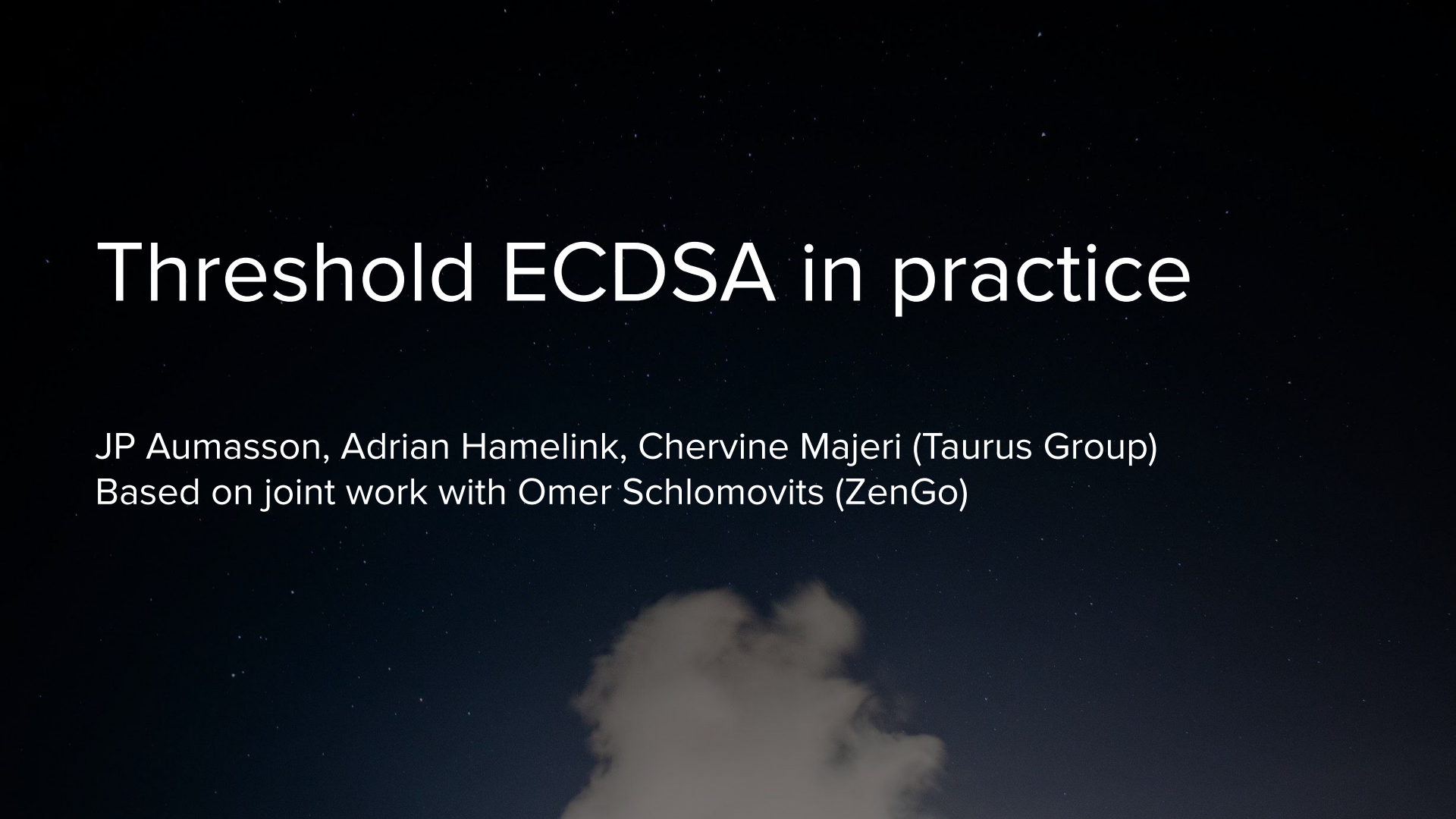
Threshold ECDSA in practice

JP Aumasson, Adrian Hamelink, Chervine Majeri (Taurus Group)
Based on joint work with Omer Schlomovits (ZenGo)



Agenda

Introduction: motivations and basic notions and formalism (JP)

ECDSA & MPC: viewing threshold ECDSA as an MPC functionality (Adrian)

Protocols: overview of the main protocols and their properties (Adrian)

Real-world: review of needs, security models, assumptions (JP/Chervine)

