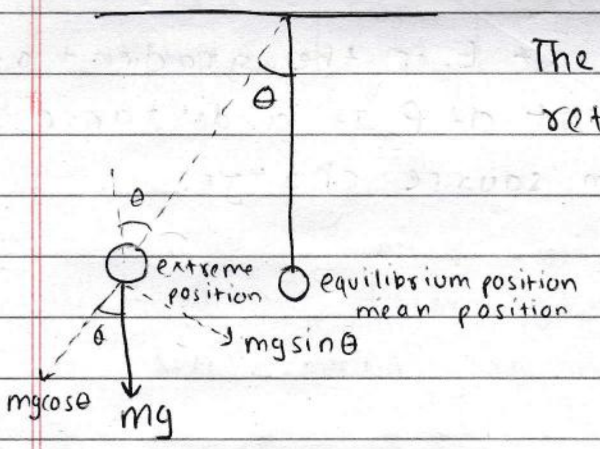
01/12/21

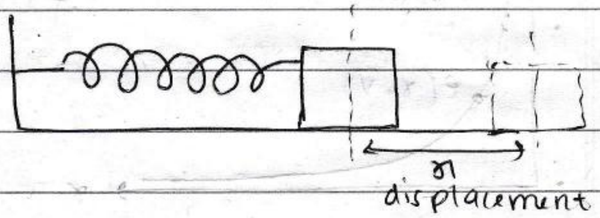
1st Dec 2021

OSCILLATIONS

- To and fro motion of an object an equilibrium position due to restoring force
- Point where object comes to rest after oscillation due to damping force is equilibrium or mean position.

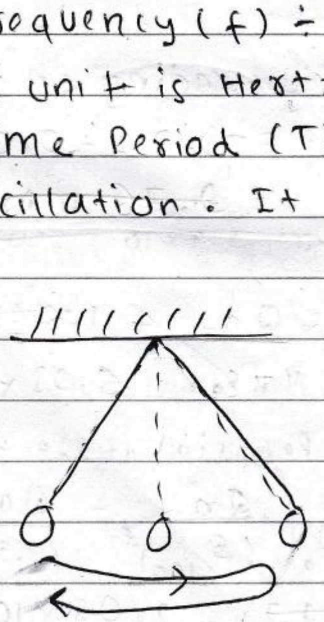


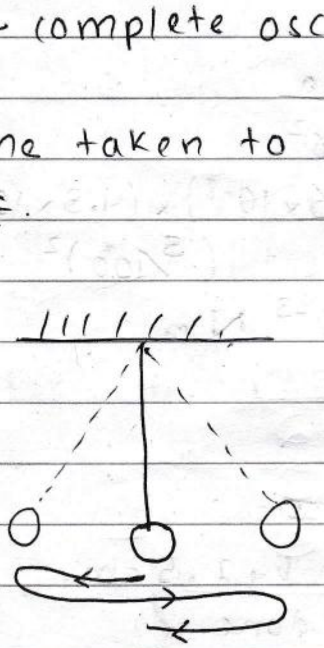
The restoring force is mgsinθ which returns pendulum to mean position



Terms related with oscillations.

- Displacement : Displacement of object from equilibrium position at any ~~position at any~~ time.
- Amplitude (x_0/A) : Max displacement from mean position. Displacement between two extreme positions is $2x_0$.
- Frequency (f) : No. of complete oscillations per unit time. SI unit is Hertz.
- Time Period (T) : Time taken to complete one oscillation. It is $1/f$.





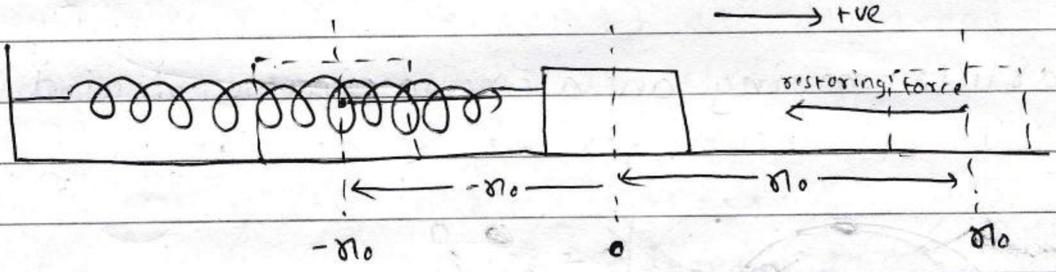
Simple Harmonic Oscillation (SHM)

- The oscillation where restoring force is directly proportional to displacement from equilibrium position and restoring force is always opposite in direction to displacement is simple harmonic oscillation.

$F \propto x$ [Restoring force always towards mean position]

$\therefore F = -kx$

Where k is proportionality constant, -ve sign indicates F and x are opposite in direction.



- An object with mass
- Displaced from mean position
- Restoring force

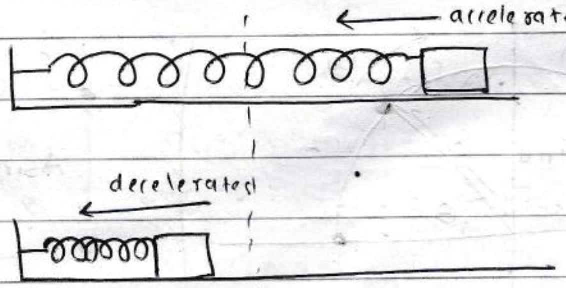
As force and acceleration are in same direction, acceleration is proportional to displacement.

$F = -kx$

or, $ma = -kx$

$\therefore a = -\frac{k}{m}x$

$\therefore a \propto x$

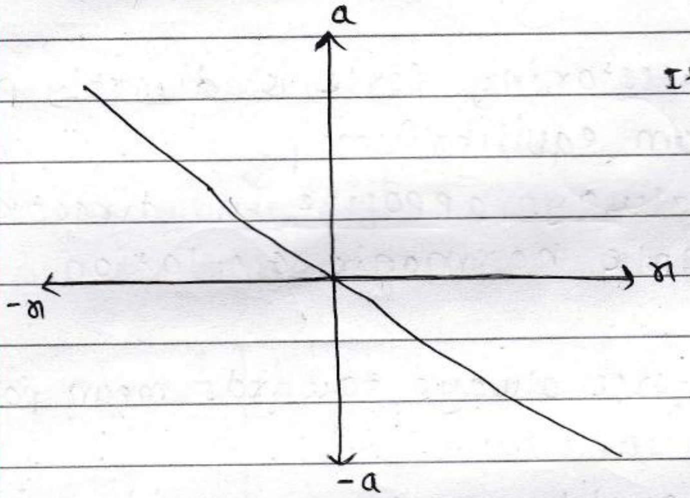


Acceleration is variable as restoring force is also variable due to variable extension.

2nd Dec 2021

Date
Page

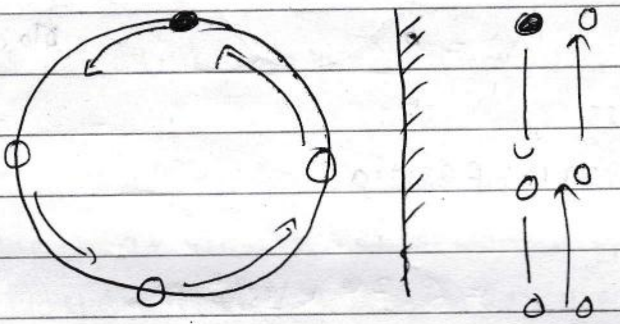
$F = -kx$, $a = -k'x$



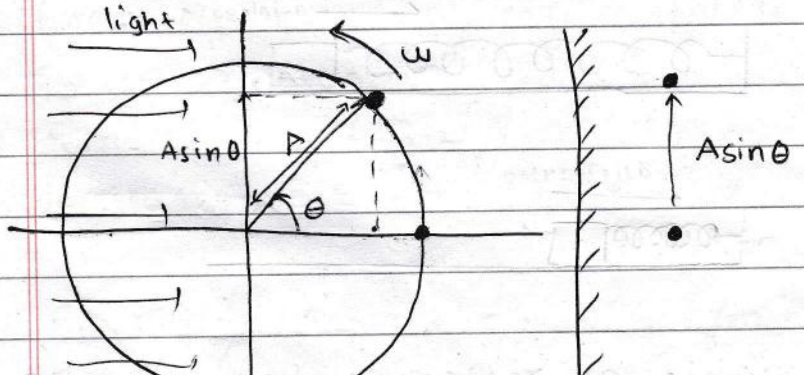
It is a line with -ve gradient.

