

## WAVES

- Mechanical waves are waves which require mechanical / material medium to travel / propagate.
- A wave / wave motion is a disturbance created in a medium which propagates energy from one point to another without net movement of particles of the medium.
- A wave that allows energy to be transferred from one point to another with <sup>movement</sup> ~~presence~~ of any particles is progressive / travelling wave.
- Mechanical waves can be classified as transverse and longitudinal waves.
- Transverse waves are waves in which motion of particles of medium are perpendicular to direction of propagation of energy / direction of wave motion

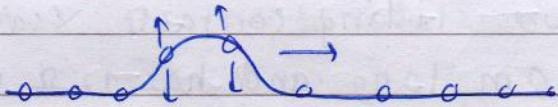


Fig. A jerked string / transverse wave

The individual particles only move up and down and not forward. It acts as an oscillating system.

- Crest is the maximum value of upward displacement within a cycle of a particle of a wave.
- Trough is the minimum value in a cycle.

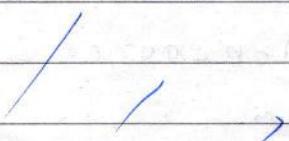
→ Longitudinal waves are waves in which motion of particles of medium are along the direction of propagation of energy.

- Displacement ( $x$ ): the distance of a point on a wave from its equilibrium position (undisturbed position).
- Amplitude ( $A$ ): max. displacement of any point on the wave from its equilibrium position.
- Wave length ( $\lambda$ ): the distance from any point on a wave to next exactly similar point. Eg: crest to nearest crest, compression to nearest compression. Distance between similar points on two nearest waves.
- Period ( $T$ ): Time taken for one complete oscillation of a point in a wave.
- Frequency ( $f$ ): Number of oscillations per unit time.  $f = \frac{1}{T}$

Transverse wave propagates via crests and troughs while longitudinal propagates via compressions and rarefactions. 5

Transverse wave can be polarized. (produced in any direction) while longitudinal & can't be polarized (fixed direction).

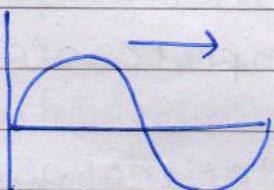
Transverse propagates in solid & liquid while longitudinal propagates in solid, liquid & gas.



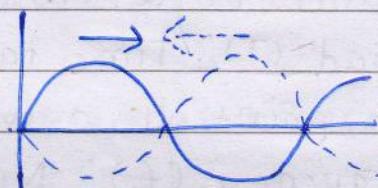
## Electromagnetic wave

- Propagate by oscillation of electric and magnetic fields.
- Radio waves, micro waves, infrared, visible light, UV rays, X-rays, Y rays.
- All of them have same velocity in vacuum.
- These show wave - particle duality.

Progressive wave is disturbance that propagates energy while stationary wave freezes energy



Progressive/ Travelling  
wave



Stationary/ Standing  
wave

In a wave, the speed of individual particles is different but wave speed is same.

Wavelength ( $\lambda$ ) is distance travelled by a wave in time equal to time period.

Wave velocity is distance travelled by a wave in one second is wave velocity.

$$v = \frac{\lambda}{T} = \lambda \times \frac{1}{T} = \lambda f$$

$$v = \lambda f \leftarrow \begin{array}{l} \text{depends on source} \\ \text{depends on medium.} \end{array}$$

## WAVES....

### Polarization

- The action of restricting the vibrations of a transverse wave wholly or partially to one direction or plane. This phenomenon is only associated with transverse waves.
- Unpolarized waves propagate more energy due to more vibrations. Thus intensity of polarized waves is lesser than that of unpolarized waves.
- The plane on which vibration occurs is called plane of vibration or plane of polarized light.
- $I = I_0 \cos^2 \theta$  → Relation between Intensity and angle of polaroid.

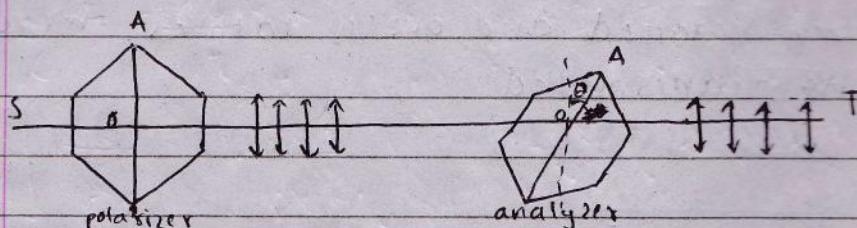
### Polaroid

- Polaroid is an artificially prepared thin, large sheet of crystalline polarizing material capable of producing plane polarized light.
- They have a long chain of molecules arranged in a particular direction.
- Consists of many microscopic crystals of iodoquinine sulfate (nerapathite) embedded in a transparent nitrocellulose polymer film.
- It only allows electric field oscillations to pass.

11.,

## Uses

- A very high intensity of light increases glare and reduces visibility. Polaroids solve this.
- Used in glass windows of an airplane to control the intensity of light.
- Used to view 3-d glasses (movies)
- Used as a filter in cameras./ pho



Analyser helps analyze change in intensity.

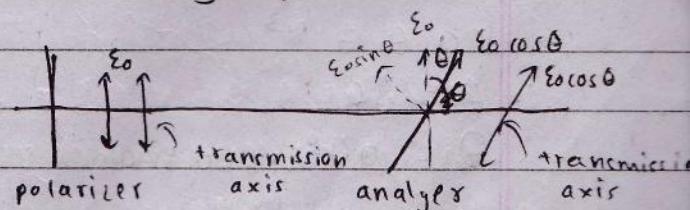
Here if  $I_0$  is source intensity, then  $I$  at T when passed through angled polaroid is  $I_0 \cos^2 \theta$ .

## Polarization by selective Absorption - Malu's Law.

Malu's law states that the intensity of plane-polarized light that passes through an analyzer varies as the square of the cosine of angle between plane of polarizer and the transmission axis of the analyzer.

$$I \propto A^2 \text{ (Amplitude)}$$

$$I \propto E^2 \text{ (Electric field intensity)}$$



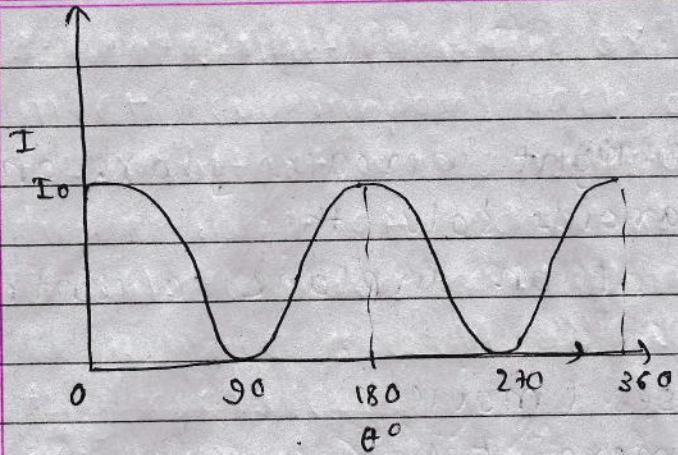
$$I \propto E^2 \text{ --- (i)}$$

$$I \propto E_0^2 \cos^2 \theta \text{ --- (ii)}$$

Dividing (ii) by (i)

$$\therefore I = I_0 \cos^2 \theta.$$

Intensity is max when transmission axes are parallel to electric field vector and min. when perpendicular.  
 $\theta = 0^\circ, 180^\circ \text{ (max)}$      $\theta = 90^\circ \text{ (min)}$