Conclusion: thoughts and open problems (JP)

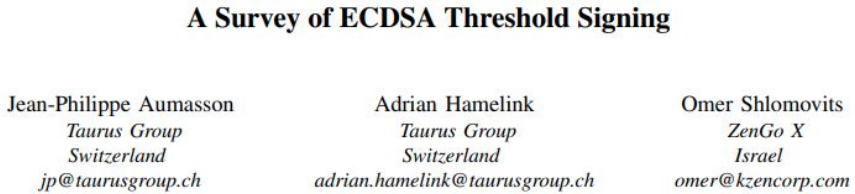
Introduction

Background & motivations

Research unit of **Taurus Group** - taurusgroup.ch

- Digital asset infrastructure for financial organizations
- Need security, reliability, maintainability, and usability
- Protocol using an **HSM**'s TEE for signing, quorum validation, security controls

How to leverage threshold crypto to offer an HSM-free alternative?
What are "real-world" security and performance needs?
What protocol and libraries are acceptable?



A Survey of ECDSA Threshold Signing

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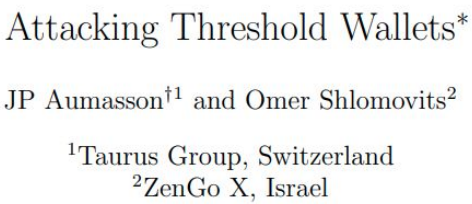

Attacking Threshold Wallets*

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eprint.iacr.org/2020/1052

Threshold signing motivations

Distributing trust, avoid a SPoF, enable “4-eye control”

Off-chain quorum mechanism, coin-agnostic, **hiding governance patterns**

Multi-signatures require multiple keys, work differently for different platforms

Alternative / complement to HSMs/TEEs, but address different needs

Efficient schemes for RSA, Schnorr sigs (including Ed25519), **ECDSA is harder**

Threshold signing?

2 protocols, although keygen can be centralized when it makes sense:

(**t**, **n**) parameters where **t**+1 shares are necessary and sufficient to sign

Distributed key generation (DKG): **n** parties obtain shares of a private key

Signing: **t+1** parties use their share to collectively sign a given message

ECDSA & MPC

Classic ECDSA

$sk \in Z_q$, $pk = sk \bullet G$, message m, $H(m) \in Z_q$

$Sign_{sk}(m)$	**$Verify_{pk}((r,s), m)$**
• Sample random k in Z^*_q • $R = k \bullet G = (r_x, r_y)$, $r = r_x \pmod q$ • $s = k^{-1} (H(m) + r \bullet sk) \pmod q$ • Output (r, s)	• $R' = [H(m)s^{-1}] \bullet G + [rs^{-1}] \bullet pk = (r'_x, r'_y)$ • Check $r'_x \pmod q = r$

Classic ECDSA to threshold ECDSA ?

$sk \in Z_q$, $pk = sk \cdot G$, message m, $H(m) \in Z_q$

Threshold ⇒ use secret sharing for sk and k

$Sign_{sk}(m)$	**$Verify_{pk}((r,s), m)$**
• Sample random k in Z^*_q • $R = k \cdot G = (r_x, r_y)$, $r = r_x \pmod q$ • $s = k^{-1} (H(m) + r \cdot sk) \pmod q$ • Output (r, s)	• $R' = [H(m)s^{-1}] \cdot G + [rs^{-1}] \cdot pk = (r'_x, r'_y)$ • Check $r'_x \pmod q = r$

