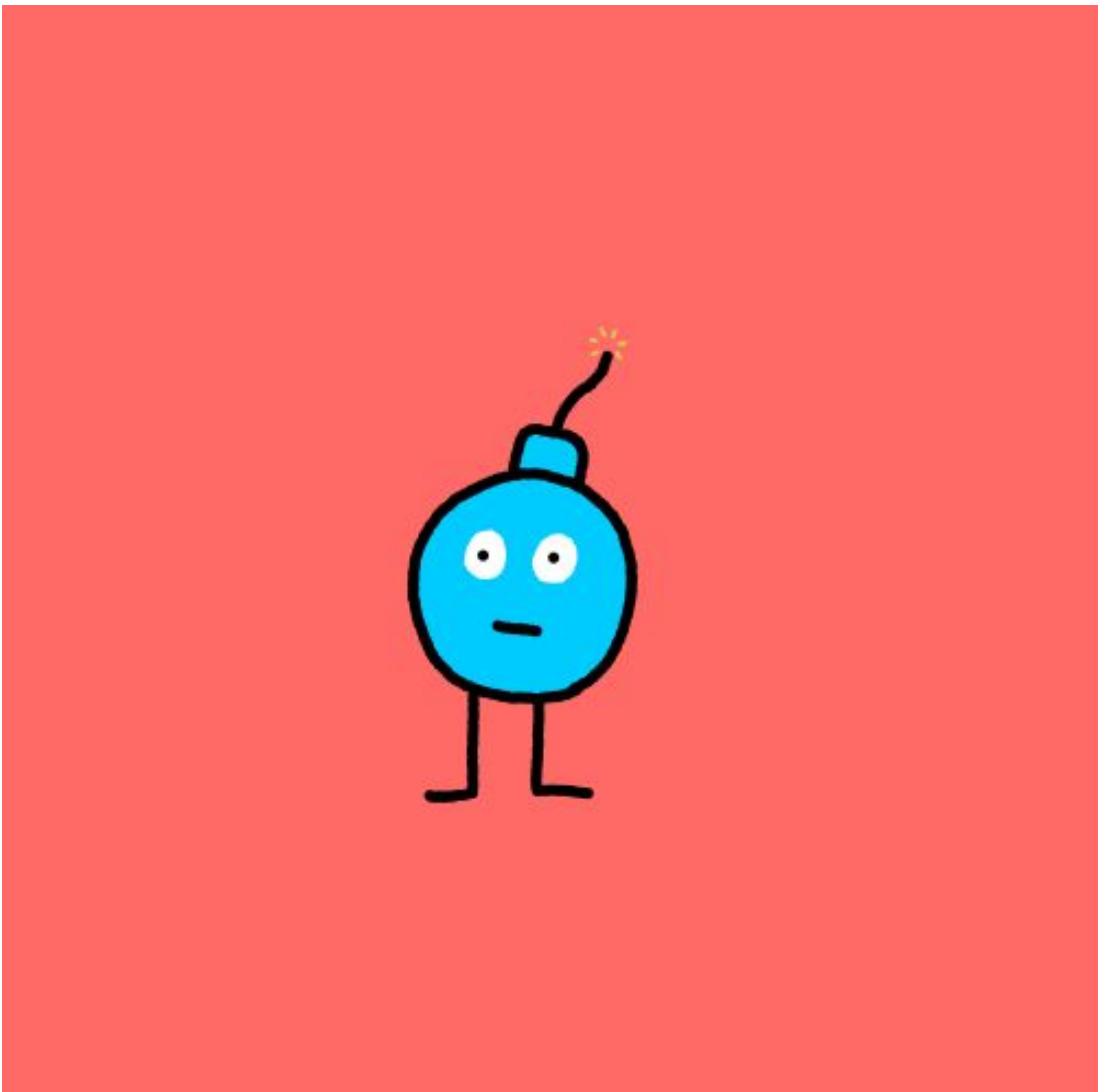
Shor's quantum algorithm

Polynomial-time algorithm for the following problems:

- Computes p given $n = pq$ → RSA dead
- Computes d given $y = x^d \text{ mod } p$ → ECC/DH dead

Practically impossible on a classical machine

#QuantumSpeedup



How bad for crypto?

Worse



Mild: Signatures (ECDSA, Ed25519, etc.)

Broken sigs can be reissued with a post-quantum algorithm



Bad: Key agreement (Diffie-Hellman, ECDH, etc.)

Partially mitigated by secret internal states and reseeding



Terrible: Encryption (RSA encryption, ECIES, etc.)

Encrypted messages compromised forever

Concretely

Worse



Mild: Signatures

PKI certificates, code signing, blockchain transactions, etc.
Migration planned, technology ready



Bad: Key agreement

TLS, IPsec, WireGuard, e2ee messaging (WhatsApp, Signal), etc.
Migration ongoing (e.g. Apple's iMessage, Cloudflare, etc.)