# Two polaroids are kept <sup>first</sup> crossed and one is rotated through  $90^\circ$ . Calculate % of light transmitted

$$\frac{I}{I_0} \times 100$$

$$= \cos^2 \theta \times 100 \quad \theta = 90 - 60^\circ = 30^\circ$$

$$= \frac{3}{4} \times 100 = 75\%$$

# Two polaroids have parallel settings and transmit max<sup>m</sup> light. Through what angle should either polaroid be rotated in order to drop the intensity  $\frac{3}{4}$ th of its maximum.

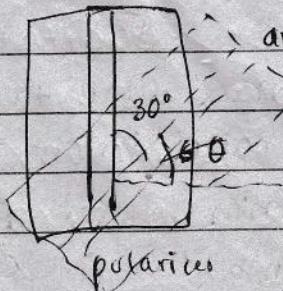
$$\frac{I}{I_0} = \frac{3}{4}$$

$$\therefore \cos^2 \theta = \frac{3}{4}$$

$$\therefore \cos \theta = \frac{\sqrt{3}}{2}$$

$$\therefore \theta = 30^\circ$$

$$\therefore \theta = 60^\circ \text{ rotation.}$$



analyzer  $I = I_0 \cos^2 \theta$

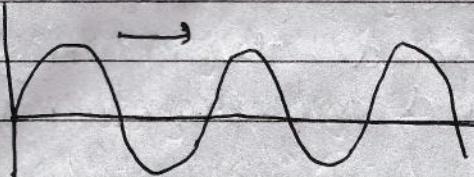
$$= I_0 \times \cos^2 30^\circ$$

$$= \frac{3}{4} I_0$$

$= 75\%$  (Amplitude is decreased)

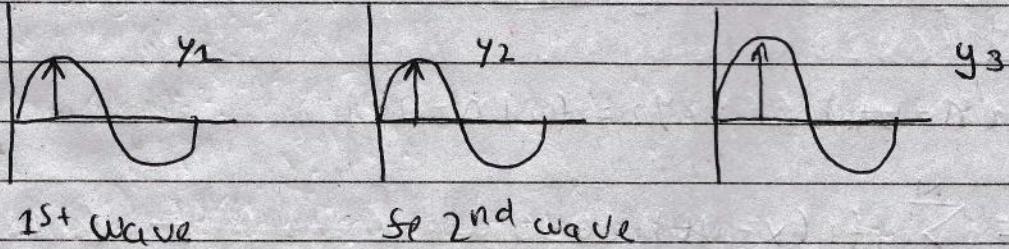
## Superposition of waves

### Progressive / travelling waves

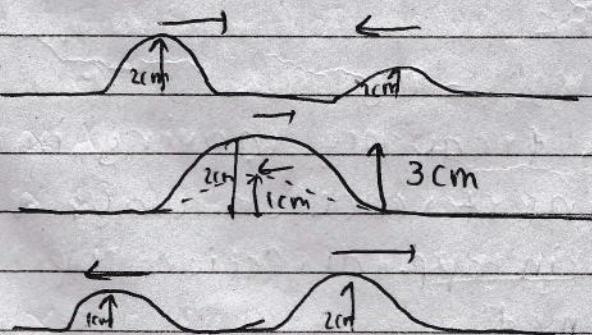


What happens when about the displacement of a particle in a medium if two or more waves propagate simultaneously in a the medium?

The resultant displacement of particle will be according to principle of superposition of waves.



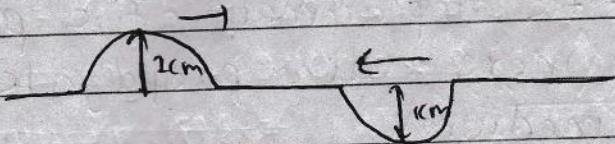
$$\text{Resultant displacement } (Y) = y_1 + y_2 + y_3$$



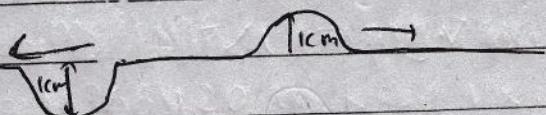
It states that when two or more waves of the same kind meet (superimpose) each other, the displacement of the resultant wave is equal to the vector sum of the displacement of individual waves.

Though the waves are superimposed, the identity of each wave is maintained and are prominent when they separate.

$$y = y_1 + y_2 + y_3 + \dots + y_n$$



$$(1\text{cm}) + (-1\text{cm}) = 0$$



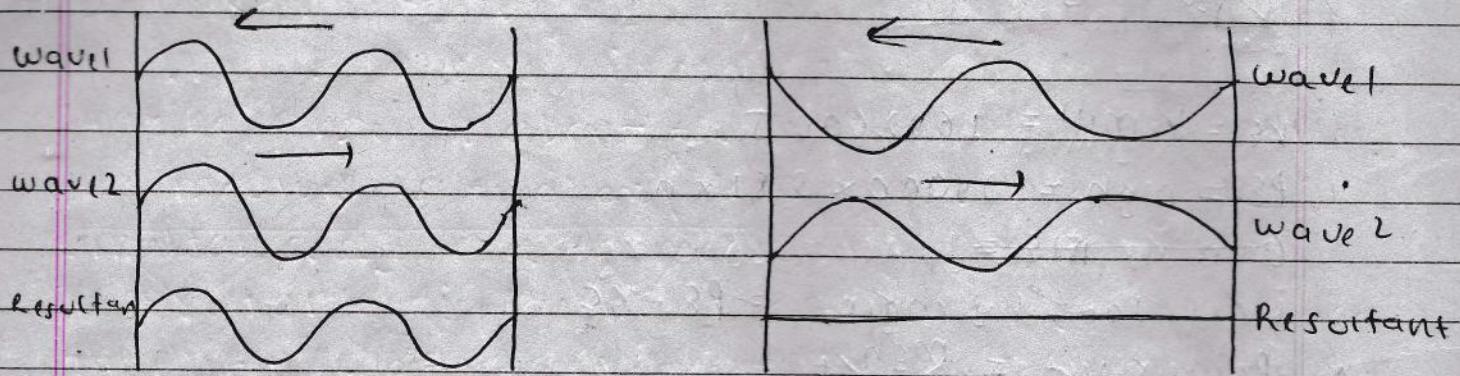
$$y_1 = f_1(n - vt) \quad y_2 = f_2(n - vt)$$

$$\therefore y = Y = \sum_{n=1}^N f_1(x - vt)$$

### Application :

- When two waves of same frequency moving in the same direction superimpose, interference of wave is formed.
- When two waves moving in opp. direction superimpose, stationary wave is formed.
- When two waves of slightly diff. frequency are travelling in same direction superimpose, beats are formed.

