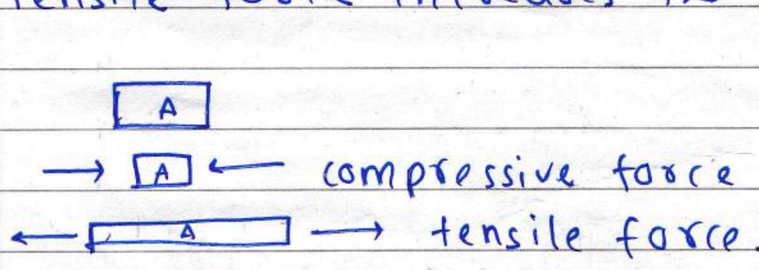
DEFORMATION OF SOLIDS

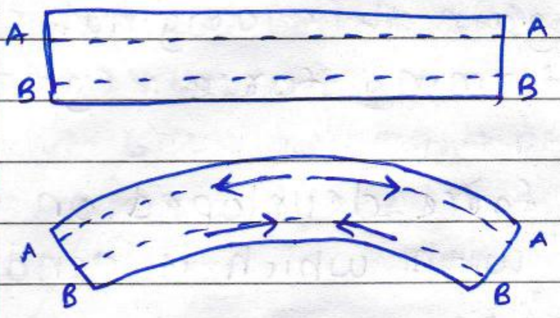
- It is the alteration in shape and size of an object.
- But it can also be in length, area & volume.
- It may be temporary or permanent.

- The deformation that lasts only unitl force is applied and the object regains its original shape and ~~z~~ size once force is stopped is temporary deformation.
- The deformation that lasts even after application of deforming force is stopped is permanent deformation.
- The force that changes the shape and/or size of an object is called deforming force.

- Deformation in one dimension / linear deformation is the change in length due to application of force.
- A pair of forces in opposite direction are required along their length to deform a body.

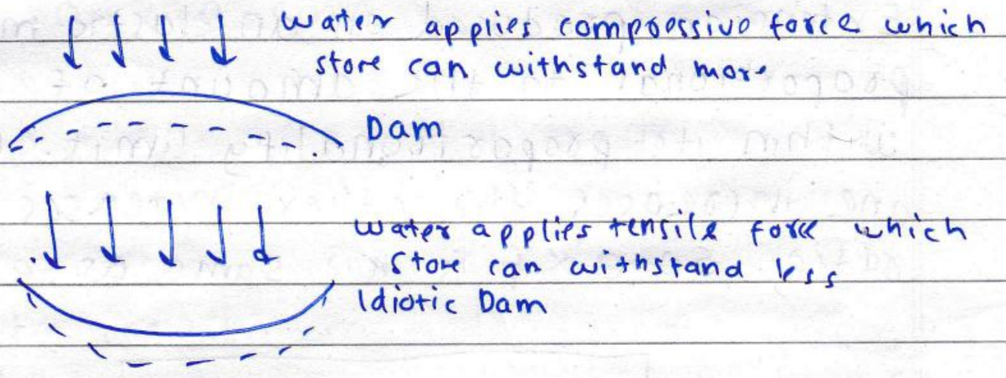
- Compressive force decreases the length of a body.
- Tensile force increases the length of body.

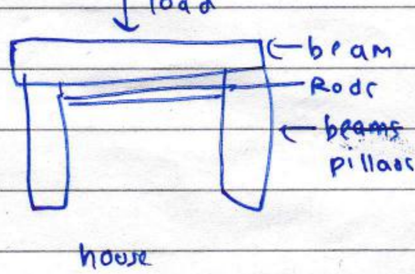




tensile forces in the upper surface AA and compress-ive forces in the lower surface BB are being applied. AA length increases while BB length decreases. (tensile force & compressive force).

+ The reason why bridges are curved downwards and dams are also curved is because they are made of stone which can withstand more compressive than tensile fore.



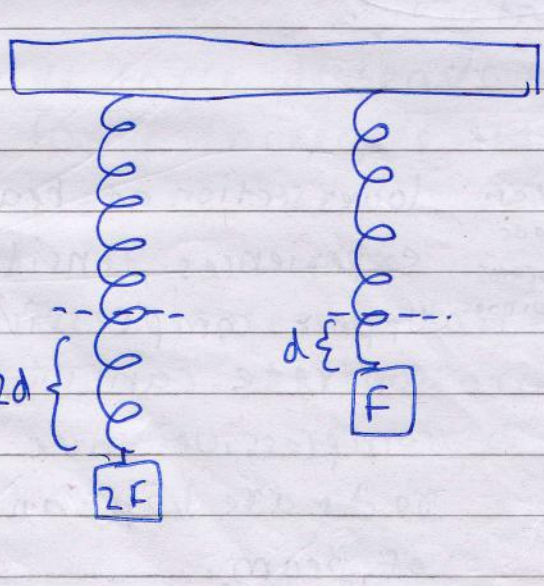


lower section of beam ~~withstand~~ experiences tensile force and upper compressive force, concrete can withstand more compressive force, so more iron rods are kept on lower section of beam.

- Elastic materials can regain their original shape after removal of deforming force. eg: spring, rubber band etc.
- Restoring force is the force developed on the elastic object (spring) with which it tends to regain its original shape.
- Restoring force is equal to deforming force if the object hasn't undergone permanent deformation when there is elastic deformation.
- When the application of deforming force is stopped then only restoring force comes to play.
- Restoring force is due to the force of attraction between the molecules.

Hooke's law

Extension produced on an elastic material is directly proportional to the amount of force applied within its proportionality limit. ($e \propto F$). [As in, if one increases the other increases in the same ratio. So $e \propto F$ is not same as $e \propto 2F$.]



e = constant × $F_{deforming}$

This law is only valid unless the spring has undergone permanent deformation.

Restoring force developed on an elastic object is directly proportional to the amount of extension produced, within ~~in~~ its proportionality limit.

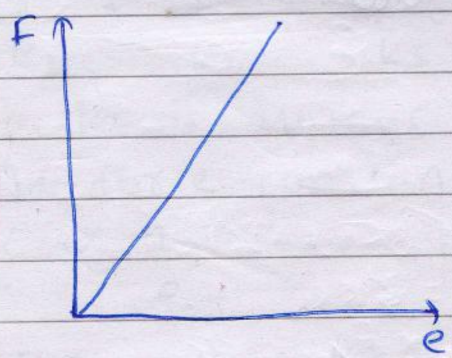
$F_{restoring} \propto e$ [In this e is independent, but in $e \propto F$, f was independent]

F = ke, where k is proportionality constant known as force constant or stiffness constant of that spring, also known as spring constant

From hooke's law, F = ke
When e = 1m,
Then, F = K
So force constant of a spring is defined as the amount of force required to produce 1 m extension within proportionality limit.
Its SI unit is Nm^{-1}.
Lets compare F = ke with y = mx

Let's assume a data

F	5	10	15	20	25
e	1	2	3	4	5



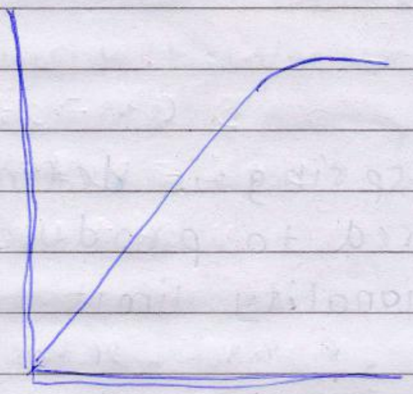
So, we can say gradient of the line is the force constant. of that material.