Terrible: Encryption

Key encapsulation, some encrypted backups, PGP messages, etc.
Migration to prioritize

Not there yet

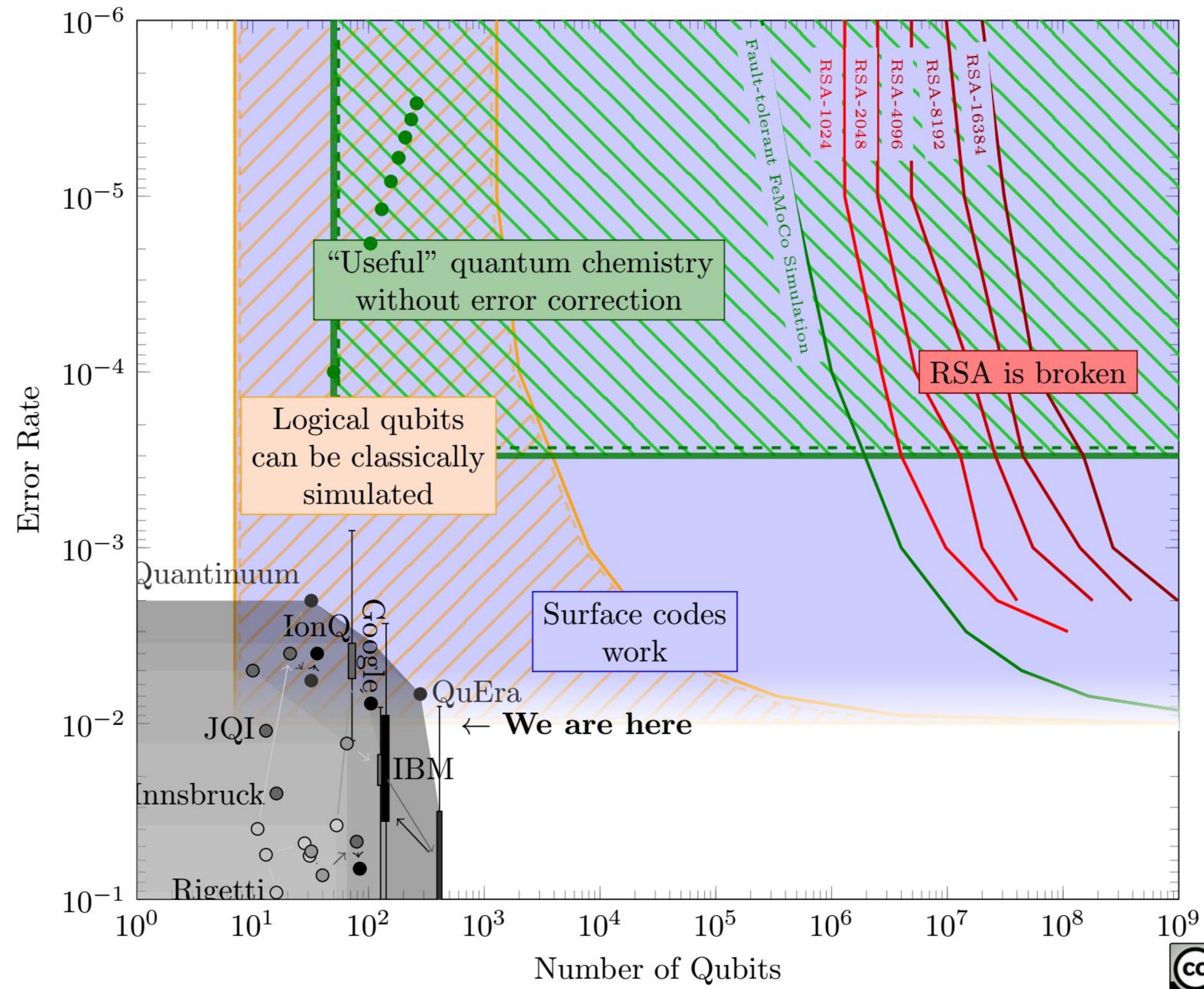
Millions of qubits to break RSA,
to implement **error correction**

QC in its infancy, only research
prototypes useless in practice

Google and IBM leading

2 main dimensions:

- Error rate
- Qubits number (physical,
logical)



https://sam-jaques.appspot.com/quantum_landscape_2024

Beware PR content

Often hyperbolic, misleading claims from QC companies



[Quantum Leap: Google Claims Its New Quantum Computer Provides Evidence That We Live in a Multiverse](#)

Google's latest refinement to its quantum computer, Willow, may represent such a moment. By achieving computational feats once thought to be confined to...

8 Jan 2025



[Google's Quantum Chip Can Do in 5 Minutes What Would Take Other Computers 10 Septillion Years](#)

Google's quantum computing division unveiled a new chip, dubbed Willow, that the tech giant says makes it infinitely faster and better than existing...

10 Dec 2024



Google's Quantum Chip Can Do in 5 Minutes What Would Take Other Computers 10 Septillion Years

Google makes a quantum leap that suggests we may live in a multiverse.



By [Kate Irwin](#) Dec 10, 2024



Harnessing a new type of material

All of today's announcements build on our team's recent breakthrough: the world's first topoconductor. This revolutionary class of materials enables us to create *topological superconductivity*, a [new state of matter](#) that previously existed only in theory. The advance stems from [Microsoft's](#) innovations in the design and fabrication of gate-defined devices that combine indium arsenide (a semiconductor) and aluminum (a superconductor). When cooled to near absolute zero and tuned with magnetic fields,

IBM's roadmap

<https://www.ibm.com/roadmaps/quantum/>

2026

**Demonstrate first example
of scientific quantum
advantage and a fault-
tolerant module.**

We will demonstrate the first examples of quantum advantage using a quantum computer with HPC.

2027

**Diversify quantum
advantage and entangle
fault-tolerant modules.**

The scale, quality, speed of the quantum computer will improve to allow executing quantum circuits at a scale of 10K gates on a 1000+ qubits.

2029

**Deliver the first fault-
tolerant quantum
computer.**

The first fault-tolerant quantum computer will be available to clients and allow execution of 100M gates on 200 qubits.

