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# Trade Dynamics with Sector-Specific Human Capital

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April 22, 2015

## Abstract

This paper develops a dynamic Heckscher Ohlin Samuelson model with sector-specific human capital and overlapping generations to characterize the dynamics and welfare implications of gradual labor market adjustment to trade. Our model is tractable enough to yield sharp analytic results, that complement and clarify an emerging empirical literature on labor market adjustment to trade. Existing generations that have accumulated specific human capital in one sector can switch sectors when the economy is hit by a trade shock. Nonetheless, the shock induces few workers to switch, generating a protracted adjustment that operates largely through the entry of new generations. This results in wages being tied to the sector of employment in the short-run but to the skill type in the long-run. Relative to a world with general human capital, welfare is improved for the skill group whose type-intensive sector shrinks. We extend the model to include physical capital and show that the transition is longer when capital is mobile. We also introduce nonpecuniary sector preferences and show that larger gross flows are associated with a longer transition.

JEL: E24, F11, F16, J24

Keywords: sector-specific human capital, trade shock, transitional dynamics, worker mobility.

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# 1 Introduction

The growth of North-South trade over the last 15 years—particularly due to the emergence of China—has sparked renewed interest in the consequences of inter-industry trade and its effects on labor reallocation and income inequality (Krugman, 2000, Autor, Dorn and Gordon, 2013, and Haskel, Lawrence, Leamer and Slaughter, 2012). In addition to the effects of trade on relative factor rewards, concern has been raised over the welfare costs of protracted labor reallocation and of the idle/lost expertise for workers whose sector is hit by import competition. More generally, the dynamics of an economy’s adjustment to trade shocks are critical to understanding the benefits and distributional consequences of both trade liberalizations and trade shocks.

Yet, most models assume perfect factor mobility or complete immobility even though, empirical results suggest that—owing to short- and medium-run adjustment costs—both assumptions are too extreme for analyzing the impact of trade shocks on the labor market. To address this issue, Matsuyama (1992) analyzes labor reallocation following a trade shock in an overlapping generations model, assuming that workers can only choose their sector once in their lives. This implies that all reallocation occurs through the entry of new generations. In contrast, we allow for labor mobility in a Heckscher Ohlin Samuelson (HOS) model augmented with sector-specific human capital. This endogenously generates little immediate reallocation of labor in response to a trade shock and leads to a protracted transition, providing a better fit with the empirical findings. This more general framework allows us to investigate additional outcomes of trade shocks, such as the share of reallocation that happens on impact and the distributional consequences of trade for workers of different cohorts.

The model is an overlapping generations HOS model in which new workers of both low- and high-skill types enter the economy each period as old generations die. Both skill types are essential in both sectors, but the sectors differ in their skill intensities. Workers accumulate human capital that is specific to the sector of their employment. The empirical relevance of sector-specific human capital has been demonstrated most notably by Neal (1995), Parent (2000), and Kletzer (2001). Because our focus is on sector-specific human capital and sectoral reallocation we keep the neoclassical assumption of perfectly competitive markets and we consider an economy with homogeneous firms. This makes our analysis complementary with recent work emphasizing within-industry reallocation, such as Helpman, Itskhoki and Redding (2010).

In steady state, workers never switch sectors and the model replicates the standard HOS model. Yet when prices and wages adjust in response to a trade shock, sector-specific human capital generates endogenous rigidities. Although all workers have the opportunity to switch sectors, not all do so and wages do not immediately equilibrate across sectors. Young workers with little accumulated sector-specific human capital find the higher relative wages of the

expanding sector attractive enough to switch, whereas older workers with more accumulated human capital find it optimal to stay.

Our main finding is that most of the adjustment occurs not through immediate labor reallocation but rather through the entry of new generations of workers. Intuitively, the wage benefits of relocating to the expanding sector diminish as the economy adjusts to its new steady state, while human capital accumulated in the sector of previous employment is permanently idled if a worker switches. Consequently, even workers with a relatively small amount of accumulated specific human capital in the shrinking sector find it optimal not to switch. Technically, we use approximation methods to prove that the number of people who switch in response to a shock is second order in the price change whereas the length of adjustment is first order in the price change. Perhaps surprisingly, we show that the transition can be slower when human capital accumulates faster. Given the small amount of labor reallocation that occurs upon impact, the immediate effect of a trade shock on factor rewards is tied to sector of employment and not (as in the standard HOS model) to skill type. As the economy moves toward the new steady state, the standard Stolper–Samuelson result emerges whereby real wage changes are tied to skill type.

To relate our model to the current debate over the consequences of imports of low-skill labor-intensive products, we consider a shock that lowers the price of goods produced by the low-skill-intensive sector. First, although sector specificity prevents some individual low-skill workers in the shrinking sector from taking advantage of the higher wages in the expanding sector, overall the slower adjustment benefits low-skill workers because factors of production are kept longer in the low-skill-intensive sector. Second, a policy, financed by high-skill workers, which subsidizes workers of both types who switch sectors reduces the welfare of some of the low-skill workers who do not move by accelerating the transition.<sup>1</sup> This general equilibrium impact can be large enough to decrease the aggregate lifetime income of all low-skill workers alive at the time of the shock. This result continues to hold if one considers a retraining program that allows workers to keep part of their sector-specific human capital when switching sectors. Finally, there are distributional consequences across generations. For instance, low-skill workers in the high-skill intensive-sector who are old enough benefit from the decrease in the price of the low-skill-intensive good.

In two extensions we include physical capital and nonpecuniary sector preferences. For both extensions, most of the adjustment still occurs through the entry of new generations. We show that the transition is slower when physical capital is general instead of sector-specific. We also show that larger gross flows (generated by nonpecuniary sector preferences) further delay the transition to the new steady state but cause more reallocation upon the shock's impact.

To illustrate the workings of our model, we calibrate two versions of the model to data

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<sup>1</sup>Because there are no inefficiencies in the economy, such a subsidy also reduces output.

from the United States. We divide US manufacturing into two sectors of similar size according to their skill intensity. First, to stay as close as possible to the theoretical setting, we ignore capital and simulate a trade shock that reduces the price of the low-skill sector's product by 1 percent. The numerical results show a relatively long transition: it takes 2.11 years for low-skill wages and 7.41 years for high-skill wages to be equalized again. Moreover, the number of workers switching sectors in response to the trade shock is very small: only low-skill (resp. high-skill) workers with experience less than 0.04 years (resp. 0.27 years) switch sectors. Yet, since the difference in skill intensity across sectors is small, the reallocation predicted in this Heckscher-Ohlin model for such a small price change seems counterfactually large. Therefore, we also calibrate the model with sector-specific capital. This allows us to study large price changes and we find that, in this case, even for a 20 percent price change, the initial reallocation of workers represent less than a quarter of the steady-state reallocation.

Our results relate to a large empirical literature, typically based on the HOS-model, on the distributional consequences of exposure to international trade, both in developing and developed countries. For developed countries, Slaughter (2000) surveys an extensive literature of the 1990s on the role of international trade in explaining rising US inequality by correlating changes in the relative producer prices of low-skill intensive goods with relative wages of low-skilled workers as predicted by the Stolper-Samuelson theorem. He documents a limited support for the Stolper-Samuelson predictions especially in the 1970s, but argues that the methodology used is too limited to make firm conclusions. Yet, other authors find that trade played a more substantial role in the increase in inequality in developed countries, and Wood (1995) argues that methodology choices in computing the factor content of trade considerably affect the estimated impact of trade on inequality. Goldberg and Pavcnik (2007) survey the literature on developing countries and document that in general labor market adjustments are sluggish and trade liberalizations have not led to the reductions in income inequality predicted by factor endowment trade models. Though the limited labor mobility seems to contradict the central tenets of HOS theory and to undermine the empirical relevance of the HOS theory, our model suggests that a lack of labor reallocation and the presence of sector wage premia on impact are fully consistent with a HOS framework that incorporates rigidities. In fact, Robertson (2004) shows that the Stolper-Samuelson predictions emerge starting 3-5 years after a trade shock in Mexico.<sup>2</sup> Similarly, Gonzaga, Menezes Filho and Terra (2006) find Stolper-Samuelson effects in Brazil.<sup>3</sup> Mayda and Rodrik (2005) show that both in developed and developing countries preferences over trade policy are in line with HOS theory, another indication that in the long-run the Stolper-Samuelson theorem holds. Our model provides some guidance for evaluating

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<sup>2</sup>In addition, Robertson uses industry-specific tariff reductions. This addresses a potential bias in the estimation of wage effects from trade liberalizations as tariff reductions are often larger for low-skill intensive industries.

<sup>3</sup>Helpman et al. (2012), however, demonstrate that within occupations inequality (for which the HOS framework is silent) increased in Brazil after trade liberalization.

the time horizon at which Stolper-Samuelson effects might become important.

The model presented here also relates to a literature that examines the short-run dynamics of trade adjustment (Matsuyama, 1992, as mentioned, and Mayer, 1974, Mussa, 1978, and Neary, 1978, who analyze limited capital mobility). Yet only recently have efforts been made to incorporate sluggish labor adjustment into theoretical trade models. Most of these efforts – some of which include sector-specific human capital – focus on structurally estimated or calibrated models. For instance, Artuç, Chaudhuri and McLaren (2010) structurally estimate a dynamic rational expectations model of labor adjustment in which nonpecuniary idiosyncratic shocks in moving costs are the sole source of rigidities. Their model does not feature entering generations and sector-specific human capital, which (as we show) can endogenously generate rigidities for pecuniary reasons. Kambourov (2009) shows in a calibrated model that, in the presence of sector-specific human capital, firing costs reduce the benefits from trade liberalization. Closer to our work, Coşar (2013) calibrates a model with overlapping generations, sector-specific human capital, and job search, and Dix-Carneiro (2014) estimates a structural model with overlapping generations, sector-specific human capital, and switching costs.<sup>4</sup> To complement this literature, we focus on deriving sharp analytical predictions from a parsimonious dynamic HOS model in which the only impediment to labor mobility is sector-specific human capital. We discuss in more detail how these two papers compare and relate to our work in the main text.

