Mathematical Programs with Equilibrium Constraints: Enhanced Fritz-John Conditions, new Constraint Qualifications and improved exact Penalty Results

Christian Kanzow, Alexandra Schwartz – September 14th, 2010

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Outline

Standard Nonlinear Problem (NLP)

We refer to the following optimization problem als NLP:

$$\min f(x) \;\text{ subject to }\; \begin{aligned} g_i(x) &\leq 0 \;\; \forall i = 1, \ldots, m, \\ h_i(x) &= 0 \;\; \forall i = 1, \ldots, p \end{aligned} \tag{1}$$

with $f, g_i, h_i : \mathbb{R}^n \to \mathbb{R}$ continuously differentiable. Its feasible set is denoted by $\mathcal{X}$ and the set of active inequalities in a point $x \in \mathcal{X}$ by

$$I_g(x) = \{i \mid g_i(x) = 0\}.$$

Standard Fritz-John Conditions

Theorem (Standard Fritz-John conditions)

Let x^* be a local minimum of (1). Then x^* is a Fritz-John point, i.e. there are multipliers $\alpha \geq 0, \lambda \geq 0$, and μ such that $(\alpha, \lambda, \mu) \neq 0$ and

$$\alpha \nabla f(x^*) + \sum_{i \in I_g(x^*)} \lambda_i \nabla g_i(x^*) + \sum_{i=1}^{p} \mu_i \nabla h_i(x^*) = 0.$$

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- If $\alpha = 0$, no information about the objective function is included.
- If $\alpha \neq 0$, we may assume $\alpha = 1$ without loss of generality.

Karush-Kuhn-Tucker Point

Definition (KKT Points)

Let x^* be feasible for (1). Then x^* is called a *KKT point* if there are multipliers $\lambda \geq 0$ and μ such that

$$\nabla f(x^*) + \sum_{i \in I_g(x^*)} \lambda_i \nabla g_i(x^*) + \sum_{i=1}^{p} \mu_i \nabla h_i(x^*) = 0.$$

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- However, not every local minimum is a KKT point.
- Additional conditions on the constraints are needed.

Standard Constraint Qualifications I

A point x^* feasible for (1) is said to satisfy

- *linear independence constraint qualification* (LICQ), if there are no multipliers λ and μ such that $(\lambda, \mu) \neq 0$ and

$$\sum_{i \in I_g(x^*)} \lambda_i \nabla g_i(x^*) + \sum_{i=1}^{p} \mu_i \nabla h_i(x^*) = 0,$$

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- *Mangasarian-Fromovitz constraint qualification* (MFCQ), if there are no multipliers $\lambda \geq 0$ and μ such that $(\lambda, \mu) \neq 0$ and

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- *Abadie constraint qualification* (ACQ), if

$$T_{\mathcal{X}}(x^*) = L_{NLP}(x^*).$$

Standard Constraint Qualifications II

The relations between these constraint qualifications are as follows:

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Theorem (KKT conditions)

Let x^* be a local minimum of (1) that satisfies ACQ. Then x^* is a KKT point.

Mathematical Program with Equilibrium Constraints (MPEC)

An MPEC is an optimization problem of the form

$$\min f(x) \;\text{ subject to }\; \begin{array}{ll} g_i(x) \leq 0 & \forall i = 1, \ldots, m, \\ h_i(x) = 0 & \forall i = 1, \ldots, p, \\ G_i(x) \geq 0 & \forall i = 1, \ldots, l, \\ H_i(x) \geq 0 & \forall i = 1, \ldots, l, \\ G_i(x) \cdot H_i(x) = 0 & \forall i = 1, \ldots, l \end{array} \tag{2}$$

with $f, g_i, h_i, G_i, H_i : \mathbb{R}^n \to \mathbb{R}$ continuously differentiable. The corresponding feasible set is denoted by X.

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with $f, g_i, h_i, G_i, H_i : \mathbb{R}^n \to \mathbb{R}$ continuously differentiable. The corresponding feasible set is denoted by X.

- The constraints G and H are called complementarity constraints.
- One also speaks of a *mathematical problem with complementarity constraints (MPCC)*.

Notation: Index Sets

We define the following index sets for an arbitrary $x \in X$:

$$\begin{aligned} I_g(x) &:= \{i \mid g_i(x) = 0\}, \\ I_{00}(x) &:= \{i \mid G_i(x) = 0, H_i(x) = 0\}, \\ I_{0+}(x) &:= \{i \mid G_i(x) = 0, H_i(x) > 0\}, \\ I_{+0}(x) &:= \{i \mid G_i(x) > 0, H_i(x) = 0\}. \end{aligned}$$

- $I_g(x)$ consists of the active inequalities.
- $I_{00}(x)$, $I_{0+}(x)$, and $I_{+0}(x)$ form a partition of the complementarity conditions.