Proportionality limit is the point on force extension graph up to where Hooke's law is valid up to where the graph is straight.
As more force is applied graph becomes curved.

Force constant = $20 Nm^{-1}$, follows hooke's law upto 5N, Draw f-e graph with 0-6 N.

$e = F/K$

N	1	2	3	4	5
e/m	0.05	0.10	0.15	0.20	0.25



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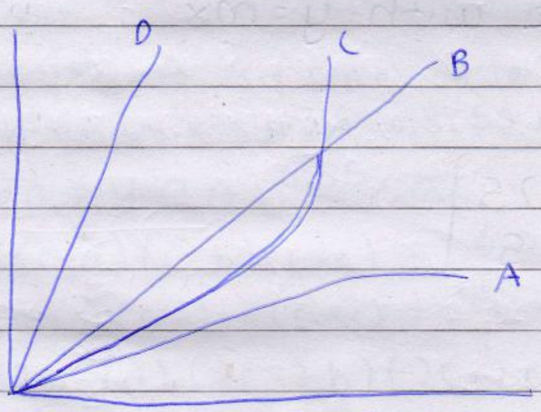


D ~~is the~~ has most force constant.
A is the least stiff.
C does not follow Hooke's law.

$F = ke$

$k = \cancel{e/F}$ $\quad F/e$

$= \cancel{\dfrac{15-10/100}{0.4}}$ $\quad = \dfrac{0.4}{15-10/100}$

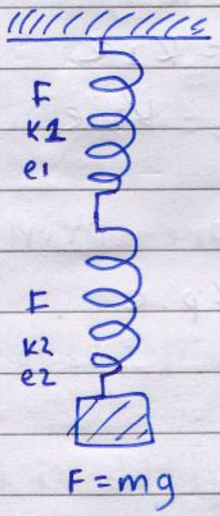
$= \cancel{5/40}$ $\quad 40/5$

$= 8 Nm^{-1}$

Combination of springs

- Series: End to end connection.
- Parallel: side by side connection.

Series combination



Let two springs have force constant k_1 and k_2
Same force is acted on them (assuming mass of spring is negligible
Total extension of combined spring is $(e = e_1 + e_2)$
For first spring, $e_1 = F/k_1$
For second spring, $e_2 = F/k_2$

If both springs can be replaced by a single spring with force constant k_s, when same force is applied, same extension is produced (equivalent spring)

For equivalent spring, $F = k_s e \rightarrow e = F/k_s$

$e_1 + e_2 = F/k_s$

or, $F/k_1 + F/k_2 = F/k_s$

$\therefore \dfrac{1}{k_1} + \dfrac{1}{k_2} = \dfrac{1}{k_s}$ // $\therefore k_s = \dfrac{k_1 . k_2}{k_1 + k_2}$

The k_s is always smaller than both k_1 & k_2

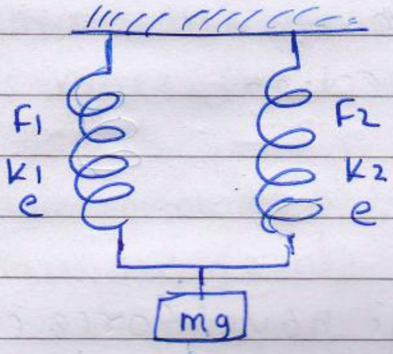
$\therefore$ The force constant decreases.

$\therefore K \propto \dfrac{1}{e}$

If $k_1 = k_2$, $k_s = \dfrac{k_1 . k_1}{k_1 + k_1} = \dfrac{k_1^2}{2k_1} = \dfrac{k_1}{2}$ //

If $k_1 = k_2 = k_3$, $\cancel{k_s = \dfrac{k_1 . k_1 . k_1}{k_1 + k_1 + k_1}}$ = $k_s = \dfrac{k_1}{3}$ //

Parallel combination



Let two springs have force constant k_1 and k_2.

$F = F_1 + F_2$

Assuming intial length of them is equal, $e = e_1 = e_2$ //.

For 1^{st} spring, $F_1 = k_1 e$

For 2^{nd} spring $F_2 = k_2 e$

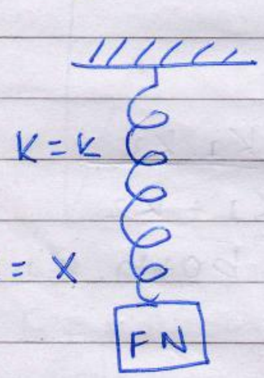
Now be replace both springs with one spring with ~~fo~~ force constant k_p and eqn $F = k_p . e$

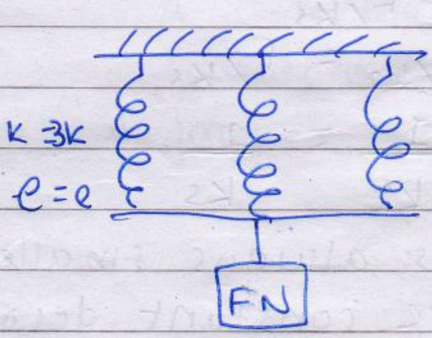
$F_1 + F_2 = k_p . e$

or, $k_1 e + k_2 e = k_p . e$

$\therefore k_p = k_1 + k_2$ //.

A spring produces extension x when certain weight is hanged. If it is cut into three pieces and connected in parallel and same weight is hanged, what will be extension?





$F = (3K + 3K + 3K)e$

or, $F = \cancel{3}9Ke$

or, $1/\cancel{3}9 \times F/K = e$

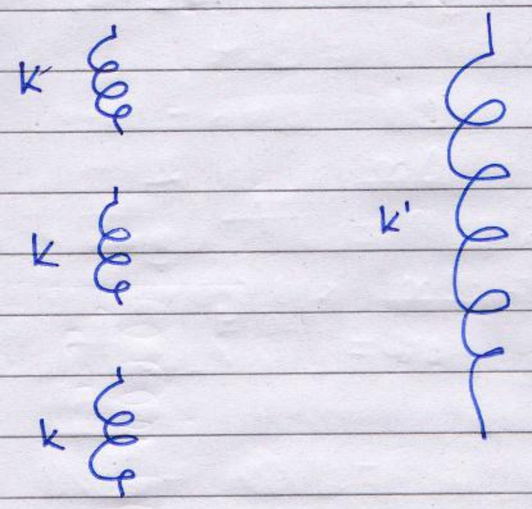
$\therefore e = 1/\cancel{3}9\, x \quad [x = F/K]$

$F = Ke$

or, $F = Kx$

$\therefore x = \frac{F}{K}$ //.

(~~on~~ Dividing spring is ~~same~~ just reverse of joining three springs in series.



$1/k' = 1/k + 1/k + 1/k$

or, $1/k' = 3/k$

$\therefore k = 3k'$

Stress (σ)

- It is defined as the restoring force per unit cross sectional area of an object, within it's proportionality limit.
- Its SI unit is ~~A~~ Nm^{-2} or Pascal(P).