The paper is organized as follows. In Section 2 we outline the model and derive the steady-state equilibrium. In Section 3 we analyze the transitional path, and in Section 4 we discuss the welfare implications of sector-specific human capital and its impact on the role of a trade adjustment policy. Section 5 presents two extensions featuring physical capital and nonpecuniary sector preferences. In Section 6, we calibrate and simulate the model, and Section 7 concludes. The main proofs and details on the calibration can be found in Appendix A, the remaining proofs are in Appendix B, which is available online.

## 2 The model

### 2.1 Production technology

We build a dynamic version of the standard, small open economy, HOS model. Time is continuous. At each point in time, two goods (indexed by  $i = 1, 2$ ) are produced competitively using two factors of production: low-skill and high-skill human capital. We denote the stock of low-skill and high-skill human capital in sector  $i$  by  $L_i$  and  $H_i$ , with per-unit wages of  $w_i$

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<sup>4</sup>Dix-Carneiro and Kovak (2015) provide empirical evidence on the slow adjustment of labor markets in Brazil following trade liberalization. Their work focuses on labor reallocation across regions and between the formal and informal sectors, instead of labor reallocation across tradeable sectors. Yet, their results are consistent with a model where factors are initially sector-specific and adjust slowly.

and  $v_i$ , respectively.<sup>5</sup> We assume that the production functions  $Y_i = F_i(L_i, H_i)$  are concave, exhibit constant returns to scale (CRS), are twice differentiable, and have weakly positive cross partial derivatives ( $\partial^2 F_i / \partial L_i \partial H_i \geq 0$ ). We use  $F_{iZ}$  to denote the derivative of the production function in sector  $i$  with respect to factor  $Z \in \{L, H\}$ .

Sector 1 is assumed to be high-skill intensive at every wage ratio. Let good 1 be the numéraire and let the price of good 2 be  $p$ , which is set exogenously at the world price. Competitive labor markets imply that human capital is paid its marginal product, and competitive goods markets imply that prices equal marginal costs.

To this standard framework we add overlapping generations of workers who accumulate nontransferable sector-specific human capital in their sector of employment. The stock of specific human capital for an individual worker in a particular sector is given by the (weakly) increasing function  $x_Z(a) \geq 0$ ,  $Z \in \{L, H\}$ , where  $a$  is the amount of time for which the worker has accumulated human capital in a given sector.<sup>6</sup> Note that the accumulation functions are different across types but the same across sectors. Our results can be generalized without affecting any of the qualitative results to accumulation functions which differ across sectors. The wage of a low-skill worker of experience  $a$  in sector  $i = 1, 2$  is thus  $w_i x_L(a)$ , while the wage of a high-skill worker of the same experience is  $v_i x_H(a)$ . Complete nontransferability implies that a sector switcher must start over from  $x_Z(0)$ , although the worker could employ human capital accumulated in his previous sector if he moved back. Labor within a given skill type is perfectly substitutable, so the total stock of human capital is the sum of the human capital for all workers employed in the sector.

This setup is motivated by findings in the labor economics literature that sector-specific human capital is important. Neal (1995) uses US data from displaced worker surveys to compare workers who are displaced and switch sectors with those who do not. He finds that the semi-elasticity of the wage loss at displacement with respect to tenure is 2–3 times as high for industry switchers. Neal also shows that workers who switch jobs but stay in the same sector are rewarded for their previous tenure as if it were seniority within their new firm, providing further evidence that an important component of human capital is sector-specific. Similarly, Parent (2000) demonstrates that much of the measured return to firm seniority loads on industry tenure when it is included in a regression, and Kletzer (2001) shows that displaced workers' earnings losses rise with tenure and age but are lower for workers who stay in the same sector. In addition, Dix-Carneiro (2014) structurally estimates that the returns to seniority

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<sup>5</sup> Although  $w_i$  and  $v_i$  technically denote the returns to a unit of human capital, we will abuse language slightly and refer to them as “low-skill wages” and “high-skill wages”, respectively.

<sup>6</sup> There is some debate in labor economics over the relative importance of sector and firm-specific human capital. If we were to include firm-specific human capital, then issues of bargaining would arise. Since we deliberately adhere closely to assumptions of the original HOS model—including that of perfect competition in the labor market—we focus solely on sector-specific human capital. Yet the effects derived here would also be present in a model with firm-specific human capital.

are imperfectly transferable across sectors in Brazil.<sup>7</sup>

Each overlapping generation lives for  $T$  periods, and the population grows at the rate of  $\eta > 0$ . Without loss of generality, we normalize the size of the population of low-skill and high-skill workers born at time  $t = 0$  to 1 and  $\bar{H}$  (respectively). For each type of worker  $Z \in \{L, H\}$ , we denote by  $Z_i(t)$  the mass of human capital of workers of skill type  $Z$  who work in sector  $i \in \{1, 2\}$ . To solve for the model in a convenient form, we defined the normalized mass of human capital in sector  $z_i(t)$  (with  $z = l$  for low-skill workers and  $z = h$  for high-skill workers) as the mass of human capital in sector  $i$  normalized by the size of the population of low-skill workers born at time  $t$ ; thus,  $z_1(t) = Z_1(t) / e^{\eta t}$ . Let  $n_Z(t)$  be the fraction of newborn workers of type  $Z$  who enter sector 1 at time  $t$ . If nobody has moved during their lifetime, then  $l_1(t) = \int_0^T n_L(t - \tau) e^{-\eta \tau} x_L(\tau) d\tau$  and  $h_1(t) = \bar{H} \int_0^T n_H(t - \tau) e^{-\eta \tau} x_H(\tau) d\tau$  and with analogous expressions for sector 2. Competitive labor markets and CRS production functions imply that we can write wages as a function of normalized factors:

$$w_1(t) = F_{1L}(l_1(t), h_1(t)) \text{ and } w_2(t) = pF_{2L}(l_2(t), h_2(t)), \quad (1)$$

$$v_1(t) = F_{1H}(l_1(t), h_1(t)) \text{ and } v_2(t) = pF_{2H}(l_2(t), h_2(t)). \quad (2)$$

## 2.2 Preferences

In a natural extension of the static HOS model, all workers have identical time-separable preferences with discount rate  $\delta$ . The lifetime utility of worker  $i$  at time  $t$  of age  $\alpha$  with consumption profile  $[C_{1i}(\tau), C_{2i}(\tau)]_{\tau=t}^{t+T-\alpha}$  is given by

$$\int_t^{t+T-\alpha} e^{-\delta(\tau-t)} u(C_{1i}(\tau), C_{2i}(\tau)) d\tau,$$

where  $u(C_1, C_2)$  is assumed to be homogeneous of degree 1 (a worker is of age 0 when he enters the labor force). The consumption profile is indexed by individual  $i$  because it can, in principle, depend on the history of an individual's sectoral employment. Let  $P(t)$  be the ideal price index associated with utility function  $u(\cdot)$  and the prices of consumption goods in period  $t$ . Workers choose their sector of employment each period to maximize lifetime utility, which with income  $[W_i(\tau)]_{\tau=t}^{t+T-\alpha}$  is

$$\int_t^{t+T-\alpha} e^{-\delta(\tau-t)} \frac{W_i(\tau)}{P(\tau)} d\tau.$$

If prices are expected to be fixed over the lifetime horizon, then this choice is equivalent to choosing the sector with the highest discounted lifetime income at labor market entry.<sup>8</sup>

<sup>7</sup>Kambourov and Manovskii (2009) find a substantial return to occupational tenure. To the extent that finely defined occupations differ across sectors, occupation-specific human capital can be reinterpreted as a form of sector-specific human capital.

<sup>8</sup>Our assumption that the utility function is homogeneous of degree 1 simplifies the analysis by pinning down the interest rate to the pure time-discount rate  $\delta$ . For small price changes, which are the focus of our analysis, this assumption is innocuous (provided there is a domestic assets market). See footnote 11.



## 2.3 Steady state

As is standard, we consider only parameters for which there is not complete specialization. Because the skill accumulation functions are identical across sectors, incomplete specialization implies that steady-state wages are equalized across sectors at  $w^{ss}$  and  $v^{ss}$  for low-skill and high-skill workers, respectively. This, in turn, means that workers never switch sectors, as doing so would result in a loss of human capital without a higher wage per effective unit (therefore, the experience of a worker in his sector of employment is the same as his age). The total stock of normalized human capital is then  $l^{\max} = \int_0^T e^{-\eta\tau} x_L(\tau) d\tau$  for low-skill workers and  $h^{\max} = \bar{H} \int_0^T e^{-\eta\tau} x_H(\tau) d\tau$  for high-skill workers. Wage equalization across sectors implies that

$$w^{ss} = F_{1L}(n_L l^{\max}, n_H h^{\max}) = p F_{2L}((1 - n_L) l^{\max}, (1 - n_H) h^{\max}), \quad (3)$$

$$v^{ss} = F_{1H}(n_L l^{\max}, n_H h^{\max}) = p F_{2H}((1 - n_L) l^{\max}, (1 - n_H) h^{\max}). \quad (4)$$

This steady state of the normalized model is isomorphic to the HOS model. Hence, the Stolper–Samuelson theorem implies that, if  $p$  falls, then the new steady state will feature an increase in production in sector 1, an increase in the relative factor rewards to high-skill workers  $v/w$ , and an increase in the relative use of low-skill workers in both sectors. These results are stated formally as follows.

