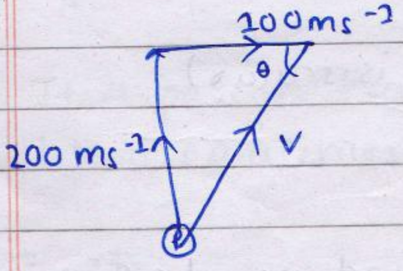
Vectors and Scalars

- Scalar is a physical quantity with only magnitude but no direction.
- Vector is a physical quantity with both magnitude and direction.
- Magnitude is a (parameter) quantity defining the amount of a quantity

- Scalar tells us how much of something is present
- Eg: distance, mass, speed, volume
- If something has mass 10 kg, 10 is the magnitude and kg is the unit.

- When we say a car is moving with a velocity of $10 ms^{-1}$ to the north direction, 10 is the magnitude ms^{-1} is the unit, and north is the direction.
- 10 N force normal to the wall means, magnitude is 10, unit is N and direction is perpendicular to the wall

A plane moves north with $200 ms^{-1}$ and an air blows east $100 ms^{-1}$. Find the ~~ve~~ final velocity of the plane.

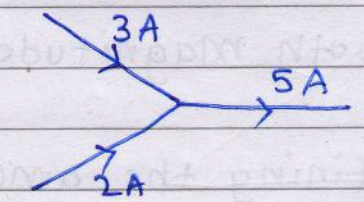


$$V = \sqrt{200^2 + 100^2}$$
$$= 223.6 \ ms^{-1}$$

$$\cos\theta = \cancel{223.6/100} \ 200/223.6$$
$$\therefore \theta = 63.43°$$

∴ The plane travels with a velocity of $223.6 ms^{-1}$, 63.43° due northeast.

- Electric current has both magnitude and direction. but still is considered as scalar quantity as it dosen't follow vector law of addition.



This only follows algebric rule of addition.

A $\xrightarrow{10N}$ □ $\longrightarrow$ 15 N net force
B $\xrightarrow{5N}$

A $\xrightarrow{10N}$ □ $\xleftarrow{5N}$ B $\longrightarrow$ 5N net force

This is vector addition.

A vector is represented by a straight line with an arrow head, the ~~a~~ arrow head represents direction while the length of the line represents magnitude

- Unit vector has magnitude 1.
- Product of scalar and vector is always vector
  $100.\vec{a} = \vec{b}$
  velocity $= \dfrac{d}{t}$, where d is vector, but $^{1}/t$ is scalar
- Product of two vectors is salar, or vector (depends)
  Energy $= \frac{1}{2} mv^2$
  $= \frac{1}{2} m (\vec{v} \cdot \vec{v})$ (which is scalar)

$\therefore$ Energy is scalar.

- Null vector has 0 magnitude and no definite direction. So, velocity of $0 ms^{-1}$ is null vector

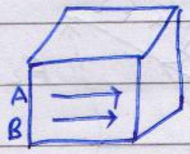
$\vec{a}.\vec{b} = |\vec{a}|.|\vec{b}|.\cos\theta$ [Dot-product becomes scalar. so it's also called scalar product

$\vec{a} \times \vec{b}$ [Result of cross product is vector, so its also called vector product.

Sum or difference of two or more vectors is also a vector.

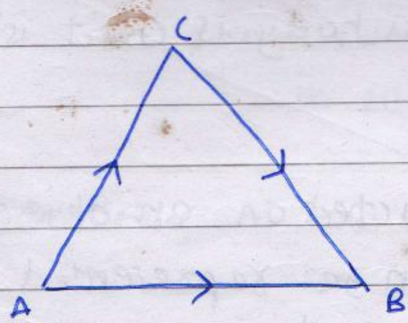
Coplanar Vectors

A pair of vectors which lie on the same plane are called coplanar vectors.



Here vectors A and B are coplanar.

- Vectors and scalars cannot be added
- Vectors and scalars of same nature can be only added. Eg mass can be added with mass only. Force can be added with force only.



$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$

This is triangle law of vector addition

If a person travels from point A to C then C to B, his displacement can be represented by AB.

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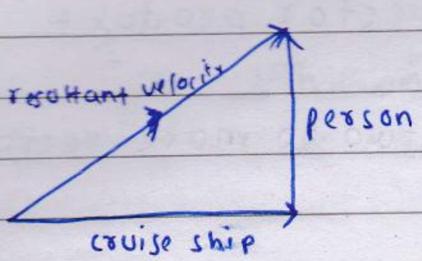
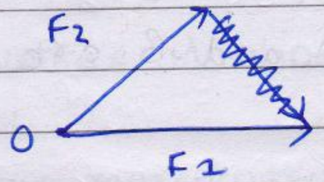
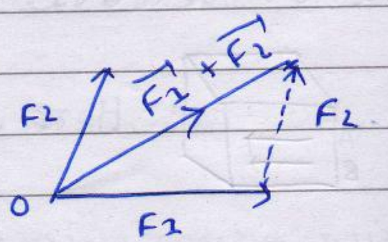
A cruise ship moves towards east and a person on the deck is moving towards north, what is the resultant velocity of the person.



Two forces are being acted on point O, draw the resultant vector.



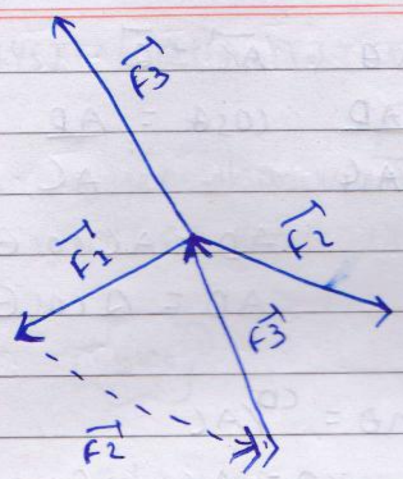


- Vectors can be transferred on parallel lines
- The real $\vec{F_2}$ and drawn $\vec{F_2}$ are equal.

If resultant forces acted on a body is 0 it is said to be at equilibrium.

If three coplalanar vectors acted on an object are at equillibrium, the forces can be represented by three sides of a Δ taken in order.

If three forces can be represented ~~o~~ by three sides of a triangle then, they are at equilibrium



To prove $\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$

$$\vec{F_1} + \vec{F_2} = -\vec{F_3}$$

$$\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$$

or $-\vec{F_3} + \vec{F_3} = 0$

$\therefore 0 = 0$

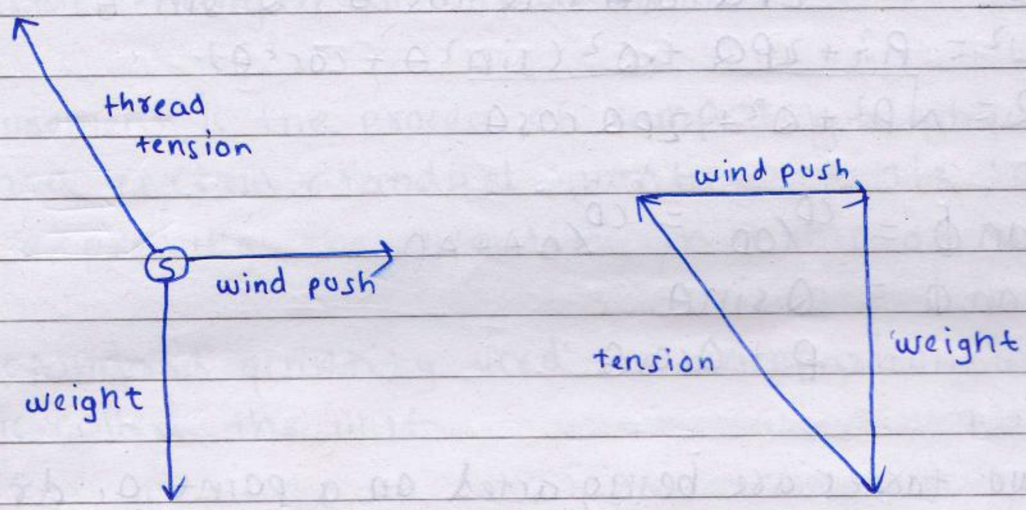
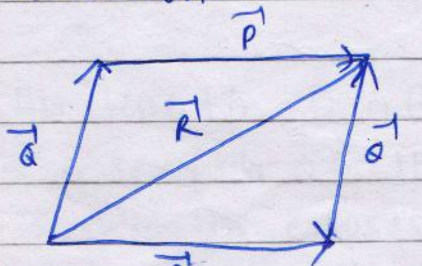


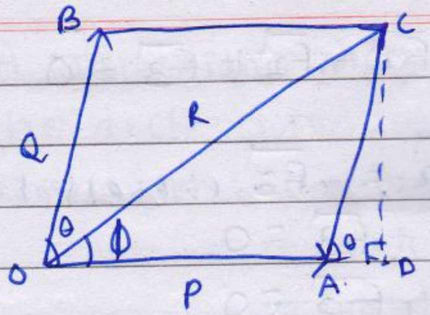
fig. Blowing in the wind - the spider hangs in equilibrium.

Parallelogram law of vector addition.