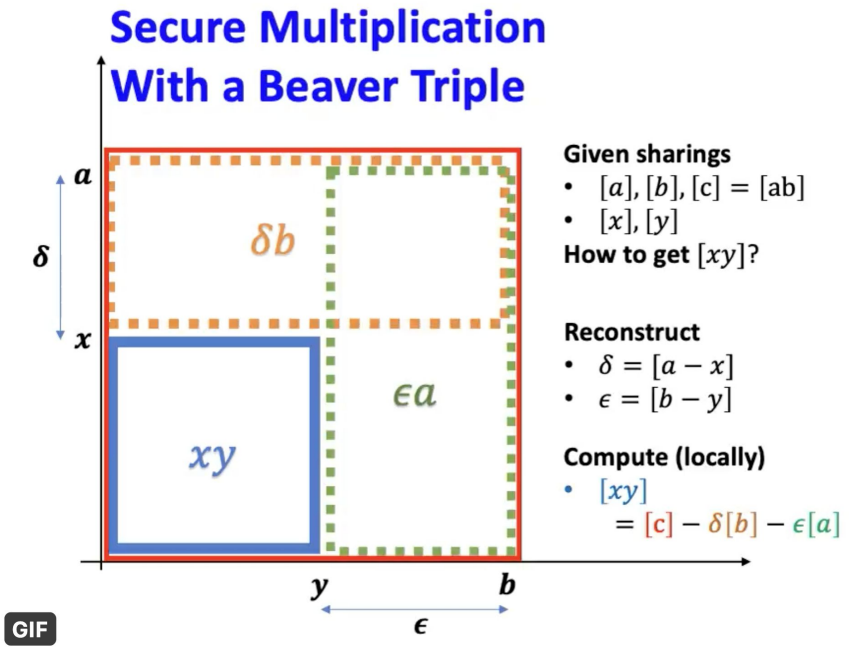
Threshold ECDSA as an MPC functionality

Secrets are now *shared*, i.e. **sk** → [**sk**] with **sk** = $sk_1 + \cdots + sk_n$, [**pk**] = [**sk**] • G

Formulate $Sign_{[sk]}$ using Arithmetic Black Box (ABB) functionality:

[r] ← Rand()	$r \in Z_q$
[c], [a], [b] ← RandMul()	$c = ab$
x ← Open([x])	$x \in Z_q$
[c] ← Mul([a], [b])	$c = ab \in Z_q$
[z] ← a • [x] + b • [y]	$z = ax + by \in Z_q$
[X] ← [x] • G	$x \in Z_q, X = x \bullet G \in \langle G \rangle$

ABB Example: Multiplication



[z] ← Mult([x], [y]):

- [c], [a], [b] ← RandMul()
- δ ← Open([a] - [x])
- ε ← Open([b] - [y])
- [z] ← [c] - δ[b] - ε[a]

Credit: A. Yanai https://twitter.com/i/status/1324424768649601024

ABB Example: Inversion

[y] ← Invert([x]):

- [b] ← Rand()
- [z] ← Mul([x], [b])
- z ← Open([z]) $z = x\ b$
- [y] ← z^{-1} [b] $y = (x\ b)^{-1}\ b = x^{-1}$

Threshold ECDSA as an MPC functionality

PreSign	Sign
- $[k] \leftarrow \text{Rand}()$ - $[b] \leftarrow \text{Rand}()$ - $[e] \leftarrow \text{Mul}([b],[k])$ $e = b\,k$ - $[f] \leftarrow \text{Mul}([b],[sk])$ $f = b\,sk$ - $e \leftarrow \text{Open}([e])$ $e = b\,k$ - $[t] \leftarrow e^{-1} \cdot [f]$ $t = (bk)^{-1} f = k^{-1} sk$ - $[k^{-1}] \leftarrow e^{-1} \cdot [b]$ k^{-1} - $R \leftarrow \text{Open}([k] \cdot G)$ $R = k \cdot G = (r_x, r_y)$ Store $(r_x, [k^{-1}], [t])$	- Retrieve $(r_x, [k^{-1}], [t])$ - $[s] \leftarrow H(M) \cdot [k^{-1}] + r_x \cdot [t]$ - $s \leftarrow \text{Open}([s])$ $s = k^{-1} \cdot H(M) + k^{-1} \cdot r_x \cdot sk$ $k^{-1}(H(M) + r_x \cdot sk)$ Output (r_x, s)

ECDSA without generic MPC?

Without MPC, we must be careful with **privacy** and **correctness**

Blinding factors helps hide secrets when revealed

Prove correctness by

- Proving computations in **zero-knowledge**
- **Commit** to values before publishing
- **Verify** algebraic relations

Shared secret multiplication

How to compute $[z] = [x] \cdot [y]$?

$$z = x \cdot y = (x_1 + \cdots + x_n) \cdot (y_1 + \cdots + y_n) = \sum_{i,j} x_i \cdot y_j \overset{?}{=} \sum_i z_i$$