**Lemma 1** *The steady state equilibrium described by (3) and (4) is isomorphic to the HOS model's equilibrium with  $l^{\max}$  and  $h^{\max}$  endowments of factors. In particular, the Stolper–Samuelson theorem is replicated such that, for a price change from  $p$  to  $p'$  with  $p' < p$ ,*

$$\frac{w^{ss'} - w^{ss}}{w^{ss}} < \frac{p' - p}{p} < 0 < \frac{v^{ss'} - v^{ss}}{v^{ss}},$$

where  $v^{ss'}$  and  $w^{ss'}$  are the respective steady-state values of high- and low-skill wages for price  $p'$ .

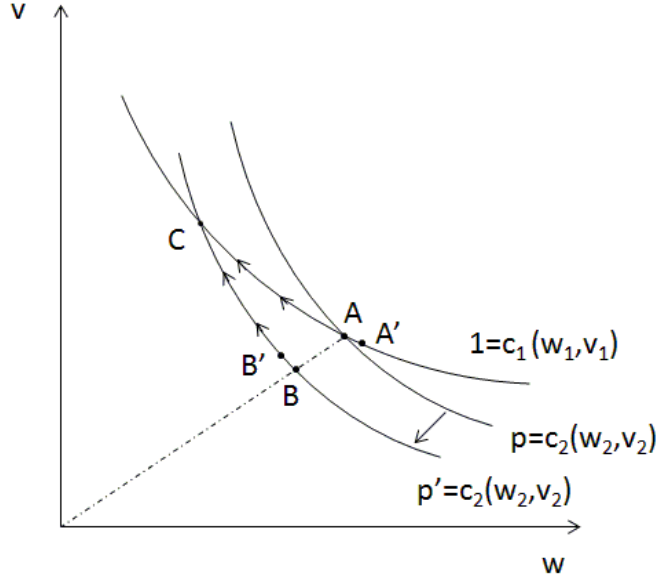
## 3 Transitional dynamics

### 3.1 Description

This paper's principal contribution consist of analyzing the transition between the two steady states. For expositional clarity, we consider an unexpected instantaneous and permanent downward shift (at time 0) in the price of the good produced by the low-skill–intensive sector (sector 2) from  $p$  to  $p'$ .

We conduct our analysis with the aid of Figure 1, which plots the isocost curves defined by  $1 = c_1(w_1, v_1)$  and  $p = c_2(w_2, v_2)$  as implied by the zero-profit conditions. The initial

Figure 1: Transition paths along the cost curves



Notes: The wage paid to a unit of low-skill human capital is  $w$  and the wage paid to a unit of high-skill human capital is  $v$ . The economy is originally in a steady state at point A, at the intersection of the two loci along which price equals marginal cost in each sector. Sector 1 is high-skill intensive. A trade shock causes the price of the good produced by sector 2 to drop. On impact, wages in sector 2 shift to point B' (and wages in sector 1 to point A'). As new generations enter, the economy transitions along the two cost curves (as indicated by the arrows) to reach the new steady state at point C.

steady state is at point A, where  $v_1 = v_2 = v^{ss}$  and  $w_1 = w_2 = w^{ss}$ . A well-known property of such cost curves is that the perpendicular vector at a given point gives the relative use of factors; the flatter slope of the vector associated with sector 2 reflects this sector's being more low-skill intensive than sector 1. A drop in prices in sector 2 to  $p'$  moves the isocost curve associated with sector 2's zero-profit condition southwest; hence, for a given allocation of labor, wages for both types in sector 2 decline proportionally. The eventual steady state is then given by point C, which implies higher relative wages for high-skill workers (i.e., the standard Stolper–Samuelson theorem described previously).

The following proposition characterizes the transitional phase for a small price change from  $p$  to  $p'$ .

**Proposition 1** *For a (small) price drop from  $p$  to  $p'$  (with  $dp \equiv p' - p < 0$ ), there exists an equilibrium fully described by the wage paths  $(w_1(t), w_2(t), v_1(t), v_2(t))_{t=0}^{\infty}$  and the tuple  $\{t_1, t_2, a_L, a_H\}$ , where the following statements hold.*

- *There is a worker of age  $a_Z$  ( $Z \in \{L, H\}$ ) in sector 2 who is indifferent between moving and*

not moving. All workers of type  $Z$  who are younger than  $a_Z$  in sector 2 move to sector 1; all older workers remain.

- Workers move only on impact of the trade shock at  $t = 0$ .
- The time at which wages are equalized first is given by  $w_1(t_1) = w_2(t_1)$  and  $v_1(t_2) = v_2(t_2)$ . Moreover,  $w_1(t) = w_2(t)$  for all  $t \geq t_1$  and  $v_1(t) = v_2(t)$  for all  $t \geq t_2$ .
- The equilibrium maximizes the present value of production.

**Proof.** See Section A.1 in the Appendix. ■

The proof is given for marginal price changes and requires a positive population growth  $\eta > 0$ .<sup>9</sup> In Section 6, we establish that the same equilibrium exists for reasonable parameter values with nonmarginal price changes. We provide intuition for the structure of the equilibrium here.

First, the transition to the new steady state cannot be immediate. If it were, then a sufficient number of workers would switch sectors for wages to equalize across sectors. In that case, some workers would experience a loss in human capital without an offsetting increase in wages and so the move for them would not have been optimal. Second, there will be some sector switching at time 0. This is because the youngest workers have little human capital to lose by switching from sector 2 to sector 1, so a difference in wages, (as implied by noninstantaneous adjustment) will lead some workers to move.

The equilibrium is efficient in the sense that it maximizes the present value of production from time 0 to infinite (if  $\delta > \eta$ , this present value of production is infinite, but the equilibrium still maximizes the present value of production from time 0 up until any time  $t \geq T$ ).

These two points can also be illustrated using Figure 1. As already mentioned, point A gives the original steady state. During the transition, the economy will be described by two points, one for each sector on its corresponding isocost curve, until wages are again equalized. If there were no immediate reallocation, then wages in sector 2 would be given by point B and there would be a proportional drop of  $dp/p$  in both low- and high-skill wages in that sector. Instead, since there is some reallocation on impact, the economy jumps to point B', which is "near" point B in a sense to be made precise shortly (although B' is to the northwest of B in figure 1, the opposite positions are possible). Wages in sector 1 are described by a point A' near A. The equilibrium wages in the two sectors eventually transition along each sector's respective isocost curve until point C, the new steady state.<sup>10</sup> As neither high-skill nor low-

<sup>9</sup> A positive growth rate is required for mostly technical reasons. After  $T - \max\{a_L, a_H\}$  time periods, when the first movers are dying out, the loss of human capital from dying generations takes a discrete jump for which new entering generations must compensate. Some population growth ensures that the entering generation is large enough (relative to the loss of human capital from dying generations) that no worker would want to switch sectors again; in our simulations, a growth rate of 2 percent was sufficient. Alternatively, some gross flow between the sectors (for instance because of stochastic nonpecuniary sector preferences of workers) could be used to circumvent the problem.

<sup>10</sup> Point C describes the wages of the new steady-state, but these wages are reached before all variables reach

skill wages can be instantaneously equalized,  $v_1(0) > v_2(0)$ ,  $w_1(0) > w_2(0)$ , and  $B'$  lies to the southwest of  $A'$ . Since we consider an equilibrium in which once wages have been equalized they remain equalized, the equilibrium point in sector 2 must always lie weakly southwest of the equilibrium point in sector 1, and  $v_1(t) \geq v_2(t)$  and  $w_1(t) \geq w_2(t)$  at all points.

Define the ages of the low-skill and high-skill workers who are indifferent to moving as  $a_L$  and  $a_H$ , respectively. These ages are given by:

$$\int_0^{T-a_L} w_1(\tau) x_L(\tau) e^{-\delta\tau} d\tau = \int_0^{T-a_L} w_2(\tau) x_L(a_L + \tau) e^{-\delta\tau} d\tau, \quad (5)$$

$$\int_0^{T-a_H} v_1(\tau) x_H(\tau) e^{-\delta\tau} d\tau = \int_0^{T-a_H} v_2(\tau) x_H(a_H + \tau) e^{-\delta\tau} d\tau; \quad (6)$$

here the left-hand (resp., right-hand) side equals the lifetime earnings associated with switching to sector 1 and (resp., staying in sector 2). A worker older than the indifferent worker will lose more sector-2-specific capital and would have fewer years to enjoy the higher wages in sector 1; hence he will remain in sector 2. Similar logic implies that all workers younger than the indifferent worker will switch.