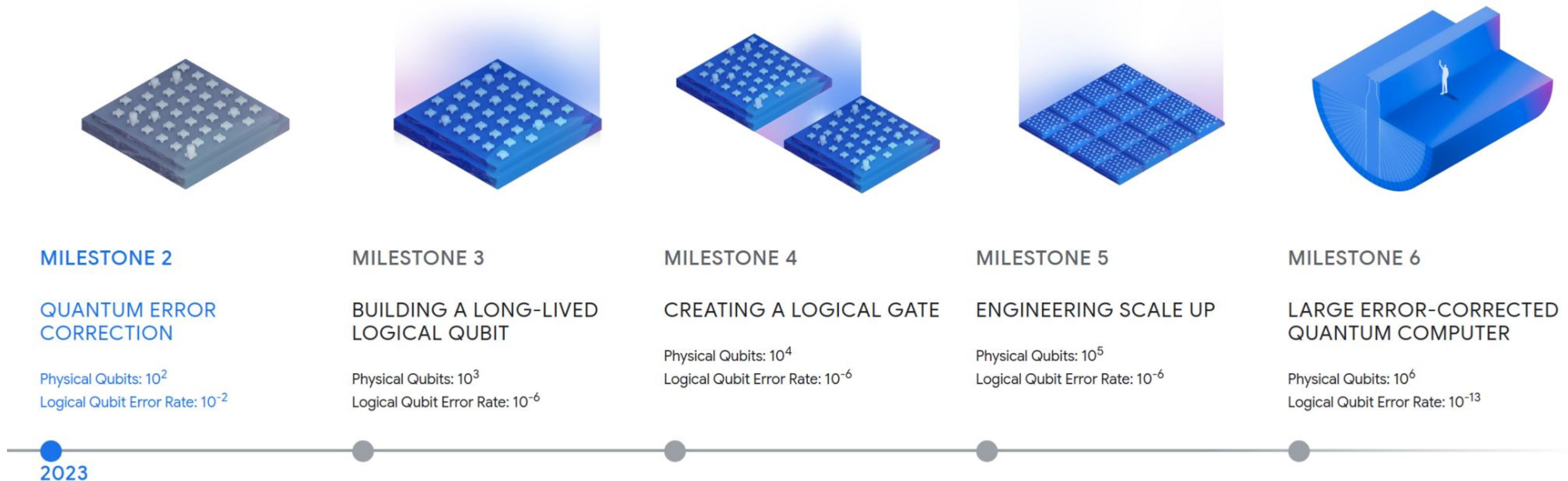
2033+

**Unlock the full power of
quantum computing at
scale.**

Scale fault-tolerant quantum computers to run circuits of 1 billion gates on up to 2000 qubits, unlocking the full power of quantum computing.

Google's roadmap

<https://quantumai.google/roadmap>



Quantum supremacy?

Google thinks it's close to "quantum supremacy." Here's what that really means.

It's not the number of qubits; it's what you do with them that counts.

by Martin Giles and Will Knight March 9, 2018

Seventy-two may not be a large number, but in quantum computing terms, it's massive. This week Google [unveiled](#) Bristlecone, a new quantum computing chip with 72 quantum bits, or qubits—the fundamental units of computation



When it Looks too Good to be True..

Factoring 2 048 RSA integers in 177 days with 13 436 qubits and a multimode memory

Élie Gouzien* and Nicolas Sangouard†

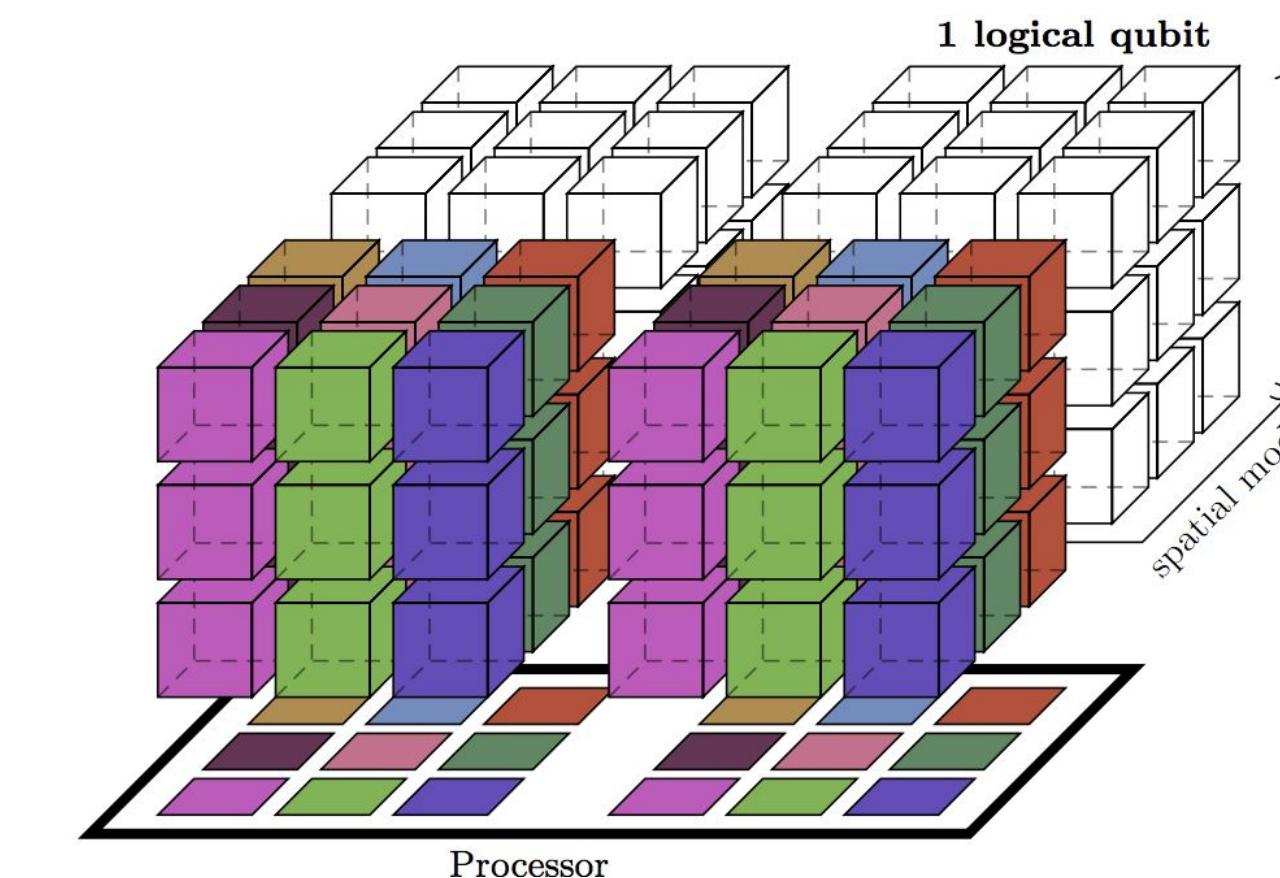
Université Paris-Saclay, CEA, CNRS, Institut de physique théorique, 91191 Gif-sur-Yvette, France