Strain (ε)

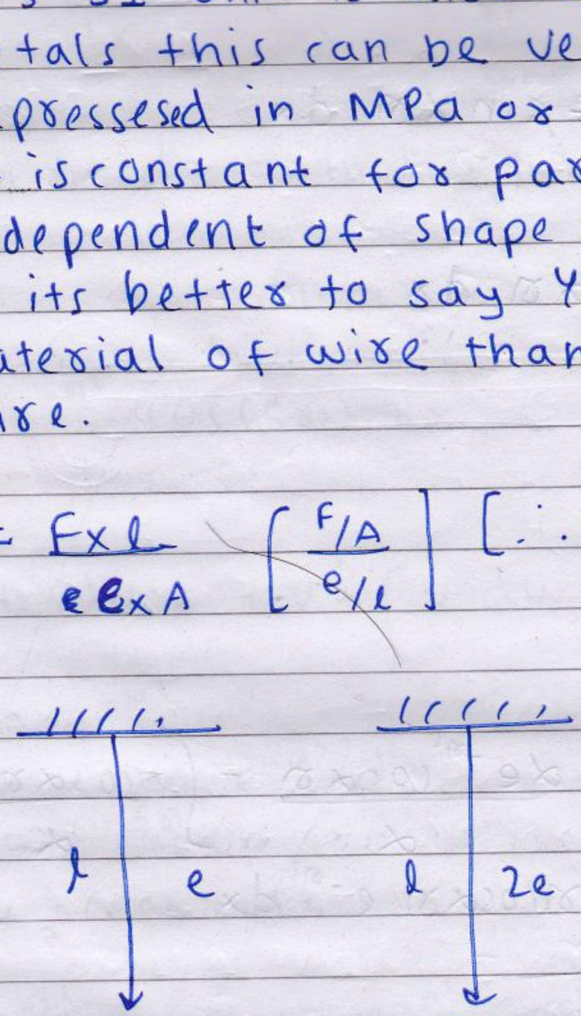
- It is defined as fractional increase in original length of object
- Extension per unit original length.
- It is unitless as it is ratio.
- Strain is about 0.1% for metals within proportionality limit. 1mm extension for 1m.

DEFORMATION OF SOLIDS . . .

Young modulus (E)

- The ratio of stress and strain within proportionality limit.
- It's SI unit is also Nm^{-2} (Pa) But for metals this can be very high so it is also expressesed in MPa or GPa.
- It is constant for particular material and independent of shape and size of the object.
- So its better to say Young modulus of material of wire than young modulus of wire.

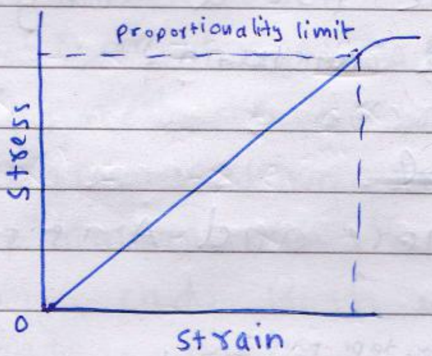
$$E = \frac{F \times l}{e \times A} \quad \left[\frac{F/A}{e/l}\right] \quad [\therefore F \propto E]$$



$$E = \frac{F \times l}{e \times A} \qquad E = \frac{\cancel{2}F \times l}{\cancel{2}e \times A}$$

Thus, it is ratio and only depends on nature of material and not on F, l, e or A.
E is higher for stiffer materials.
Thus $E \propto F$, $E \propto l$ is incorrect.

- Higher the young modulus of a material higher will be its stiffness.
- Eg : $E_{steel} > E_{copper}$
- stress = young modulus x strain
Comparing this with $y = mx$



Slope of this line will be the young modulus.

-

Material	Young modulus (GPa
Aluminium	70
Brass	90 - 110
Brick	7 - 20
Concrete	40
Copper	130
Wood	10 approx.

- Thus higher young modulus also means the substance is more elastic.

Suppose a wire of Young modulus E, A, l, e extension when force F is applied. What's its force constant

$F = E \times e \times A/l$

or, $F = EA/l \times e$

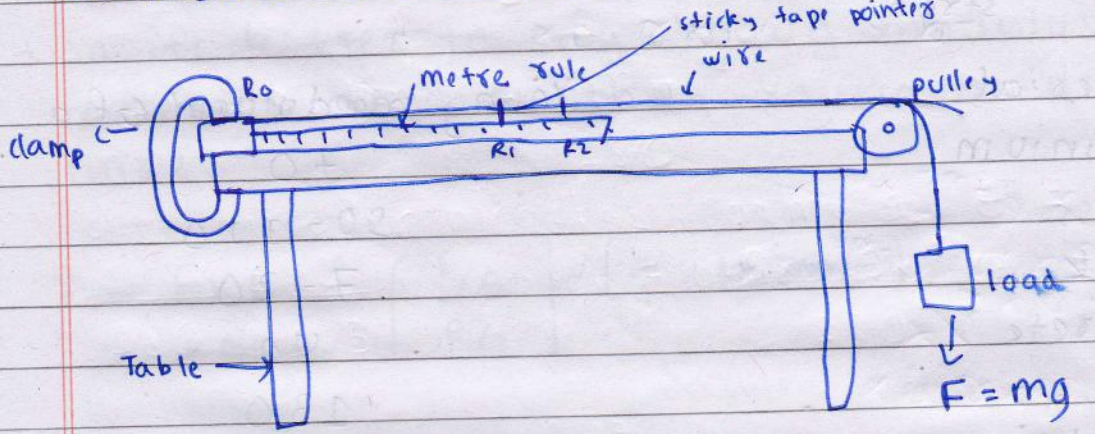
Comparing with $F = Ke$

$\therefore$ Force constant $= EA/l$

Date
Page

Experimental determination of young modulus of material of wire

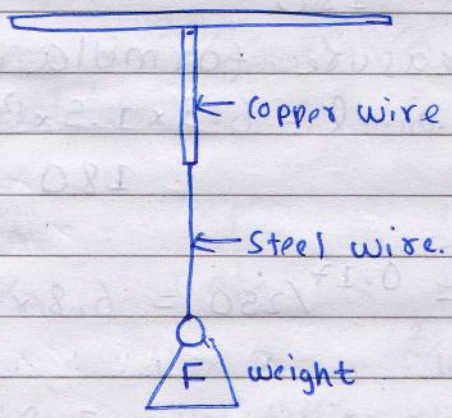
- We need a long wire with small diameter.
- When force is applied from both ends, the extension will be notable for a long wire
- $e = \frac{F \times l}{E \times A}$ (where — are constant)
- $e \propto F$, $e \propto 1/A$, $e \propto l$
- So when length is ~~lon~~ more and A is small extension is notable.



We put a mark on the wire and take it as a reference point. As load increases, the reference point moves. We can use a microscope to observe this. If initial reading of reference point is R_1 and final is R_2,
$e = R_2 - R_1$ //
As load increases reference point moves more.
Now,
$$E = \frac{F \times l}{e \times A}$$
Length of wire is only read from fixed point to reference point R_1 (R_0 to R_1), ~~R_2~~
$l = R_1 - R_0$ //

To make sure we haven't exceeded the proportionalit limit we gradually remove weight until there is no weight and we absorve if R1 is back to its original position.

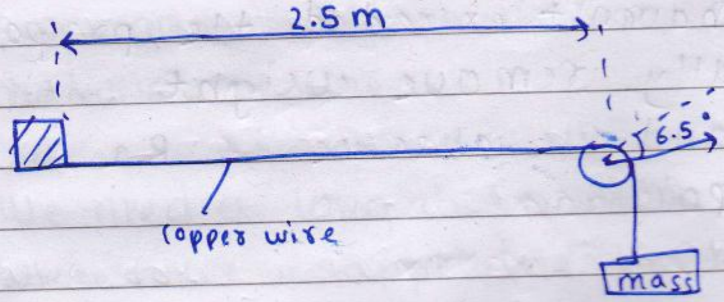
The Young modulus of steel is twice that of copper. A 50 cm length of Cu wire of diameter 2mm is joined to a 50 cm steel wire of diameter 1.0mm making combination wire of length 1m. The combination is stretched by a weight added to its end. Both wires obey Hooke's law. What is ratio $\frac{\text{extension of steel wire}}{\text{extension of copper wire}}$.