- Resultant of superposition waves is stationary waves.
- Wave doesn't travel but it is a wave.
- Requirement is two progressive waves and travelling in opp. direction, also called standing waves.
- formed when two waves having equal frequency, amplitude and velocity but travelling in opp. direction



$$\text{A} + \text{A} = \text{A}$$

Constructive interference

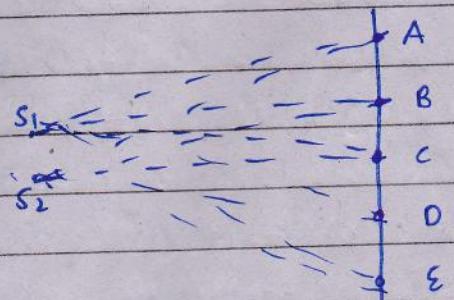
$$\text{A} + \text{U} = -$$

Destructive interference

## WAVES...

### Superposition of waves

### Interference pattern

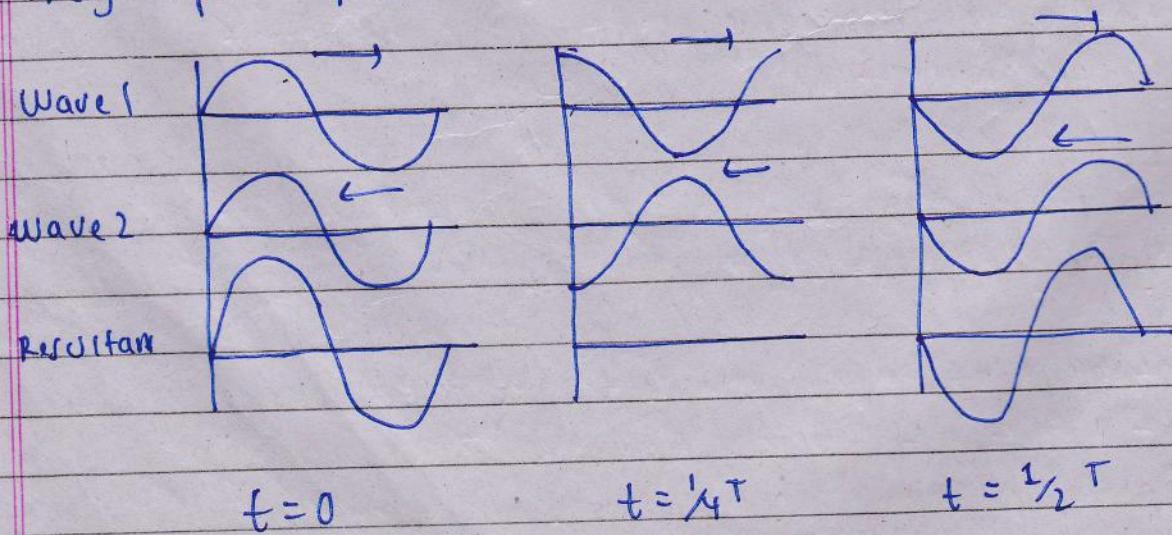


Here  $S_1$  and  $S_2$  are two sources with waves of same frequency, direction and amplitude. The propagation/superposition of waves in diff. region forms region with high max.

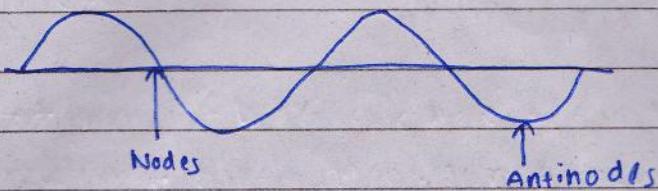
intensity and min. intensity.

The resultant wave has no direction of transfer of energy. i.e. it localizes energy.

Stationary wave (standing wave) is formed when two waves having equal frequency, amplitudes and velocity are travelling in opposite directions to each other and they superimpose.



Standing waves are the result of two travelling waves.



In a standing wave, particles in nodes don't oscillate at all while antinodes oscillate the most. At nodes difference between amplitudes of two travelling waves is always 0.

Velocity of a standing wave is the velocity of the travelling waves that make it up.

### Characteristics of stationary waves

- Nodes and antinodes are alternately formed in the wave.
- Node is the pos. of particle whose amplitude of oscillation is minimum (zero) means particles are at rest.
- Antinode is the pos. of particle whose amplitude of oscillation is maximum.
- All particles (except nodes) vibrate with SHM whose period is equal to period of wave.
- Amplitude of vibration gradually increases from 0 at node to max. at antinode.
- Energy of wave does not transfer from one point to another; it remains localized in medium.
- Distance between two adjacent nodes is  $\frac{1}{2}$  and two adjacent antinodes is also  $\frac{1}{2}$ .
- Velocity and acceleration of any two particles separated by  $d$  is same at any instant of time.

## Situations where stationary waves are formed

- Guitar string
- Resonance tube
- Both ends open tube
- Stretched rubber string.

### Progressive wave

- Energy is transferred
- All particles oscillate with same amplitude.
- At every point, there is change in pressure.
- Regular phase diff. exists between successive particles.
- The value of max. velocity for all particles of medium is same.

### Stationary wave

- Energy is localized
- Particles have different amplitudes
- Pressure variation is max. at nodes at 0 at antinodes.
- All particles between two successive nodes have the same phase.
- The value for max. velocity for particles is diff. 0 at nodes. max. at antinodes

### Progressive

Wavelength  
Frequency  
Speed

$\lambda$

$f$

$v$

### Stationary

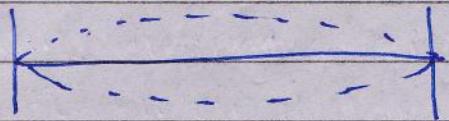
$\lambda$

$f$

$0$

## # Stationary waves in stretched strings fixed at two ends.

Transverse wave can be produced in a stretched string by plucking it. If both ends are fixed, then stationary wave is produced. Nodes are always formed on the fixed points. L is the length of string.



If the min. frequency can be produced at the stationary wave shown.

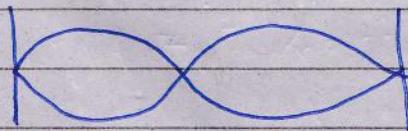
It is fundamental frequency,  $F_1$ .