(Dated: March 11, 2021)

We analyze the performance of a quantum computer architecture combining a small processor and a storage unit. By focusing on integer factorization, we show a reduction by several orders of magnitude of the number of processing qubits compared to a standard architecture using a planar grid of qubits with nearest-neighbor connectivity. This is achieved by taking benefit of a temporally and spatially multiplexed memory to store the qubit states between processing steps. Concretely, for a characteristic physical gate error rate of 10^{-3} , a processor cycle time of 1 microsecond, factoring a 2 048 bits RSA integer is shown possible in 177 days with a processor made with 13 436 physical qubits and a multimode memory with 2 hours storage time. By inserting additional error-correction steps, storage times of 1 second are shown to be sufficient at the cost of increasing the runtime by about 23 %. Shorter runtimes (and storage times) are achievable by increasing the number of qubits in the processing unit. We suggest realizing such an architecture using a microwave interface between a processor made with superconducting qubits and a multiplexed memory using the principle of photon echo in solids doped with rare-earth ions.

Introduction — Superconducting qubits form the building blocks of one of the most advanced platforms for realizing quantum computers [1]. The standard architecture consists in laying superconducting qubits in a 2D grid and making the computation using only neighboring interactions. Recent estimations showed however that fault-tolerant realizations of various quantum algorithms with this architecture would require millions physical qubits [2–4]. These performance analyses naturally raise the question of an architecture better exploiting the potential of superconducting qubits.

In developing a quantum computer architecture we have much to learn from classical computer architectures



Sam Jaques

@sejaques

Replying to @veorq

Very important caveat: it needs 430 million "memory qubits"

Craig Gidney @CraigGidney · Mar 15

Replying to @quantumVerd @KikeSolanoPhys and 4 others

The paper uses a cost model where quantum memory is comparatively cheap. I'd have included the mem qubit count in the title (at n=2048 there's 13K compute qubits and 430M mem qubits) but don't see anything wrong with considering a world where mem ends up cheaper than cpu.

Quantum search

Impacts symmetric cryptography

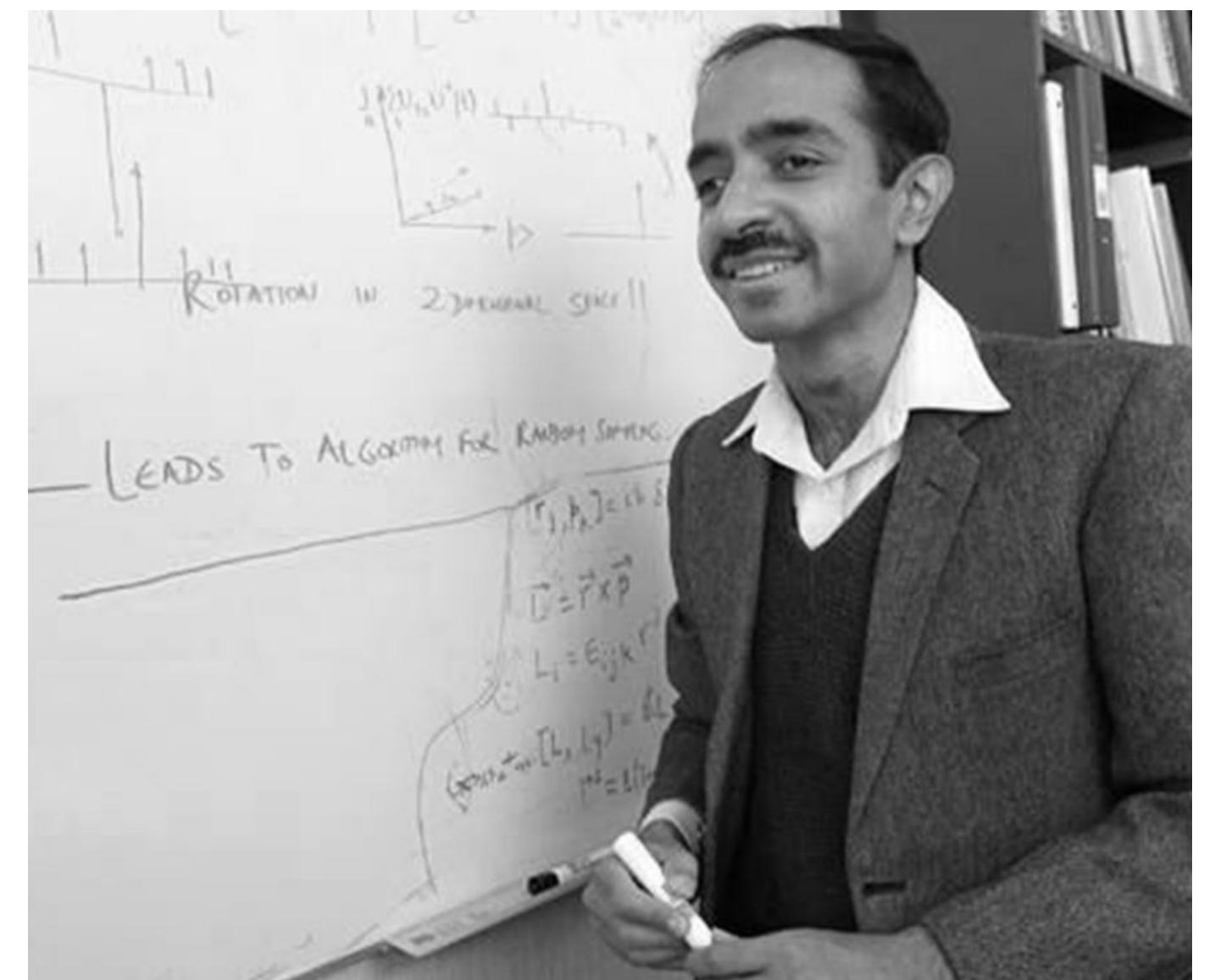
Grover's algorithm (1996)