If two vectors can be represented as two adjacent sides of a ||gm, then diagonal passing through their common point represents the resultant in both magnitude & direction



$$\vec{P} + \vec{Q} = \vec{R}$$



$$\cancel{\vec{AC} = \vec{Q}\cos\theta}$$
$$= \cancel{\vec{Q} \times \frac{AD}{AC}}$$

$$\vec{AC} = \vec{Q}$$
$$\cos\theta = \frac{AD}{AC}$$
$$\therefore AD = AC\cos\theta$$
$$\therefore AD = Q\cos\theta$$

$$OC^2 = OD^2 + DC^2$$

$$\sin\theta = CD/AC$$
$$CD = AC\sin\theta$$
$$\therefore CD = Q\sin\theta$$

Or, $OC^2 = (OA + AD)^2 + DC^2$

Or, $R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$

Or, $R^2 = P^2 + 2PQ\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta$

Or, $R^2 = P^2 + 2PQ + Q^2(\sin^2\theta + \cos^2\theta)$

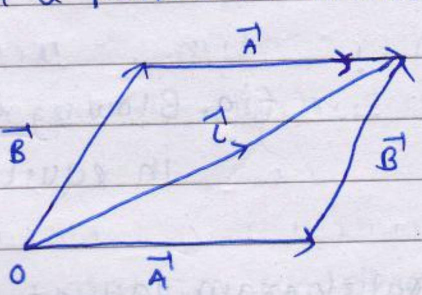
$\therefore R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$$\tan\phi = CD/OD = CD/OA + AD$$

Or, $\tan\phi = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

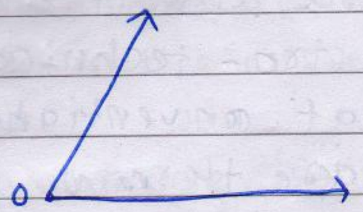
Two forces are being acted on a point O, draw the resultant vector.

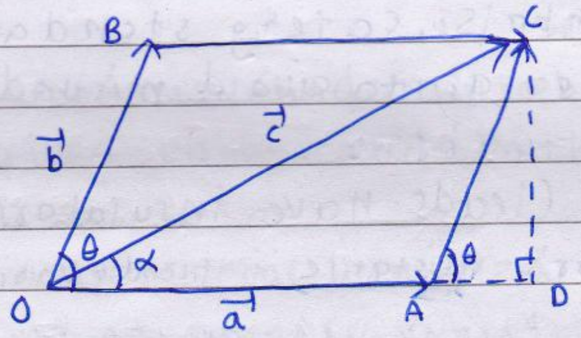
$\vec{C} = \vec{A} + \vec{B}$



VECTORS AND SCALARS

Two forces are being acted on a point O, draw the resultant vector.





$\cos\theta = AD/AC$

or, $AC\cos\theta = AD$

or, $b\cos\theta = AD$

$\sin\theta = CD/AC$

or, $AC\sin\theta = CD$

or, $b\sin\theta = CD$

$OC^2 = OD^2 + DC^2$

or, $OC^2 = (OA + AD)^2 + (b\sin\theta)^2$

or, $OC^2 = (a + b\cos\theta)^2 + b^2\sin^2\theta$

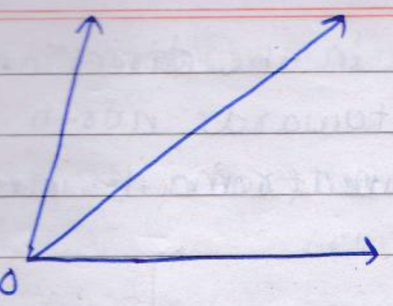
or, $OC^2 = \underline{a^2 + 2ab\cos\theta + b^2}(\cos^2\theta + \sin^2\theta)$

$\therefore OC = \sqrt{a^2 + 2ab\cos\theta + b^2}$

$\tan\alpha = CD/OD$

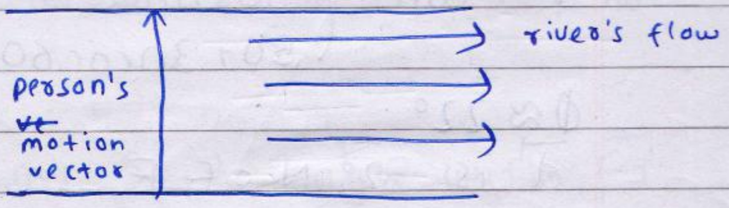
or, $\tan\alpha = b\sin\theta / a + b\cos\theta$

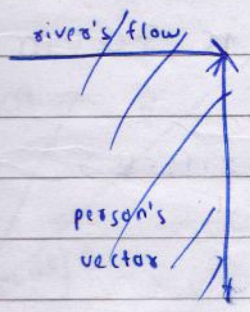
$\therefore \alpha = \tan^{-1}\left(\dfrac{b\sin\theta}{a + b\cos\theta}\right)$ //

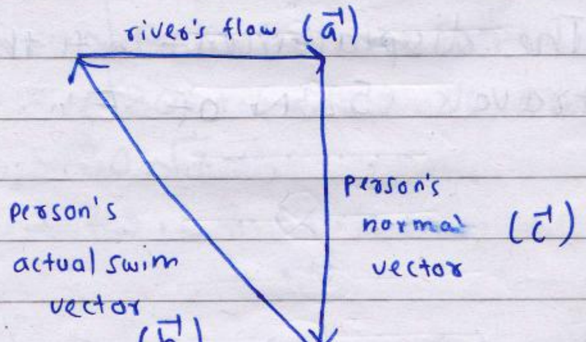


The resultant vector has a magnitude of $\sqrt{a^2+2ab\cos\theta+b^2}$, where a and b represents the magnitude of the two vectors and θ is the angle between them, and the vector is at an angle of $\tan^{-1}\left(\frac{b\sin\theta}{a+b\cos\theta}\right)$ from the base vector.

A river is flowing toward right, a person on the bank of the river wants to cross the river normally, how should he swim?

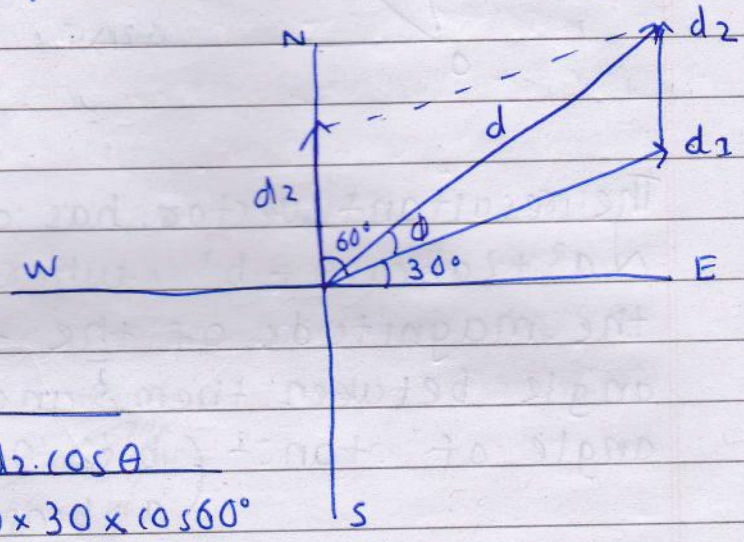






Here since, $\vec{b}+\vec{a}=\vec{c}$, the person should ~~mov~~ swim some degrees north-west to cross the river normally.

An airplane flies 50 km in the direction 30° N of east and then 30 km towards north. Calculate the displacement of the plane from its initial position.



$|\vec{d_2}| = 50\text{ km}$
$|\vec{d_2}| = 30\text{ km}$

Angle between the vectors $= 60°$

$d = \sqrt{d_1^2 + d_2 + 2d_1 d_2 \cdot \cos\theta}$
$= \sqrt{50^2 + 30^2 + 2\times 50\times 30\times \cos 60°}$
$= 70\text{ km}$ //.

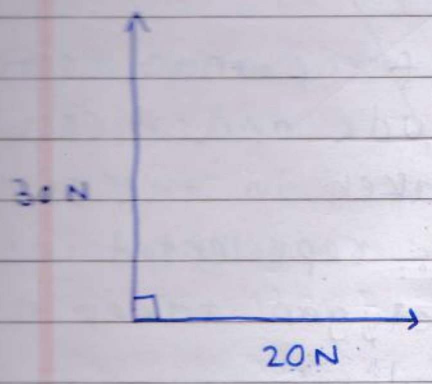
$\tan\phi = \dfrac{d_2 \sin\theta}{d_1 + d_2 \cos\theta}$

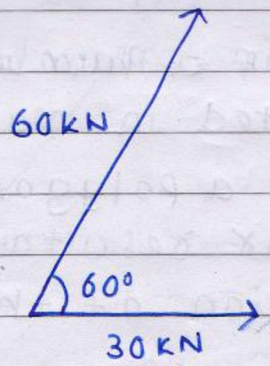
or, $\phi = \tan^{-1}\left(\dfrac{30\sin 60}{50 + 30\cos 60}\right)$

$\therefore \phi \approx 22°$
$\vec{d}$ is $52°$ N of E

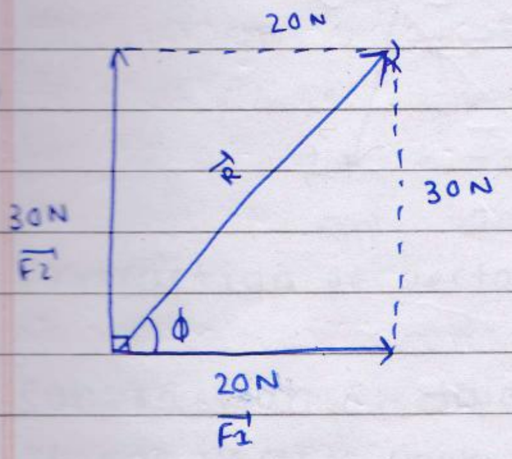
$\therefore$ The displacement of the plane is 70 km and it travels 52° N of E.

= Calculate the vector sum of two forces.