Standard Fritz-John Conditions are too weak for MPECs

Consider the 2-dimensional MPEC

$$\min f(x) \text{ subject to } x_1 \geq 0, x_2 \geq 0, x_1 x_2 = 0.$$

All feasible points are of the form $(a, 0)^T$ or $(0, a)^T$ with $a \geq 0$. Consider a feasible point $x = (a, 0)^T$. Then we have

$$0 \cdot \nabla f(x) - 0 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} - a \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 1 \cdot \begin{pmatrix} 0 \\ a \end{pmatrix} = 0,$$

i.e. x is a Fritz-John point.

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Theorem

Let x be feasible for (2). Then x is a Fritz-John point.

KKT conditions are too strong for MPECs

Consider the 3-dimensional MPEC

$$\min x_1 + x_2 - x_3 \ \text{ subject to } \ \begin{aligned} &-4x_1 + x_3 \leq 0, \\ &-4x_2 + x_3 \leq 0, \\ &x_1 \geq 0, x_2 \geq 0, x_1 x_2 = 0. \end{aligned}$$

The global minimum $(0,0,0)^T$ is not a KKT point since elementary calculations yield that

$$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + \lambda_1 \begin{pmatrix} -4 \\ 0 \\ 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix} - \lambda_3 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - \lambda_4 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \mu_1 \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0$$

is only possible if $\lambda_3 + \lambda_4 = -2$, i.e. at least one of them has to be negative.

Standard CQs are usually not satisfied

Consider again the 2-dimensional MPEC

$$\min f(x) \text{ subject to } x_1 \geq 0, x_2 \geq 0, x_1 x_2 = 0$$

and a feasible point $(a, 0)^T$ with $a \geq 0$. Then we have

$$-0 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} - a \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 1 \cdot \begin{pmatrix} 0 \\ a \end{pmatrix} = 0,$$

i.e. MFCQ is violated in x.

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i.e. MFCQ is violated in x.

Theorem

Let x be feasible for (2). Then both LICQ and MFCQ are violated in x.

Summary

Standard Fritz-John conditions are too weak

KKT conditions are often violated

Standard constraint qualifications are too strong

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Summary

Standard Fritz-John conditions are too weak	KKT conditions are often violated	Standard constraint qualifications are too strong
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Preliminary result by Jane Ye	Weaker stationarity concepts have been developed, e.g. M-stationarity	Special MPEC constraint qualifications have been introduced, e.g. MPEC-MFCQ, MPEC-ACQ

MPEC Constraint Qualifications

A point x^* feasible for (2) is said to satisfy

- *MPEC-MFCQ*, if there are no multipliers $\lambda \geq 0$ and μ, γ, ν such that $(\lambda, \mu, \gamma, \nu) \neq 0$ and

$$\sum_{i:g_i(x^*)=0} \lambda_i \nabla g_i(x^*) + \sum_{i=1}^{p} \mu_i \nabla h_i(x^*) - \sum_{i:G_i(x^*)=0} \gamma_i \nabla G_i(x^*) - \sum_{i:H_i(x^*)=0} \nu_i \nabla H_i(x^*) = 0.$$

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- *MPEC-ACQ* if $T_X(x^*) = L_{MPEC}(x^*)$, where

$$\begin{aligned}
T_X(x^*) &:= \{d \mid \exists \{t_k\} \downarrow 0, \{x^k\} \to_X x^* : \tfrac{x^k - x^*}{t_k} \to d\}, \\
L_{MPEC}(x^*) &:= \{d \mid \nabla g_i(x^*)^T d \leq 0 \ (i \in I_g(x^*)), \\
&\qquad \nabla h_i(x^*)^T d = 0 \ (i = 1, \ldots, p), \\
&\qquad \nabla G_i(x^*)^T d = 0 \ (i \in I_{0+}(x^*)), \\
&\qquad \nabla H_i(x^*)^T d = 0 \ (i \in I_{+0}(x^*)), \\
&\qquad \nabla G_i(x^*)^T d \geq 0, \nabla H_i(x^*)^T d \geq 0 \ (i \in I_{00}(x^*)), \\
&\qquad (\nabla G_i(x^*)^T d)(\nabla H_i(x^*)^T d) = 0 \ (i \in I_{00}(x^*))\}.
\end{aligned}$$