SHM as projection of uniform circular motion onto a diameter.

A particle moving on a circular path, viewed from the side of circumference is oscillation.



Displacement of particle at any time 't'



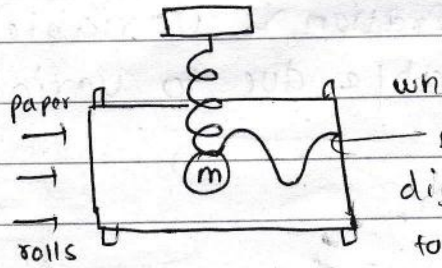
Let x = displacement

$x = A \sin\theta$

$x = A \sin \omega t$

$\therefore x = x_0 \sin \omega t$

Let's say $A = x_0$



where m has ink,

sine function

displacement related to sine function

8th Dec 2021

Velocity of particle in SHM

$x = x_0 \sin \omega t$

$v = \frac{dx}{dt} = x_0 \omega \cos \omega t$

Max velocity is attained when $\cos \omega t = 1$

$\therefore$ when $\theta = 0°$, $V = \omega x_0$, $\omega t = 0°$

If displacement is a sine function, velocity is a cosine function.

velocity is ~~extreme~~ max. at the mean position.

$v_{max} = \omega x_0$ $[\cos \omega t = 1]$ at mean pos.

$v_{min} = 0$ $[\cos \omega t = 0]$ at extreme pos.

$\cancel{a = v\omega}$

$\cancel{a_{max} = v_{max}\omega = \omega^2 x_0}$ ~~[At ~~extreme~~ mean pos.]~~

$\cancel{a_{min} = v_{min}\omega = 0}$ ~~[At extreme pos.]~~

$\boxed{V = V_{max} \cos \omega t}$

Acceleration of particle in SHM

$V = \omega x_0 \cos \omega t$

$\cancel{a = \omega v}$

$\therefore \cancel{a = \omega^2 x_0 \cos \omega t}$

$\therefore \cancel{a_{max} = \omega^2 x_0}$ ~~– Mean~~

$\therefore \cancel{a_{min} = 0}$ ~~– Extreme~~

~~dv~~ $a = \frac{dv}{dt} = -\omega^2 x_0 \sin \omega t$

$= -\omega^2 x$

$a_{max} = -\omega^2 x_0$

$a_{min} = 0$

$a = -\frac{k}{m} x$, $a = -\omega^2 x$

$\frac{kx}{m} = \omega^2 x$

$\therefore K = m\omega^2$

$a_{max} = -\omega^2 x_0$, when $\omega t = \pi/2$

$a_{max} = \omega^2 x_0$, when $\omega t = 3\pi/2$

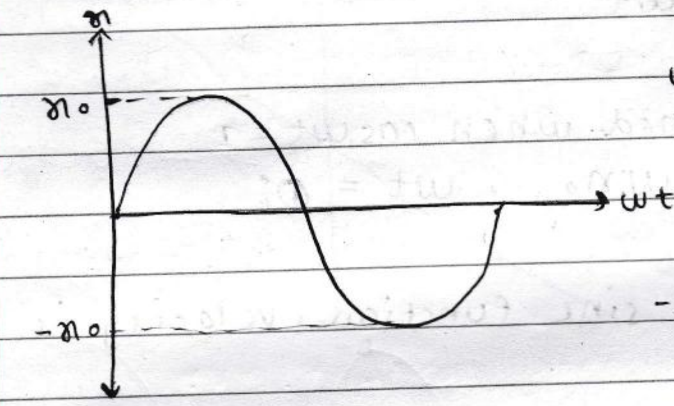
$a_{min} = 0$, when $\omega t = 0, \pi$

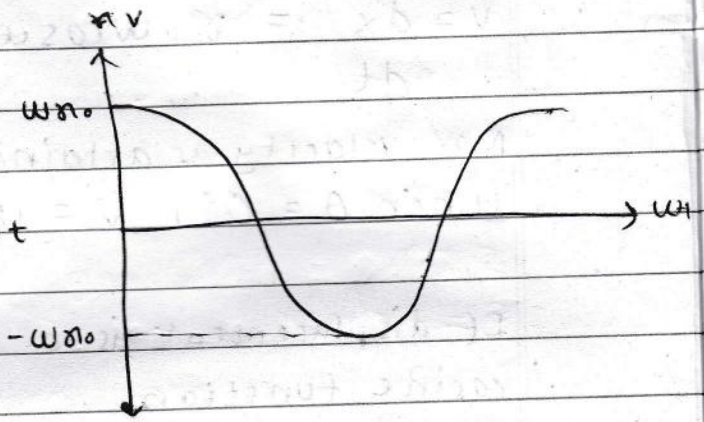
$\boxed{a = a_{max} \sin \omega t}$

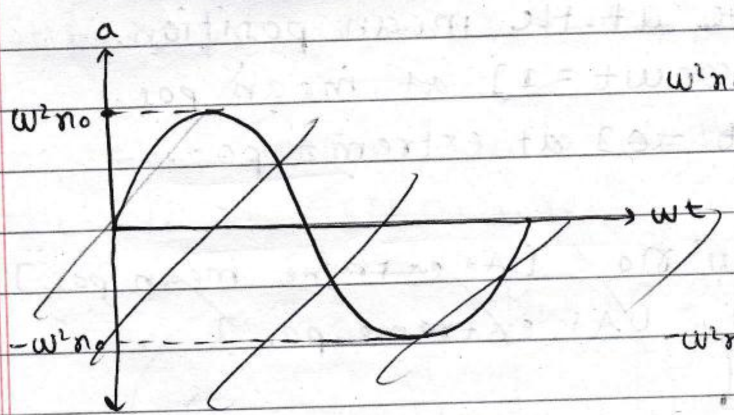
$x = x_0 \sin\omega t$

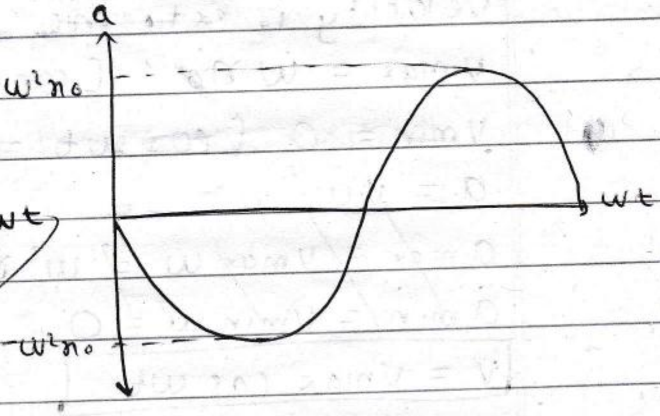
$v = \omega x_0 \cos\omega t$

$a = -\omega^2 x_0 \sin\omega t$









Magnitude of velocity at anytime

$v = \omega x_0 \cos\omega t$

or, $v^2 = \omega^2 x_0^2 \cos^2\omega t$

or, $v^2 = \omega^2 x_0^2 (1 - \sin^2\omega t)$

or, $v^2 = \omega^2 (x_0^2 - x_0^2 \sin^2\omega t)$

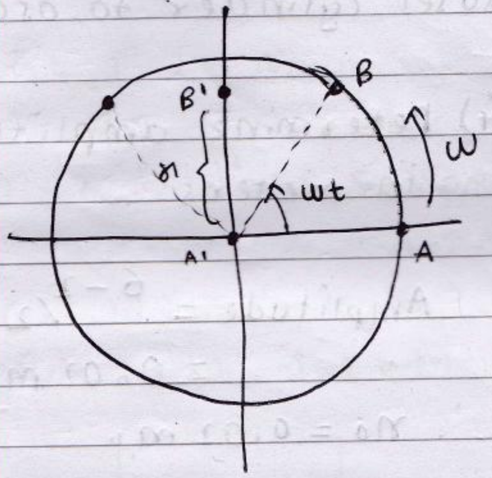
or, $v^2 = \omega^2 (x_0^2 - x^2)$

$\therefore v = \pm\omega\sqrt{x_0^2 - x^2}$

So, $v = 0$, when $x = x_0$ at extreme position.