Searches in N items in \sqrt{N} queries!

- AES-128 broken in $\sqrt{(2^{128})} = 2^{64}$ operations?
- Applications in machine learning models

Caveats:

- Constant factor in **$O(\sqrt{N})$** may be huge
- Doesn't parallelize as classical search does



Quantum-searching AES keys

k	T	#gates Clifford	depth		#qubits
			T	overall	
128	$1.19 \cdot 2^{86}$	$1.55 \cdot 2^{86}$	$1.06 \cdot 2^{80}$	$1.16 \cdot 2^{81}$	2,953
192	$1.81 \cdot 2^{118}$	$1.17 \cdot 2^{119}$	$1.21 \cdot 2^{112}$	$1.33 \cdot 2^{113}$	4,449
256	$1.41 \cdot 2^{151}$	$1.83 \cdot 2^{151}$	$1.44 \cdot 2^{144}$	$1.57 \cdot 2^{145}$	6,681

Table 5. Quantum resource estimates for Grover's algorithm to attack AES- k , where $k \in \{128, 192, 256\}$.

<https://arxiv.org/pdf/1512.04965v1.pdf>

If gates are the size of a hydrogen atom (12pm) this depth is the **diameter of the solar system** ($\sim 10^{13}$ m), yet less than 5 grams

More efficient circuits will be designed...

Quantum-searching AES keys

From February 2020, better circuits found

Implementing Grover oracles for quantum key search on AES and LowMC

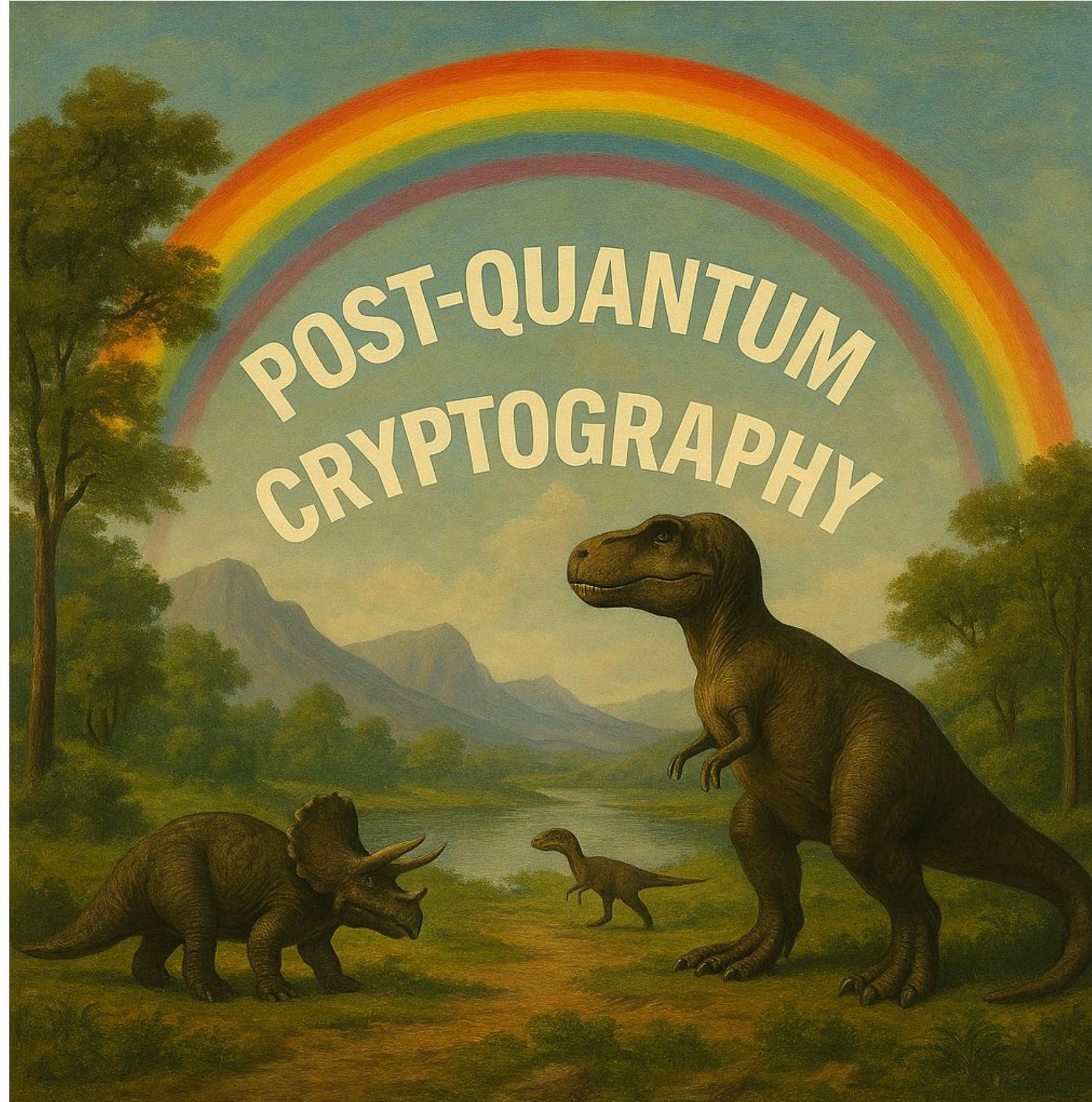
Samuel Jaques^{1*†}, Michael Naehrig², Martin Roetteler³, and Fernando Virdia^{4†‡}