Because wages are not completely equalized on impact, all new workers will enter sector 1 for some time. Low-skill (resp. high-skill) workers will do so until  $w_1(t) = w_2(t)$  (resp.  $v_1(t) = v_2(t)$ ) which by definition occurs first at  $t = t_1$  (resp.  $t = t_2$ ). Without loss of generality, we consider parameter values for which  $t_1 < t_2$ . Doing so implies that the normalized stock of low-skill human capital in sector 1 at time  $t \leq t_1$  can be written as the sum of the mass of existing workers prior to time 0 (term 1), the mass of workers that move at time 0 (term 2), and newly born workers who all enter sector 1 until wages are equalized at time  $t_1$  (term 3):

$$l_1(t) = \underbrace{n_L \int_t^T e^{-\eta\tau} x_L(\tau) d\tau}_{\text{term 1}} + \underbrace{e^{-\eta t} (1 - n_L) x_L(t) \int_0^{a_L} e^{-\eta\tau} d\tau}_{\text{term 2}} + \underbrace{\int_0^t e^{-\eta\tau} x_L(\tau) d\tau}_{\text{term 3}}, \quad (7)$$

for  $0 \leq t \leq t_1$ . The mass of low-skill human capital in sector 2 is given by the mass of those who stay,  $l_2 = (1 - n_L) \int_{t+a_L}^T e^{-\eta\tau} x_L(\tau) d\tau$ . Equivalent expressions hold for high-skill workers whose wages are equalized at  $t = t_2$ .

The transition can therefore be split into three phases. In phase I ( $t < t_1$ ) we have  $w_1(t) > w_2(t)$  and  $v_1(t) > v_2(t)$ , and new workers enter only sector 1. In phase II ( $t_1 \leq t < t_2$ ),  $w_1(t) = w_2(t)$  and  $v_1(t) > v_2(t)$ ; in this phase, low-skill workers enter both sectors (and so keep wages equal across sectors) while high-skill workers enter only sector 1. In phase III ( $t_2 \leq t$ ) we have  $w_1(t) = w_2(t)$  and  $v_1(t) = v_2(t)$ , and the allocation of entering workers across sectors ensures that wages remain equalized for both types.

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their new steady state levels. The Rybczynski theorem guarantees that, once wages are equalized, they will remain so until human capital reaches its maximum level and the new steady state is reached.

It is worth noting that, in each phase, the model is isomorphic to a series of well-studied models in trade theory. During phase I, the model is isomorphic to a series of models with completely sector-specific factors. During phase II, it is isomorphic to a series of Jones (1971) models. Finally, in phase III it is isomorphic to a series of HOS models.

### 3.2 Adjustment through new generations

To assess the extent to which the adjustment to the new steady state occurs by workers switching sectors versus new generations entering the workforce in one sector only, we use a Taylor expansion to obtain explicit expressions for the age of the indifferent workers as well as the time until wages are equalized. We formalize the results as follows.

**Proposition 2** *Given the price change described in Proposition 1, the following statements hold.*

- *The times until equalization of wages  $t_1$  and  $t_2$  are of first order in  $dp$ . If  $t_1 < t_2$ , then  $t_1$  is given by*

$$t_1 = \frac{w^{ss}}{(1 - n_L) x_L(0) [w_{1L} + w_{2L}] + (1 - n_H) x_H(0) \bar{H} [w_{1H} + w_{2H}]} \frac{dp}{p} + o(dp), \quad (8)$$

where  $w_{iZ}$  denotes the derivative of low-skill wages in sector  $i = 1, 2$  with respect to labor type  $Z \in \{L, H\}$ .

- *The ages of the indifferent workers  $a_L$  and  $a_H$  are of second order in  $dp$ . If  $t_1 < t_2$ , then  $a_L$  is given by:*

$$a_L = -\frac{x_L(0) t_1}{2 \int_0^T e^{-\delta\tau} x'_L(\tau) d\tau} \frac{dp}{p} + o(dp^2). \quad (9)$$

**Proof.** See Section A.1. ■

Similar expressions hold for the age  $a_H$  of the indifferent high-skill worker and the time  $t_2$  at which high-skill wages are equalized; these expressions are derived in the Appendix.<sup>11</sup> Symmetric expressions hold when  $t_1 > t_2$ . The age of the indifferent worker is of second order whereas the time until wages are equalized again is of first order, which implies that most of the adjustment is driven by entry. Formally, the total amount of low-skill human capital reallocated in steady-state is first order in the price change and can be written as  $dl_1^{ss} = \frac{(v_{1H} + v_{2H})w^{ss} - v^{ss}(w_{1H} + w_{2H})}{(v_{1H} + v_{2H})(w_{1L} + w_{2L}) - (w_{1H} + w_{2H})^2} \frac{dp}{p}$ , while the mass of low-skill human capital moving upon shock is given by  $a_L (1 - n_L) x_L(0)$ . Hence the initial adjustment's share of total labor reallocation for low-skill workers,  $\chi_L$ , is given by

$$\chi_L = \frac{-x_L(0) w^{ss} \left( (v_{1H} + v_{2H}) (w_{1L} + w_{2L}) - (w_{1H} + w_{2H})^2 \right) \frac{dp}{p} + o(dp)}{2 \int_0^T e^{-\delta\tau} x'_L(\tau) d\tau \left( (v_{1H} + v_{2H}) w^{ss} - v^{ss}(w_{1H} + w_{2H}) \right) \left( w_{1L} + w_{2L} + \frac{(1 - n_H) x_H(0)}{(1 - n_L) x_L(0)} \bar{H} (w_{1H} + w_{2H}) \right)}, \quad (10)$$

<sup>11</sup>With a more general homothetic function and domestic asset markets, the proposition still holds if one replaces  $\delta$  with the steady-state interest rate in equation (9).

which is first-order and increasing in the price change (and so is the equivalent term for high-skill workers). In other words, whereas Matsuyama (1992) exogenously imposes that no workers can reallocate, we endogenously derive that few will do so. The endogenous choice of reallocation has the additional benefit of enabling us to analyze policy designed to increase the number of workers that reallocate on impact (see Section 4.3).

To understand the intuition behind this result, consider the indifferent low-skill worker's costs and benefits of moving to sector 1, which are plotted in Figure 2. The benefits are a higher wage per unit of human capital until time  $t_1$ , when wages are again equalized. Because the wage difference and the time until wages equalize are both first order in the price change, these benefits will be second order in that price change. The costs are a lower level of sector-specific human capital, and—since a worker has no incentive to switch back—they represent, in effect, a permanent loss of this human capital. The costs are thus first order in the age of the worker at the time of the trade shock. The age of the indifferent worker equates costs and benefits; therefore, whereas  $t_1$  is of first order in the price drop, the age of the indifferent worker is second order in that price change.<sup>12</sup> The assumption of rational expectation plays a crucial role here: it is because workers correctly anticipate that the wage gap will quickly close that very few workers move. Alternative assumptions about expectations could make the mass of switchers first order.

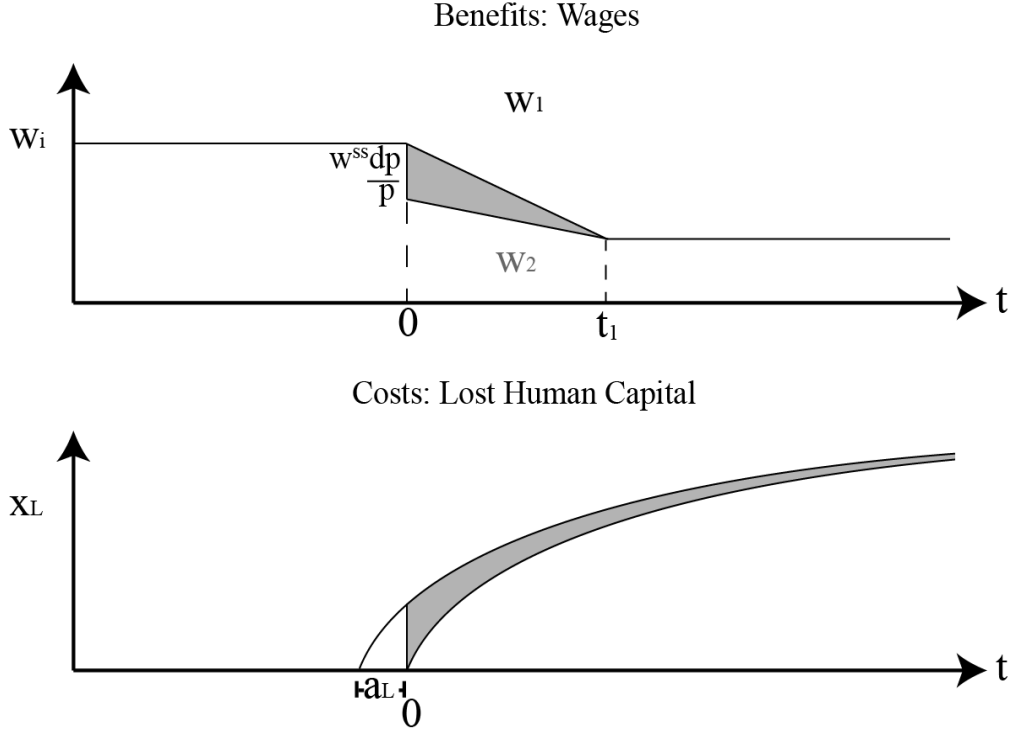
Equation (8) follows from noting that the low-skill worker wage differential created on impact is given by  $w^{ss} dp/p$  and that the denominator in (8) captures the effect on this wage differential of the inflow of new generations. The adjustment time depends on the share of people already allocated to sector 2, the production function, and the human capital accumulation function.