So, $v = \pm\omega x_0$, when $x = 0$ at mean position.

Phase of a particle in SHM (ϕ)

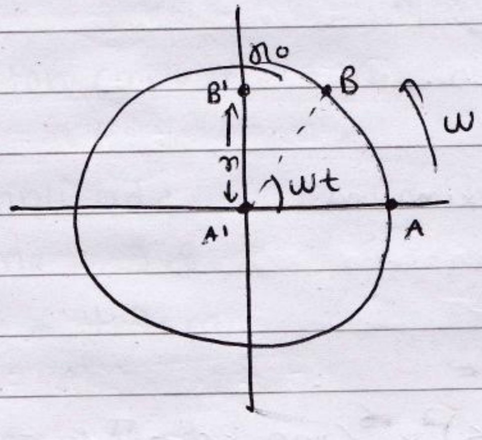


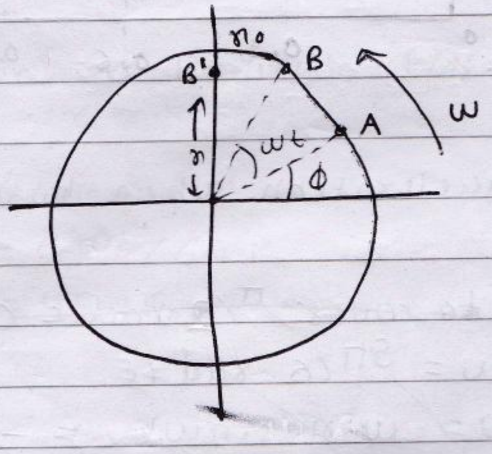
Phase of a particle in SHM can be defined as the angle subtended at the centre with initial position by its equivalent point in a circle.

$$\phi_{B'} = \omega t$$

Phase of B' is angle subtended by B with A to the center, which is ωt.

Phase difference





$x_1 = x_0 \sin \omega t$

Initial phase = 0

$x_2 = x_0 \sin(\omega t + \phi)$

Initial phase = ϕ

β

$\therefore$ Phase difference = ϕ

$x_1 = x_0 \sin \omega t$

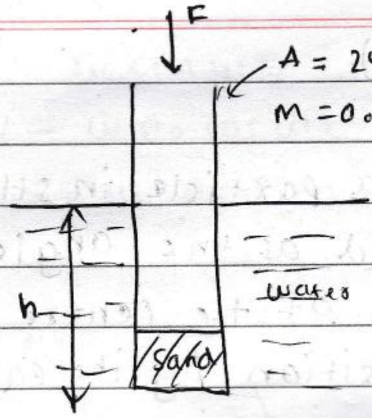
$x_2 = x_0 \sin(\omega t + \pi/2)$

$= x_0 \cos \omega t$

Phase diff (ϕ) = $\pi/2$

So, $x_1 = x_0 \sin \omega t$ and $x = x_0 \cos \omega t$ are two waves with phase difference $\pi/2$.

#



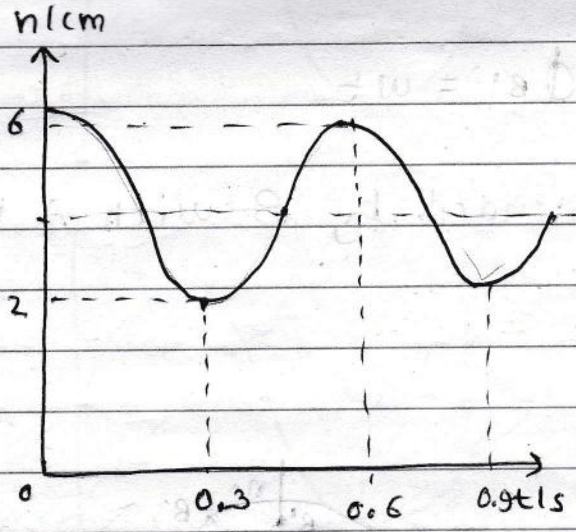
A = 24cm² (0.24m²)
m = 0.23 kg

F is applied and upthrust causes cylinder to oscillate.

i) Determine amplitude in meters

Amplitude = $6-2/2 = 2cm$
$= 0.02\,m$

$\therefore x_0 = 0.02\,m$



ii) Frequency of oscillation

$T = 0.6s$

$f = 1/T = 1.67\,Hz$

iii) Acceleration when h is maximum

~~T to cover π/2 rad = 0.3s~~

$\omega = 5\pi/6$ rad/s

$a = -\omega^2 x_0 \sin\omega t = -(5\pi/6)^2 \times 6/100 \times \sin\pi/4 = -0.41\ ms^{-2}$

$\omega = 2\pi/T = 2\pi/0.6 = 10\pi/3$ rad/s

$a = -\omega^2 x_0 =$

~~$T = 2\pi/$~~

$\omega = 2\pi/T = 2\pi/0.6 = 10\pi/3$ rad/s

$a = -\omega^2 x_0 = -(10\pi/3)^2 \times 2/100 = -2.19\ ms^{-2}$

iv) Frequency is given by, calculate ρ.

$$f = \frac{1}{2\pi}\sqrt{\frac{A\rho g}{M}}$$

or $(1.67 \times 2\pi)^2 = \dfrac{9.81 \times \rho \times {}^{24}/_{100 \times 100}}{0.23}$

$\therefore \rho = 1075.57 \ kg\,m^{-3}$

$\therefore \rho \approx \cancel{1 \times 10^2} \ 1.1 \times 10^3 \ kg\,m^{-3}$.

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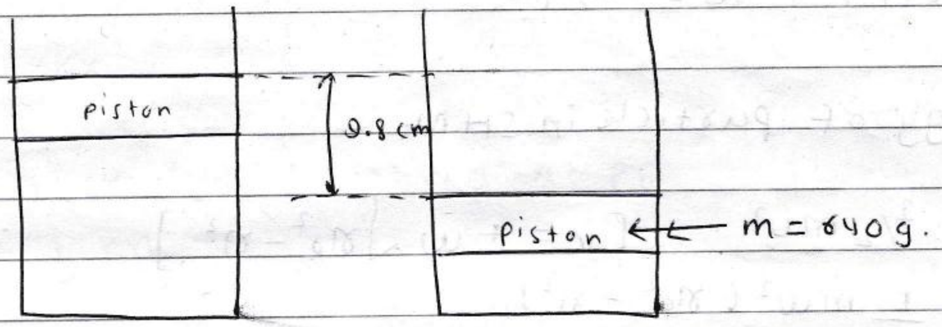


Piston covers 2700 oscillations in 1 minute. For this find:

i) amplitude

$2x_0 = 9.8$

$\therefore x_0 = 4.9 cm$.

ii) frequency

$f = 2700/60$

$= 45 \ Hz$

iii) max. speed

Attained at mean pos.