(i)



$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 . F_2 . \cos\theta}$$

$$= \sqrt{20^2 + 30^2 + 2 . 20 . 30 . \cos 90}$$

$$= 10\sqrt{13}\ N$$

$$\approx 36.05\ N$$

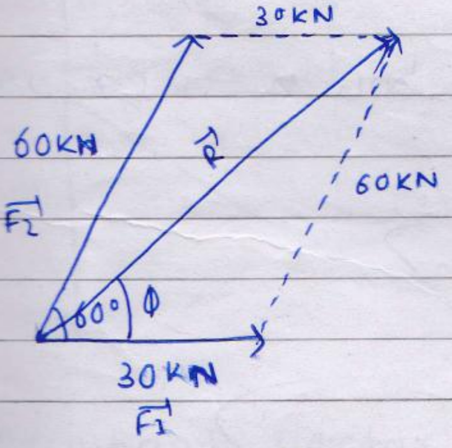
$$\phi = \tan^{-1}\left(30 \times \sin 90 / 20 + 30 \cos 90\right)$$

$$= \tan^{-1}(3/2)$$

$$= 56.3°$$

$\therefore \vec{R} = 36.05$ N and forms 56.3° angle with $\vec{F_1}$.

(ii)



$$R = \sqrt{F_1^2 + F_2^2 + 2F_1.F_2.\cos\cancel{60°}\,\theta}$$

$$= \sqrt{30^2 + 60^2 + 2.30.60.\cos 60°}$$

$$= 30\sqrt{7}$$

$$= 79.37\ kN$$

$$\phi = \tan^{-1}\left(60\sin 60 / 30 + 60\cos 60\right)$$

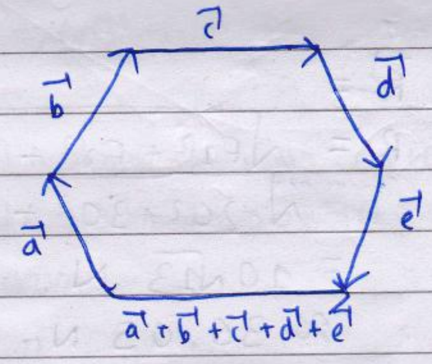
$$= \tan^{-1}(\sqrt{3}/2)$$

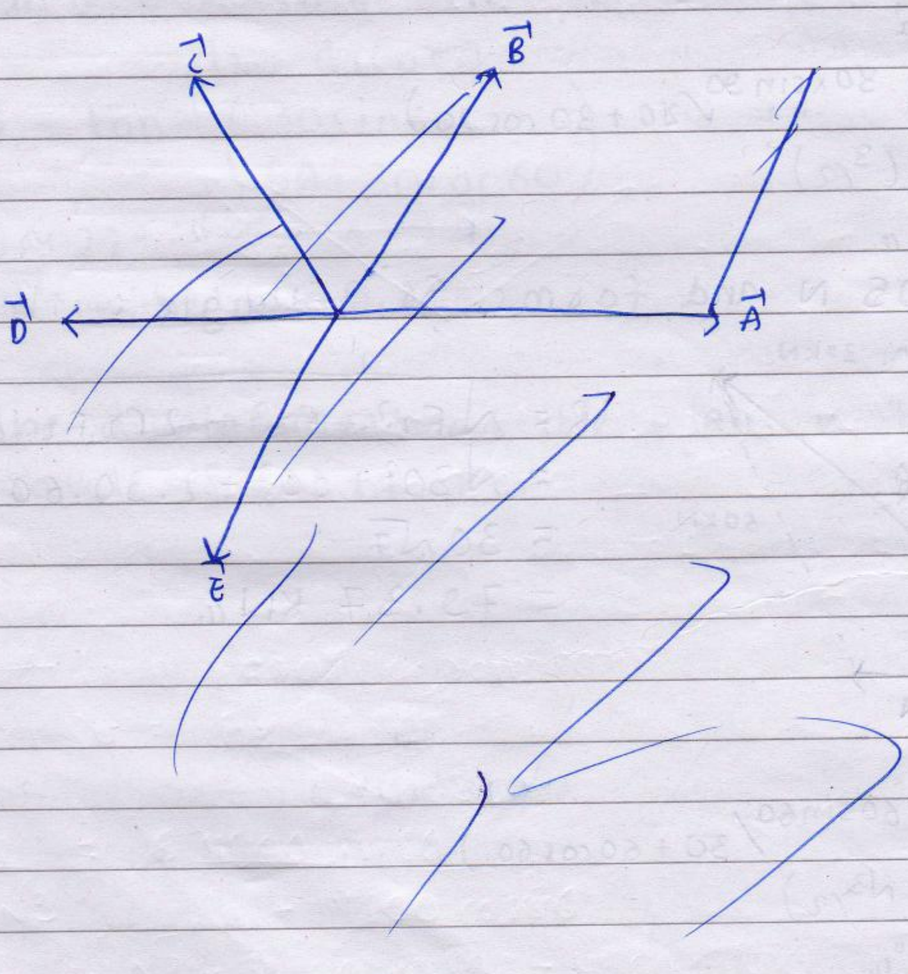
$$= 40.89°$$

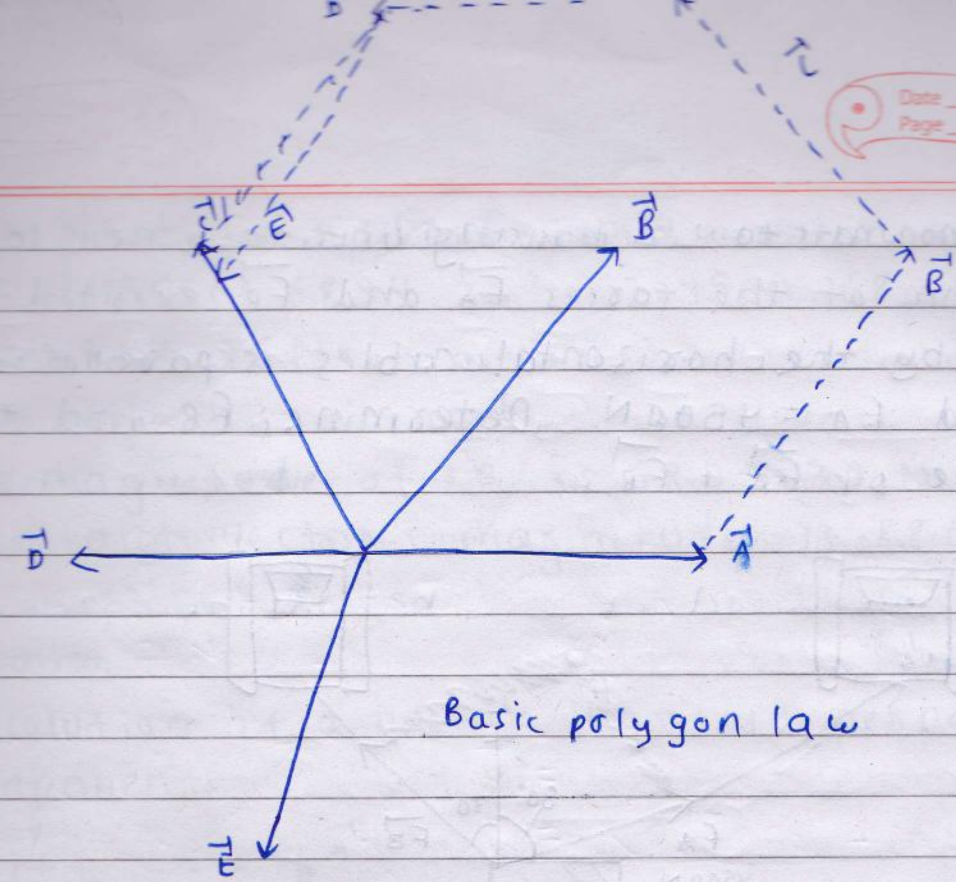
$\therefore \vec{R} = 79.37$ KN and forms 40.89° angle with $\vec{F_1}$.

Polygon law of vector addition.

It states that if a number of coplanar vectors can be represented in magnitude and direction by the sides of a polygon taken in the same order, then their resultant is represented in mag. and direction of the polygon taken in the opposite order.



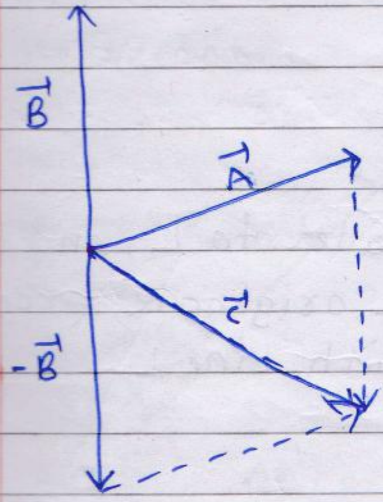




Basic polygon law

Subtraction of vectors.

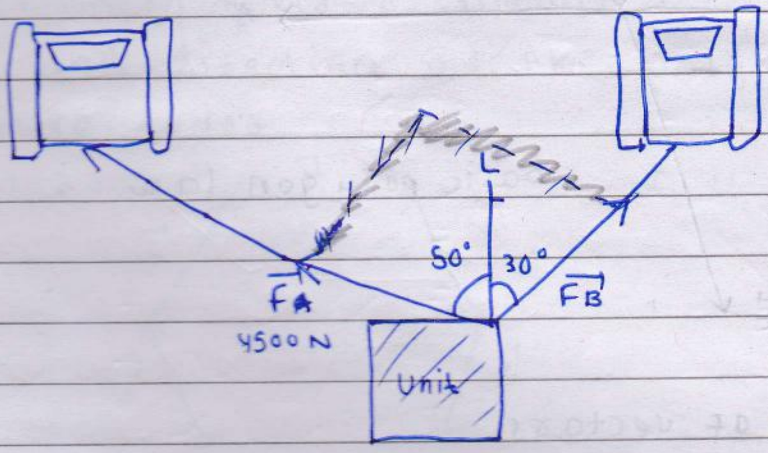
- Subtraction of two vectors is addition of negative of one vector with another.
- $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$



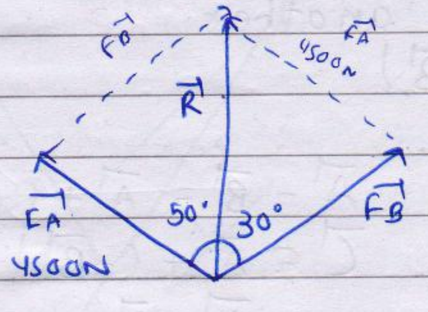
$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$
$$\vec{C} = \vec{A} + (-\vec{B})$$
$$= \vec{A} - \vec{B}.$$

Resultant vector is the sum of two vectors.