Perhaps surprisingly, more rapid sector specific human capital accumulation can have a negative effect on the speed of adjustment. This follows because a faster accumulation of human capital has two opposing effects on the speed of transition. After a move, switchers accumulate new human capital more quickly. Yet, since all workers accumulate human capital faster, the total stock of human capital in the economy is higher, such that any given change in human capital has a smaller impact on relative wages. As most of the adjustment occurs through entry, the transition period must be longer. To see that the second effect can dominate, consider the special case in which the high-skill and low-skill capital accumulation functions are proportional—that is,  $x_H = \gamma x_L$  for  $\gamma$  a constant; then replace the low-skill capital accumulation function with some  $\hat{x}_L(a) \geq x_L(a)$ , where  $\hat{x}_L(0) = x_L(0)$  and  $\hat{x}_L(T) = x_L(T)$ , and replace the high-skill capital accumulation function with  $\hat{x}_H = \gamma \hat{x}_L$ . Such a function implies the same initial and terminal levels of human capital, but faster accumulation. Since

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<sup>12</sup>The result that few people move does not depend on the permanent and unanticipated nature of the price shock we are considering. If the price change were perceived to be temporary then the incentive to move would be even lower.

Figure 2: Intuition For  $a_L$  Being of Second Order



Notes: The shaded area in the upper panel shows the benefits of switching (a higher wage in sector 1 relative to sector 2) while the shaded area in the lower panel shows the costs (forgone accumulated human capital). This explains why few individuals move on impact: the benefits are second order in the price change but the costs are first order. For the shaded areas to be equal, the number of switchers must be small.

$w_{1L} = F_{1LL}(n_L l^{\max}, n_H h^{\max})$  is the second derivative of a CRS production function, it is homogenous of degree  $-1$ . Hence, the change in capital accumulation function from  $(x_L, x_H)$  to  $(\hat{x}_L, \hat{x}_H)$  will increase  $l^{\max}$  and  $h^{\max}$  proportionally and thereby decrease  $w_{1L}, w_{2L}, w_{1H}, w_{2H}$  and *increase* the time until wages are equalized. This comparative static extends to  $t_2$ , the time at which wages of high-skill workers are equalized.<sup>13</sup>

Equation (9) results from noting that a first-order approximation to the accumulated wage difference is  $1/2 \times t_1 w^{ss} dp/p$  per unit of human capital (which is close to  $x_L(0)$  for the indifferent worker) while a first-order approximation to the loss is given by  $a_L w^{ss} \int_0^T e^{-\delta\tau} x'_L(\tau) d\tau$  since, for every subsequent period, the worker's human capital will be lower by  $x'_L(t) a_L$ . Faster accumulation of human capital has an ambiguous effect on the number of people moving. As explained previously, it increases  $t_1$  but also increases the denominator of (8) if the time

<sup>13</sup>This result is formally proved in Appendix B.2. An analogous argument demonstrates that the length of the transition is decreasing in the rate of population growth  $\eta$  when the accumulation function is identical across sectors.

discount rate is positive:  $\int_0^T e^{-\delta\tau} \widehat{x}_L(\tau)' d\tau > \int_0^T e^{-\delta\tau} x_L(\tau)' d\tau$  if  $\delta > 0$ . When human capital increases faster, losing a given level of experience represents a bigger loss of human capital in the short run and a smaller loss in the long run; with positive discounting, the initial bigger loss matters more. Even so, the initial adjustment's share of total labor reallocation increases when the learning curve becomes steeper.<sup>14</sup>

Further insight into the transition process can be gained by considering the special case of CES production functions. When the elasticity of substitution,  $\sigma$ , is the same in both sectors, equation (8) can be written as

$$t_1 = \frac{-\sigma \frac{dp}{p}}{\frac{(1-n_L)x_L(0)}{l^{\max}} \left[ \frac{\theta_{1H}}{n_L} + \frac{\theta_{2H}}{1-n_L} \right] - \frac{(1-n_H)x_H(0)\bar{H}}{h^{\max}} \left[ \frac{\theta_{1H}}{n_H} + \frac{\theta_{2H}}{1-n_H} \right]} + o(dp), \quad (11)$$

where  $\theta_{iH}$  is the factor share of high-skill workers in sector  $i = 1, 2$ . Consider the first term in the denominator. Each period, a fraction  $(1 - n_L)x_L(0)/l^{\max}$  of low-skill human capital is reallocated from sector 2 to sector 1 through the death of old and the entry of new generations. This reduces low-skill wages in sector 1 and increases them in sector 2. The relative importance of these two effects in closing the low-skill wage gap across sectors is captured by the relative importance of  $\theta_{1H}/n_L$  and  $\theta_{2H}/(1 - n_L)$ . As is standard, the effect on low-skill wages from changes in relative factors depends on the factor share of high skill workers,  $\theta_{iH}$ , but the original allocation of low-skill workers is crucial: if  $n_L$  is close enough to 1 that most low-skill labor was initially allocated to sector 1, then the reallocation from sector 2 has little effect on sector-1 wages and so most of the adjustment in the wage gap comes from sector 2. The second term in the denominator captures the reallocation of high-skill workers. The interpretation is analogous except that the term is negative because the reallocation of high-skill workers to sector 1 widens the low-skill wage gap. Since the adjustment transpires through changes in factor intensity, the elasticity of substitution,  $\sigma$ , plays a crucial role. For higher  $\sigma$ , any change in factor intensity is associated with a smaller change in wages and thus, a longer adjustment time.

Our model shows that the absence of short-run labor reallocation does not mean that Heckscher–Ohlin forces are unimportant. This is consistent with empirical research such as Revenga (1992) and Artuc et al. (2010) who find substantial inter-industry labor reallocation for the United States at a 5 years horizon. In developing countries, labor reallocation tends to take more time, as additional sources of rigidities are likely to play a larger role.<sup>15</sup>

<sup>14</sup>The transformation of  $x_L$  into  $\widehat{x}_L$  does not affect the second fraction in (10), so  $\chi_L$  increases. One can easily demonstrate that  $\chi_H$  also increases when the learning curve becomes steeper.

<sup>15</sup>For instance, Wacziarg and Wallack (2004) consider 25 episodes of liberalization across many countries and find, over 2–5 year horizons, no evidence of labor reallocation at the 1-digit level and only weak evidence at the 3-digit level. Topalova (2010) finds very limited labor reallocation in India, partly because of very rigid labor laws. Menezes Filho and Muendler (2011) study Brazil's trade liberalization using linked employer–employee data and find that trade liberalization induces job displacement; however, exporters in comparative advantage



In addition, the model predicts that young workers are more responsive to trade shocks, which is consistent with the data. Kletzer (2001) shows that workers with low tenure are considerably more likely to be displaced as a result of product competition from imports. More generally, our model predicts that the net flows of workers between sectors is more sensitive to wage differentials for younger workers. Artuc et al. (2010) structurally estimate a model which does not feature sector-specific human capital; they find cost of switching sectors that are around 30 per cent lower for young workers than old workers.<sup>16</sup>

Closer to our work, Dix-Carneiro (2014) structurally estimates a dynamic Roy model with high- and low-skill labor, multiple sectors, capital, a labor supply decision, moving costs, and human capital that is imperfectly transferable across sectors. He matches gross flows across sectors by including idiosyncratic productivity and taste shocks. The model is estimated using matched employer–employee data from Brazil. He finds a substantial role for sector-specific human capital with yearly accumulation rates of around 4 to 9 per cent. He simulates a trade shock of a 30 percent reduction in tariffs on the high-tech industry and finds a relatively large and fast labor reallocation, with 80 percent adjustment after only three years. Relative to our analysis, two reasons can explain this quick reallocation. First, the price shock is large (the reallocation is slower when he considers a 10 percent price shock). Second, the trade experiment is performed on the sector for which human capital is most easily transferred to other sectors.

Similarly, Coşar (2013) builds an overlapping generations model, which features both search frictions and sector-specific human capital, and then calibrates this model using aggregate data from Brazil. Coşar’s quantitative results suggest that sector-specific human capital is critical to explaining the sluggishness of the transition.

Note that the result that reallocation on impact is second order in the price change would apply in more general settings than that of a HOS model. Any model with overlapping generations and perfect foresight where reallocation costs are not proportional to the price change would feature this result.

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sectors hire fewer workers in the short term, which results in a slow labor reallocation process. Goldberg and Pavcnik (2007) review the literature on trade liberalization in the developing world and show that, in almost every case, labor reallocation in the short run was extremely limited.

<sup>16</sup>Kambourov (2009) finds that industry mobility decline sharply with age in the United States: over the time period 1969-1997, they estimate a probability of switching industry at the 2-digit level of 21.3% per year for 23-28 year old high-skill workers (that is workers with at least some college education), but a probability of switching of 4% for 47-69 year old high-skill workers (for unskilled workers the corresponding numbers are 25% and 4.8%). These estimates are for steady-state gross flows though and are therefore not directly comparable with our analysis.

## 4 Welfare implications

The structure of the model allows us to conduct welfare analysis across sectors, skill types, and generations. We begin in Section 4.1 by analyzing the effects on real factor rewards. In Section 4.2 we turn to a welfare analysis that compares our model economy to one in which human capital is not sector specific. Finally, in Section 4.3, we consider the role of trade adjustment assistance.

### 4.1 Real factor rewards

Since few workers switch sectors in the immediate aftermath of the trade shock and since our model replicates the HOS model in the long run, the following corollary holds.