Multiplicative-to-Additive conversion (MtA) protocol:

$$a \cdot b = c \rightarrow \alpha + \beta = c$$

$$z_i = x_i \cdot y_i + \sum_{i \neq j} a_{i,j} + b_{i,j}$$

Multiplicative-to-Additive share conversion

Using Homomorphic encryption

Alice: x		**Bob**: y
$c \leftarrow Enc_A(x)$	$c \rightarrow$	$b \leftarrow\$ Z_q$
	$\leftarrow c'$	$c' \leftarrow (c \odot Enc_A(y)) \oplus Enc_A(-b)$
$a \leftarrow Dec_A(c')$		$c' = Enc_A(x \cdot y - b)$

$$a + b = (x \cdot y - b) + b = x \cdot y$$

Security concerns

Same **unforgeability** property wanted

Threshold optimal ⇒ t = n - 1 ⇒ dishonest majority ⇒ no **robustness**

Parties must **abort** when checks go wrong

Identification helps eject misbehaving parties

Type of adversary (active vs. passive)

Universal Composability (UC) proof

Protocols

2018, first *efficient* constructions

Fast Multiparty Threshold ECDSA with Fast Trustless Setup

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Paillier as homomorphic scheme

Explicitly verify all computations

Fast Secure Multiparty ECDSA with Practical Distributed Key Generation and Applications to Cryptocurrency Custody*

Yehuda Lindell[†] Ariel Nof[†] Samuel Ranellucci[‡]

October 14, 2018

UC security

Paillier or OT based MtA

Verify result “in-the-exponent” with ElGamal commitments

Gennaro and Goldfeder ‘20

- $[k], [b] \leftarrow \text{Rand}()$, $[B] \leftarrow [b] \cdot G$, commit to B_i
- $[v] \leftarrow \text{Mult}([k], [b])$, $[t] \leftarrow \text{Mult}([k], [sk])$, check with pk_i
- $v \leftarrow \text{Open}([v])$, $v = k\,b$
- $B \leftarrow \text{Open}([B])$, verify decommit
- $R \leftarrow v^{-1} \cdot B = (k^{-1}\, b^{-1})\,(b \cdot G) = k^{-1} \cdot G$, $[V] \leftarrow [k] \cdot B$, prove using MtA msgs
- $V \leftarrow \text{Open}([V])$, verify $V = v \cdot G$
- $[s] \leftarrow m \cdot [k] + r \cdot [t]$,
- $s \leftarrow \text{Open}([s])$

2019-2020,design *trade-offs*

Threshold ECDSA from ECDSA Assumptions:
The Multiparty Case

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OT based Multiplication

Logarithmic rounds *but*
constant number of operations

Bandwidth-efficient threshold EC-DSA

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GG18

~~Paillier~~ → Class group MtA

2020, more security *features*

One Round Threshold ECDSA with Identifiable Abort

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Detection and attribution of misbehavior via secure “aborts”

UC Non-Interactive, Proactive, Threshold ECDSA

Ran Canetti* Nikolaos Makriyannis† Udi Peled†

May 8, 2020

Use Paillier as commitments

Proactive key refresh

Only 4 rounds (3 offline)

Abort with identification

Detect misbehaving users:

- Failure to **verify ZK proof**
- Consistency of **R** using Enc($\mathbf{k}_i$) from MtA in Mul([k],[sk])
- Valid **decommitment**
- **Algebraically** [k]•R = G, and pk = [t]•R *where $R = k^{-1} \bullet G$ and $t = k\ sk$*
- Verify **signature**

Identification protocol needed for last two checks.

Proactive key refresh

If [0] is a secret sharing of 0 with shares 0_i for party i,
then [z] = [x] + [0] is a new secret sharing of [x] but with different shares.

In **Refresh+AuxInfo** protocol, each party distributes shares of $[0^{(i)}]$

$$[sk'] = [sk] + \sum_i [0^{(i)}]$$

Auxiliary parameters such as Paillier keys, and Pedersen are also regenerated.

Old shares are no longer valid.

GG ‘20 + CMP ‘20 = CGGMP ‘20

UC Security

Proactive key refresh in 3 rounds

Non interactive signing

Two protocols for presigning & identification

- 3 offline rounds + $O(n^2)$ cost for identification
- 6 offline rounds + $O(n)$ cost for identification

Protocols comparison (from CGGMP ‘20)