Two snowcats tow a housing unit to a new location. The sum of the forces $\vec{F_A}$ and $\vec{F_B}$ exerted on the unit by the horizontal cables is parallel to the line L, and $F_A = 4500N$. Determine F_B and the magnitude of $\vec{F_A} + \vec{F_B}$.



Angle between $\vec{F_A}$ and $\vec{F_B}$ is 80°



Since, the resultant vector is parallel to L, and both the resultant vector and Line L originate from the same point, $\vec{R}$ must overlap with Line L

~~Let~~ From parallelogram law,

$$\tan 30° = \frac{F_A \sin 80°}{F_B + F_A \cos 80°}$$

or, $$\frac{1}{\sqrt{3}} = \frac{\cancel{F}\ 4500 \sin 80°}{F_B + 4500 \cos 80°}$$

or, $F_B = \sqrt{3} \times 4500 \sin 80° - 4500 \cos 80°$

$\therefore F_B = 6894.39\ N$ //

$$R = \sqrt{F_A^2 + F_B^2 + 2 \cdot F_A \cdot F_B \cos\theta}$$

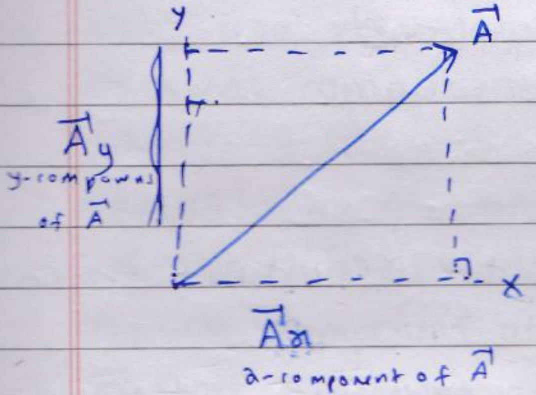
$$= \sqrt{4500^2 + 6894.39^2 + 2 \cdot 4500 \cdot 6894.39 \cdot \cos 80^\circ}$$

$$= 8863.26\ N$$

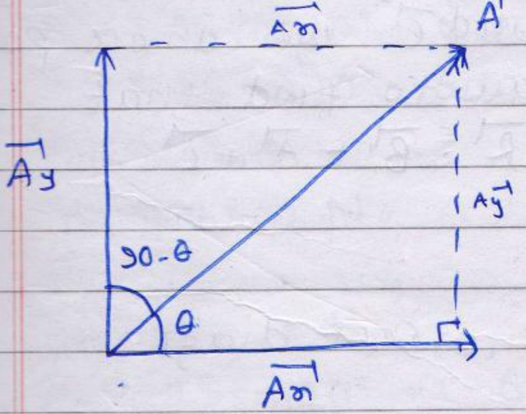
$\therefore$ The magnitude of $\vec{F_B}$ is 6894.39 N and the resultant vector's magnitude is 8863.26 N.

~~Resultar~~

Resolution of a vector into two perpendicular components.



These two vectors are called component vectors of the vector.



$\cos\theta = \vec{A_x} / \vec{A}$

$\therefore \vec{A_x} = \vec{A} \cos\theta$

$\cos(90-\theta) = \vec{A_y} / \vec{A}$

$\therefore \vec{A_y} = \vec{A} \sin\theta$

$\vec{A} = \vec{A_x} + \vec{A_y}$

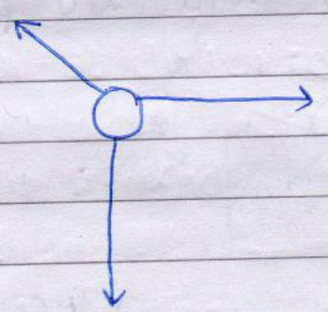
$\cancel{A =}\ R = \sqrt{(A\cos\theta)^2 + (A\sin\theta)^2}$

$=$

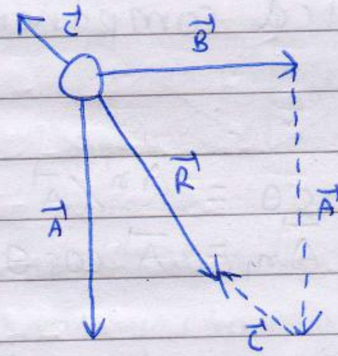
Vector addition by resolution method.

If we want to find $\vec{A}+\vec{B}$, we add $\vec{A_x}+\vec{B_x}$ to get $\vec{R_x}$, and then $\vec{A_y}+\vec{B_y}$ to get $\vec{R_y}$ then find $\vec{R}$ using $\sqrt{\vec{R_x}^2 + R_y^2}$.

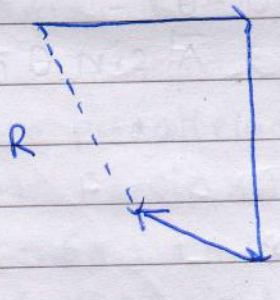
Three wires each exist a horizantal force on a vertical pole as shown.



Which vector diagram shows the resultant force R acting on the pole?

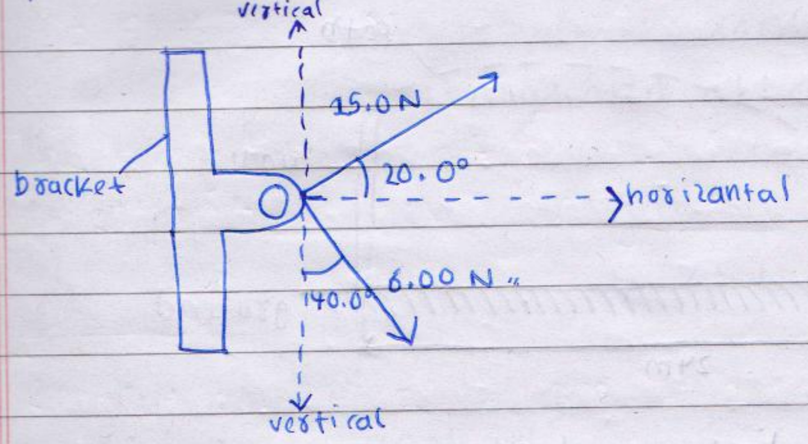


Here by ~~bo~~ re-producing $\vec{A}$ and $\vec{C}$ we can use polygon law to find that $\vec{R} = \vec{B} + \vec{A} + \vec{C}$



is the correct diagram.

Two cables are attached to a bracket and exert the forces as shown:



What are the magnitudes of the horizantal and vertical components of the resultant of the two forces

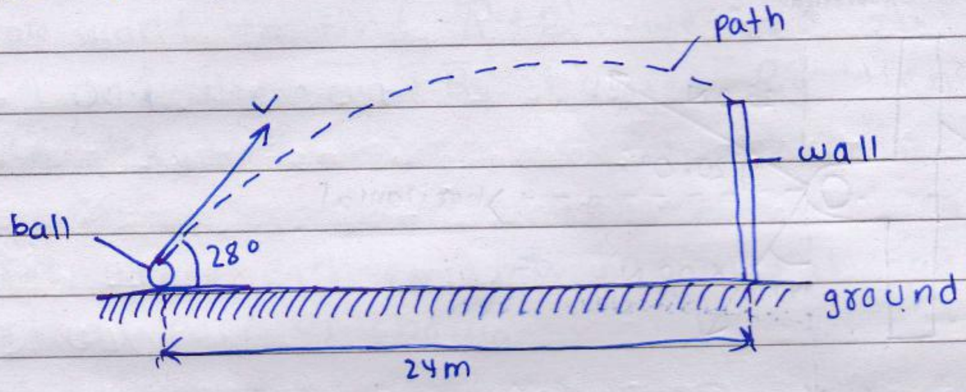
Let 15 N be ~~F_1 an~~ $\vec{A}$ and 6 N be $\vec{B}$

Horizantal component of resultant vector = $\vec{A_x} + \vec{B_x}$
Vertical component of resultant vector = $\vec{A_y} + \vec{B_y}$

$$\vec{R_x} = \vec{A_x} + \vec{B_x}$$
$$= 15\cos 20° + 6\cos(90-40)°$$
$$= 15\cos 20° + 6\cos 50°$$
$$= 17.9$$
$$\approx 18.0 \text{ N}$$

$$\vec{R_y} = \vec{A_y} + \vec{B_y}$$
$$= 15\sin 20° + (-6\cos 40°)$$ [∵ vertical components are opposit
$$= 15\sin 20° - 6\cos 40°$$
$$= 0.534 \text{ N}$$

A ball is kicked from horizantal ground towards the top of a vertical wall,



The horizontal distance between the initial position of the ball and the base of the wall is 24m. The ball is kicked with an initial velocity v at an angle of 28° to the horizantal. The ball hits the top of the ~~ball~~ wall after 1.5 s. Air resistanle may be assumed to be negligible

(i) ~~Calcult~~ Calculate the initial horizantal component V_x of the velocity of the ball.