$e_{steel} = \frac{F\times l}{E\times A} = \frac{\frac{1}{2}\times F}{\cancel{2}E\times \pi d^2/\cancel{4}2} = \frac{F/\cancel{2}}{\pi E\times 1^2/\cancel{2}} = \frac{F}{\pi E}$ //

$e_{copper} = \frac{F\times l}{E\times A} = \frac{\frac{1}{2}\times F}{E\times \pi d^2/4} = \frac{F/2}{\pi E\times \cancel{2^2}/\cancel{4}} = \frac{F}{2\pi E}$ //

$\text{Ratio} = \frac{e_{steel}}{e_{copper}} = \frac{F/\pi E}{F/2\pi E} = \frac{\cancel{F}}{\cancel{\pi E}}\times\frac{2\cancel{\pi E}}{\cancel{F}} = 2$ //



When mass is increased by ~~6x~~ 6 kg, a pointer attached to the pulley rotates through angle $6.5°$. The diameter of ~~wire~~ pulley is 3.0 cm. For this increase in mass, show that the wire extends by 0.17 cm.

$6.5°$ in radian $= \dfrac{6.5\pi}{180}^c$ //.

Using circular measure, formula, $\theta^c = l/r$,

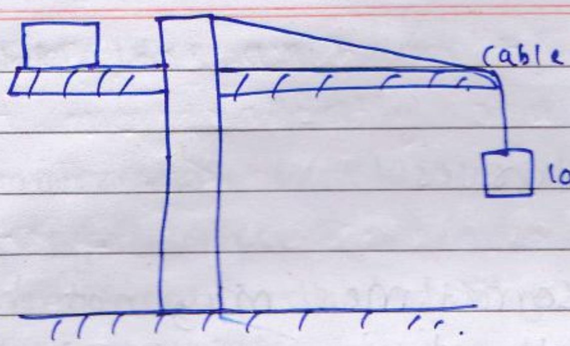
$\dfrac{6.5\pi}{180} = \dfrac{l}{1.5}$ $\therefore l = \dfrac{6.5 \times 1.5 \times 22/7}{180} = 0.17$ cm //.

Strain in wire $= 0.17/250 = 6.8 \times 10^{-4}$ //.

The area of cross section is 7.9×10^{-7} m^2. Calculate increase in stress produced by increase in load of 6 kg.

Stress $= mg/A = 6 \times 9.81 / 7.9 \times 10^{-7} = 74506329$ Pa //
$= 7.4 \times 10^7$ Pa //

Young's Modulus $= \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{7.4 \times 10^7}{6.8 \times 10^{-4}}$
$= 1.08 \times 10^{11}$ Pa //



This is a model of a crane which is exact. But the model is $1/10$th full size in linear dimensions. What is ratio of $\frac{\text{stress}_{\text{full-size}}}{\text{stress}_{\text{model}}}$

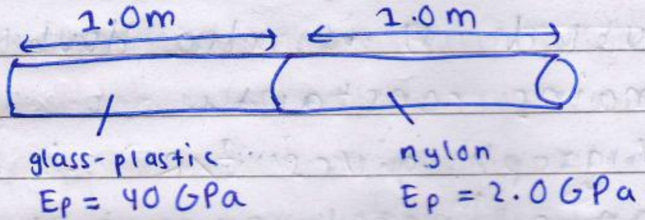
$$\text{Stress}_{\text{full size}} = F/A = \frac{mg}{\pi d^2/4} = \frac{4mg}{\pi d^2}$$

$$\text{Stress model} = \frac{4 \times 1/10\, mg}{\pi \times 1/100\, d^2} = \frac{2/5\, mg}{\pi d^2/100}$$

$$\text{Ratio} = \frac{4mg}{\pi d^2} \times \frac{\pi d^2/100}{2/5\, mg} = 0.1$$

$$\text{Ratio} = \frac{mg/A}{m'g/A'} = \frac{\dfrac{V \times \rho}{\pi d^2/4}}{\dfrac{V' \times \rho}{\pi d'^2/4}} = \frac{\dfrac{\ell^3}{\pi d^2}}{\dfrac{1/1000\, \ell^3}{\pi \times 1/100\, d^2}} = \frac{1}{1/10} = 10$$

DEFORMATION OF SOLIDS . . . .



The rod will break when total extension reaches 3.0 mm. What is the greatest tensile strength that can be applied to the composite rod before it breaks,

$e_{glass\text{-}plastic} + e_{nylon} = \Sigma e_{total}$

or, $\dfrac{F \times l}{E \times A} + \dfrac{F \times l}{E \times A} = \dfrac{3}{1000}$

or, $\dfrac{F}{(4\times10^{10}) A} + \dfrac{F}{(2\times10^{9}) A} = \dfrac{3}{1000}$

Since F/A is stress

$\dfrac{1}{4\times10^{10}} S + \dfrac{1}{2\times10^{9}} S = \dfrac{3}{1000}$

$\therefore S_{max} = 5.7 \times 10^{6}$ Pa

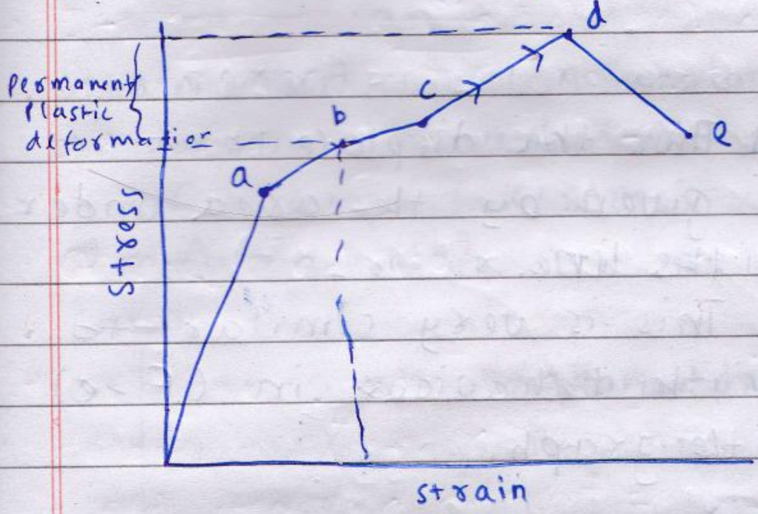
Proportionality limit.

Point on (F-e) or stress strain graph upto where graph is linear, where Hooke's law is valid.

Elastic limit is the point ~~to~~ on stress strain graph upto where object can regain its original shape. ~~That~~ After this point there is permanent / plastic deformation. Beyond this point strain raises sharply so it is also called yield point.

Ultimate tensile stress (UTS) / Breaking stress

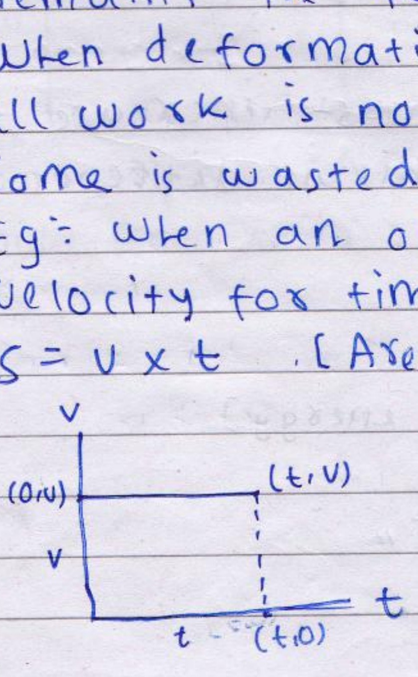
- It is the maximum stress the material can withstand, if stress increases the object can break at any point.