**Corollary 1** *Consider a price change and equilibrium as described in Proposition 1. Then, for small price changes:*

- *in the short run, wages are tied to sector of employment and move proportionally with price;*
- *in the long run, wages are tied to the type of skill and so the Stolper–Samuelson theorem applies.*

Consequently immediately after a price shock, the real wage of workers in sector 1 (including those who moved) will be higher than without the shock, irrespective of their skill level. Similarly, all workers in sector 2 will have a lower real wage than without the trade shock. This occurs because the wage change is first order in the price change, while the reallocation of workers is second order. Therefore, for small price changes, the direct effect of the price change dominates the indirect effect going through workers' reallocation. Once wages are equalized, however, the Stolper–Samuelson result applies; therefore starting at some time before wages are equalized for both skill types, all high-skill workers will have a higher real wage than without the price shock and the opposite will hold for low-skill workers. Hence, old workers from sector 1 benefit from the price change and old workers from sector 2 lose from it; whereas whether young workers lose or gain depends on their skill type. In fact, several papers have reported that real wages do not follow Stolper–Samuelson's prediction in the short run (see the survey by Goldberg and Pavcnik, 2007). Our results show that this finding does not preclude the Stolper–Samuelson theorem from accounting accurately for the welfare consequences of trade liberalization for most of the population (as mentioned in the introduction, this is in line with Robertson, 2004, who find Stolper–Samuelson effects at a 3–5 years horizon, and with Mayda and Rodrik, 2005, who find that political preferences regarding trade policies fall along Stolper–Samuelson's predictions).

It also follows from the corollary that the skill premium controlling for the industry of employment does not change on impact, yet the economy's overall skill premium increases since the trade shock favors the high-skill intensive sector. As the economy moves towards

steady-state, the aggregate skill premium increases further (the within sector dynamics are quite complicated, and we will return to them in Section 6).

Here the supply of skills has been kept exogenous. In a HOS model where the proportion of high-skill and low-skill workers is endogenous, for instance because of heterogeneous costs of schooling, an increase in the skill premium would be associated with an increase in the share of high-skill workers. This, however, would not affect the steady-state skill premium, which is entirely determined by international prices, and therefore is the same whether the supply of skills is endogenous or not. In our model with sector-specific human capital, similar dynamics would apply, but, since the skill premium increases gradually, the share of high-skill workers would also increase gradually until  $\max(t_1, t_2)$ .

## 4.2 Comparison with a model of general human capital

In order to identify the winners and losers from the nontransferability of human capital, we compare our economy with one in which any accumulated human capital is general and can costlessly be utilized in both sectors. Such an economy features instantaneous adjustment to the new steady state and, since all human capital is fully transferable, the model is isomorphic to the standard HOS model at all times. We define the aggregate welfare of a generation as the sum of the discounted lifetime income of all its members. Then, following an unanticipated and permanent price shock, the aggregate welfare of a given generation born before the price change must be lower under sector-specific than under general human capital. In the context of a model with general human capital, the allocation of the sector-specific human capital model is equivalent to a misallocation of factors.<sup>17</sup>

**Proposition 3** *For an unanticipated permanent price drop in the low-skill-intensive sector's product:*

- *all low-skill workers are better-off in the economy with sector-specific human capital than in the economy with general human capital; and*
- *all high-skill workers are worse-off in the sector-specific human capital economy than in the economy with general human capital.*

**Proof.** See Section A.2. ■

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<sup>17</sup>It further holds that the difference in total welfare is third order in the price change. By the envelope theorem, a small factor misallocation has only a second-order effect on the total value of production, and the factors are misallocated for only a short period of time (until wages are equalized); hence the overall effect is of third order. The sector-specific human capital economy suffers also from the loss of effective units of human capital that results when workers who switch sectors must begin anew to accumulate human capital in their new sector. However, this loss is only fourth order because the mass of switchers is second order and each switcher will have accumulated only a second-order amount of human capital. Therefore, it is the indirect consequence—namely, the lack of mobility across sectors—that explains most of the cost of human capital's nontransferability.

The transition created by the nontransferability of human capital “protects” low-skill workers.<sup>18</sup> It is noteworthy that even the low-skill workers who switch sectors (and therefore lose sector-specific human capital) are better-off with sector-specific than with general human capital. The logic behind this result is based on Corollary 1: in the sector-specific human capital economy, wages for low-skill workers are at their lowest point in the long run (when the Stolper–Samuelson theorem applies); in a general human capital economy, however, the steady state is reached immediately. Proposition 3 suggests a qualification to the typical argument that slow adjustment is costly for those in a sector adversely affected by trade shocks. If we seek to make this argument for the low-skill workers, then sector-specific human capital and other factor rigidities are insufficient. One would need to add other elements—such as unemployment and search frictions, from which this model abstracts—in order to generate a decline in the welfare of low-skill workers due to a slow transition.

### 4.3 Trade adjustment assistance policy

To build further on this point, we next consider the impact of a relocation program for workers willing to switch to sector 1; similar programs, which aim at accelerating the transition, are studied in Coşar (2013) and Dix-Carneiro (2014). More specifically, we assume that the government distributes some income with a present value  $S_Z$  to all workers of type  $Z$  who permanently switch from sector 2 to sector 1 at time  $t = 0$ .<sup>19</sup> For simplicity, we focus on the case where the accumulation functions are proportional to each other (i.e.  $x_H(t) = \gamma x_L(t)$ ), and we assume that the sum received by both groups is proportional to their steady-state wages, that is, there is a subsidy coefficient  $s$  such that  $S_L = s x_L(0) w^{ss}$  and  $S_H = s x_H(0) v^{ss}$ . The program is financed through lump-sum taxation on high-skill workers. Because there are no inefficiencies in our economy, such a program has a negative impact on output and so will hurt the economy as a whole.

Assuming that  $s$  is first order in the price change and small enough that full adjustment is not reached on impact, the structure of the equilibrium will be conserved. However, the number of workers switching on impact is now first order and determined by the cut-off wages

$$a_L = a_H + o(dp) = \xi s + o(dp), \quad (12)$$

with  $\xi \equiv x_L(0) \left( \int_0^T x_L(\tau)' e^{-\delta\tau} d\tau \right)^{-1}$ . At first order, the age of the indifferent worker is the same for high-skill and low-skill workers. As a result, such a subsidy program simply shifts the

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<sup>18</sup>For a specific generation of a given skill type, the difference between its welfare in the sector-specific human capital economy and its welfare in the general human capital economy is second order.

<sup>19</sup>For simplicity we assume that the government does not differentiate between workers of different ages, although doing so would not change our results. We assume that workers who switch back to sector 2 must reimburse the government; we could instead assume that payments are distributed over time in such a way that workers never want to move back. Finally, we assume that the income is distributed over time so that old workers do not move simply to benefit from the subsidy just before dying.

transition process, so that the economy at time  $t$  is identical (at first order) to the economy without the subsidy at time  $t + \xi s$ . Since low-skill wages are higher during the transition than in steady-state, this subsidy program hurts the group of low-skill workers who do not directly benefit from it.<sup>20</sup> For a small subsidy only few low-skill workers move and although the recipients of the program benefit, the direct impact of the program on aggregate low-skill income is small. In spite of the subsidy program being financed entirely by high-skill workers, the aggregate income of low-skill workers can suffer from it, as stipulated in the following proposition.<sup>21</sup>

**Proposition 4** *Consider a small price change, and assume that the subsidy coefficient  $s$  is first order in the price change and that full adjustment is not reached. Then the aggregate present value of lifetime income of all low-skill workers alive at  $t = 0$  is reduced by the subsidy if  $s$  is small enough.*

**Proof.** See Section B.3. ■

For CES production functions, with the same elasticity between high-skill and low-skill workers,<sup>22</sup> the proposition can be extended to a program which subsidizes the reallocation of low-skill workers relatively more than that of high-skill workers, so that the subsidy coefficients obey  $s_H < s_L$ . In this case, the subsidy program is not equivalent to a shift in the transition process, and low-skill workers who remain in sector 2 might benefit from it (as high-skill workers do not leave their sector as fast as low-skill workers). Yet, the negative impact on low-skill workers from sector 1 outweigh the possibly positive impact on low-skill workers from sector 2.

It should be clear that, although this analysis depends crucially on the relative skill intensities of the two sectors, it does *not* hinge on sector-specific human capital being the source of the rigid adjustment. In a Heckscher–Ohlin framework, any subsidy program that succeeds in more rapidly shifting the economy’s resources to the skill-intensive sector entails a general equilibrium effect that is detrimental to the welfare of low-skill workers. More generally, this result demonstrates the importance of bearing in mind the long-term effects of trade shocks when assessing the implications of a subsidy for switching sectors. In the United States, where trade shocks are usually considered to be detrimental to low-skill workers in the long-run, such program might have negative distributional consequences.

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<sup>20</sup> A small number of old low-skill workers who retire during the transition may benefit from the program, as wages in sector 2 may be increasing during  $(0, \min(t_1, t_2))$ .

<sup>21</sup> Interestingly, the logic of this analysis can be extended to a retraining program that allows workers to transform their sector 2-human capital into sector 1-human capital up to some experience level  $\bar{a}$ . In contrast to the subsidy, the retraining program increases the present value of production. However, beneficiaries from the program themselves might lose from it since in the limit where  $\bar{a}$  is large enough, the economy is identical to the general human capital case.

<sup>22</sup> A weaker sufficient condition is that  $(1 - n_L)w_{2L} - n_L w_{1L} > 0$  for  $t_1 \leq t_2$  or  $(1 - n_L)v_{2L} - n_L v_{1L} < 0$  otherwise.