Signing Protocol	*Rounds*	*Group Ops*	*Ring Ops*	*Communication*	*Proactive*	*ID Abort*	*UC*
Gennaro and Goldfeder [26]	9	$10n$	$50n$	$10\kappa + 20N$ (7 KiB)	✗	✗	✗
Lindell et al. [37] (Paillier)†‡	8	$80n$	$50n$	$50\kappa + 20N$ (7.5 KiB)	✗	✗	✓
Lindell et al. [37] (OT)†	8	$80n$	0	50κ (190 KiB)	✗	✗	✓
Doerner et al. [23]	$\log(n) + 6$	5	0	$10 \cdot \kappa^2$ (90 KiB)	✗	✗	✓
Castagnos et al. [16]*	8	$15n$	0	$100 \cdot \kappa$ (4.5 KiB)	✗	✗	✗
This Work: *Interactive*§	4 *or* 7	$10n$	$90n$	$10\kappa + 50N$ (15 KiB)	✓	✓	✓
This Work: *Non-Int. Presign*§	3 *or* 6	$10n$	$90n$	$10\kappa + 50N$ (15 KiB)	✓	✓	✓
This Work: *Non-Int. Sign*	1	0	0	κ (256 bits)	✓	✓	✓

Real-world crypto

Use cases

Shares should be controlled by different entities

“Simple” cases:

- B2C company with 2-party wallet shared control
- Multiple organizations sharing control of a wallet

Less simple cases (operation segregation needs):

- Cold wallet of a single organization (few addresses, high latency ok)
- Hot wallet a single organization (many addresses, need low latency)
    - How to do BIP32/44 efficiently?
    - Depends on the pooling model (relation between addresses and owners)

Security models vs. reality

Corruption:

- **Static** more realistic most of the time, so adaptive security safer
- Rarely the ability to “corrupt” one part, “uncorrupt” it to “corrupt” another
- Either a system is to be trusted (for some time), or it’s not and then it’s forever

Protocol obedience:

- **Malicious** models a compromised system
- Honest-but-curious makes little sense

Majority:

- It depends: with 2-2 you need **dishonest majority**
- Might be cases with a large **n** and small **t** where honest majority makes sense

Software implementing TSS

2 main open-source libraries used in production:

Binance's https://github.com/binance-chain/tss-lib (Go)

Zengo's https://github.com/ZenGo-X/multi-party-ecdsa (Rust)

Reviewed in detail in our survey, we did paid security audits of both

Recent lib by **ING bank** https://github.com/ing-bank/threshold-signatures/
(With Omer we found and reported some bugs)

Some organizations have non-OSS code, partially relying on OSS (arithmetic, etc.)

Deployment constraints

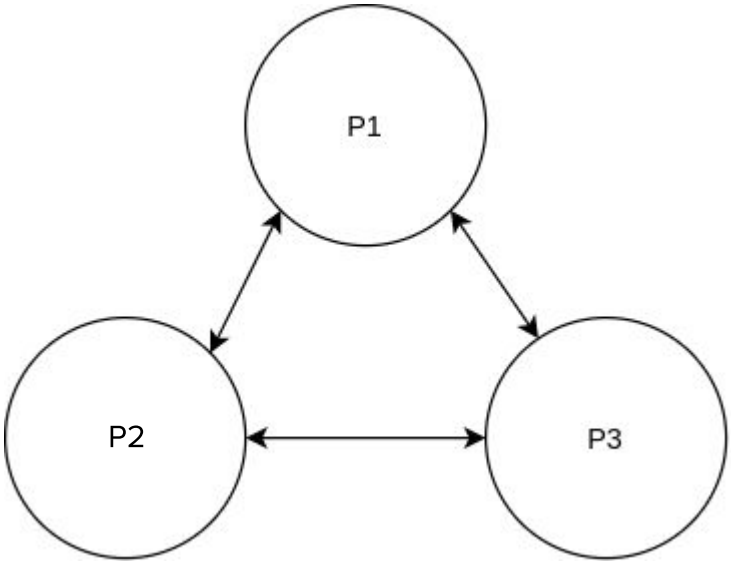
Most papers describing protocols assume that

- **Key distribution** is in place and safe (enabling secure channels)
  - Usually fine
- Communication is **lossless and in-order** (thus reliable) and low-latency
  - Not always the case

- **Practical** aspects of deployment often missing
  - Protocol limitations (GG18/20) may impact deployment potential
  - Slower signatures means scalability at infrastructure-level is important