scheme	r	#Clifford	# T	# M	T -depth	full depth	width	G -cost	DW -cost	p_s
AES-128	1	$1.13 \cdot 2^{82}$	$1.32 \cdot 2^{79}$	$1.32 \cdot 2^{77}$	$1.48 \cdot 2^{70}$	$1.08 \cdot 2^{75}$	1665	$1.33 \cdot 2^{82}$	$1.76 \cdot 2^{85}$	$1/e$
AES-128	2	$1.13 \cdot 2^{83}$	$1.32 \cdot 2^{80}$	$1.32 \cdot 2^{78}$	$1.48 \cdot 2^{70}$	$1.08 \cdot 2^{75}$	3329	$1.34 \cdot 2^{83}$	$1.75 \cdot 2^{86}$	1
AES-192	2	$1.27 \cdot 2^{115}$	$1.47 \cdot 2^{112}$	$1.47 \cdot 2^{110}$	$1.47 \cdot 2^{102}$	$1.14 \cdot 2^{107}$	3969	$1.50 \cdot 2^{115}$	$1.11 \cdot 2^{119}$	1
AES-256	2	$1.56 \cdot 2^{147}$	$1.81 \cdot 2^{144}$	$1.81 \cdot 2^{142}$	$1.55 \cdot 2^{134}$	$1.29 \cdot 2^{139}$	4609	$1.84 \cdot 2^{147}$	$1.45 \cdot 2^{151}$	$1/e$
AES-256	3	$1.17 \cdot 2^{148}$	$1.36 \cdot 2^{145}$	$1.36 \cdot 2^{143}$	$1.55 \cdot 2^{134}$	$1.28 \cdot 2^{139}$	6913	$1.38 \cdot 2^{148}$	$1.08 \cdot 2^{152}$	1

Eliminating the Problem: 256-bit Keys



Defeating Quantum Algorithms



A.k.a. “quantum-safe”, “quantum-resilient”

Must not rely on factoring or discrete logarithm

Why bother?

Insurance against QC threat:

- “QC has a probability p work in year X and the impact would be \$N for us”
- “I’d like to eliminate this risk and I’m ready to spend \$M for it”

Supposedly the motivation of USG/NSA:

“we anticipate a need to shift to quantum-resistant cryptography in the near future.” – NSA in CNSS advisory 02-2015



NSA's Take (Aug 2021)

Q: Is NSA worried about the threat posed by a potential quantum computer because a CRQC exists?

A: NSA does not know when or even if a quantum computer of sufficient size and power to exploit public key cryptography (a CRQC) will exist.

Q: Why does NSA care about quantum computing today? Isn't quantum computing a long way off?

A: The cryptographic systems that NSA produces, certifies, and supports often have very long lifecycles. NSA has to produce requirements today for systems that will be used for many decades in the future, and data protected by these systems will still require cryptographic protection for decades after these solutions are replaced. There is growing research in the area of quantum computing, and global interest in its pursuit have provoked NSA to ensure the enduring protection of NSS by encouraging the development of post-quantum cryptographic standards and planning for an eventual transition.

Q: What are the timeframes in NSS for deployment of new algorithms, use of equipment, and national security information intelligence value?

A: New cryptography can take 20 years or more to be fully deployed to all National Security Systems. NSS equipment is often used for decades after deployment. National security information intelligence value varies depending on classification, sensitivity, and subject, but it can require protection for many decades.

https://media.defense.gov/2021/Aug/04/2002821837/-1/-1/1/Quantum_FAQs_20210804.pdf