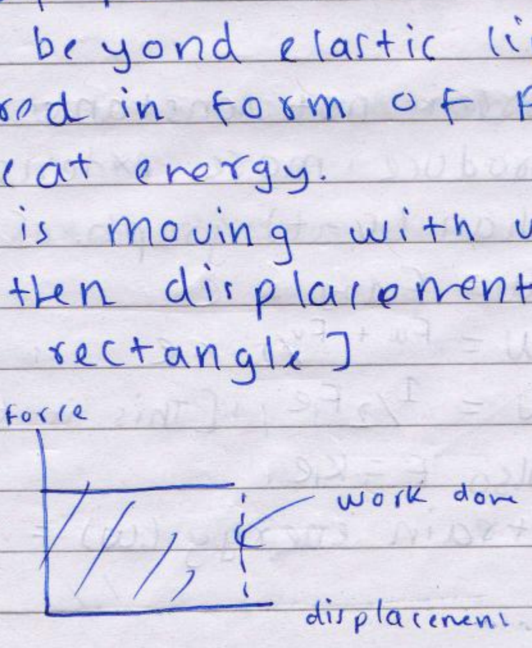


Here,
a is proportionality lim
b is elastic limit.
~~c is yield point.~~
d is UTS
e is fracture point.

Elastic Potential Energy / Strain energy

- Energy stored on an elastically deformed object
- Eg: Potential energy of a compressed spring
- Whenever force is applied, work is done and thus there must be presence of energy. This remains in form of PE.
- When deformation is beyond elastic limit, all work is not stored in form of PE. Some is wasted in heat energy.
- Eg: when an object is moving with uniform velocity for time t, then displacement is,
$s = v \times t$. [Area of rectangle]



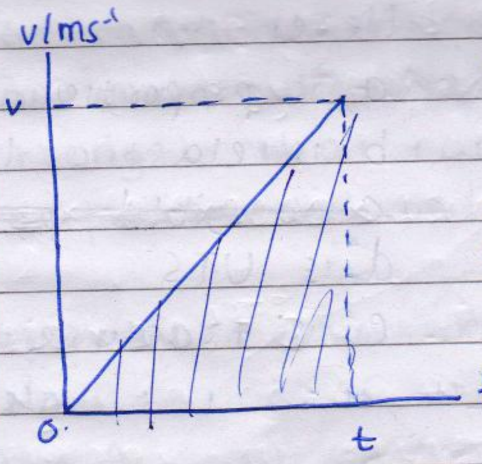


Eg:- when object starts from rest and accelerates uniformly to velocity v in ts. Then displacement is given by,

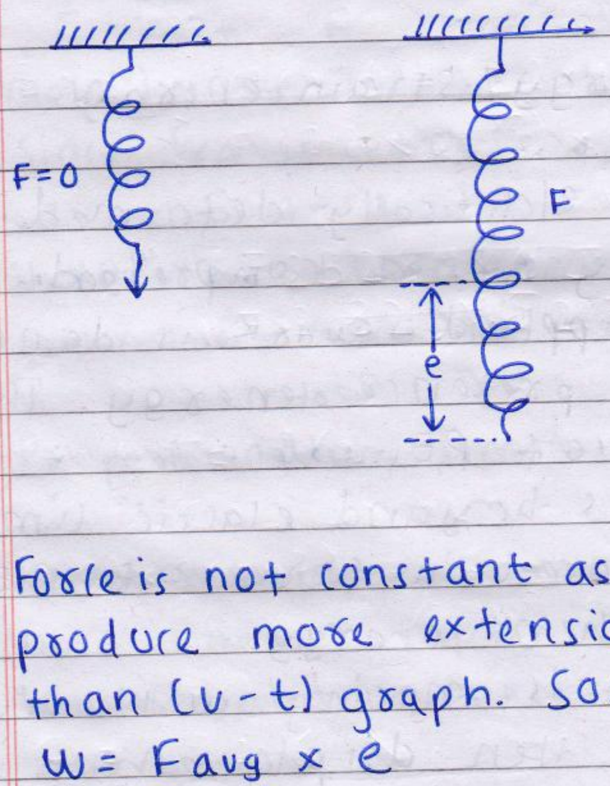
$S = V_{avg} \times t$

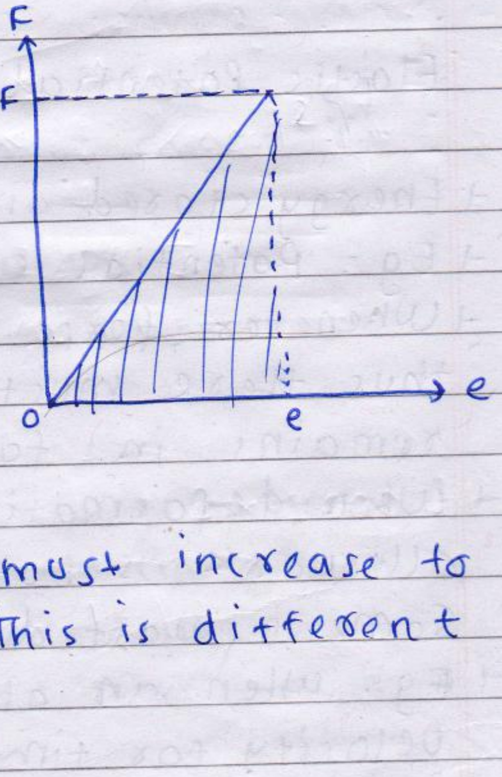
or, $S = \frac{u+v}{2} \times t$

$\therefore S = \frac{1}{2} v t$



Thus the displacement is given by the area under the line.
This is very similar to the behaviour in (F-e) graph.





Force is not constant as it must increase to produce more extension. This is different than (v-t) graph. So,

$W = F_{avg} \times e$

or, $W = \frac{F_u + F_v}{2} \times e$

$\therefore W = \frac{1}{2} F e$ [This is strain energy]

Also $F = ke$

$\therefore$ strain energy $(W) = \frac{1}{2} k e^2$ "

///

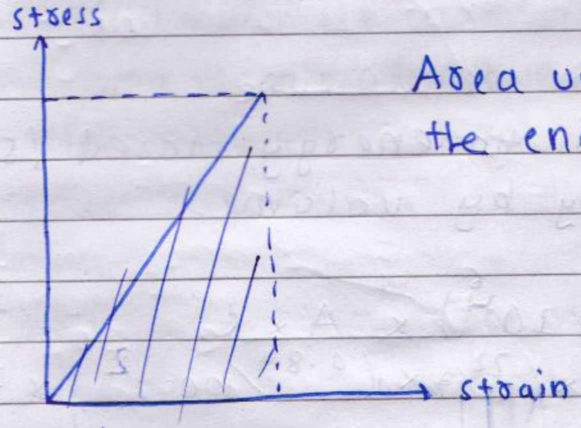
Also, $E = \frac{F \times L}{e \times A}$ $\therefore F = \frac{E \times e \times A}{l}$

$\therefore$ Strain energy $= \frac{1}{2} \times \frac{E \times e \times A}{l} \times e = \frac{1}{2} \times \frac{EA}{l} \times e^2$

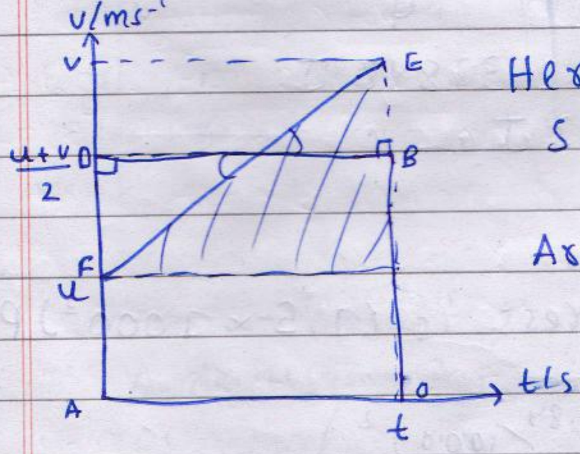
Where EA/l is the force constant k.