$V_x = V\cos 28°$

or, $24/1.5 = V\cos 28°$

or, $V = 16/\cos 28°$

$\therefore$ $V = 18.12\ ms^{-1}$,,

$\therefore$ $V_x = 24/1.5 = 16\ ms^{-1}$,,

(ii) Show that the initial vertical component V_y of the velocity is $8.5\ ms^{-1}$,,

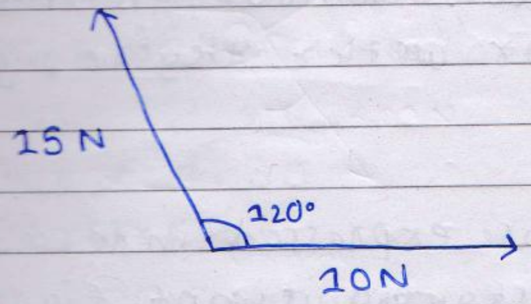
$V_y = V\sin 28°$

or, $V_y = 18.12 \times 0.46$

$\therefore$ $V_y = 8.50\ ms^{-1}$,,

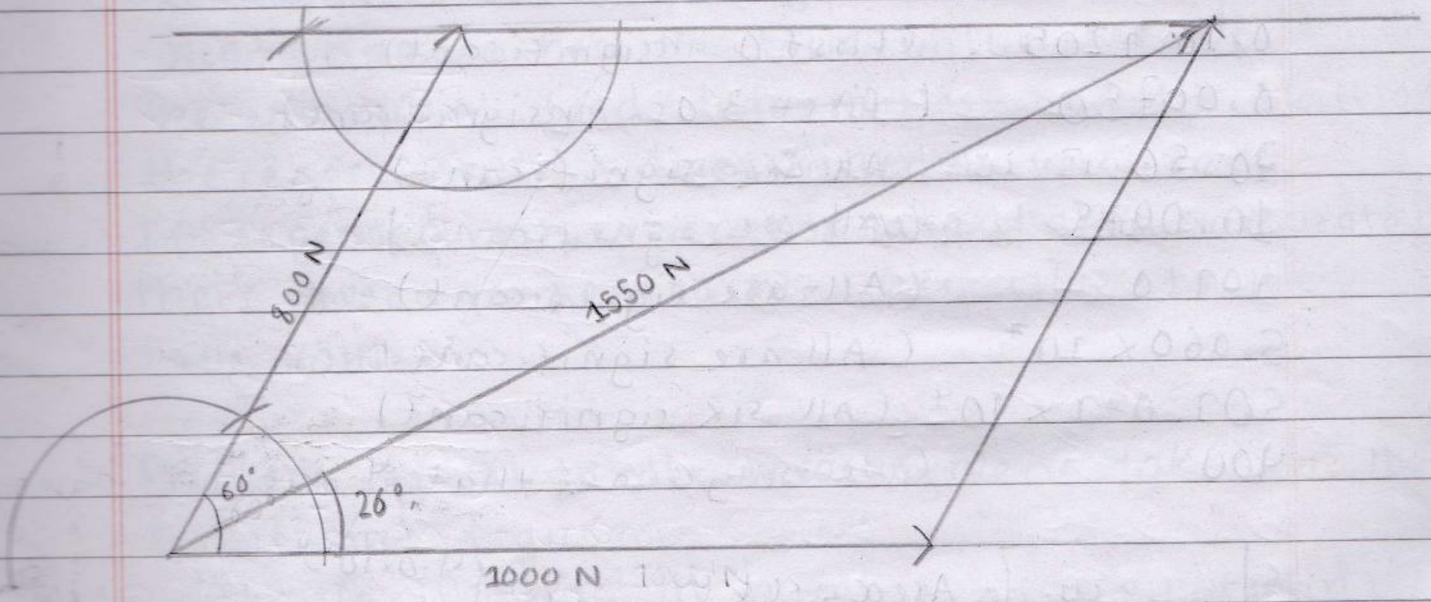
Vector scale diagram

= Calculate the ~~magnitude of~~ resultant $ of two forces 10N & 15 N, acting and at angle 120°.



Calculate the resultant of 1000 N and 800 N at an angle of 60°, using vector-scale diagram.

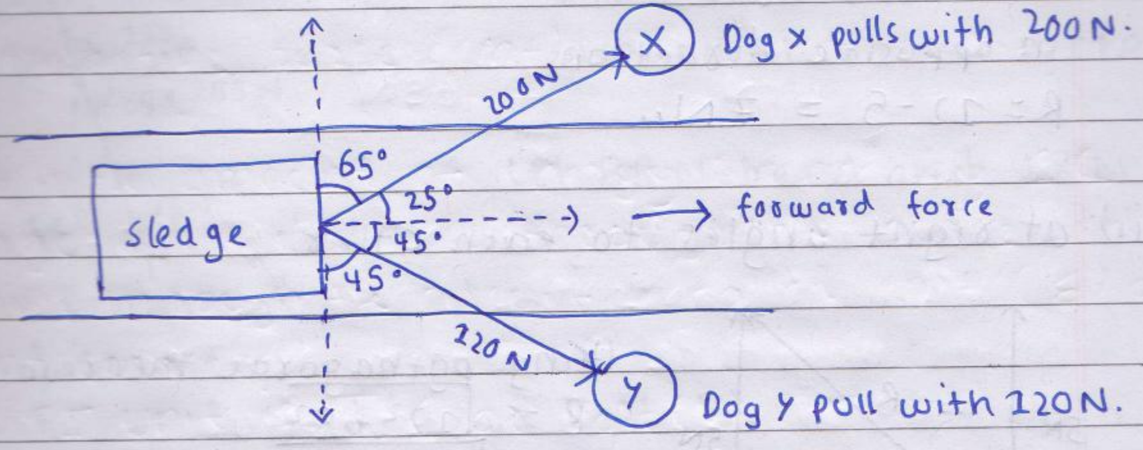
Let, scale be, 1cm = 10 N.



The magnitude of the resultant is 1550 N and the angle it forms is 26° with the base vector.

VECTORS AND SCALARS...

= Two dogs pull a sledge along an icy track.



what is the resultant forward force exerted on the sledge by the two dogs?

$\vec{R_x} = \vec{X_x} + \vec{Y_x}$
$= 200\cos 25° + 120\cos 45°$
$= 266.11\ N$

$\vec{R_y} = \vec{X_y} + \vec{Y_y}$
$= 200\sin 25° - 120\cos 45°$
$= \cancel{266.11\ N}\ -0.32\ N$

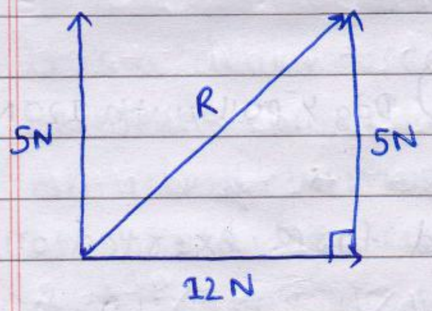
$\vec{R} = \sqrt{R_x^2 + R_y^2}$
$\therefore \vec{R} = \sqrt{266.11^2 + (-0.32)^2}$
$\therefore \vec{R} = 266.11\ N$
$\cancel{\therefore \vec{R} =}$
$\therefore R \approx 270\ N$

Two forces with magnitude 5.0N and 12N, act from the same point on an object. Calculate the magnitude of the resultant force R for the force acting

i) in opposite directions

$R = 12 - 5 = 7\,N$ //

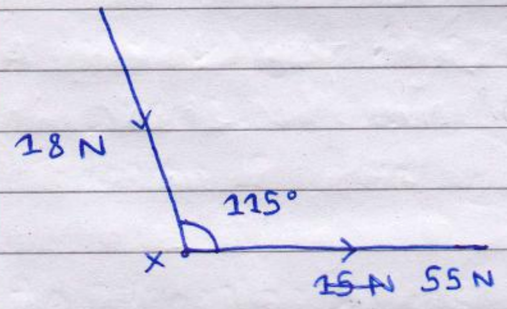
ii) at right angles to each other.



Using pythagoras theorem,

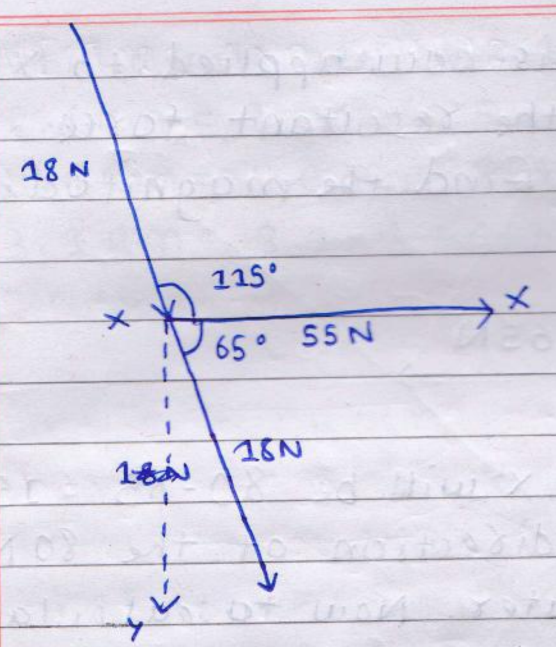
$$R = \sqrt{12^2 + 5^2}$$
$$= \sqrt{144 + 25}$$
$$= \sqrt{169}$$
$$= \sqrt{13^2}$$
$$= 13\,N$$ //

An object X rests on a smooth horizantal surface. Two horizantal forces act on X as shown.



i) Use the resolution method or scale diagram to show that the magnitude of the resultant force acting on X is 65 N.

PTO =)



Using resolution method,
Let the dotted line be the y-component of the vectors and 55 N be the x-component of the vectors.

Let 55 N be A and 18 N be B.