Weaker Stationarity Concept for MPECs

Definition (M-stationarity)

A vector x^* feasible for (2) is called *M-stationary* if there are multipliers $\lambda \geq 0$ and μ, γ, ν such that

$$\nabla f(x^*) + \sum_{i: g_i(x^*)=0} \lambda_i \nabla g_i(x^*) + \sum_{i=1}^{p} \mu_i \nabla h_i(x^*) - \sum_{i: G_i(x^*)=0} \gamma_i \nabla G_i(x^*) - \sum_{i: H_i(x^*)=0} \nu_i \nabla H_i(x^*) = 0$$

and either $\gamma_i > 0$, $\nu_i > 0$ or $\gamma_i \nu_i = 0$ for all $i \in I_{00}(x^*)$.

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and either $\gamma_i > 0$, $\nu_i > 0$ or $\gamma_i \nu_i = 0$ for all $i \in I_{00}(x^*)$.

Theorem

Let x^* be a local minimum of (2) that satisfies MPEC-ACQ. Then x^* is M-stationary.

New Fritz-John Result

Theorem (MPEC Fritz-John conditions)

Let x^* be a local minimum of (2). Then there are multipliers $\alpha \geq 0, \lambda \geq 0$, and μ, γ, ν such that $(\alpha, \lambda, \mu, \gamma, \nu) \neq 0$,

$$\alpha \nabla f(x^*) + \sum_{i: g_i(x^*)=0} \lambda_i \nabla g_i(x^*) + \sum_{i=1}^{p} \mu_i \nabla h_i(x^*) - \sum_{i: G_i(x^*)=0} \gamma_i \nabla G_i(x^*) - \sum_{i: H_i(x^*)=0} \nu_i \nabla H_i(x^*) = 0,$$

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and either $\gamma_i > 0$, $\nu_i > 0$ or $\gamma_i \nu_i = 0$ for all $i \in I_{00}(x^*)$.
If $(\lambda, \mu, \gamma, \nu) \neq 0$, there is a sequence $\{x^k\} \to x^*$ such that for all $k \in \mathbb{N}$ we have $f(x^k) < f(x^*)$ and

$$\lambda_i > 0 \Longrightarrow \lambda_i g_i(x^k) > 0 \ \text{ and } \ \mu_i \neq 0 \Longrightarrow \mu_i h_i(x^k) > 0,$$
$$\gamma_i \neq 0 \Longrightarrow \gamma_i G_i(x^k) < 0 \ \text{ and } \ \nu_i \neq 0 \Longrightarrow \nu_i H_i(x^k) < 0.$$

MPEC-generalized quasinormality

Definition

A vector x^* feasible for (2) is said to satisfy *MPEC generalized quasinormality*, if there are no multipliers $\lambda \geq 0$ and μ, γ, ν such that $(\lambda, \mu, \gamma, \nu) \neq 0$,

$$\sum_{i:g_i(x^*)=0} \lambda_i \nabla g_i(x^*) + \sum_{i=1}^{p} \mu_i \nabla h_i(x^*) - \sum_{i:G_i(x^*)=0} \gamma_i \nabla G_i(x^*) - \sum_{i:H_i(x^*)=0} \nu_i \nabla H_i(x^*) = 0$$

and either $\gamma_i > 0$, $\nu_i > 0$ or $\gamma_i \nu_i = 0$ for all $i \in I_{00}(x^*)$ and there is a sequence $\{x^k\} \to x^*$ such that for all $k \in \mathbb{N}$

$$\begin{aligned}
\lambda_i > 0 \Longrightarrow \lambda_i g_i(x^k) > 0 \;\text{ and }\; \mu_i \neq 0 \Longrightarrow \mu_i h_i(x^k) > 0,\\
\gamma_i \neq 0 \Longrightarrow \gamma_i G_i(x^k) < 0 \;\text{ and }\; \nu_i \neq 0 \Longrightarrow \nu_i H_i(x^k) < 0.
\end{aligned}$$

MPEC-generalized pseudonormality

Definition

A vector x^* feasible for (2) is said to satisfy *MPEC generalized pseudonormality*, if there are no multipliers $\lambda \geq 0$ and μ, γ, ν such that $(\lambda, \mu, \gamma, \nu) \neq 0$,