Strain energy per unit volume $= \frac{1}{2} \frac{F \times e}{V}$

$= \frac{1}{2} \times \frac{F}{A} \times \frac{e}{l} = \frac{1}{2} \times$ stress $\times$ strain



Area under this graph gives us the energy per unit volume.



Here area or displacement is,

$s = \frac{u+v}{2} \times t$

Area of rectangle $= \frac{u+v}{2} \times t$

Thus, Area AOBD = Area AOBEF

Calculate force which causes the wire to break and energy stored when wire has 0.6 GPa stress, assuming cross-sectional area remains constant. Given that diameter is 0.84 mm and length 3.5 m.

When stress = 0.6 GPa, strain = 4×10^{3}

Strain energy per unit volume = $\frac{1}{2}\times$ stress $\times$ strain

$= \frac{1}{2}\times(0.6\times1000\times1000\times1000)\times(4\times10^{-3})$

$= 1.2\times10^{6}$ J

Now to convert this to energy stored / strain energy, we multiply by volume.

Energy stored $= (1.2\times10^{6})\times A\times l$

$= (1.2\times10^{6})\times\left(\frac{22/7\times(0.84/1000)^{2}}{4}\right)\times3.5$

$= \cancel{2328480\text{ J}}\ 2.32848$ J

$= \cancel{2.32\text{ MJ}}\ 2.32$ J

Stress = F/A

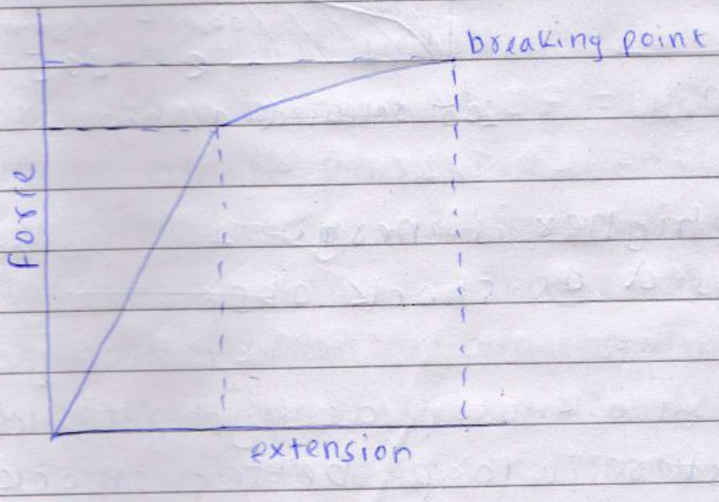
When wire breaks, stress is (1.5×1000^{3}) Pa.

$(1.5\times1000^{3}) = \frac{F}{\left(\frac{22/7\times(0.84/1000)^{2}}{4}\right)}$

$\therefore F = 831.6$ N

Ductile and Brittle objects.

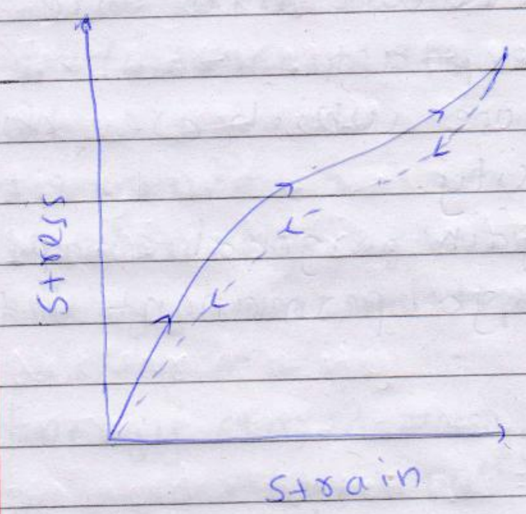
- Those objects which can be drawn into wire like structure permanently by application of deforming force.
- Plastic deformation occurs between elastic limit and breaking point.
- Ductility is the property of materials by virtue of which they can be drawn into wire like structure, eg. metals, plastic.
- Brittle objects are those which are hard but can shatter / break easily.
- Brittle objects either regain original shape or break but never undergo permanent deformatio

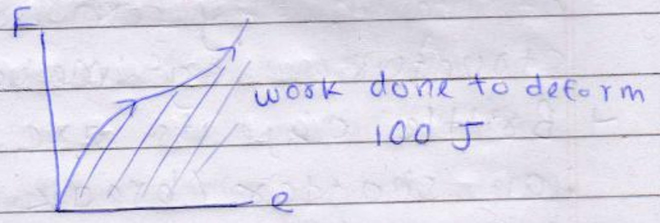


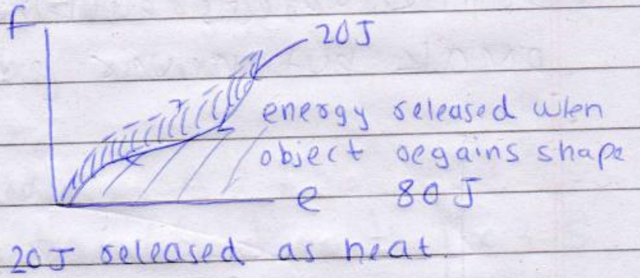
A ductile substance

Elastic Hysteresis

- Certain materials dosen't follow Hooke's law but show elastic properties. eg :- vulcanized rubber
- Elastic Hysteresis is property of certain materials by virtue of which they follow different path, when stress is increased and released in stress-strain graph.







Higher the area, higher is energy dissipated, and is better used as shock o~~bserver~~.
absorber.
The 20J region is also known as hysteresis loop.
The more the hysteresis loop, better shock absorber.

DEFORMATION OF SOLIDS EXERCISE

1a) Define, for a wire,

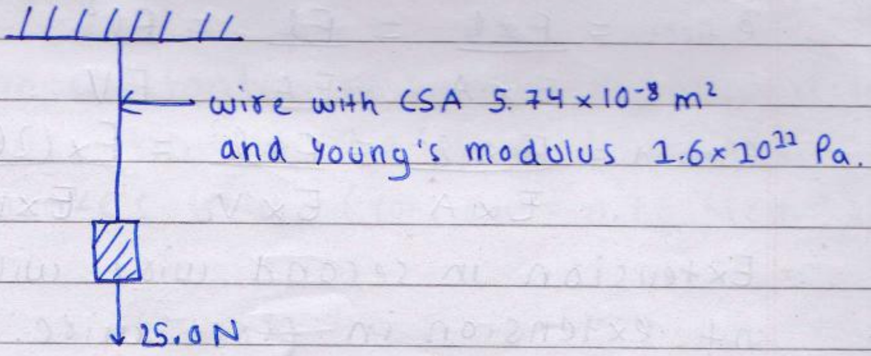
i) stress

→ Deforming force per unit cross-sectional area of that wire.

ii) strain

→ ~~Extension~~ Ratio of extension and original length of the wire, when force is applied.

b) A wire of length 1.70 m hangs vertically from a fixed point.



i) Calculate extension of wire.

$$E = \frac{F/A}{e/l}$$

or, $$1.6\times10^{11} = \frac{25/5.74\times10^{-8}}{e/1.7}$$

or, $$1.6\times10^{11} = \frac{25\times1.7}{(5.74\times10^{-8})\times e}$$

$\therefore e = 4.6\times10^{-3}$ m.

ii) The same load is hung from second wire of same material, but with twice length and same volume. State and explain how extension of second wire compares with that of first wire.