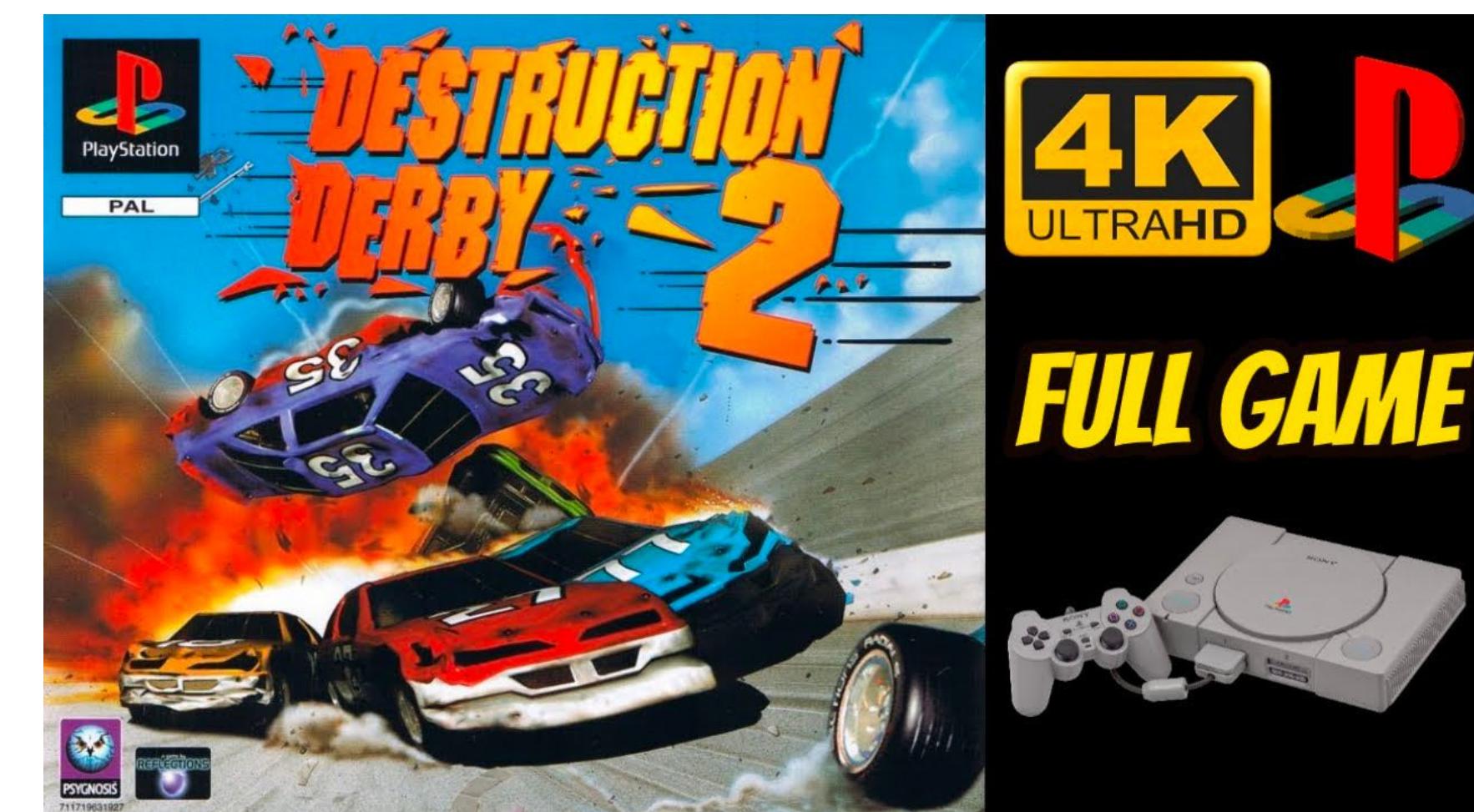
The NIST competition

CSRC HOME > GROUPS > CT > POST-QUANTUM CRYPTOGRAPHY PROJECT

POST-QUANTUM CRYPTO PROJECT

NEWS -- August 2, 2016: The National Institute of Standards and Technology (NIST) is requesting comments on a new process to solicit, evaluate, and standardize one or more quantum-resistant public-key cryptographic algorithms. Please see the Post-Quantum Cryptography Standardization menu at left.

Fall 2016	Formal Call for Proposals
Nov 2017	Deadline for submissions
Early 2018	Workshop - Submitter's Presentations
3-5 years	Analysis Phase - NIST will report findings <i>1-2 workshops during this phase</i>
2 years later	Draft Standards ready



NIST standards and round 4

Standards announced in 2022:

- Encryption/KEM: **Kyber** (ML-KEM, FIPS 203)
- Signature:
 - **Dilithium** (ML-DSA, FIPS 204)
 - **SPHINCS+** (SLH-DSA, FIPS 205)
 - **Falcon** (*TBD*)

All *latticed-based* except SPHINCS+

Round 4 only for encryption/KEM, all *code-based*:

~~BIKE, Classic McEliece, HQC~~ selected as the winner in 2025

FIPS 205

Federal Information Processing Standards Publication

Stateless Hash-Based Digital Signature Standard

Category: Computer Security

Subcategory: Cryptography

Information Technology Laboratory
National Institute of Standards and Technology
Gaithersburg, MD 20899-8900

This publication is available free of charge from:
<https://doi.org/10.6028/NIST.FIPS.205>

Published: August 13, 2024



Lattice-based crypto intuition

Based on problems such as **learning with errors** (LWE):

S a secret vector of numbers

The attacker receives pairs of vectors (**A**, **B**)

- **A** = (**A₀**, ..., **A_{n-1}**) is a vector of uniformly random numbers
- **B** = **S**, **A** + **E**, a vector of **B_i** = **S_i*****A_i** + **E_i**
- **E** = (**E₀**, ..., **E_{n-1}**) is an **unknown** vector or *normal-random* numbers

Attacker's goal: find **S** given many pairs (**A**, **B**)

Lattice-based crypto intuition

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Attacker's goal: find **S** given many pairs (**A**, **B**)

Without the errors **E**: trivial to solve (just a linear systems of equations)

With the errors **E**: **NP-hard**

PQC performance

Algorithm	Public key (bytes)	Ciphertext (bytes)	Key gen. (ms)	Encaps. (ms)	Decaps. (ms)	
ECDH NIST P-256	64	64	0.072	0.072	0.072	Elliptic curve key agreement
SIKE p434	330	346	13.763	22.120	23.734	
Kyber512-90s	800	736	0.007	0.009	0.006	Post-quantum standard

Algorithm	Public key (bytes)	Signature (bytes)	Sign (ms)	Verify (ms)	
ECDSA NIST P-256	64	64	0.031	0.096	Elliptic curve signature
Dilithium2	1,184	2,044	0.050	0.036	Post-quantum standard

From "Benchmarking Post-Quantum Cryptography in TLS" <https://eprint.iacr.org/2019/1447>