Both Dix-Carneiro (2014) and Coşar (2013) consider the welfare effects of a similar subsidy program in numerical models. Dix-Carneiro (2014) finds that, although a switching subsidy reduces overall welfare by introducing distortions, it does increase the welfare of low-skill workers. This is line with our analysis because he considers a negative price shock to high-tech manufacturing—a sector that is relatively high-skill intensive—whereas we consider a shock to the low-skill-intensive sector. If we had considered a negative price shock to the skill-intensive sector, then the analogue of Proposition 4 would likewise have carried through; we would have found a negative effect on high-skill workers and a positive effect on low-skill workers, just as Dix-Carneiro does.

Coşar (2013) finds *positive* welfare effects for two reasons. First, he does not consider the distinction between low- and high-skill workers, so the unintended distributional consequences at the heart of our model are absent. Second, his model features an externality whereby workers do not capture the full social benefit of their human capital. This makes workers inefficiently reluctant to accept jobs, which slows down the adjustment period following a trade shock. This inefficiency can be partly overcome by the switching subsidy, which implies an increase in overall efficiency. This rationale for a subsidy is not, however, specific to trade-displaced workers; it applies equally to any subsidy that encourages more search, a general point that is emphasized by Kletzer (2001).

## 5 Extensions

In this section we show that our results are robust to introducing an additional factor of production, or nonpecuniary sector preferences that generate bilateral flows of workers across sectors.

### 5.1 Physical capital

In this extension we allow for physical capital as a factor of production. The production functions are now given by

$$Y_i = F_i(L_i, H_i, K_i) \text{ for } i \in \{1, 2\},$$

where  $K_i$  is the physical capital employed in sector  $i$ . We assume that both functions are CRS with positive cross partial derivatives. The total amount of physical capital increases proportionally with population. We study two different cases, one where physical capital is entirely sector specific and one where it is fully transferable. In addition, Appendix A.4 studies the case where capital is slowly movable between the two sectors.

An equilibrium analogous to the one studied so far still exists in both cases. In particular, the number of workers switching sectors upon impact is second order whereas the time at which wages are equalized is first order. When physical capital is sector specific, the expressions

derived in the case with no physical capital for the time  $t_1$  of adjustment (8) and for the mass  $a_L$  of low-skill workers who switch sectors (9) are still valid, and so are the expressions derived for  $t_2$  and  $a_H$ . When capital is fully transferable, these expressions become (respectively)

$$t_1 = \frac{\left(1 - \frac{(w_{1K} + w_{2K})}{(r_{1K} + r_{2K})} \frac{r^{ss}}{w^{ss}}\right) w^{ss} \frac{dp}{p} + o(dp)}{\left( \begin{aligned} &\left(w_{1L} + w_{2L} - \frac{(w_{1K} + w_{2K})}{(r_{1K} + r_{2K})} (r_{1L} + r_{2L})\right) (1 - n_L) x_L(0) \\ &+ \left(w_{1H} + w_{2H} - \frac{(w_{1K} + w_{2K})}{(r_{1K} + r_{2K})} (r_{1H} + r_{2H})\right) (1 - n_H) x_H(0) \bar{H} \end{aligned} \right)}, \quad (13)$$

$$a_L = - \frac{x_L(0) \left(1 - \frac{(w_{1K} + w_{2K})}{(r_{1K} + r_{2K})} \frac{r^{ss}}{w^{ss}}\right) t_1 \frac{dp}{p} + o(dp^2)}{2 \int_0^T x'_L(\tau) e^{-\delta\tau} d\tau}. \quad (14)$$

Here  $r^{ss}$  denotes the rental rate of capital in steady state, and  $r_{1X} = \frac{\partial^2 F_1}{\partial K_1 \partial X_1}$  and  $r_{2X} = \frac{p \partial^2 F_2}{\partial K_2 \partial X_2}$  for  $X \in \{L, H, K\}$  are the derivatives of the rental rate of capital in each sector.

When the allocation of capital *prior* to the price shock is identical in the sector-specific and fully transferable cases, comparing (8) and (13) shows that the transition time is longer when capital is transferable. Since  $r_{1K} + r_{2K} < 0$  and  $w_{1K} + w_{2K} > 0$ , the term in parenthesis in the numerator of (13) is greater than 1. Moreover, as  $r_{1L} + r_{2L}, r_{1H} + r_{2H} > 0$ , the denominator is less negative in (13). Both imply a higher  $t_1$ . Comparing (9) and (14) also shows that more low-skill workers switch sectors immediately after the trade shock (and one can similarly show that more time is required for high-skill wages to equalize). It is intuitive that the transfer of capital from sector 2 to sector 1 increases the marginal product of the other factors, so more high-skill and low-skill workers need to reallocate in order to equalize wages. This directly increases the length of the transition period and the number of low-skill workers who switch sectors upon impact of the trade shock.<sup>23</sup>

With more inputs than goods, the strict version of the Stolper–Samuelson theorem (as in Lemma 1) no longer holds. However, when physical capital is sector specific, we still get that low-skill workers are hurt relatively more than high-skill workers when the price of good 2 decreases ( $w^{ss}/v^{ss}$  changes in the same direction as  $p$ ). When physical capital is mobile, then, if its intensity is the same in both sectors, low-skill workers lose relative to high-skill workers when the price of good 2 decreases.

## 5.2 Nonpecuniary sectoral preferences

There is empirical evidence that gross flows across sectors—that is, flows in both directions between sectors in the absence of trade shocks—significantly outweigh net flows (see e.g. Davis

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<sup>23</sup>Dix-Carneiro (2014) finds that the adjustment takes about the same amount of time whether physical capital is mobile or sector specific. This case, however, involves full specialization. In an exercise with a lower price change of 10 per cent - Web Appendix M - he finds that perfect capital mobility implies a longer transition, as we do here.

and Haltiwanger, 1992). In this section we augment our model with nonpecuniary sector preferences that generate gross flows and show that doing so does not significantly affect our qualitative results.

Workers of both types can be in one of three different “states”: biased states 1 and 2 and a normal state 0. Workers in the biased state  $i$  receive a nonpecuniary benefit  $b > 0$  per unit of time from working in sector  $i$  (these nonpecuniary benefit may originate, for instance, from geographical preferences if the two goods are produced in different places). We assume that workers in state 0 move to either of the biased states according to a Poisson process at a rate of  $\lambda/2$  for each state; workers in a biased state (1 or 2) move to the normal state at a Poisson rate  $1/2$ . To keep the problem tractable, we change the specification slightly. First, workers do not have a fixed lifetime and do not discount the future, but die at the Poisson rate  $\delta$ . Second, workers who switch sectors lose all the sector-specific human capital accumulated so far; for example, if a worker from sector 2 with an accumulated experience equal to  $a$  in that sector moves to sector 1 and later moves *back* to sector 2 then his sector-2 human capital reverts to nothing. There is no population growth, the flow of newborn workers is of size 1, and the accumulation functions are such that  $x_Z(t) e^{-\delta t}$  has a finite integral over  $[0, \infty)$  for  $Z \in \{H, L\}$ . We also assume that the economy is initially in a steady state in which the fraction of people in each state (normal or biased) is the same across ages. This assumption implies that the share of entrants in the normal state 0 is given by  $1/(1 + 2\lambda)$  and the share in each of the biased states by  $\lambda/(1 + 2\lambda)$ . Finally, we assume that the nonpecuniary benefits  $b$  are large enough that—in the equilibrium considered—all workers in state 1 work in sector 1 and all workers in state 2 work in sector 2.

In steady state, workers in the normal state choose a sector at birth and remain in that same sector until they die or reach the biased state corresponding to the other sector. The share of workers switching sectors per unit of time is given by  $\lambda/(2(1 + 2\lambda))$ , so a higher  $\lambda$  is associated with larger gross flows. Following a small one-time, unanticipated price change, the equilibrium described in Proposition 1 still exists but with  $a_L$  and  $a_H$  now referring to the *experience* of the indifferent worker (instead of to the age, since the two can now differ). It is still the case that if workers in the normal state switch they will do so only at the time of the shock. The full solution to the problem is given in Appendix B.4. When  $t_1 < t_2$ , the time until wages are equalized is given by

$$t_1 = \frac{w^{ss}(1 + 2\lambda)}{(1 - n_L)x_L(0)[w_{1L} + w_{2L}] + (1 - n_H)x_H(0)\bar{H}[w_{1H} + w_{2H}]} \frac{dp}{p} + o(dp). \quad (15)$$

This expression is identical to (8) except for the term  $(1 + 2\lambda)$ . The direct effect of a greater likelihood of moving from one sector to another for nonpecuniary reasons is an increase in the transition time  $t_1$ . The reason is that only workers in the normal state can respond to the incentive of a wage differential, and the share of such workers decreases with the rate  $\lambda$  at



which workers leave the normal state.<sup>24</sup> This analysis also applies to the expression for  $t_2$  and the case where  $t_2 < t_1$ .

When  $t_1 < t_2$ , the experience of the indifferent low-skill worker is given by

$$a_L = - \frac{x_L(0) t_1}{\int_0^\infty x'_L(s) \left( \left(1 + (1 + 4\lambda^2)^{-\frac{1}{2}}\right) e^{-\lambda_1 s} + \left(1 - (1 + 4\lambda^2)^{-\frac{1}{2}}\right) e^{-\lambda_2 s} \right) ds} \frac{dp}{p} + o(dp^2), \quad (16)$$

where  $\lambda_1 \equiv \frac{1}{4} + \delta + \frac{1}{2}\lambda - \frac{1}{4}(1 + 4\lambda^2)^{\frac{1}{2}}$  and  $\lambda_2 \equiv \frac{1}{4} + \delta + \frac{1}{2}\lambda + \frac{1}{4}(1 + 4\lambda^2)^{\frac{1}{2}}$ . The denominator of this expression is decreasing in  $\lambda$ , so a higher probability of switching sectors plays a role similar to a higher death rate (or discount rate in the previous exercise): the loss of human capital resulting from a sector switch is less costly when the worker is likely to switch sectors again for nonpecuniary reasons. Thus the experience of an indifferent worker necessarily increases in the frequency of nonpecuniary shocks when  $t_1$  is increasing in  $\lambda$ . Overall, this exercise suggests positive associations between larger gross flows (a large  $\lambda$ ), slower transitions (higher  $t_1$  and  $t_2$ ), and more workers reallocating upon impact (greater  $a_L$  and  $a_H$ ). Crucially, however, this extension does not alter our conclusion that the number of workers who switch sectors due to the price change is of second order in the price change.