$A_x = A\cos 0° = 55\cos 0° = 55 \times 1 = 55\ N$
$A_y = A\sin 0° = 55\sin 0° = 55 \times 0 = 0\ N$
$B_x = B\cos 65° = 18\cos 65° = 18 \times 0.42 = 7.5\ N$
$B_y = B\sin 65° = 18\sin 65° = 18 \times 0.90 = 16.2\ N$

$R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$
$= \sqrt{(55+7)^2 + (0+16)^2}$
$= \sqrt{62^2 + 16^2}$
$= \sqrt{3844 + 256}$
$= \sqrt{4100}$
$= 64.03\ N$
$\approx 65\ N$

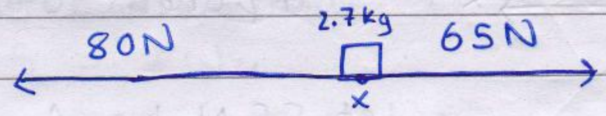
ii) Determine the angle between the resultant force and the 55 N force.

Let the angle be ϕ

$\phi = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$, where θ is the angle between the vectors $\vec{A}$ and $\vec{B}$.

$= \tan^{-1}\left(\frac{18\sin 65}{55 + 18\cos 65}\right)$
$= \tan^{-1}(0.26)$
$\approx 14.6°$

c) A third force of 80N is now applied to X in the opposite direction of the resultant force. The mass of X is 2.7 Kg. Find the magnitude of acceleration of X.



Resultant force acting on X will be 80-65 = 15 N and it will be towards the direction of the 80 N force. as its magnitude is greater. Now to calculate acceleration, we use the derivative of momentum with respect to time.

$F = dp/dt = d/dt\,(m.v) = m\,d/dt(v) = ma$

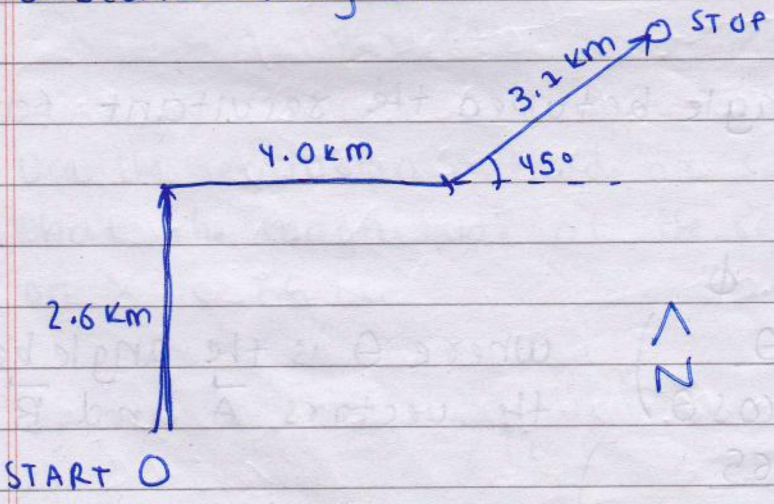
or, $F = ma$

or, $15 = 2.7\,a$

or, $a = 15/2.7$

$\therefore a = 5.56\ ms^{-2}$ //.

A postal employee drives a delivery truck along the route shown. Determine the magnitude and direction of the resultant displacement by drawing a scale diagram.

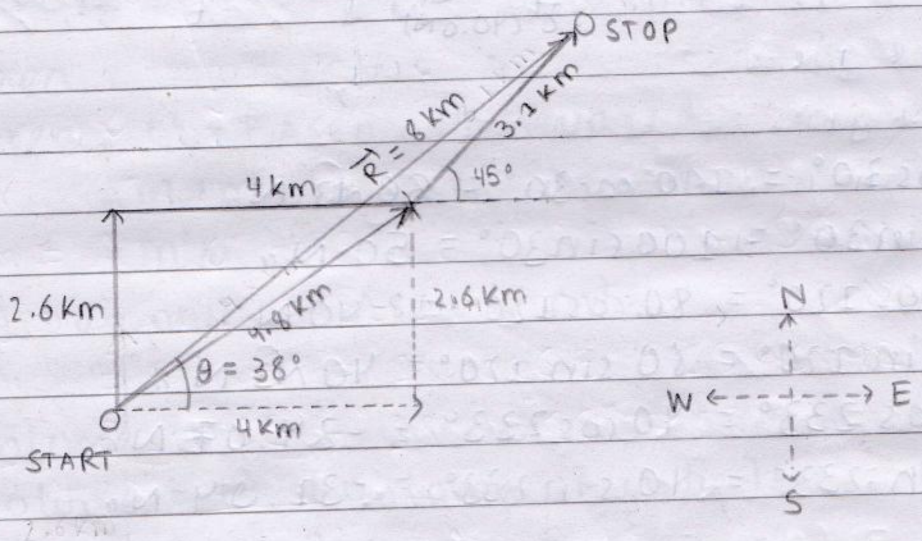


Let the scale be ~~2cm = 1km~~ .. 1cm = 1km

2.6 km = ~~2.6 × 2 = 5.2 cm~~ 2.6 cm ..

4.0 km = ~~4.0 × 2 = 8.0 cm~~ 4.0 cm ..

3.1 km = ~~3.1 × 2 = 6.2 cm~~ 3.1 cm ..



Here $\vec{R}$ is the resultant vector and θ is the angle it forms with respect to the West - East horizontal line in the directions.

∴ The magnitude of the resultant vector is 8 km and its direction is 38° east of north.

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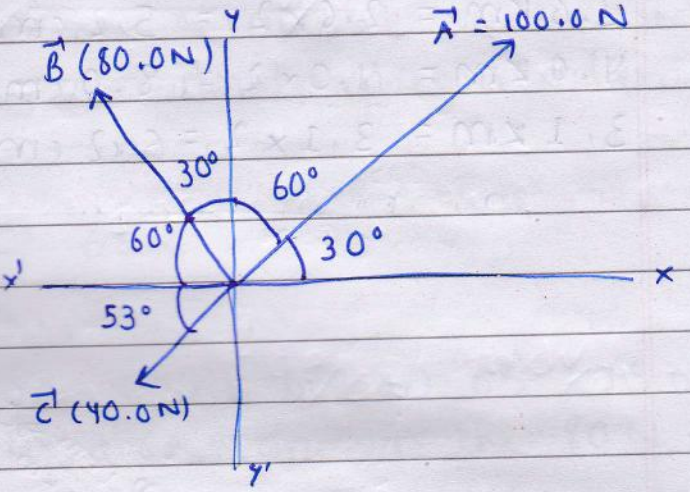
Calculate resultant with resolution.



$A_x = A\cos 30° = 100\cos 30° = 50\sqrt{3}$ N

$A_y = A\sin 30° = 100\sin 30° = 50$ N

$B_x = B\cos 120° = 80\cos 120° = -40$ N

$B_y = B\sin 120° = 80\sin 120° = 40\sqrt{3}$ N

$C_x = C\cos 233° = 40\cos 233° = -24.07$ N

$C_y = C\sin 233° = 40\sin 233° = -31.94$ N

$R = \sqrt{(A_x + B_x + C_x)^2 + (A_y + B_y + C_y)^2}$

$= \cancel{\sqrt{(50\sqrt{3} + 5 - 40 - 24.07)^2 + \sqrt{}}}$

$= \sqrt{(50\sqrt{3} - 40 - 24.07)^2 + (50 + 40\sqrt{3} - 31.94)^2}$

$= 90.2$ N

$\cancel{\phi = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)}$ $\qquad \tan^{-1}\left(\frac{R_y}{R_x}\right)$

$\cancel{= \tan^{-1}\left(\frac{80\sin 90}{100 + 80\cos 90}\right)}$ $\qquad \tan^{-1}\left(\frac{(50 + 40\sqrt{3} - 31.94)}{(50\sqrt{3} - 40 - 24.07)}\right)$

$\cancel{= \tan^{-1} 4/5}$

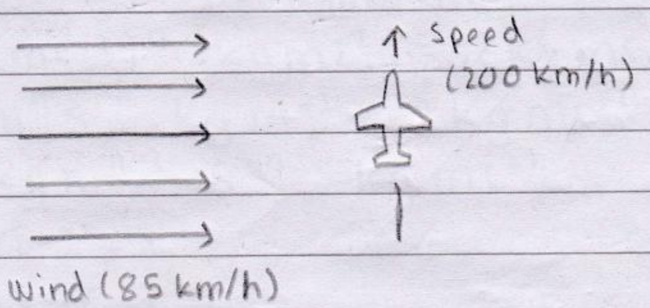
$\cancel{= 38.65°}$ $\qquad \approx 75.5°$

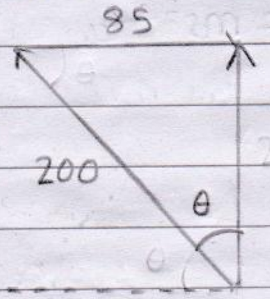
$\therefore$ Angle is ~~68.65°~~ 75.5° with +ve x-axis and magnitude is 90.2 N

VECTOR EXERCISE

1. which two quantities are both vector quantities?
B. Force and momentum.

2. The speed of an aircraft in still air is 200 km/h. The wind blows from the west at a 85.0 km/h speed. In which direction must the pilot steer the aircraft to fly due north.



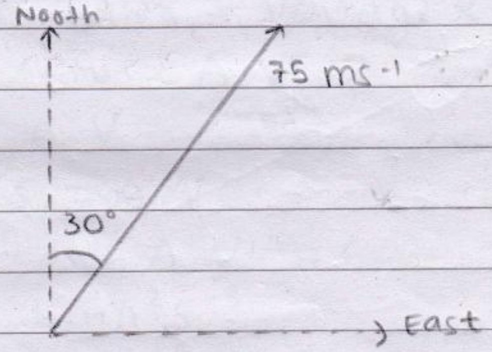


$\theta = \sin^{-1}(85/200)$

$\theta \approx 25.15^\circ$

$\therefore$ $\theta \approx 25.2^\circ$ west of north.