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and either $\gamma_i > 0$, $\nu_i > 0$ or $\gamma_i \nu_i = 0$ for all $i \in I_{00}(x^*)$ and there is a sequence $\{x^k\} \to x^*$ such that for all $k \in \mathbb{N}$

$$\sum_{i:g_i(x^*)=0} \lambda_i g_i(x^k) + \sum_{i=1}^{p} \mu_i h_i(x^k) - \sum_{i:G_i(x^*)=0} \gamma_i G_i(x^k) - \sum_{i:H_i(x^*)=0} \nu_i H_i(x^k) > 0.$$

MPEC-generalized MFCQ

Definition

A vector x^* feasible for (2) is said to satisfy *MPEC generalized MFCQ*, if there are no multipliers $\lambda \geq 0$ and μ, γ, ν such that $(\lambda, \mu, \gamma, \nu) \neq 0$,

$$\sum_{i:g_i(x^*)=0} \lambda_i \nabla g_i(x^*) + \sum_{i=1}^{p} \mu_i \nabla h_i(x^*) - \sum_{i:G_i(x^*)=0} \gamma_i \nabla G_i(x^*) - \sum_{i:H_i(x^*)=0} \nu_i \nabla H_i(x^*) = 0$$

and either $\gamma_i > 0$, $\nu_i > 0$ or $\gamma_i \nu_i = 0$ for all $i \in I_{00}(x^*)$.

Relations to Existing Results

$$
\begin{array}{ccc}
 & \text{MPEC-MFCQ} & \\
 & \Downarrow & \\
 & \text{MPEC generalized MFCQ} & \\
 & \Downarrow & \\
 & \text{MPEC generalized pseudonormality} & \\
\swarrow & & \searrow \\
\text{MPEC-ACQ} & & \text{MPEC generalized quasinormality}
\end{array}
$$

Relations to Existing Results

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\begin{tikzcd}[column sep=small]
 & \text{MPEC-MFCQ} \arrow[d, Rightarrow] & \\
 & \text{MPEC generalized MFCQ} \arrow[d, Rightarrow] & \\
 & \text{MPEC generalized pseudonormality} \arrow[dl, Rightarrow] \arrow[dr, Rightarrow] & \\
\text{MPEC-ACQ} & & \text{MPEC generalized quasinormality}
\end{tikzcd}
$$

Theorem

Let x^* be a local minimum of (2) that satisfies MPEC generalized quasinormality. Then x^* is M-stationary.

Definition of the Penalty Function

Consider the penalty function

$$P_\alpha(x) := f(x) + \alpha \left[\sum_{i=1}^{m} \mathrm{dist}_{(-\infty,0]} g_i(x) + \sum_{i=1}^{p} \mathrm{dist}_{\{0\}} h_i(x) + \sum_{i=1}^{l} \mathrm{dist}_C(G_i(x), H_i(x)) \right],$$

where distances are measured in the l_1-norm and

$$C = \{(a,b) \in \mathbb{R}^2 \mid a \geq 0, b \geq 0, ab = 0\}.$$

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where distances are measured in the l_1-norm and

$$C = \{(a,b) \in \mathbb{R}^2 \mid a \geq 0, b \geq 0, ab = 0\}.$$

Its explicit representation is

$$P_\alpha(x) = f(x) + \alpha \left[\sum_{i=1}^{m} \max\{g_i(x), 0\} + \sum_{i=1}^{p} |h_i(x)| \right.$$

$$\left. + \sum_{i=1}^{l} \max\{-G_i(x), -H_i(x), -(G_i(x) + H_i(x)), \min\{G_i(x), H_i(x)\}\} \right].$$

Exactness Result

Theorem (Exactness of the Penalty Function)

Let x^* be a local minimum of (2) that satisfies MPEC generalized pseudonormality. Then P_α is exact in x^*, i.e. there is an $\bar{\alpha} > 0$ such that x^* is an unconstrained local minimum of P_α for all $\alpha \geq \bar{\alpha}$.

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If instead distances are measured in the l_∞-norm, one obtains

$$\tilde{P}_\alpha(x) = f(x) + \alpha \left[\sum_{i=1}^{m} \max\{g_i(x), 0\} + \sum_{i=1}^{p} |h_i(x)| + \sum_{i=1}^{l} |\min\{G_i(x), H_i(x)\}| \right],$$

which has the same exactness properties as P_α thanks to the equivalence of norms in finite dimensional spaces.

Conclusion

All in all we have

- presented a new Fritz-John result specialized for MPECs,
- derived some new constraint qualifications guaranteeing M-stationarity of local minima,
- used one of these new CQs to prove exactness of two penalty functions.

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All in all we have

- presented a new Fritz-John result specialized for MPECs,
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Thank you very much for your attention!