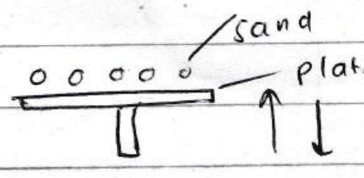
$v = \omega x_0 \cos\omega t$

$= 2\pi f \times {}^{4.9}/_{100} \times 1$

$= 13.85 \ ms^{-2}$

$= 14 \ ms^{-2}$

\#



sand loses contact with plate coming down from ~~at~~ upper extreme pos. when v is maximum

Angular frequency (ω)

- One complete Oscillation = 2π phase.
- When a particle moves 2π radian in a circle, its image completes one oscillation.
- f = frequency, f oscillations in 1 second.
- So in 1 sec, its phase changes by $2\pi f$
- This phase change in angular frequency.
- $\omega = 2\pi f$, $\omega = 2\pi/T$

Energy of particle in SHM

$KE = \frac{1}{2} m v^2 \quad [v = \pm \omega \sqrt{x_0^2 - x^2}]$

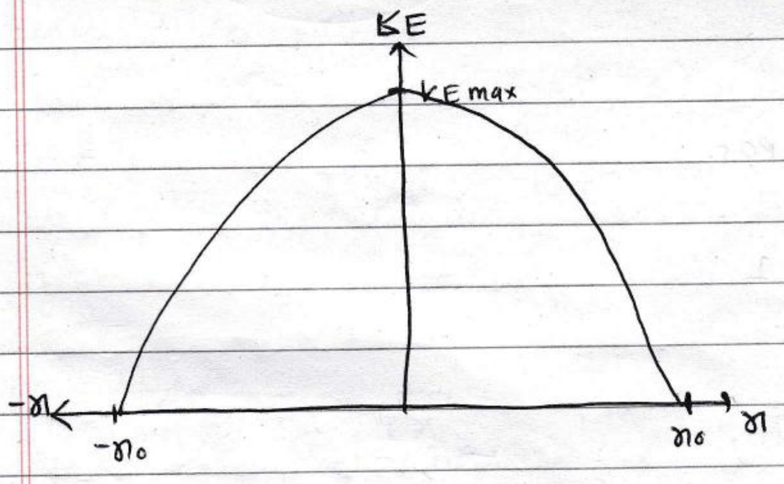
$\therefore KE = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$

KE = max when $x = 0$ at mean pos.

$\therefore KE_{max} = \frac{1}{2} m\omega^2 x_0^2$

KE = min when $x = x_0$ at extreme pos.

$\therefore KE_{min} = 0$



PE = work done when displacement is x.

$dW = F \times dx$

$\therefore \int dW = \int_0^x F\,dx$ [We integrate because F is not constant and is also a function of x.]

$\therefore W = \int_0^x kx\,dx$

$\therefore W = k\left[\frac{x^2}{2}\right]_0^x$

$\therefore W = \frac{1}{2}kx^2$ //.

This work done is stored as PE

$\therefore PE = \frac{1}{2}kx^2$

$a = {-k}/{m}\,x$, $a = -\omega^2 x$, $\therefore k = m\omega^2$

$\therefore PE = \frac{1}{2}m\omega^2 x^2$ //. PE = max when $x = x_0$

PE = min when $x = 0$.

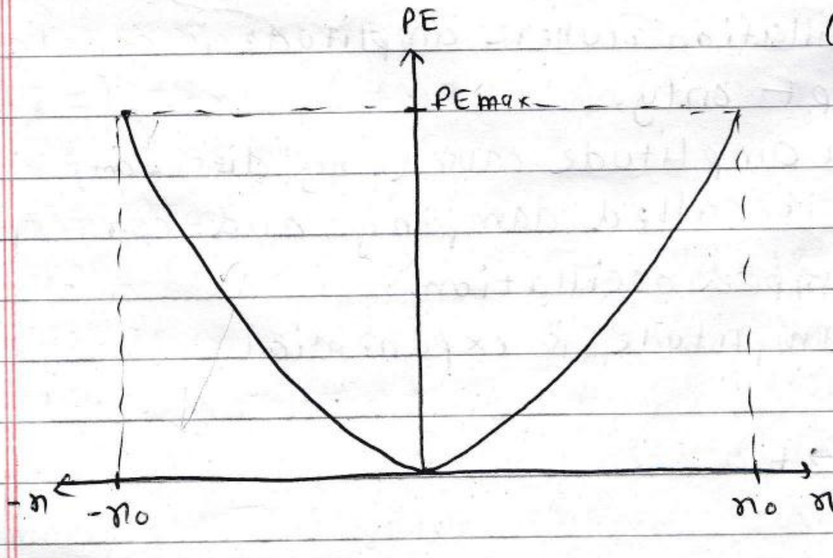
Total energy = KE + PE

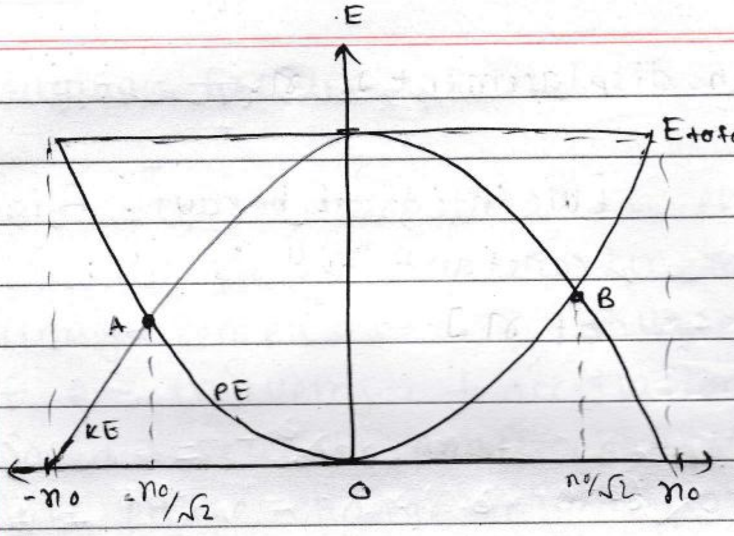
$= \frac{1}{2}m\omega^2(x_0^2 - x^2) + \frac{1}{2}m\omega^2 x^2$

$= \frac{1}{2}m\omega^2 x_0^2$ //.

$PE_{max} = KE_{max}$

Consv. of energy



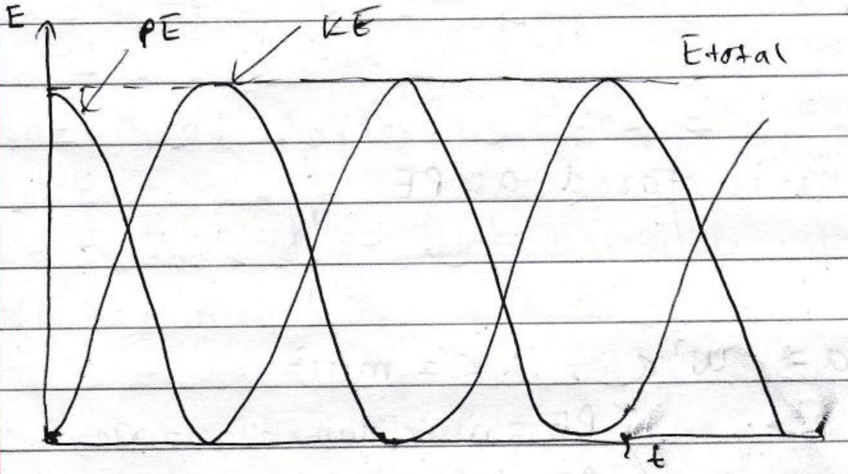


A and B are points where $KE = PE$

When $x = \frac{x_0}{\sqrt{2}}$ ..

$x_0^2 - x^2 = x^2$

$\therefore x = \frac{x_0}{\sqrt{2}}$ ...