3. A velocity vector is shown.



What are the components of the vector in the north and east directions.

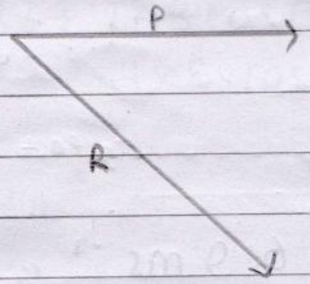
Let north component be V_n
Let east component be V_e

$V_n = V\cos 30°$
$= 75 \cos 30°$
$= 64.95$
$\approx 65 \text{ ms}^{-1}$

$V_e = V\sin 30°$
$= 75 \sin 30°$
$= 37.5$
$\approx 38 \text{ ms}^{-1}$

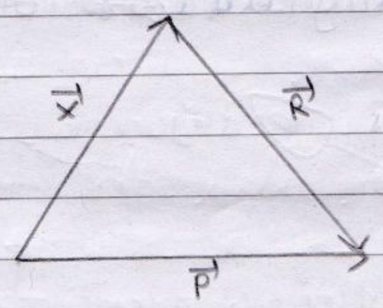
$\therefore$ Answer = C [65, 38]

4. P and R are coplanar vectors.

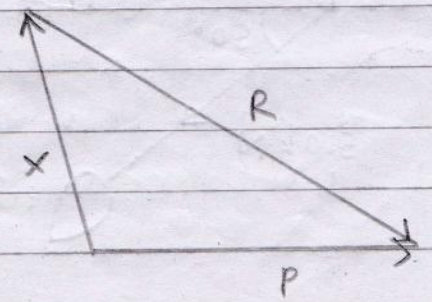


If X = P − R, which diagram best represents vector X

$\vec{X} = \vec{P} - \vec{R} \quad \therefore \vec{X} + \vec{R} = \vec{P}$ [Can be represented as a Δ]

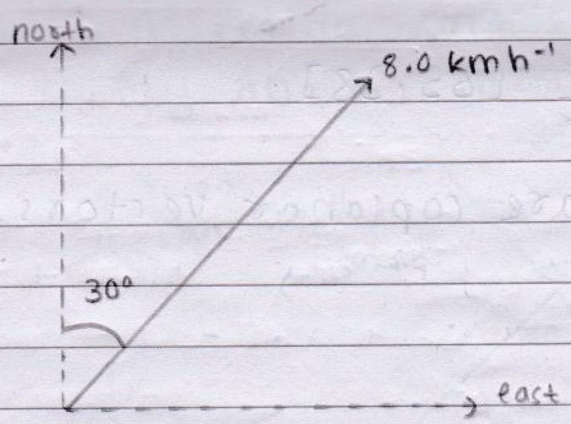


$\therefore$ Answer = A



5. A ship is travelling with a velocity of 8.0 km/h in a direction 30° east of north. What are the ship's velocity components in east and north directions.

Let V_n be the north component of the ship's velocity.
Let V_e be the east component of the ship's velocity.

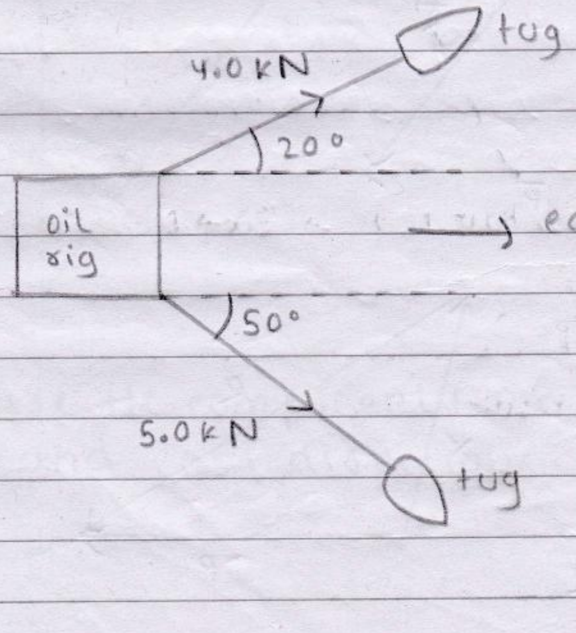


$V_n = V\cos 30° = 8\cos 30° = 6.9\ ms^{-1}$ "

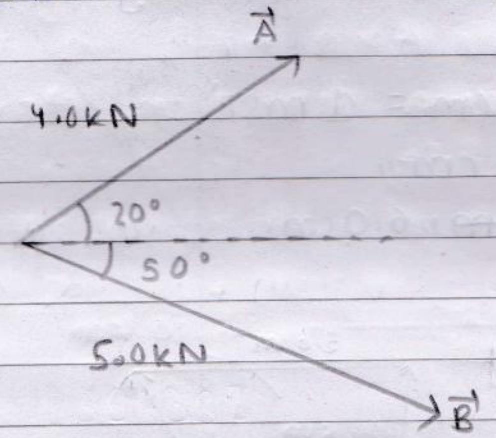
$V_e = V\sin 30° = 8\sin 30° = \cancel{4\ ms^{-1}}$ " $4.0\ ms^{-1}$

∴ Answer = B [$V_e = 4.0$, $V_n = 6.9$] "

6. Two tugs are towing an ~~oir~~ oil rig as shown.



What is the total force acting on the rig, in the east direction.



Let the total force be R

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ [where θ is angle between vectors]

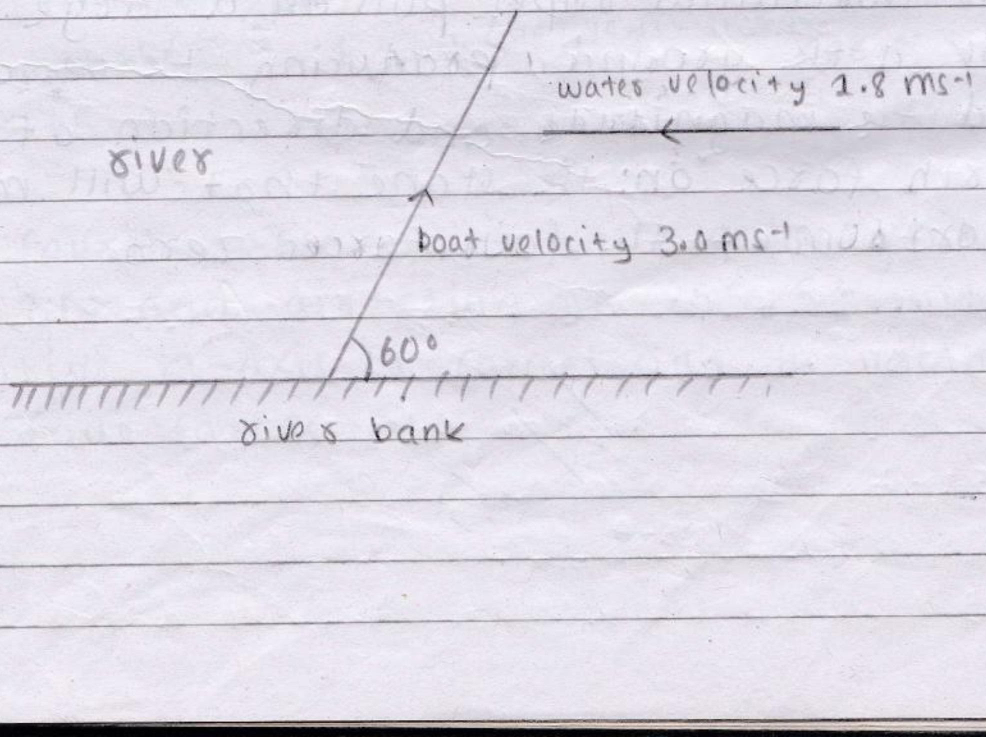
$= \sqrt{4^2 + 5^2 + 2 \times 4 \times 5 . \cos 70}$

$= \sqrt{54.6}$

$= 7.3\,kN$

∴ ~~Asw~~ Answer = D (7.3 kN)

7. A boat travels across a river in which the water is moving at 1.8 ms^{-1} speed. The velocity vectors for the boat and river are shown.

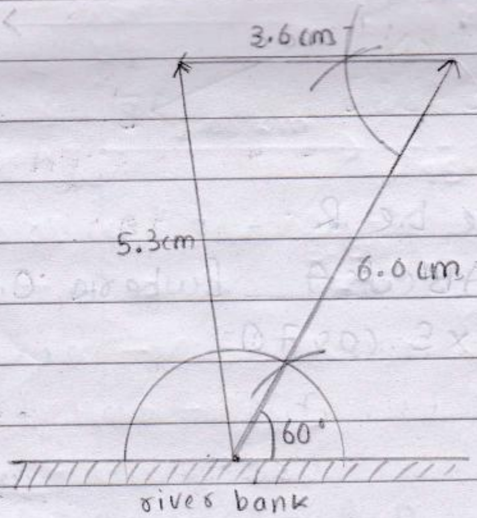


i) Draw a vector ~~or~~ triangle or scale diagram to show the resultant velocity of the boat.

Let scale be 2cm = $1 ms^{-1}$

$1.8 ms^{-1}$ = 3.6 cm //.