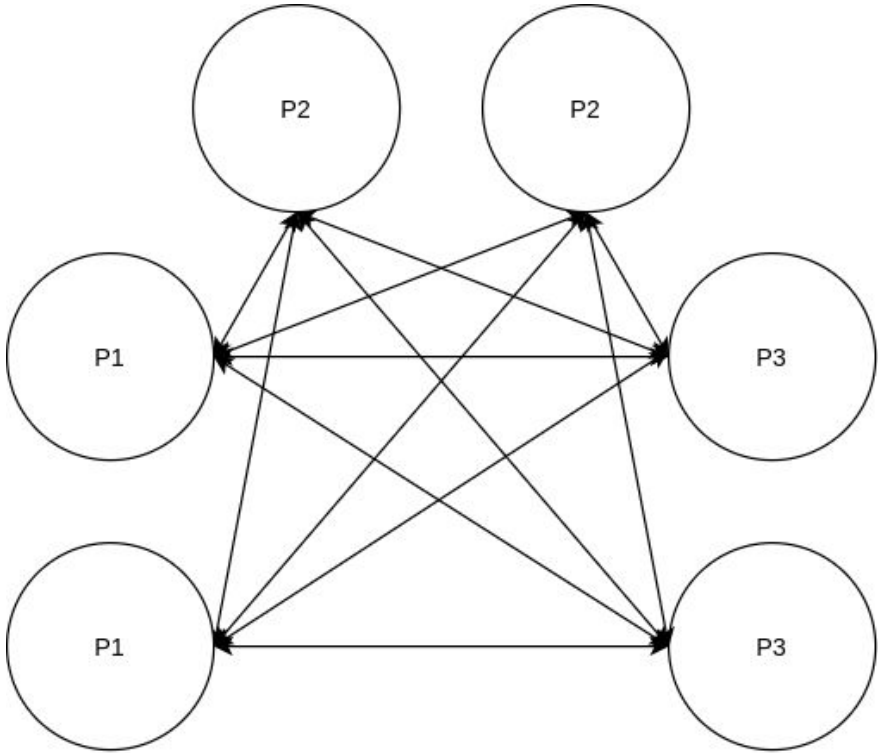
How would “less simple” deployment work in practice?

Naive deployment



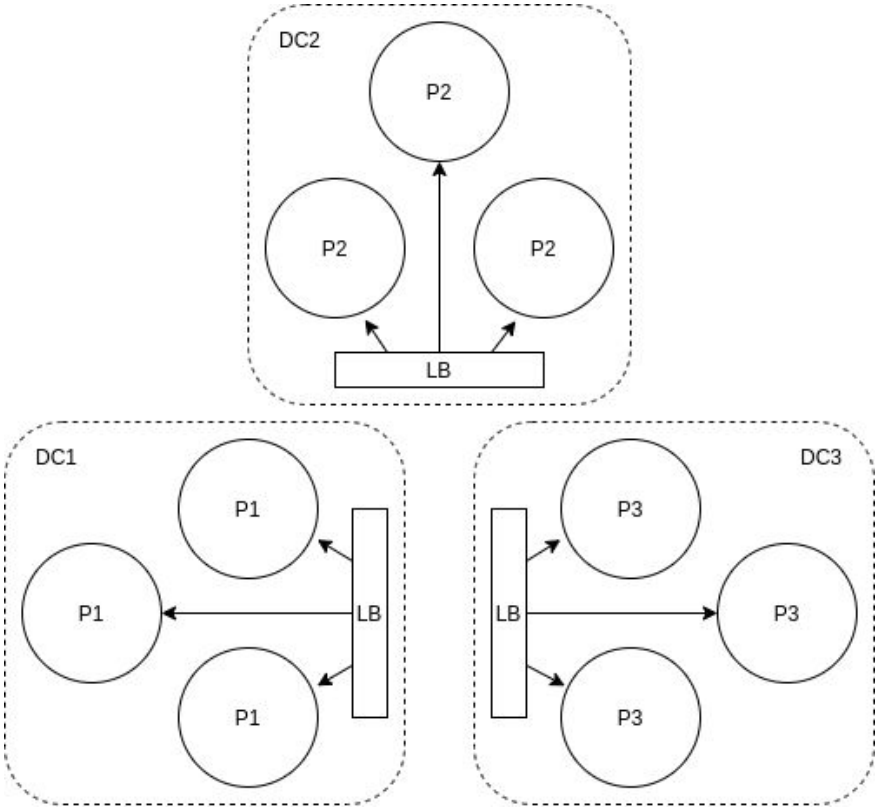
- Exactly mirrors crypto diagrams
- Doesn’t hold up in practice
  - No scalability
  - No failover
- Improvement: Support rolling out multiple instance of the same party.