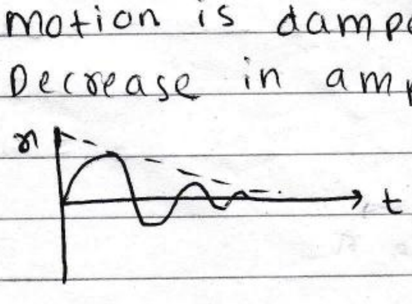
1 complete oscillation,

KE = 0, KE = max, KE = 0, KE = max, KE = 0

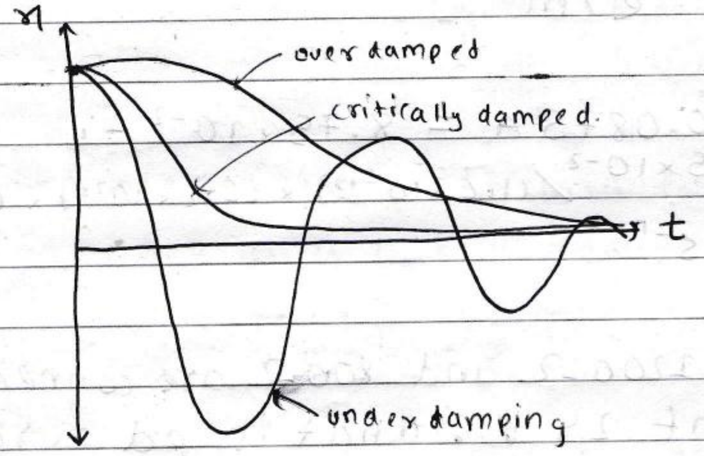
Extreme, Mean, Extreme, Mean, Extreme

Damped oscillation

- Undamped oscillation (where amplitude is constant) is an ideal concept only.
- The decrease in amplitude caused by dissipative / resistive forces is called damping, and corresponding motion is damped oscillation.
- Decrease in amplitude is exponential.



- However time period remains same for free oscillation with damping medium.
- Damping can be useful to get rid of vibrations.
- Under damping results in unwanted oscillations.
- Over damping results in slower return to equilibrium
- Critical damping is min. amount of damping required to return an oscillator to its equilibrium position without oscillating.



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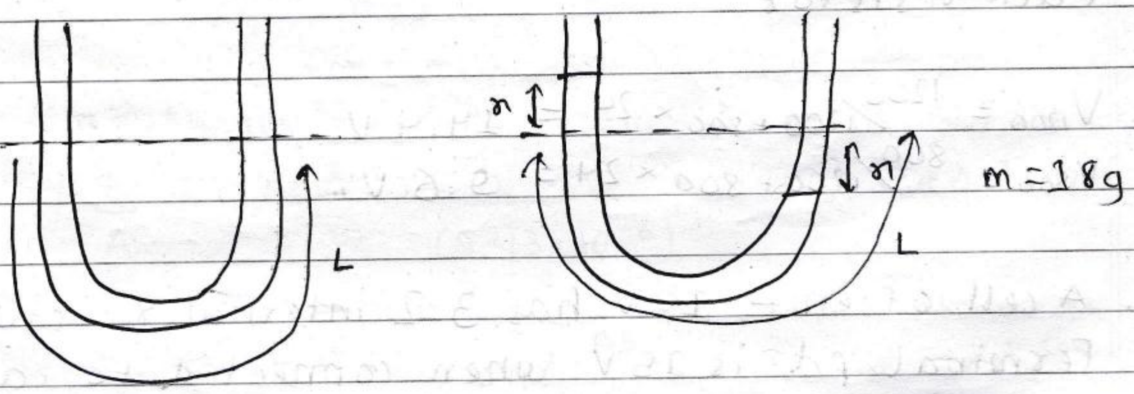


$a = (-2g/L)x$. find a for L = 19 cm and E lost for 2.5T

$a = -\omega^2 x$

or $\omega^2 = 2g/L$

~~or $\omega =$~~ $2\pi/T = \sqrt{2g/L}$

$\therefore T = 0.618\,s$

$E = \frac{1}{2} m \omega^2 x_0^2$

$E_1 = \frac{1}{2} \times 18/1000 \times (2\pi/0.618)^2 \times (\cancel{1.5}\ 2/100$

$= 3.7 \times 10^{-4}$

$E_2 = \frac{1}{2} \times 18/1000 \times (2\pi/\cancel{2.5\times}\,0.618)^2 \times (-0.95/100$

$= 1.\cancel{3 \times 10^{-5}}$ 8.94×10^{-5}

Damped due to viscous force. $\therefore \Delta E = \cancel{3.67}\ 2.86 \times 10^{-4}\,J$

29th Dec 2021

Date
Page

OSCILLATIONS...

Free oscillation

+ When an object is displaced from its equilibrium position and object oscillates with restoring force normally, frequency of oscillation is equal to its natural frequency.
+ Vibration of guitar string, prong of tuning fork, mass attached on a spring

Forced oscillation

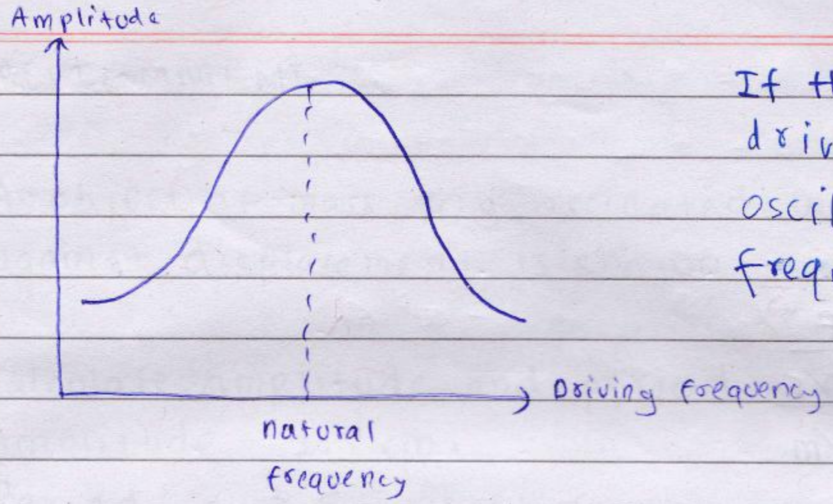
+ Oscillation in which external force is repeatedly applied to oscillating system.
+ Oscillating swing, alternate picking

Resonance

+ When frequency of driving force and natural frequency of object matches, then object vibrates with max. amplitude, this phenomenon is known as resonance.
+ When frequency of driving force is close to the natural frequency of the system.

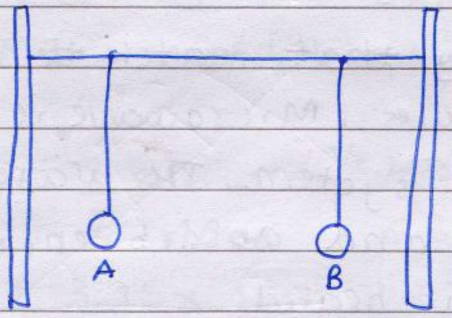
Statements applied to system in resonance.

+ Natural frequency equal to frequency of driver
+ Its amplitude is maximum.
+ Absorbs greatest possible energy from driver



If there is removal of driving force, system oscillates with its natural frequency.

Barton's pendulum



If pendulum A and pendulum B are similar, if pendulum A is oscillated, ~~as it's amplitude~~ pendulum B also starts to oscillate.

This is because pendulum B has same natural frequency as that of A and driving force of A resonates with natural frequency of B.

This is the same physics behind the shattering of wine glass by singing.

Houses are destroyed during earthquakes when frequency of earthquake and natural frequency of building is the same. Tall buildings will respond to low frequency earthquakes while short ones will respond to high frequency oscillations.