Relation between wavelength and length of wire.

$$L = \lambda_1 / 2, v = \lambda F_1$$

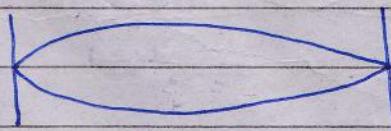
$$\text{So, } F_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

Similarly if frequency of vibration is gradually increased, two loops are formed.

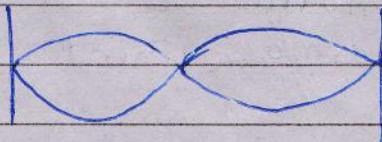


$$\lambda_2 = L$$

$$\therefore F_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2 \left( \frac{v}{2L} \right) = 2F_1,$$



First harmonic



Second harmonic

$$f_0 = \frac{1}{2L} \sqrt{\frac{T}{m}}, \text{ where } T = \text{tension in string and } m = \text{mass per unit length.}$$

When an external vibrator is used to resonate with the string, the min. frequency required to bring it to max. amplitude with just one anti-node is fundamental frequency or first harmonic ( $f_1$ ). This mode of vibration is also called first mode of vibration.

$$f_1 = \frac{V}{\lambda_1} = \frac{V}{2L}$$

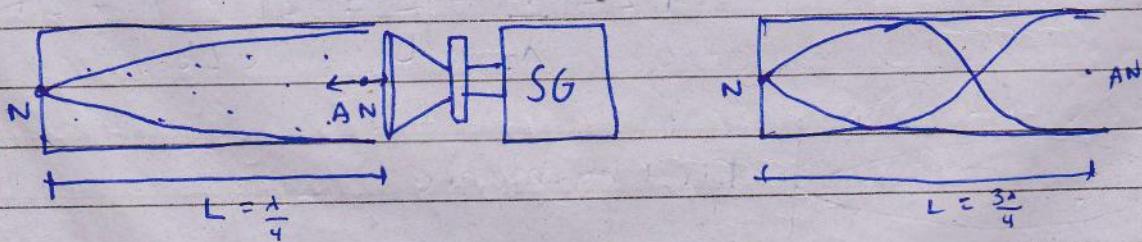
$$f_2 = \frac{V}{\lambda_2} = \frac{V}{L} = 2 \left( \frac{V}{2L} \right) = 2f_1$$

$$f_3 = \frac{V}{\lambda_3} = \frac{3V}{2L} = 3f_1$$

$$\therefore f_1 : f_2 : f_3 : f_4 : \dots = 1 : 2 : 3 : 4 : \dots$$

### Formation of stationary wave in a pipe

(i) One-end closed



$$L = \lambda_1/4, f_1 = V/\lambda_1 = V/4L \rightarrow \text{first harmonic}$$

$$f_2 = V/\lambda_2 = 3V/4L = 3f_1 \rightarrow \cancel{\text{second harmonic}} \text{ Third harmonic}$$

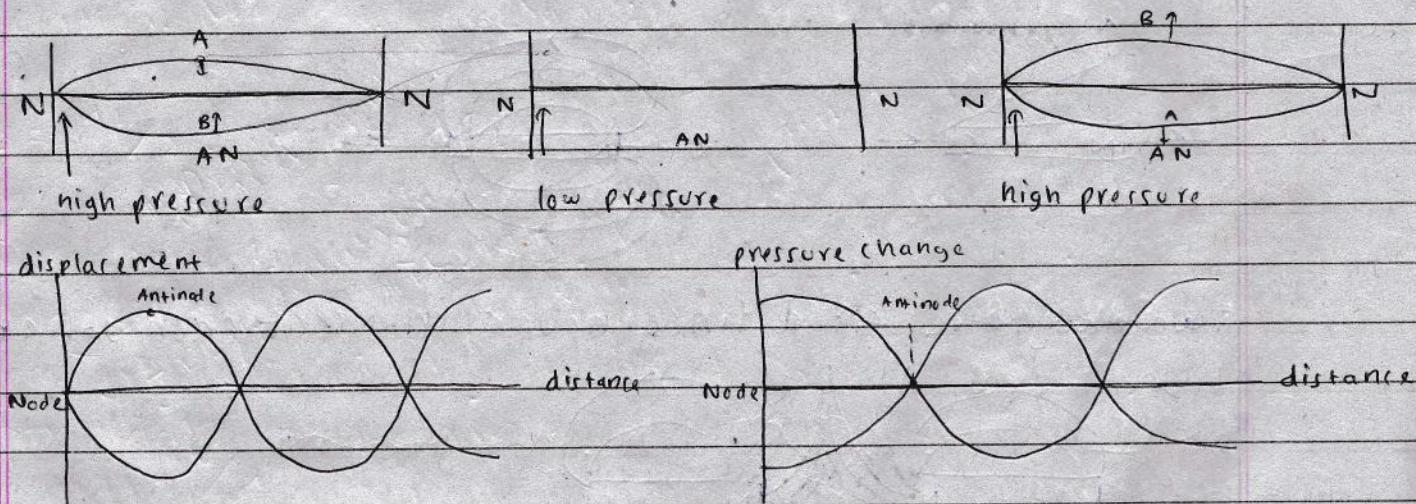
$$f_3 = 5f_1 \rightarrow \text{fifth harmonic}$$

$$\therefore f_1 : f_2 : f_3 : \dots = 1 : 3 : 5 : \dots$$

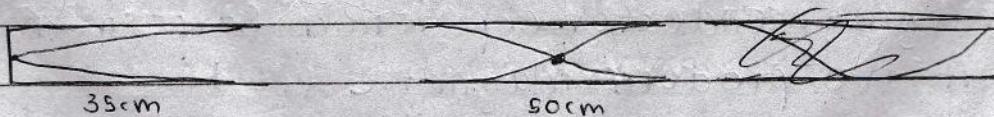
A harmonic only occurs when frequency of source and vibration matches as well as stationary wave is formed.

### Pressure variation in pipes

At the sites of antinode all molecules are moving in same direction so the spacing among the molecules remains constant that causes no change in pressure but at nodes particles on both sides sometimes come closer (compact) and sometimes go apart (rarefied) which causes large change in pressure.



# In stationary travelling waves have to wavelength 20 cm, in which tubes can st. waves form.



$$\text{Min wavelength} = \lambda/4$$

$$3\frac{1}{4}$$

$$5\frac{1}{4}$$

$$7\frac{1}{4} = 35$$

Possible.

$$\text{Min wavelength} = \lambda/2$$

$$\lambda$$

$$3\frac{1}{2}$$

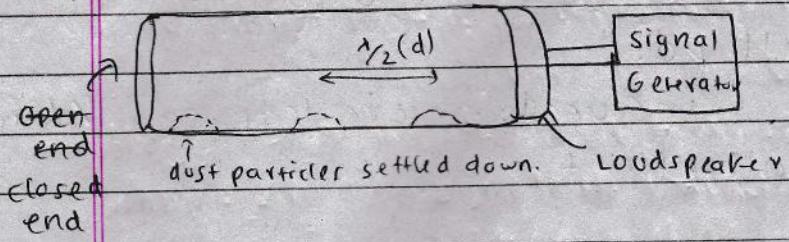
$$5\frac{1}{2} = 50$$

possible

In both,

## WAVES...

(i) Using Kundt's tube to determine wavelength and speed of sound.