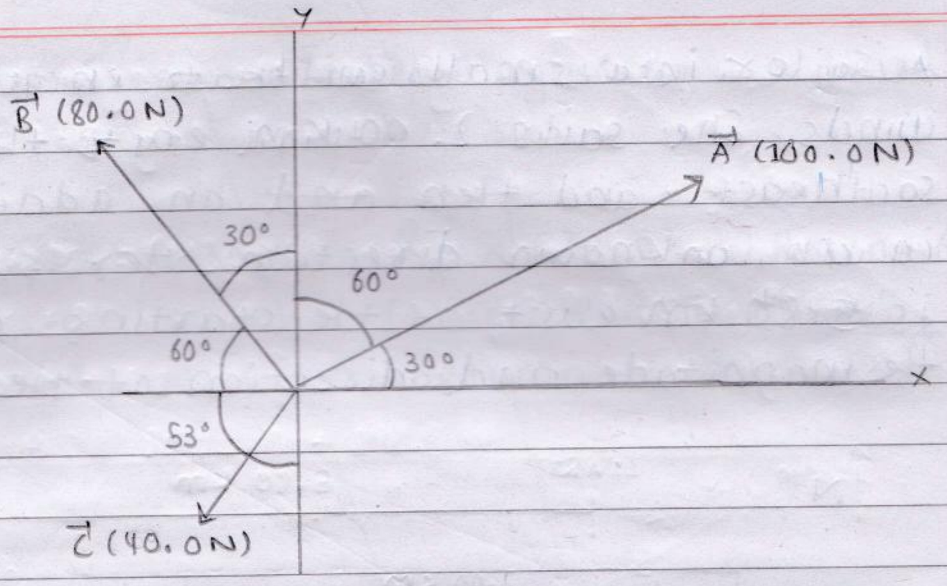
$3.0 ms^{-1}$ = ~~6 cm~~ //. 6.0 cm



ii) Determine the magnitude of the boat's resultant velocity.

velocity = $^{5.3}/_2$ = 2.6 ms^{-1} //.

8. Three horizantal ropes pull on a large stone stuck in the ground, producing the vector forces. Find the magnitude and direction of a fourth force on the stone that will make the vector sum of the four forces zero.



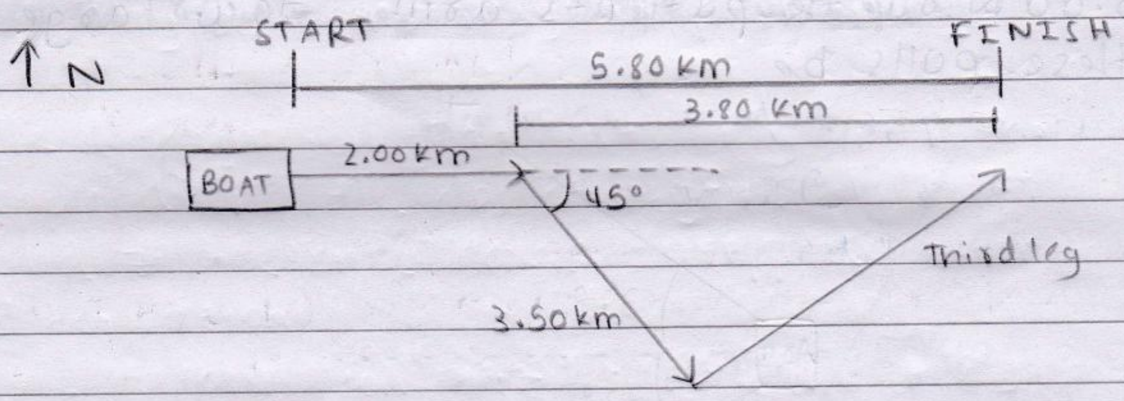
$A_x = A\cos 30° = 100\cos 30° = 50\sqrt{3}\ N$
$B_x = B\cos 120° = 80\cos 120° = -40\ N$
$C_x = C\cos 233° = 40\cos 233° = -24.07\ N$
$A_y = A\sin 30° = 100\sin 30° = 50\ N$
$B_y = B\sin 120° = 80\sin 120° = 40\sqrt{3}\ N$
$C_y = C\sin 233° = 40\sin 233° = -31.94\ N$

$$R = \sqrt{(A_x + B_x + C_x)^2 + (A_y + B_y + C_y)^2}$$
$$= \sqrt{(50\sqrt{3} - 40 - 24.07)^2 + (50 + 40\sqrt{3} - 31.94)^2}$$
$$= 90.2\ N$$

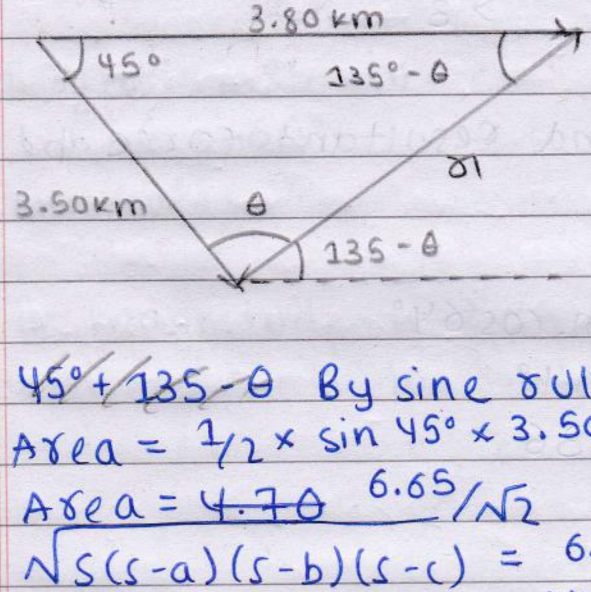
$$\phi = \tan^{-1}(R_y / R_x)$$
$$= \cancel{\tan^{-1}(50\sqrt{3}}\ \tan^{-1}\left(\frac{50 + 40\sqrt{3} - 31.94}{50\sqrt{3} - 40 - 24.07}\right)$$
$$\approx 75.5°$$

The fourth force must have a magnitude of 90.2N and direction of 255.5° with the positive x-axis to cancel out the vector sum of the four forces.

9. A sailor in a small sailboat encounters shifting winds. She sails as shown in the diagram. Find the magnitude and direction of the third leg of the journey.



Simplyfing the vector diagram,



Let the third leg be 'x'.

~~45° + 135 − θ~~ By sine rule,

Area $= \frac{1}{2} \times \sin 45° \times 3.50 \times 3.80$

or, Area $= \cancel{4.70}\ 6.65/\sqrt{2}$

or, $\sqrt{s(s-a)(s-b)(s-c)} = 6.65/\sqrt{2}$

or, $s(s-a)(s-b)(s-c) = 44.22/2$

or, $2s(2s-2a)(2s-2b)(2s-2c) = 353.78$ $[2s = 7.3 + x]$

or, $x+7.3\,(x+7.3-7.6)(x+7.3-7)(x+7.3-2x) = 353.78$

or, $(x+7.3)(x-0.3)(x+0.3)(7.3-x) = 353.78$

or, $-(x+7.3)(x-7.3)(x-0.3)(x+0.3) = 353.78$

or, $-(x^2-53.29)(x^2-0.09) = 353.78$

or, $x^2(53.29-x^2) - 0.09(53.29-x^2) = 353.78$

or, $53.29x^2 - x^4 - 4.79 + 0.09x^2 - 353.78 = 0$

or, $x^4 - 53.38x^2 + 358.57 = 0$

or $(r^2 - 45.49)(r^2 - 7.88) = 0$

$\therefore r = 6.74$ km, 2.80 km

~~(Value 6.74 km can't exist as using it, we get~~

~~$135 - \theta = \sin^{-1}(1.7)$, which can't exist)~~

~~$\therefore r = 2.80$ km~~

~~Area $= \frac{1}{2} \times 3.80 \times 2.80 \times \sin\theta$~~

~~or $44.22/2 = 4.8 \sin\theta$~~

~~or $\theta = \sin^{-1} 4.51$~~

~~et~~

(Value 6.74 km can't exist as it dosent satisfy cosine rule)

$\therefore r = 2.80$ km

Area $= \frac{1}{2} \times 3.50 \times 2.80 \times \sin\theta$

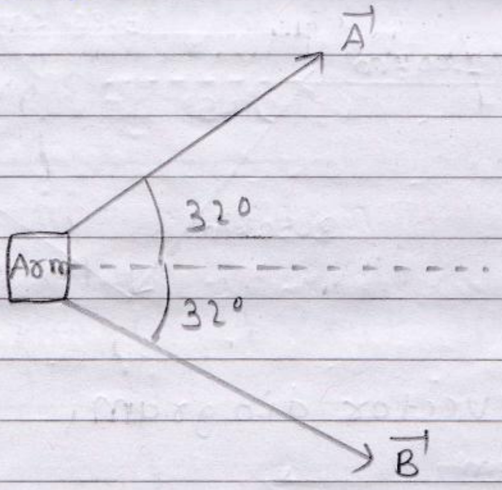
or $6.65/\sqrt{2} = 4.9 \sin\theta$

or $\theta = \sin^{-1} 0.99$

$\therefore \theta = 73.66°$

$\therefore$ The magnitude is 2.80 km and direction is 61.33° north of east.

10. A patient with a dislocated shoulder is put into a traction apparatus as shown. The pulls $\vec{A}$ and $\vec{B}$ have equal magnitudes and must combine to produce an outward traction force of 5.60 N on the patient's arm. How large should these pulls be.



Let's take $A = B = x$ and resultant force be R

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

or, $5.60 = \sqrt{x^2 + x^2 + 2.x.x.\cos 64°}$

or, $5.60^2 = 2x^2 + 2x^2.\cos 64°$

or, $2x^2 (1 + \cos 64°) = 31.36$

or, $x^2 = \dfrac{31.36}{2(1+\cos 64°)}$

or, $x = \sqrt{10.90}$

$\therefore x = 3.30$ N

$\therefore$ The pulls should be 3.30 N large each.