A guitar has a sound box, so more volume of air is disturbed and oscillated, which increases sound intensity, amplitude. intensity $\propto$ amplitude^2

$F = -Kx$

$a = -\omega^2 x$

$F/K = a/\omega^2$

or, $\omega^2 = aK/F$

or $4\pi^2 f^2 = aK/F$

or, $f = 1/2\pi \sqrt{aK/F}$

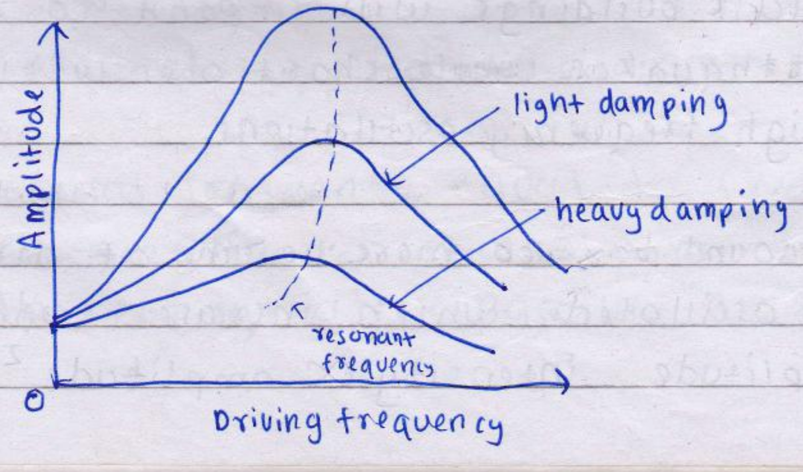
$\therefore f = 1/2\pi \sqrt{K/m}$

Useful applications of resonance.

- Microwaves uses a frequency that match the natural frequency of water molecules. Microwave is driver and molecule is resonating system. The water molecules in food vibrate and absorb energy of radiation. Thus food is heated.
- MRI scan (Magnetic resonance imaging). Uses 1-2 Tesla of magnetic field (Tesla is unit of magnetic strength) This is much stronger than the 3.4×10^{-5} T of earth.
- Resonance in tuning TV and radio.

Variation in amplitude in resonance with driving medium.

- Damping is useful if we want to reduce the damp~~ing~~ aging effects of resonance. Damping reduces amplitude



ASSIGNMENT

1. An object of mass 80 g oscillated with SHM and time against displacement is shown.

a) Calculate amplitude and period of oscillation.
Amplitude = 1.8 cm.
Period = 0.3 s.

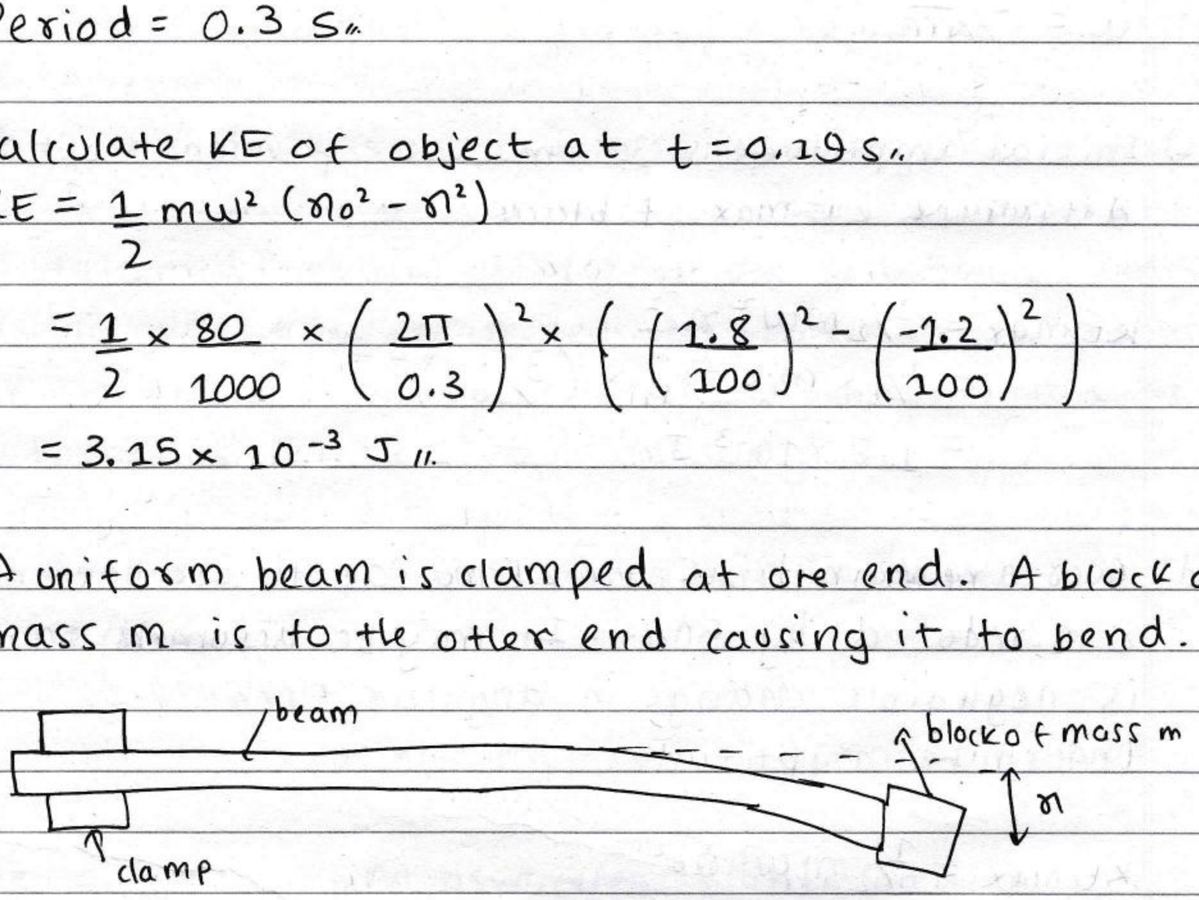
b) Calculate KE of object at $t = 0.19$ s.

$$KE = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$$

$$= \frac{1}{2} \times \frac{80}{1000} \times \left(\frac{2\pi}{0.3}\right)^2 \times \left\{\left(\frac{1.8}{100}\right)^2 - \left(\frac{-1.2}{100}\right)^2\right\}$$

$$= 3.15 \times 10^{-3}\ J$$

2. A uniform beam is clamped at one end. A block of mass m is to the other end causing it to bend.



The block is oscillated and such and acceleration is given by $a = -\frac{k}{m} x$.

a) Explain how the eqⁿ deduces that the block is in SHM
→ Rearranging the expression we get $F = -kx$.
This is the basic principle of SHM, where restoring force is directly proportional to displacement from mean position and is opposite in direction.

b) For the beam, $k = 4.0\ kgs^{-2}$. Show that angular frequency ω of the oscillations is given by $\omega = 2/\sqrt{m}$.

$\cancel{F = -kx}$ $a = -k/m\ x$, $a = -\omega^2 x$

$k/m = \omega^2$

or $4/m = \omega^2$

$\therefore \omega = 2/\sqrt{m}$ //.

c) Initial amplitude is 3.0 cm. Use expression in b to determine KE max of block.

$KE_{max} = 1/2\, m\omega^2 x_0^2$

$= 1/2 \times \cancel{m} \times 4/\cancel{m} \times (3/100)^2$

$= 1.8 \times 10^{-3}\ J$ //.