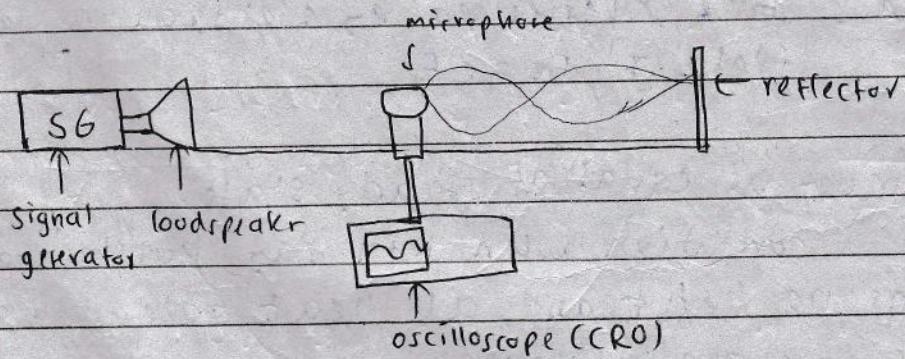
When a stationary wave is formed in the glass tube, all dust particles inside Kundt's tube vibrate / oscillate, so they come and settle down at places where particles don't oscillate i.e. at nodal points.

Frequency is adjusted manually to  $f$ .

$$\text{Wavelength of sound} = 2d \dots$$

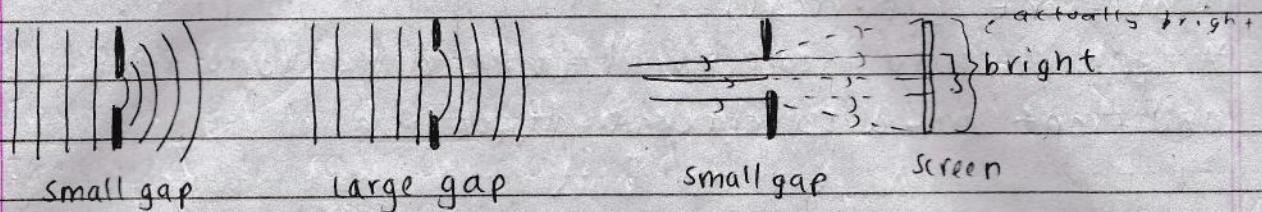
$$\text{Speed of sound} = 1f = 2df \dots$$

(ii) Using reflection



Oscilloscope reading can be used to determine wavelength and speed of sound wave.

## Diffraktion of waves



When light is passed through the small gap, we only expect the small region same as the gap to be bright. But actually a much larger region is found to be bright.

Thus, the spreading of wave when it is passed through a small gap is diffraction of waves.

The smaller the gap, the more circular the wave becomes.

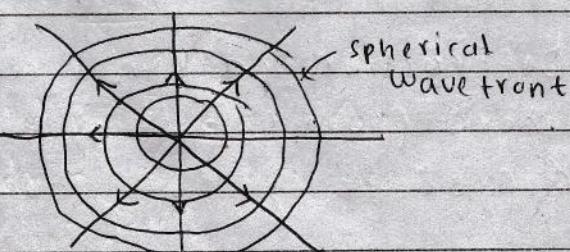
If diffraction doesn't occur, we wouldn't get cell signal inside a closed room.

The phenomenon of spreading of wave when it passes through a small gap or sharp edge is called diffraction of wave.

It is associated with all kinds of waves.

## Huygens' principle

Wavefronts - It is the set (locus) of all points around the source which oscillate in the same phase.



spherical  
wavefront

Points  $sphere$  source  
→ Spherical (circular wave front)  
→ Plane wavefront - linear source

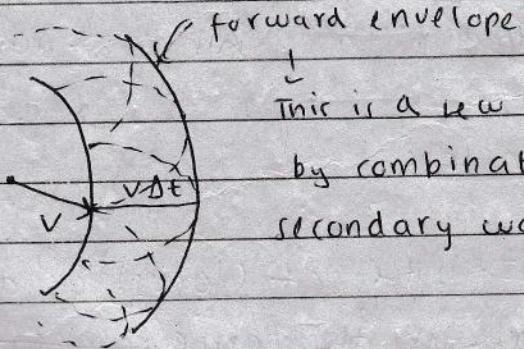
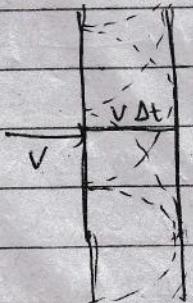
A plane wavefront is also a very small part of a big wavefront (spherical)

A spherical wavefront has infinite directions while plane wavefront has a particular direction.

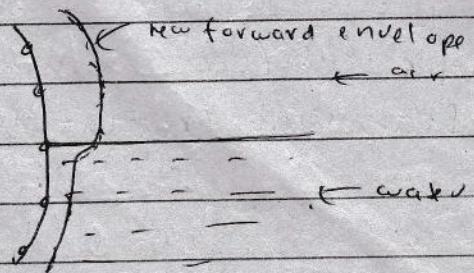
Huygen's principle / construction gives the idea how wave propagates away from source. Using this, we can find the new position and shape of wave fronts after certain time of known wave front.

Assumptions of Huygen's principle.

- Each point in primary (known) wavefront behaves as a source for secondary wavelengths and sends the disturbances in a similar manner as the original source does.
- The new pos. and shape of the wave front at any instant is given by the forward envelope of secondary wavelets at that instant.



forward envelope  
This is a new wave front formed by combination of individual secondary wavelets.



Diffraction pattern becomes more prominent if size of gap approaches size of wavelength.

Maximum diffraction occurs if size of hole equals size of wavelength. If size of hole is smaller, wave will not propagate.