Using PQC today

Integrated by most hyperscalers

Cloudflare now uses post-quantum cryptography to talk to your origin server

2023-09-29

AWS Security Blog

Post-quantum TLS now supported in AWS KMS

by Andrew Hopkins | on 04 NOV 2019 | in Advanced (300), AWS Key Management Service, Security, Identity, & Compliance | Permalink | Comments | Share

Security & Identity

Announcing quantum-safe digital signatures in Cloud KMS

February 21, 2025

Software libraries

OpenSSL 3.5.0 now contains post-quantum procedures

With the new LTS version 3.5.0, OpenSSL adds the post-quantum methods ML-KEM, ML-DSA and SLH-DSA to its library.

open-quantum-safe / liboqs

Code

Issues 19

Pull requests 4

Actions

Projects 0

W

C library for quantum-safe cryptography. <https://openquantumsafe.org/>

mupq / pqm4

Code

Issues 3

Pull requests 0

Actions

Post-quantum crypto library for the ARM Cortex-M4

More about post-quantum crypto

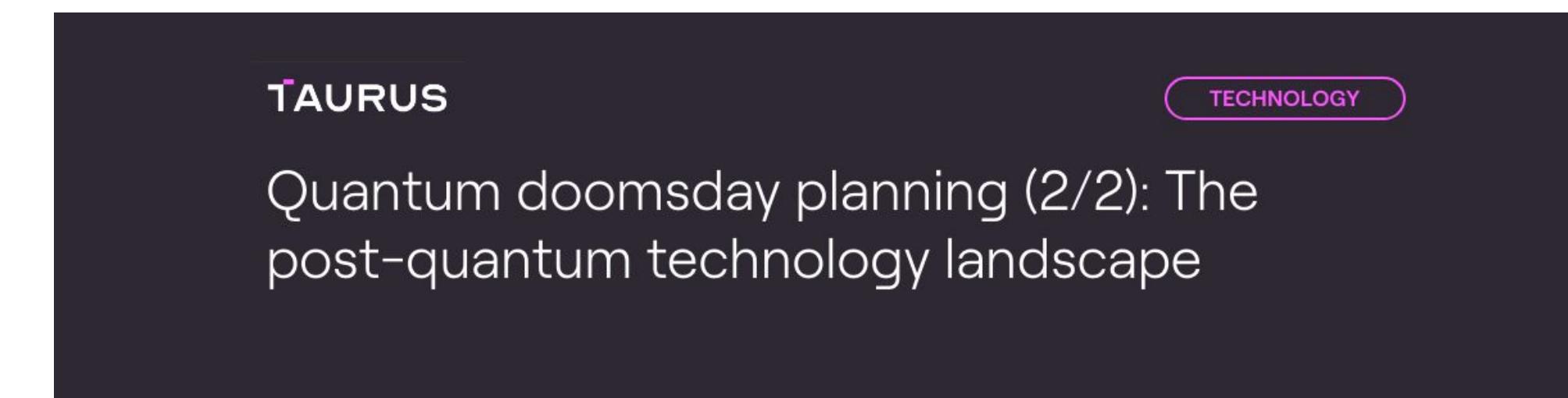
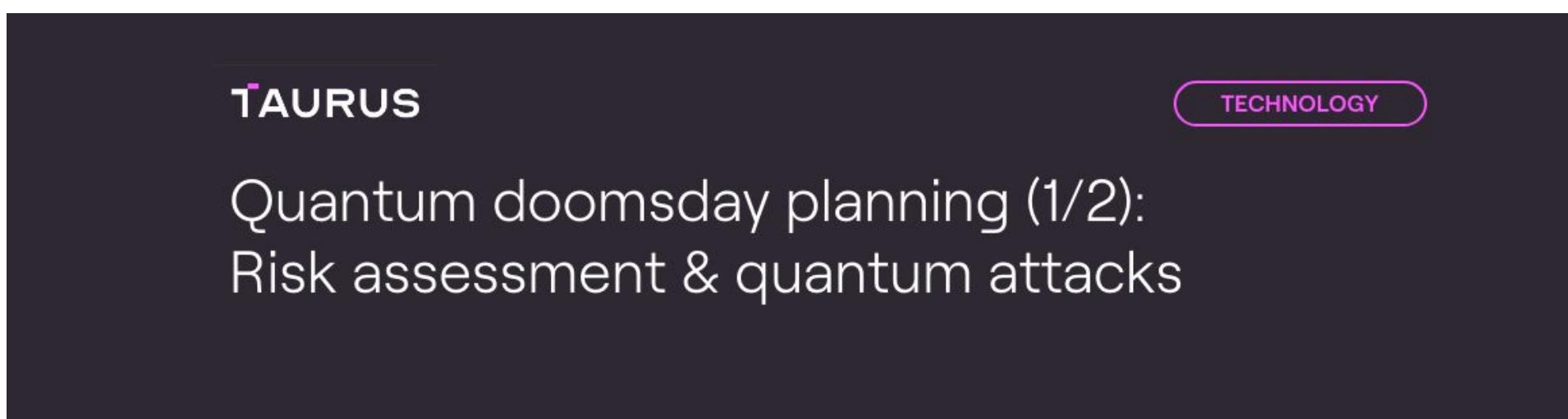
<https://github.com/veorq/awesome-post-quantum>

<https://github.com/qosf/awesome-quantum-software>

<https://csrc.nist.gov/projects/post-quantum-cryptography/post-quantum-cryptography-standardization>

IETF RFC 8391 (XMSS), RFC 8554 (LM)

May 2023 articles on <https://blog.taurushq.com/>, on how to prepare for the transition in an enterprise IT environment (inventory, risk management, etc.)



TAURUS



شكراً جزيلاً

شكراً جزيلاً

Thank you!

jp@taurushq.com