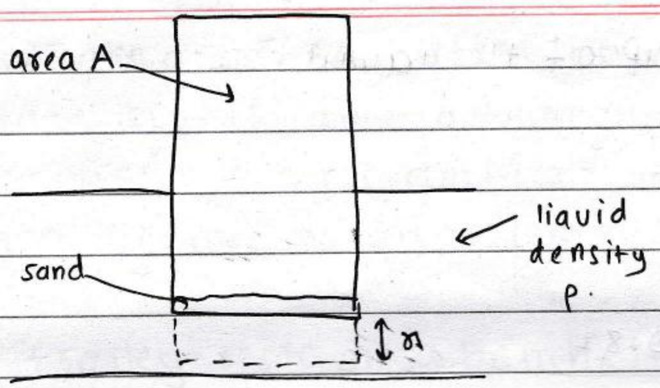
d) Over a certain time, the KE max of the oscillations in c is reduced by 50%. It may be assumed that there is negligible change in angular frequency. Determine amplitude.

$KE_{max} = 1/2\, m\omega^2 x_0^2$

or $1.8 \times 10^{-3}/2 = 1/2 \times \cancel{m} \times 4/\cancel{m} \times x_0^2$

$\therefore x_0 = 0.021\ m$ // (2.1 cm)..

3. A cylindrical tube, sealed at one end has cross-sectional area A and contains some ~~used~~ sand. Total mass of tube and sand in M. Tube floats uprigt in liquid of density ρ.
The tube is pushed a short distance into the liquid.



a) i) State the two forces acting on tube immediately after its release.

→ the upthrust due to liquid and its weight.

ii) State and explain direction of resultant force acting on the tube immediately after its release.

→ the direction is towards equilibrium position as upthrust is greater than its weight.

b) The acceleration of tube is given by

$$a = -\left(\frac{A\rho g}{M}\right)x$$

Use this to explain why tube undergoes SHM?

→ The tube is in SHM as acceleration of the tube is directly proportional and opposite in direction to its displacement in mean position.

c) For $A = 4.5\,cm^2$ and $M = 0.17\,kg$, period of oscillation of tube is 1.3 s.

i) Determine angular frequency ω.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.3} = 4.8\ rad\,s^{-1}$$

ii) Determine density ρ of the liquid.

$$a = -\left(\frac{A\rho g}{M}\right)x$$

or $\cancel{-}\omega^2 \cancel{x} = \cancel{-}\left(\frac{A\rho g}{M}\right)\cancel{x}$

or $(4.8)^2 = \dfrac{\frac{4.5}{100\times100} \times \rho \times 9.81}{0.17}$

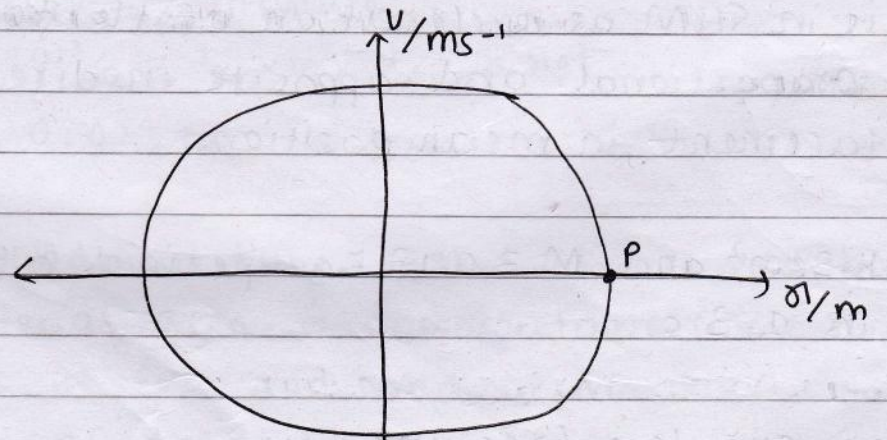
$\therefore$ ~~or~~ $\rho = 900\ kg\,m^{-3}$

4a) A body undergoes SHM.
Variation of ~~displaction~~ displacement with velocity is shown.

i) State amplitude of the oscillations.
$\therefore x_0 = 0.05\ m$

ii) Calculate period T of the oscillations.
$v_{max} = \omega x_0$
or $0.42 = 2\pi/T \times 0.05$
$\therefore T = 0.75\ s$

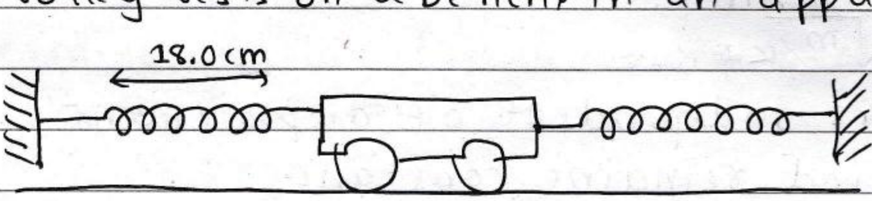
iii) On figure label with a P a point where body has PE max.



/ / / /

5a) Defining eq^n for SHM is $a = -\omega^2 x$
State significance of minus sign.
→ The minus sign signifies that acceleration is opposite in direction to displacement.

b) A trolley rests on a bench, in an apparatus as shown.



Unstretched length of spring is 12.0 cm. Spring constant is $8.0\ Nm^{-1}$. Trolley is at equilibrium when length of each spring is 18.0 cm. Trolley is displaced 4.8 cm to a side.

i) Show that resultant force on trolley at moment of release is 0.77 N.

$F = ke$

or, $F = 8 \times 4.8/100$

$\therefore F = 0.384\ N$

Since there are two springs,

$F_{net} = 2F$

$\therefore F_{net} = 0.768\ N\ [0.77\ N]$

ii) Mass of trolley is 250 g. Calculate max acceleration of the trolley.

$F = ma$

or, $0.768 = 250/1000\ a$

$\therefore a = 3.07\ ms^{-2}$

iii) Calculate period T of oscillation.

$a = -\omega^2 x$

or, $3.07 = -4\pi^2/T^2 \times 4.8/100$

$\therefore T = 0.78\ s$

iv) The trolley is displaced 2.4 cm. State the effect it has on period of oscillation.

$F = -ke$ $\quad\quad a = -\omega^2 x$

or $a = -k/m \, x$

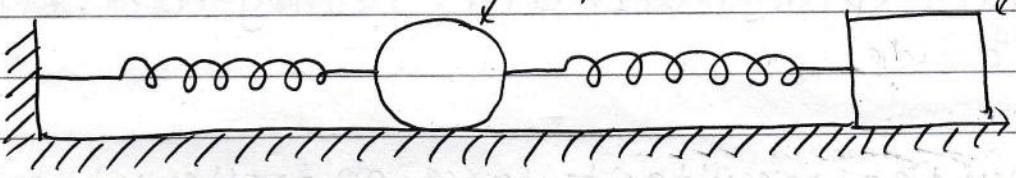
$k/m = \omega^2$

or $\cancel{\omega}\ 4\pi^2/T^2 = k/m$

$\therefore T = 2\pi\sqrt{m/k}$

Since T is independent of displacemento, ~~here~~ the time period remains constant.

6. A ball of mass M is held on a surface by two springs as shown.



The oscillator is switched off and displacement against time is shown (there is damping)

a) State

i) What is meant by damping

→ Damping is the decrease in amplitude of an oscillator due to the loss of energy it experiences because of resistive forces.

ii) evidence provided by s-t graph for damping.

→ There is decreare in amplitude with time but time period remains the same.