Impractical deployment



- Multiple instances of each party with a mesh network
- Addresses scalability issues
- Mesh networks across different clouds is impractical/hard.

Robust deployment



- One party in each DC means segregation of duty is possible
- Access via load-balancer is standard
- DCs can scale their instances horizontally w/o any change to configuration
- Some engineering tricks to perform multiple rounds with the same participant for each party

Performance

Benchmark notes from a **robust deployment**:

- Keygen: 7-8s
  - 5s to generate safe primes
  - 1s for the other operations
  - WAN overhead 700ms

- Signature: 2-3s
  - 400ms for each of the two (optional*) MtAwc proof
  - 300ms for each of the two (optional*) MtAwc verifications
  - WAN overhead 900ms

Conclusions

Theory + practice = 🙌

Non-trivial protocols driven by practical needs

- Lindell and GG18 were game-changers
- CMP with aborts checking most of the boxes

Some protocols in production and battle-tested

Crypto addresses part of the problem, main real-world risks related to

- Implementation matching of described provensecure protocol
- Platform trust, software assurance, access control, back-ups, etc.

Open problems and challenges

NIST standardization ongoing https://csrc.nist.gov/Projects/threshold-cryptography

Formal verification of TSS-based protocols and software?

BTC supports Schnorr, BLS getting adoption; TSS ECDSA soon useless? :)

BIP32/HKD is not easy to threshold

Extras

Gennaro and Goldfeder ‘20

- $[k], [\gamma] \leftarrow \text{Rand}()$, $[\Gamma] \leftarrow [\gamma] \cdot G$, commit to Γ_i
- $[\delta] \leftarrow \text{Mult}([k], [\gamma])$, $[\sigma] \leftarrow \text{Mult}([k], [sk])$, check with pk_i
- $\delta \leftarrow \text{Open}([\delta])$,
- $\Gamma \leftarrow \text{Open}([\Gamma])$, verify decommit
- $R \leftarrow \delta^{-1} \cdot \Gamma = (k^{-1} \gamma^{-1}) (\gamma \cdot G) = k^{-1} \cdot G$, $[\Delta] \leftarrow [k] \cdot \Gamma$, prove using MtA msgs
- $\Delta \leftarrow \text{Open}([\Delta])$, verify $\Delta = \delta \cdot G$
- $[s] \leftarrow m \cdot [k] + r \cdot [\sigma]$,
- $s \leftarrow \text{Open}([s])$

Gennaro and Goldfeder ‘20

Improve upon GG18 by optimizing number of rounds and enabling presignatures.

Also adds protocol for identifying misbehaving users before aborting.

- **ZK proof** fails verification ⇒ Prover misbehaved
- **Signature** (r, s) is invalid ⇒ Check each share s_i in the exponent
- Inconsistency detected through **nonce** :

$$G \neq [k^{-1}] \bullet R \quad \text{or} \quad pk \neq [k^{-1}\, sk] \bullet R \qquad \textit{(where R should be [k]•G)}$$

⇒ each party must re-prove that MtA was performed correctly for [k][sk]

Security of the MPC functionality

k must never be revealed.

- Using δ as *blinding* factor while inverting
- Special care Multiplication

Parties must check for consistency between R and s.

- Zero-knowledge to prove computations are correct
- Perform computations “in the exponent” (ElGamal)
- Algebraic verification

Need weaker ECDSA assumption when R is revealed before.

Aborting in case of inconsistencies (with identification?)

Shared secret multiplication

How to compute [x] • [y] ?

$$x \bullet y = (x_1 + \cdots + x_n) \bullet (y_1 + \cdots + y_n) = \sum_{i,j} x_i \bullet y_j$$

Multiplicative-to-Additive conversion (MtA) protocol:

$$x_i \bullet y_j = c_{ij} = a_{ij} + b_{ij}$$

$$x \bullet y = \sum_{i,j} x_i \bullet y_j = \sum_i \{ x_i \bullet y_i + \sum_{j \neq i} a_{ij} + b_{ji} \} = \sum_i c_i$$

Protocols comparison

	LNC 18	GG 18	DKLS 19	CCLST 20	CGGMP 20
DKG rounds	5	4	5	5	3 + 3
Signing rounds	8	9	log(t) + 6	8	3 or 6 + 1
Proof type	UC (ECDSA)	Game	UC (ECDSA)	Game	UC (Sign)
Communication[1]	7.5 KiB/190 KiB	7 KiB	90 KiB	4.5 KiB	15 KiB
EC & ring ops[1]	80n & 50n	10n & 50 n	5 & 0	15n - 0	10n & 90n