The Young modulus will remain the same as it is independent of dimensions of wire.

Volume $= (5.74 \times 10^{-8}) \times 1.70 = 9.75 \times 10^{-8}\ m^3$

$\text{length}_{second} = 2 \times 1.70 = 3.4\ m$ //

$\text{Area}_{second} = \text{Volume}/\text{Length}_{second} = 9.75 \times 10^{-8}/3.4$

$= 2.86 \times 10^{-8}\ m^2$ //

$$e_{second} = \frac{F \times l}{E \times A} = \frac{25 \times 3.4}{(1.6 \times 10^{11}) \times (2.86 \times 10^{-8})} = 0.018 m$$ //

$$e_{first} = \frac{F \times l}{E \times A} = \frac{Fl}{EA} = \frac{Fl^2}{EV}$$

$$e_{second} = \frac{F \times l^2}{E \times A} = \frac{F \times l^2}{E \times V} = \frac{F \times (2l)^2}{E \times V} = 4\frac{Fl^2}{EV}$$

∴ Extension in second wire will be four times that of extension in first wire.

2a) State what is meant by elastic potential energy.

→ Elastic potential energy is the potential energy stored in an elastically deformed object due to the ~~applica~~ work being done to deform the object.

b) A spring is extended by applying force. The F-e graph is shown.

(i) Use data to show the spring obeys hooke's law for this range of extensions.

To do this, we take two different points on the graph and check for the spring constant.

~~Spring constant~~
Taking reference pt. as (20, 5.2) and taking two other points (30, 7.8) and (40, 10.4)

Spring constant $_{(30,7.8)}$ = $\frac{7.8-5.2}{30-20} = \frac{2.6}{10} = 0.26\ \mathrm{Nm^{-1}}$
Spring constant $_{(40,10.4)}$ = $\frac{10.4-5.2}{40-20} = \frac{5.2}{20} = 0.26\ \mathrm{Nm^{-1}}$

Since, the spring constants are same in two different points, it shows that $F \propto e$ and thus the spring obey's hooke's law. ((constant = 0.26 $\mathrm{N\,cm^{-1}}$)

(ii) Calculate

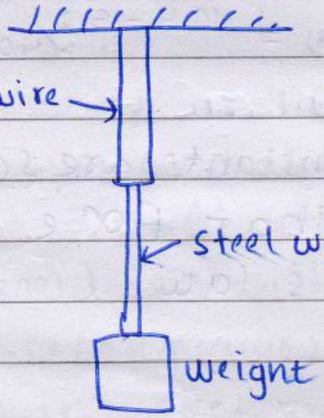
- the spring constant
Spring constant = $\frac{10.4-5.2}{40-20} = \frac{5.2}{20} = 0.26\ \mathrm{Nm^{-1}}$ //
$= 26\ \mathrm{Nm^{-1}}$ //

- the work done extending spring from 20 cm to 40 cm
$e = 40\,\mathrm{cm} - 20\,\mathrm{cm} = 20\,\mathrm{cm} = 0.2\,\mathrm{m}$ //
$F = 10.4 - 5.2 = 5.2\,\mathrm{N}$ //
work done = Area under graph with true origin.
$= \frac{1}{2} \times 5.2 \times 0.2$
$= 0.52\,\mathrm{J}$ //
~~= 0.5~~

c) A force is applied to the spring to give an extension of 50 cm. State how you would check that the spring has not exceeded its elastic limit.

→ We can check it by stopping the application of force, then measuring the length of the wire. If the measured length is equal to original length, the spring has not exceeded its elastic limit, if not it has exceeded its elastic limit.

3. Young modulus of steel is twice that of copper. A 50 cm Cu wire with diameter 2.0 mm is joined to 50 cm steel wire of diameter 1.0 mm, making a combination



What is ratio, $\frac{\text{extension steel wire}}{\text{extension copper wire}}$

Here,

$l_{steel} = l_{copper}$, $E_{steel} = 2E_{copper}$, $d_{steel} = \frac{1}{2} d_{copper}$

$e_{copper} = \cancel{E\times} \frac{F \times l}{E \times A} = \frac{Fl}{E \times \pi d^2/4}$

$e_{steel} = \frac{F \times l}{E \times A} = \frac{Fl}{2E \times \pi(\frac{1}{2}d)^2/4}$

$$\cancel{\text{Ratio}} = \frac{F\ell}{E\pi d^2/4} \times \frac{\frac{1}{4}E\pi d^2/2}{F\ell}$$

$$= \frac{4}{E\pi d^2} \times \frac{E\pi d^2}{8} = \frac{4}{8} = \frac{1}{2} \text{ //.}$$

$$\text{Ratio} = \frac{F\ell}{E\pi d^2/8} \times \frac{E\pi d^2/4}{F\ell}$$

$$= \frac{8}{\cancel{E\pi d^2}} \times \frac{\cancel{E\pi d^2}}{4} = 2 \text{ //.}$$

4. Two wires X and Y are made of different ~~wires~~ metals. Young modolus of X is twice that of Y. Diameter of X is half of Y. wires are extended with same strain and obey Hooke's law find ratio of $\frac{\text{tension wire X}}{\text{tension wire Y}}$

$\text{Stress}_{\text{wire } x} = 2E \times \text{strain}$

$\text{Stress}_{\text{wire } y} = E \times \text{strain}$

$$\frac{\text{Stress}_x}{\text{stress}_y} = \frac{2\cancel{E} \times \cancel{\text{strain}}}{\cancel{E} \times \cancel{\text{strain}}}$$

$$\frac{F_x/A_x}{F_y/A_y} = 2$$

$$\text{or, } \frac{F_x}{\pi d^2/4} \times \frac{\pi (2d)^2/4}{F_y} = 2$$

$$\text{or, } \frac{4F_x}{\cancel{\pi d^2}} \times \frac{\cancel{\pi d^2}}{F_y} = 2$$

$$\text{or, } \frac{F_x}{F_y} = \frac{2}{4}$$

$$\therefore \text{Ratio} = \frac{1}{2} \text{ //.}$$

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5. A wire stretches 8mm under 60N load. A second wire of same material, with half diameter and quarter length is stretched by same load. What is extension in this wire.