///.

b) Acceleration and displacement of ball are related.

$$a = -\left(\frac{2K}{m}\right) x$$

Where K is spring constant of one spring. Mass M of ball is 1.2 Kg.

i) Determine angular frequency of oscillations of ball

$$\omega = 2\pi/T = 2\pi/0.8 = 7.85\ rad\ s^{-1}$$

ii) Determine spring constant K.

$$a = -\omega^2 x \qquad a = -(2K/m)x$$

$$\omega^2 = 2K/M$$

$$\cancel{4\pi^2/T^2} \quad K = M\omega^2/2$$

$$\therefore K = 37.0\ Nm^{-1}$$

iii) Calculate energy dissipation after 3 oscillations.

$$KE_{max\,1} = \tfrac{1}{2} m\omega^2 x_0^2 = \tfrac{1}{2}\times 1.2\times 7.85^2\times(1.5/100)^2$$

$$KE_{max\,3} = \tfrac{1}{2} m\omega^2 x_0^2 = \tfrac{1}{2}\times 1.2\times 7.85^2\times(0.65/100)^2$$

$$\therefore \text{Energy dissipated} = KE_{max1} - KE_{max2}$$

$$= \tfrac{1}{2}\times 1.2\times 7.85^2\times\left((1.5/100)^2 - (0.65/100)^2\right)$$

$$= 6.8\times 10^{-3}\ J$$

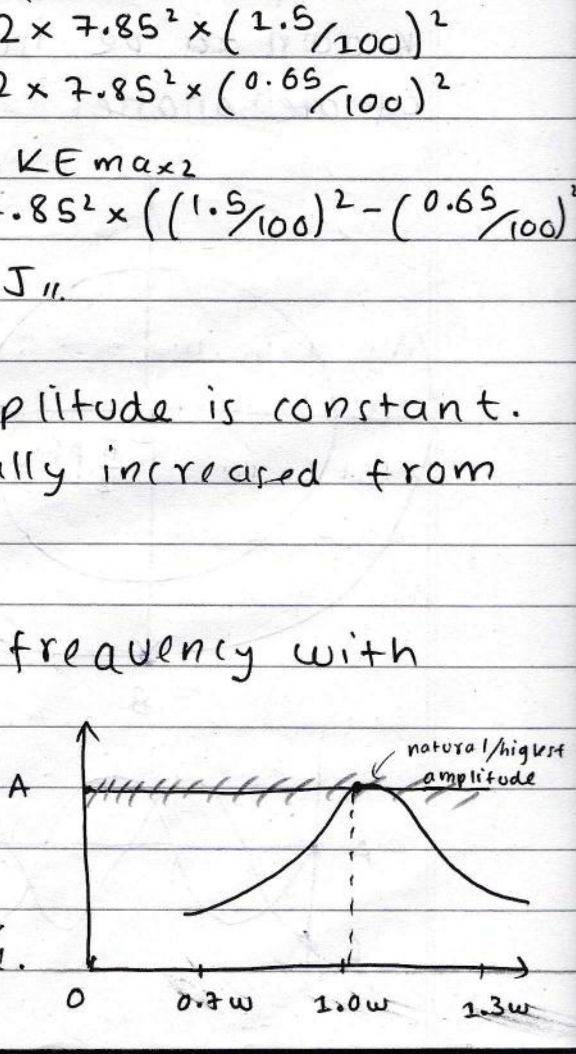
c) Oscillator is switched on. Amplitude is constant. Angular frequency is gradually increased from 0.7ω to ~~0.~~ 1.3ω.

i) Draw variation of angular frequency with amplitude of ball.

$$\cancel{\omega^2 = 2K/m}$$

$$\cancel{\therefore \omega = \sqrt{2K/m}}$$

~~∴ ω is independent of amplitude.~~

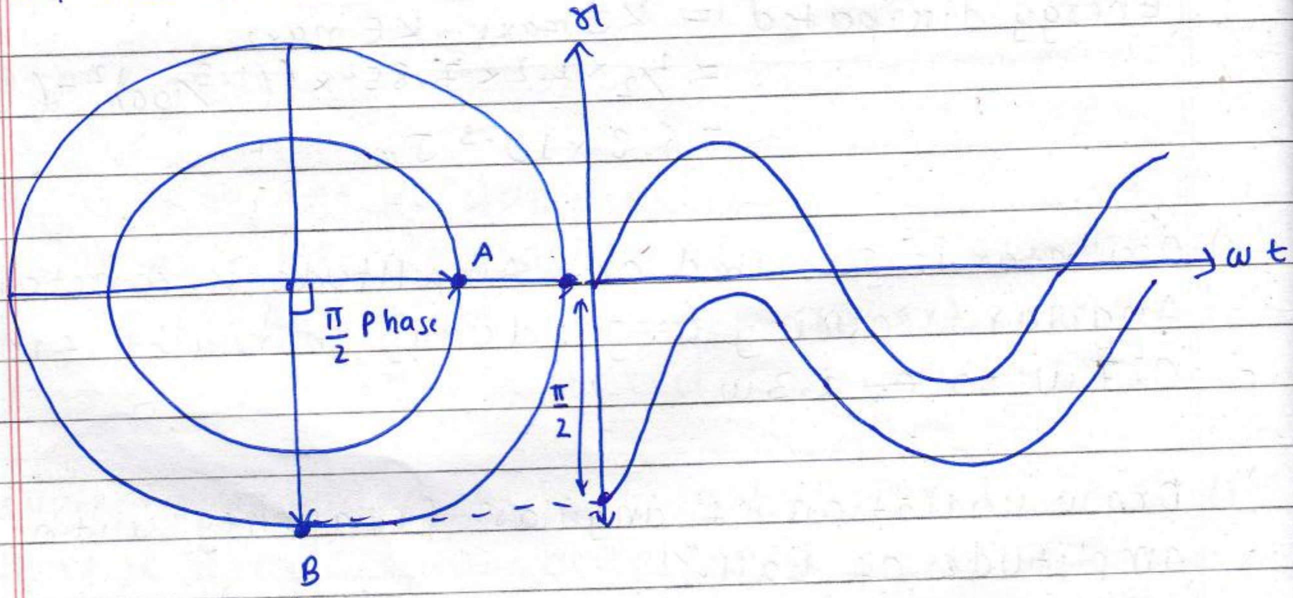


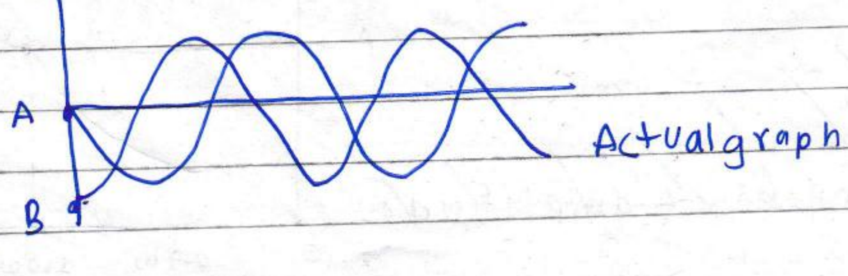
ii) Some sand is sprinkled on the horizantal surface. The angular frequency of oscillations is again gradually increased.
State two changes that occur to line drawn.

→ ~~Amplitude decreases as now~~
→ The graph shifts down due to increased friction
→ Highest amplitude ~~dt~~ is attained before 1.0 ω as the friction decreases natural frequency. so highest amplitude is attained earlier.

PHASE DIFFERENCE BETWEEN TWO POINTS FROM (s-t), (v-t), (a-t) GRAPH.

→ Difference in 1 Time period is 2π phase.
→ Difference in $\frac{1}{2}T$ is π phase.
→ Two particles with π phase difference are also known to be in anti phase, as they're opposite of one another.





Quantity	Formula	At mean pos	At extreme po
Displacement	$x = x_0 \sin\omega t$	$x = 0$	$x = x_0$
Velocity	$v = x_0\omega\cos\omega t$ $v = \pm\omega\sqrt{x_0^2 - x^2}$	$v = \omega x_0$	$v = 0$
Acceleration	$a = -\omega^2 x_0 \sin\omega t$	$a = 0$	$a = -\omega^2 x_0$
KE	$\frac{1}{2} m\omega^2 (x_0^2 - x^2)$	$\frac{1}{2} m\omega^2 x_0^2$	0
PE	$\frac{1}{2} m\omega^2 x^2$	0	$\frac{1}{2} m\omega^2 x_0^2$
Total energy	$\frac{1}{2} m\omega^2 x_0^2$	$\frac{1}{2} m\omega^2 x_0^2$	$\frac{1}{2} m\omega^2 x_0^2$