1. source: CGGMP20, with t = n-1

Distinguishing features

Threshold optimal ($n = t + 1$)

Non interactive signing

Identifiable aborts

Key refreshing

Efficiency (bandwidth, rounds, computation)

Gennaro and Goldfeder ‘18

MtA with Paillier

Alice: x		**Bob**: y
$c \leftarrow Enc_A(x)$	$c \rightarrow$	$b \leftarrow\$ Z_q$
	$\leftarrow c'$	$c' \leftarrow (c \odot Enc_A(y)) \oplus Enc_A(-b)$
$a \leftarrow Dec_A(c')$		$c' = Enc_A(x \cdot y - b)$

$$a + b = (x \cdot y - b) + b = x \cdot y$$

Gennaro and Goldfeder ‘18

MtA with Paillier

ZK proofs that ciphertexts are in correct range for Z_q, and correct values

Game-based security proof

Consistency checked “in the exponent”

Assume DDH, strong RSA

Lindell et al. ‘18

Simultaneously performs computations over ElGamal ciphertexts

Full UC proof of ECDSA functionality

MtA must be private, but consistency is checked with ElGamal

Use OT or HE for MtA

Assume DDH, Paillier indistinguishability

Canetti, Makriyannis, Peled ‘20

Builds upon GG18, adding pre-signatures

UC security, threshold sign functionality

Commitments by encrypting under own Paillier key ⇒ prove consistency of MtA

Proactive key refresh

What is a threshold?

Among **n** participants P_1, ... , P_n , a *threshold* **t** is the maximum number of parties that can be corrupted while keeping things safe.

- Keeping a secret among a group
- Generating signatures on behalf of a group

Secret Sharing

Secret [**x**] is split into **n** *shares* x_1, ..., x_n and **t+1** are needed to reconstruct **x**.

Additive: $\mathbf{x} = x_1 + \ldots + x_n$. $t = n-1$

Shamir: $f(u) = \mathbf{x} + a_1u + \ldots + a_tu^t$, $\mathbf{x} = f(0)$, $x_i = f(i)$,

- Lagrange interpolation with t+1 points
- Can convert Shamir to additive: $\mathbf{x} = x'_1 + \ldots + x'_{t+1}$

ECDSA timeline

2018: **First efficient** threshold ECDSA
- Gennaro, Goldfeder - ACM CCS 18
- Lindell, Nof, Ranellucci ACM CCS 18

2019: Without Paillier, but OT-based sampling, CDH instead of DDH:
- Doerner, Kondi, Lee, Shelat - IEEE S&P 19

2020:
- Bandwidth-efficient threshold EC-DSA
  Castagnos, Catalano, Laguillaumie, Savasta & Tucker (PKC 20)
- UC Non-Interactive, Proactive, Threshold ECDSA
  Canetti, Makriyannis & Peled
- One Round Threshold ECDSA with Identifiable Abort
  Gennaro & Goldfeder
- UC Non-Interactive, Proactive, Threshold ECDSA with Identifiable Aborts
  Canetti, Gennaro, Goldfeder, Makriyannis & Peled (ACM CCS 20)

Doerner, Kondi, Lee, Shelat 2019

Builds upon 2-n TSS

Prove UC of ECDSA functionality

Use OT based MtA

log(t) rounds but constant group operations

Consistency check “in the exponent”

Assume CDH + UC commitment/ZK

Benchmarks with n=256 over WAN

Castagnos, Catalano, Laguillaumie, Savasta, Tucker 2020

Build upon GG20

Replace Paillier HE with class group HE

More efficient in higher security

Less bandwidth

Assume low order + strong root

TSS as an MPC functionality

[x] is a representation of a shared secret. Each party has share x_i .
<x> is a representation of x•G, each party has share $X_i = x_i \cdot G$ computed locally.

- [x] ← Rand()
- [z], [x], [y] ← RandMul() s.t. z = x y
- [z] ← Mult([x], [y]) s.t. z = x y
- [z] ← a[x] + b[y] s.t. z = ax + by
- <x> ← Convert([x])
- x ← Open([x])
- X ← Open(<x>) s.t. X = x•G
- <z> ← a<x> + b<y> s.t. z•G = (ax + by)•G