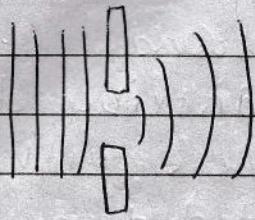


Fig 1

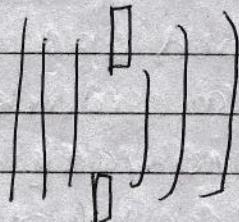
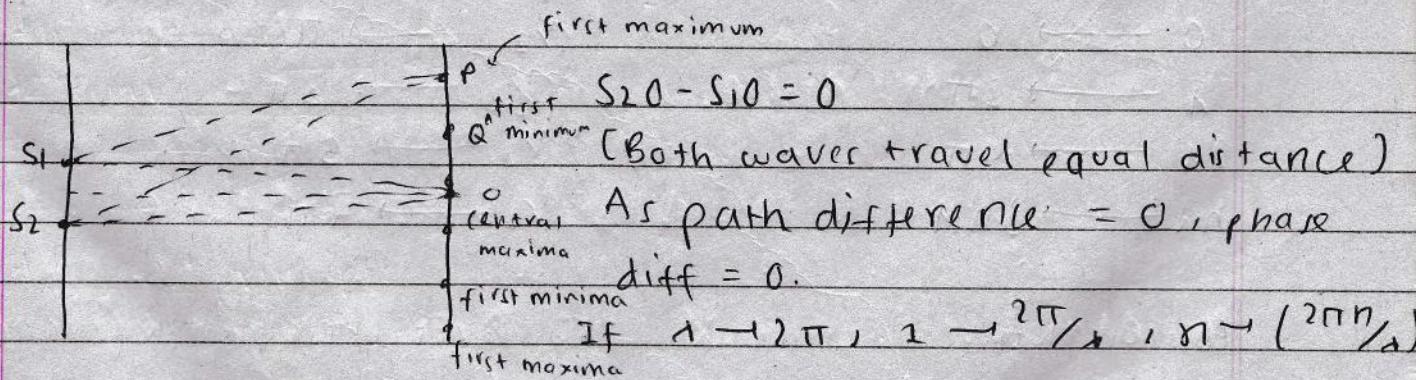


Fig 2

To make Fig 1 resemble Fig 2, we can increase frequency in Fig 1 as  $v = \lambda f$ ,  $\lambda = \frac{v}{f}$ , so  $\lambda$  will decrease. When  $\lambda$  decreases, the <sup>difference</sup> gap between 1 and hole is more, and thus diffraction is less.

## INTERFERENCE



So in O, when  $S_1$  sends crest,  $S_1$  also sends crest and now amplitude doubles and energy quadruples, so O has max intensity/brightness.

$$S_2P - S_1P \neq 0 [S_2P > S_1P]$$

This path difference can be  $^{\wedge}100$ , or  $^{\wedge}150$ ; or even  $1$ .

Now if path difference is  $\lambda$ , at Q, path difference is  $\frac{\lambda}{2}$ .  
 Now in  $\theta$  at Q, when  $S_1$  sends crest,  $S_2$  will send trough and they cancel out, so there is a min. intensity and that area is dark.

In case of sound it is loud and soft sound.

Now a pattern of light and dark is distributed.

This is called interference pattern where we have a uniform distribution of constructive and destructive interference.

The phenomenon in which two waves of same kind are superimposed and resultant wave is formed is called interference.

This occurs in all type of waves, unlike polarization.

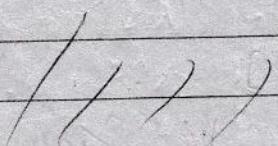
$$\begin{array}{ll} n & \phi \\ 0 & 0 \\ 1 & 2\pi \\ 2 & 4\pi \\ n & 2n\pi \end{array}$$

$$\begin{array}{ll} n & \phi \\ \frac{1}{2} & \pi \\ \frac{3}{2} & 3\pi \\ \frac{5}{2} & 5\pi \\ \frac{7}{2} & 7\pi \end{array}$$

For constructive interference

For destructive interference

At places with constructive interference, antinodes are formed and at places with destructive interference, nodes are formed.





If the screen is farther from source, gap between max. and min. is more & interference pattern is more prominent.

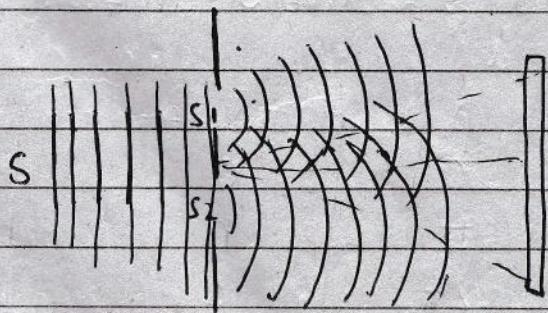
If sources are not coherent, at the same point, phase difference can be 0 and at same point later it can be  $\pi$ . So, we get light and dark at same point.

Coherent sources are those which send light of constant phase difference. Thus coherent sources are required for interference.

We can never get two independent sources which are coherent as we cannot calibrate it to quantum level. But, we can use the same source to produce two sources (Huygen's principle.)

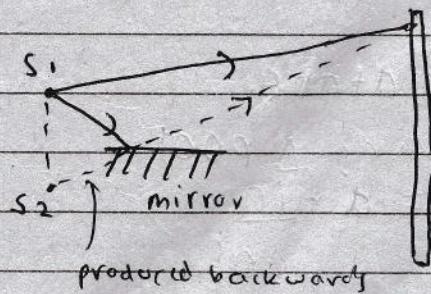
We can produce coherent sources from single source (i.e.)

(i) Using Double slit



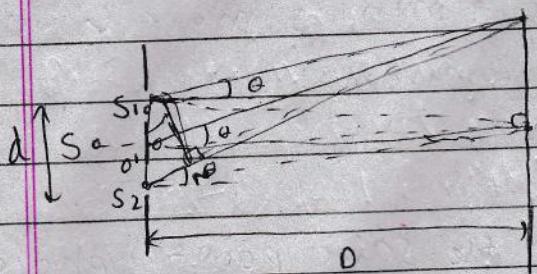
Young's double slit experiment.

ii) Using Lloyd's mirror



Mirror used to reflect source.

## Young's Double slit experiment



At point O (centre)

$$S_1O - S_2O = 0$$

Path difference = 0

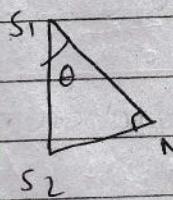
So constructive interference takes

place and we get max intensity at O. That is; we get local maxima at point O (brightest)

At point P,  $S_2P - S_1P > 0$

As wavelength in light is very small order of magnitude, we can assume  $S_1P$  and  $S_2P$  are parallel, because  $S_1$  and  $S_2$  are very close to each other.

In  $\Delta S_1S_2N$ ,  $\angle S_2S_1N = \theta$



$$\sin\theta = \frac{S_2N}{S_1S_2}$$

$$S_1S_2$$

$$\therefore n = d \sin\theta$$

$$\sin\theta = \frac{n}{d}, \text{ where } n \text{ is path diff.}$$

between waves

In experiment it is found that  $\theta \approx \sin\theta \approx \tan\theta$ .

$$n = d \tan\theta$$

Taking  $\Delta POO'$

$$n = d \times \frac{PO}{OO'}$$

$$OO'$$

$$\therefore n = d \times \frac{y}{D}$$

$$\therefore n = \frac{dy}{D}$$

$$y_n = \frac{n \lambda D}{d} \leftarrow n^{\text{th}} \text{ maxima}$$

X	$\phi$	Interference
0	0	C
$\lambda/2$	$\pi$	D
$\lambda$	$2\pi$	C
$3\lambda/2$	$3\pi$	D
$2\lambda$	$4\pi$	C
$5\lambda/2$	$5\pi$	D

$$y_2 - y_1 = y_3 - y_2 = y_4 - y_3 \dots = y_n - y_{n-1}$$

$\beta = \frac{\lambda D}{d}$  = separation between any two consecutive maxima.