$e_{first\ wire} = \frac{F \times l}{E \times A}$ or $\frac{8}{1000} = \frac{60l}{EA}$ $\therefore \frac{l}{EA} = \frac{1}{7500}$

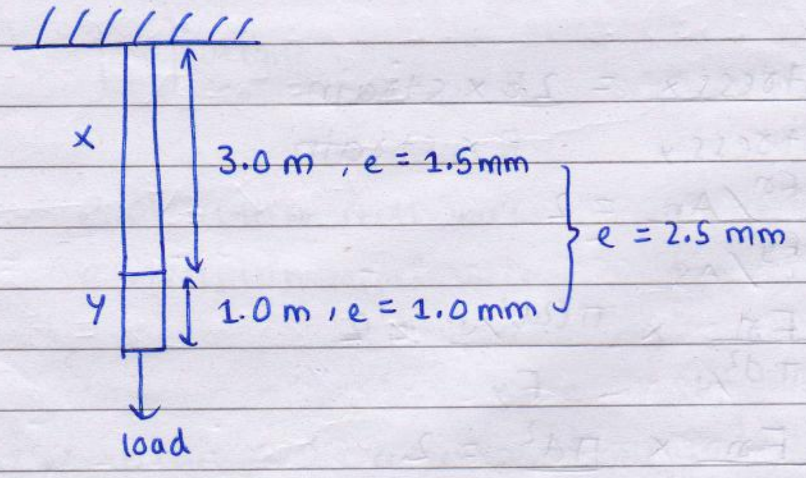
$e_{second\ wire} = \frac{F \times l}{E \times A} = \frac{60 \times l/4}{E \times \pi(\frac{1}{2}d)^2/4} = \frac{15l}{E \times \pi d^2/16}$

$= \frac{15l}{E \times \pi d^2/4 \times 1/4} = \frac{15l}{EA \times 1/4} = \frac{60l}{EA}$

$= 60 \times \frac{1}{7500} = 8 \times 10^{-3}\ m = 8mm$

$\therefore$ The extension is the same as the first wire, 8mm

6. A wire consists of 3m length of metal X & joined to a 1m length of metal Y, with uniform cross-sectional area.



A load hung causes metal X to stretch by 1.5 mm and metal Y to stretch by 1.0 mm. The same load is then hung from a second wire of same cross-section, consisting 1 m of metal X and 3 m of metal Y. What is total extension of this second wire.

$$e_{\text{first wire}} = \frac{F \times l}{E \times A} + \frac{F \times l}{E \times A}$$

or $$\frac{2.5}{1000} = \frac{3F}{E_x.A} + \frac{F}{E_y.A}$$

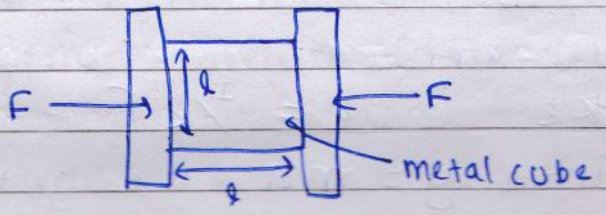
$$\frac{3F}{E_x A} = \frac{1.5}{1000} \quad \therefore \frac{F}{E_x A} = \frac{1}{2000}$$

$$\frac{F}{E_y A} = \frac{1}{1000}$$

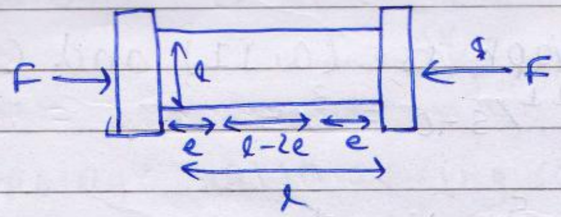
$$e_{\text{second wire}} = \frac{F}{E_x A} + \frac{3F}{E_y A} = \frac{1}{2000} + \frac{3}{1000} = \frac{7}{2000}$$

$$= 3.5 \times 10^{-3}\ m = 3.5\ mm$$

7. Show with necessary explanation.
A metal cube of side l is placed in a vice and compressed elastically by two ~~for~~ opposing forces



How will Δl, the amount of compression relate to l?



$$\Delta l = 2e$$

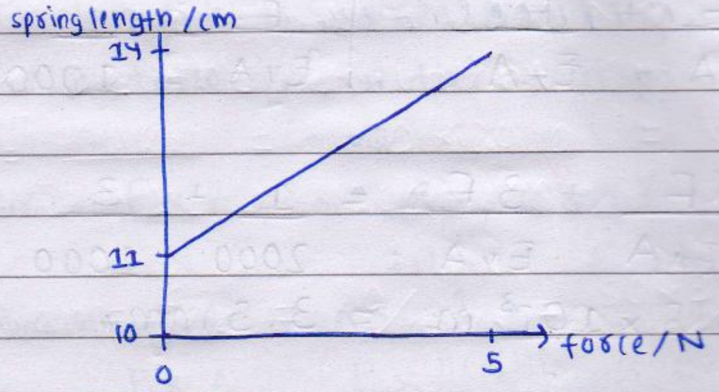
or $$\Delta l = 2 \times \left(\cancel{E \times} \frac{F \times l}{E \times A} \right)$$

or $$\Delta l = 2 \times \frac{F \times l}{E \times l^2}$$

$$\therefore \Delta l = \frac{2F}{El}$$

$$\therefore \text{Answer} = B \left(\Delta l \propto \frac{1}{l} \right)$$

8. The graph shows effect of applying ~~f~~ force of upto 5N to a spring.



What is the total increase in length produced by a 7N force, assuming the spring obeys Hooke's law.

We know, $F = ke$ $\therefore e = \frac{1}{k} F$ ---(i)

comparing (i) with $y = mx$, we get the given graph.

So, gradient of above graph gives $1/k$

Two points on graph are (0, 11) and (5, 14)

Gradient $= \frac{14-11}{5-0} = 3/5$

or, $1/k = 3/5$

$\therefore k =$ ~~5/3 Ncm~~ $5/3\ Ncm^{-1}$

Now, using $F = ke$ for 7N, to find e.

$7 = 5/3\, e$

$\therefore e = 4.2$ cm //

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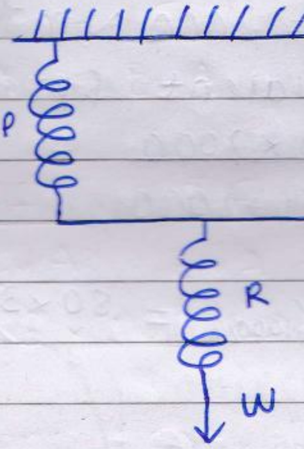
9. Three springs are arranged as shown.



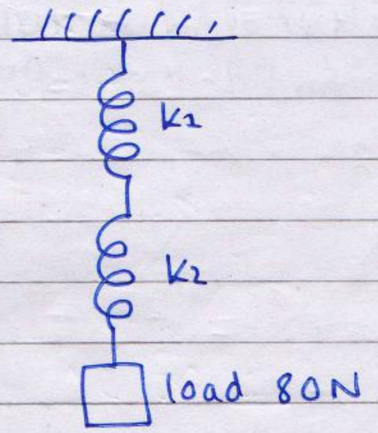
Springs P and Q are identical and have spring constant K, Spring R has spring constant 3K. What is the increase in overall length of the arrangement when force W is applied as shown.

Spring constant $_{PQ}$ = $K + K = 2K$

Spring constant $_{PQR}$ = $\frac{2K \cdot 3K}{2K+3K} = \frac{6K^2}{5K} = \frac{6}{5}K$ //

Extension = $F/K = \frac{W}{6K/5} = \frac{5W}{6K}$ //

10. two springs, with spring constants $K_1 = 4kNm^{-1}$ and $K_2 = 2kNm^{-1}$ are connected as shown.



What is the total extension of the springs?

$k_1 = 4000\ Nm^{-1}$, $k_2 = 2000\ Nm^{-1}$

Let k_{eq} be the equivalent spring constant.

$$k_{eq} = \frac{k_1 . k_2}{k_1 + k_2} = \frac{4000 \times 2000}{4000 + 2000} = \frac{4000}{3}\ Nm^{-1}$$

$$\text{Extension} = F/k = 80/_{4000/3} = 80 \times 3/4000 = 0.06\ m = 6\,cm$$