Minima at P

$$y_1 = \frac{\lambda D}{2d}, y_2 = \frac{3\lambda D}{2d}, y_3 = \frac{5\lambda D}{2d}$$

$$\therefore y_n = \frac{(2n-1)\lambda D}{2d} \rightarrow n^{\text{th}} \text{ minima}$$

$\beta' = \frac{\lambda D}{d}$  = separation between two consecutive minima.

$\therefore \beta = \beta' = \frac{\lambda D}{d}$  = separation between two consecutive maxima/minima  
This is fringe separation / fringe width

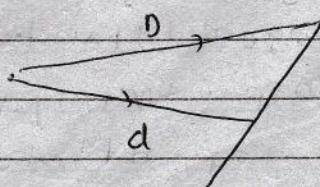
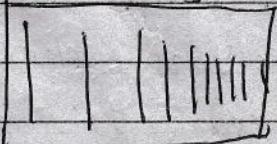
For interference pattern to be clear, the separation  $y$  must be larger as it makes pattern more distinct.

$$\beta \propto \lambda, \beta \propto D, \beta \propto \frac{1}{d}$$

So, as red has more wavelength than violet, red light gives more distinct interference pattern.

If white is used instead of monochromatic, 7 colours of white diffract and we get multicolored band like a rainbow.

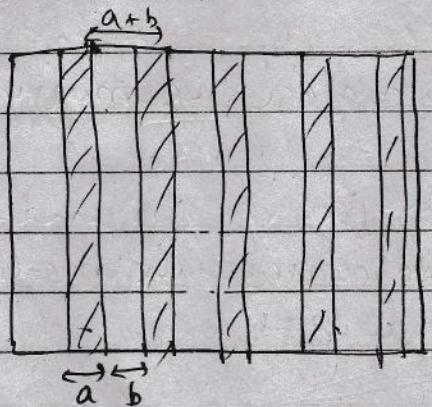
If screen is not parallel to slits,  $\beta$  increases and decreases, also intensity difference can be observed.



$d < D$  so  $d$  has more intensity

## # Diffraction at multi slits (Diffraction gratings)

Diffraction gratings (multislits) consists of large no. of equally spaced parallel slits. It is made by drawing lines of equal widths and gaps on surface of transparent plastic or glass. The lines are opaque and gaps are transparent. The widths of lines and gaps are very small so light can easily be diffracted. Generally 300 lines /mm or 1000 lines /mm.



If it has 300 lines / mm

$$300(a+b) = 1 \text{ mm} = 10^{-3} \text{ m}$$

$$\therefore a+b = 3.33 \times 10^{-6} \text{ m.}$$

Now separation bet<sup>n</sup> two gaps is  $a+b$

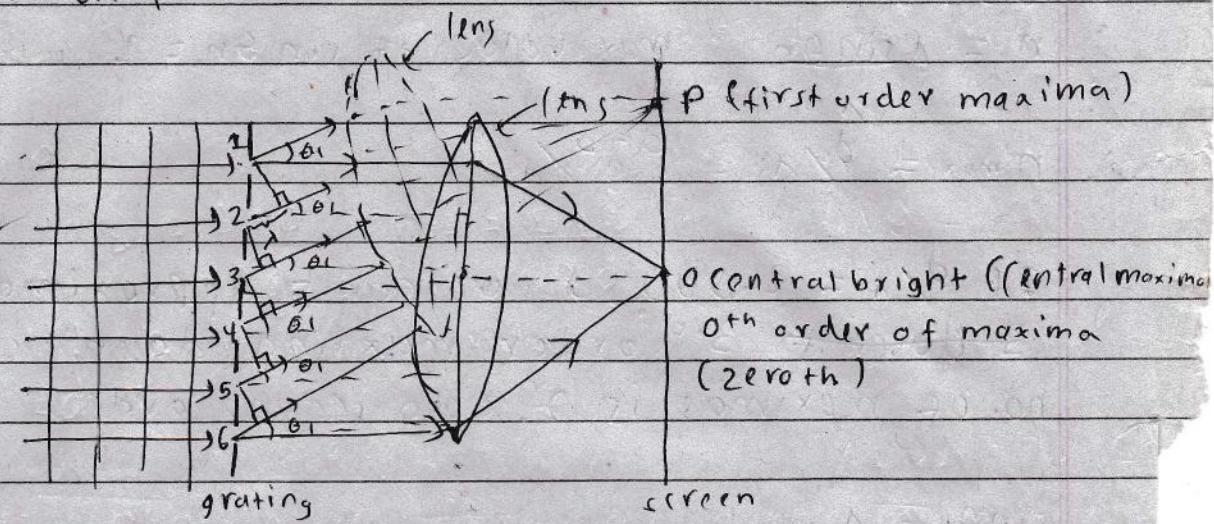
$$\therefore d = a+b = 3.33 \times 10^{-6} \text{ m.}$$

A reflection diffraction grating consists of lines made on the reflection surface so light is both reflected and diffracted by the grating. The example can be DVD surface. The color bands are observed on its surface due to refraction diffraction.

## Transmission diffraction grating

A transmission grating consists of a large no. of fine, closely spaced, equidistant parallel lines of equal width ruled on a glass (transparent space). The width and gap bet<sup>n</sup> two lines are very less comparable.

to wavelength  $\lambda_0$  of light rays, so they are easily diffraction from it.

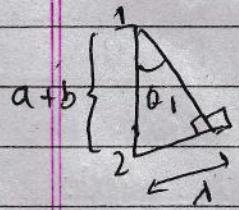


At O, we get constructive interference.

As light passes from small gaps, it spreads out (i.e. diffracts). So, we have light rays angled at  $\theta_1$ . As well, we can then adjust lens such that rays of light inclined at  $\theta_1$  are perp. to lens.

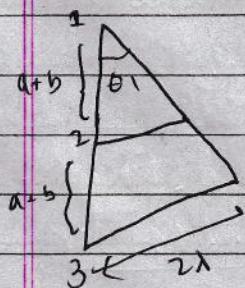
At P all waves meet due to constructive interference.

For the waves inclined at  $\theta_1$ , if path difference between them is  $\lambda$ . (constructive interference)



$$\sin \theta_1 = \frac{1}{a+b}$$

$\therefore (a+b) \sin \theta_1 = 1 \text{ m. (constructive)}$



$$\sin \theta_1 = \frac{2\lambda}{(a+b)}$$

$\therefore (a+b) \sin \theta_1 = 2\lambda \text{ (constructive)}$

For second order maxima,  $(a+b) \sin \theta_1 = 2\lambda$   
For third

For  $n^{th}$  order maxima,  $(a+b) \sin \theta_n = n\lambda$ ,

$$(a+b) \sin \theta_n = n\lambda$$

a.  $d \sin \theta_n = n\lambda$

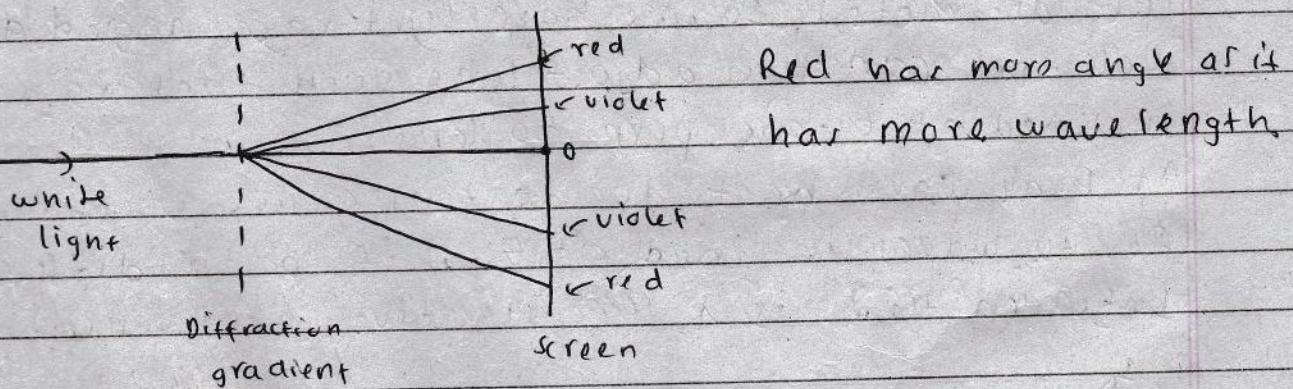
a.  $n = d \sin \theta_n$  max value of  $\sin \theta_n = 1$ , when  $\theta_n = 90^\circ$

i.  $n_{\max} = \frac{d}{\lambda} = \frac{a+b}{\lambda}$

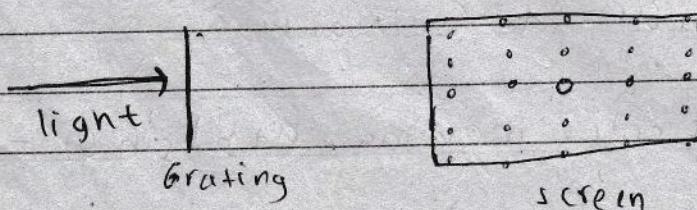
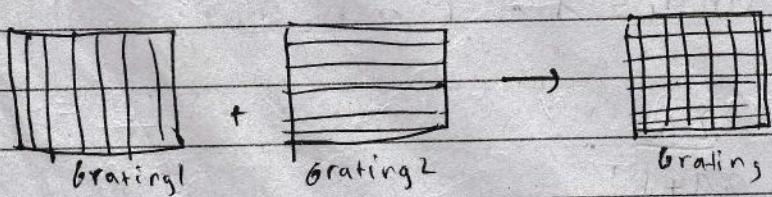
If  $d/\lambda = 2.99999$ , we can't approximate to 3 as 3 doesn't 3rd order maxima, doesn't come. So max no. of maxima is 2, upto second order.

$$\sin \theta_1 = \frac{1}{a+b}$$

As wavelength increases, value of  $\theta_1$  also increases.

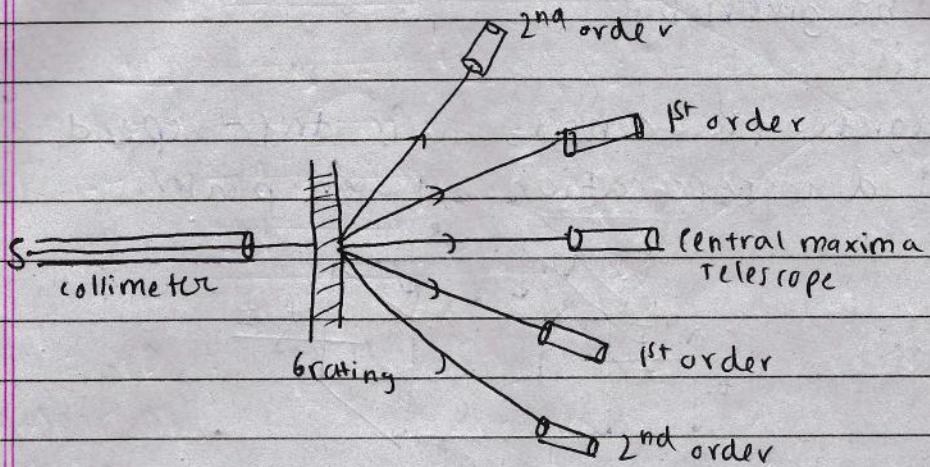


# When two diffraction gratings are crossed,



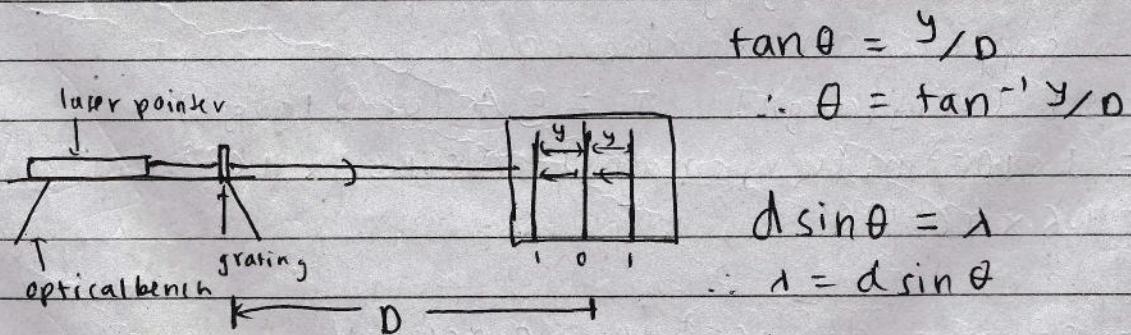
# Diffraction grating can be used in determination of wavelength of visible light.

### 1. Using spectrometer



The telescopes are calibrated such that we can read the angular position of the telescope and this angle can be used to determine  $\lambda$ .

### 2. without using spectrometer.



$$\tan \theta = y/D$$

$$\therefore \theta = \tan^{-1} y/D$$

$$d \sin \theta = \lambda$$

$$\therefore \lambda = d \sin \theta$$

We can use the given measurements to determine value of the wavelength  $\lambda$ .