In our set-up, an increase in  $\lambda$  is associated with a larger likelihood of being in a biased state, which leads to larger intersectoral gross flows. It is possible to dissociate the two by assuming that workers leave a biased state at a Poisson rate  $\lambda/2$ , in which case, the share of workers in each state is independent of  $\lambda$  in steady-state. Equation (15) holds but replacing  $(1 + 2\lambda)$  by 3, so that an increase in  $\lambda$  only affects the time until which wages are equalized indirectly through its impact on  $n_L$  and  $n_H$ .

## 6 Simulations

### 6.1 Calibration

We supplement our analysis of a marginal price change by parameterizing the model to fit US data and then simulating the effects of a non-marginal price change. For the quantitative analysis, it is important to recognize that not all human capital is sector specific. Therefore, in this section, we incorporate accumulation of general human capital into the model by generalizing the human capital accumulation functions to  $x_L(a, \theta)$  and  $x_H(a, \theta)$ , both of which are functions of  $a$  (a worker's experience in her current sector) and  $\theta$  (her general experience). Both  $x_L$  and  $x_H$  are weakly increasing in both arguments. An equilibrium analogous to the one described previously exists and is characterized by a cutoff value of sector-specific human

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<sup>24</sup>There are indirect effects of  $\lambda$  as well which operate through its influence on the steady-state allocation  $n_L$  and  $n_H$  of newborn workers and on the steady-state mass of workers in each sector. Typically, an increase in  $\lambda$  means that a larger share of entrants needs to be allocated to the larger sector so as to compensate for the future sector switches caused by preference shocks.

capital  $a_L$  (i.e., for low-skill workers) given by

$$\int_0^{T-a_L} w_1(\tau) x_L(\tau, \tau + a_L) e^{-\delta\tau} d\tau = \int_0^{T-a_L} w_2(\tau) x_L(\tau + a_L, \tau + a_L) e^{-\delta\tau} d\tau;$$

this equality replaces (5), with a similar expression for  $a_H$  (the cutoff for high-skill workers). Our previous analysis generalizes to this case.<sup>25</sup>

Before proceeding, we briefly describe our calibration (see Appendix A.3 for additional details). To identify a high-skill-intensive and a low-skill-intensive sectors we rank each two-digit-SIC US manufacturing industry by the share of its total wage bill accruing to college-educated workers (which we identify as high-skill workers) based on data from EU KLEMS (March 2008 release) for 2000. We define the industries with the highest wage bill share for high-skill workers as sector 1 and those with the lowest share as sector 2; the cutoff is chosen so that the two sectors generate approximately the same value added. The gross output of sector 1 is \$2.27 trillion (US) with an average wage bill share of 0.49 for high-skill workers, while the total output of sector 2 is \$2.19 trillion with an average high-skill wage bill share of 0.24. We obtain the ratio of high-skill workers to low-skill workers ( $\bar{H}$ ) from the same data.

First, to remain close to the theory, we calibrate the initial model without physical capital. However, with no long-run source of rigidity and since the difference in skill intensity between the two sectors is limited, this model (or a version with general physical capital) would predict a counterfactually large amount of labor reallocation following moderate price-shocks, quickly leading to full specialization. Therefore to study price shocks of 10 or 20 percents, we also calibrate a version with sector-specific physical capital. The production functions are assumed to be CES between low-skill and high-skill workers in both sectors with the same elasticity of substitution but different factor intensities, and, when present, physical capital is combined with the labor aggregate using a Cobb-Douglas aggregator, that is  $F_1 = A_1 K_1^{\beta_1} \left( \alpha_1 H_1^{\frac{\sigma-1}{\sigma}} + (1-\alpha_1) L_1^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}(1-\beta_1)}$  and  $F_2 = A_2 K_2^{\beta_2} \left( \alpha_2 H_2^{\frac{\sigma-1}{\sigma}} + (1-\alpha_2) L_2^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}(1-\beta_2)}$ , with  $\alpha_1 > \alpha_2$  since we have assumed that sector 1 is high-skill intensive, and  $\beta_i$  denotes the capital share in production (0 in the baseline model). Without loss of generality, we normalize  $A_1$  such that  $A_1 (K_1(0))^{\beta_1} = 1$ , assume that the total capital stock at time 0 is equal to 2 and imposes that the rental rates of capital are equal at time 0 between the two sectors. We choose  $\sigma = 2$ , which is in the range of commonly estimated values for the elasticity of substitution between high-skill and low-skill workers in the United States (for instance, Card and Lemieux,

<sup>25</sup>The analytical expressions given in (8) and (9) for the time until the wages of low-skill workers are equalized again (at  $t_1$ ) and for the sector-specific human capital of the indifferent worker ( $a_L$ ) must be updated as follows:

$$t_1 = \frac{w^{ss}}{(1-n_L) x_L(0,0) [w_{1L} + w_{2L}] + (1-n_H) x_H(0,0) \bar{H} [w_{1H} + w_{2H}]} \frac{dp}{p} + o(dp),$$

$$a_L = - \frac{x_L(0,0) t_1}{2 \int_0^T e^{-\delta\tau} \frac{\partial x_L}{\partial a}(\tau, \tau) d\tau} \frac{dp}{p} + o(dp^2).$$

2001, estimate an elasticity of substitution between 2 and 2.5 for the US and the UK for men only and between 1.1 and 1.6 when men and women are combined).

We assume that the flow utility function is Cobb–Douglas,  $u(C_1, C_2) = C_1^\nu C_2^{1-\nu}$ , and define a unit of good as what can be purchased for \$1 trillion dollar in each sector (therefore  $p = 1$  initially). We identify  $\nu$  from the ratio of these two goods’ consumption (we derive consumption in each sector by combining output data in EU KLEMS with trade data for 2000 by SIC from Schott, 2010). A unit of time corresponds to a year, and we fix  $T$  (the length of a lifetime of work) to be 40 years. Finally, we set the discount rate  $\delta = 0.05$  and the growth rate  $\eta = 0.02$ .

For the human capital accumulation functions, we assume that  $x_L(a, \theta)$  and  $x_H(a, \theta)$  are proportional to each other; we base our parameterization on Neal (1995), who estimates wages both for displaced workers who stay in the same industry and for those who switch industries based on worker experience and tenure. More specifically, we assume that

$$x_Z(a, \theta) = x_{Z0} \exp \left( \varphi_a \min(a, m_a) + \varphi_{a^2} \min(a, m_a)^2 + \varphi_\theta \min(\theta, m_\theta) + \varphi_{\theta^2} \min(\theta, m_\theta)^2 \right) \text{ for } Z \in \{L, H\}, \quad (17)$$

where  $\varphi_a$ ,  $\varphi_{a^2}$ ,  $\varphi_\theta$  and  $\varphi_{\theta^2}$  are obtained using Neal (1995)’s regressions as described in the Appendix.  $m_a$  is the sector-specific experience level for which  $\varphi_a a + \varphi_{a^2} a^2$  is maximized (and similarly for  $m_\theta$ ), so that we flatten the accumulation functions once they reach their maxima. This defines the accumulation functions up to the constants  $x_{L0}$  and  $x_{H0}$ .

To derive  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_1$ ,  $\beta_2$ ,  $A_2 K_2(0)^{\beta_2}$ ,  $x_L(0, 0)$ , and  $x_H(0, 0)$ , we use the following moments and constraints: the share of capital costs in the sum of capital and labor costs for both sectors (only in the case with capital), the share of labor costs associated with high-skill workers in the total labor costs for both sectors, the share of high-skill and of low-skill workers in sector 1 (both from EU-KLEMS), the output in each sector, and the constraints of wage equality for both high-skill and low-skill workers. There are ten moments and constraints for nine unknowns (the parameters plus the initial values for  $n_L$  and  $n_H$ ), so we choose the parameters that come closest to fulfilling the constraints as measured by an equally weighted distance function (there are 8 moments for 7 unknowns in the case with no capital). Table 1 reports the parameters.

Parameters	$\eta$	$\delta$	$\nu$	$\sigma$	$\bar{H}$	$\varphi_a$	$\varphi_{a^2}$	$\varphi_\theta$	$\varphi_{\theta^2}$
Both cases	0.02	0.05	0.5087	2	0.286	0.03	−0.0007	0.027	−0.0007
Parameters	$\alpha_1$	$\alpha_2$	$x_{L0}$	$x_{H0}$	$A_1$	$A_2$	$\beta_1$	$\beta_2$	
Baseline	0.705	0.5779	0.2537	0.0878	1	0.7089	0	0	
With capital	0.5952	0.4567	0.3262	0.2985	0.9705	0.8222	0.3678	0.3212	

Table 1: Parameters

## 6.2 Simulation results without physical capital

We simulate a trade shock by considering a 1 percent price drop in sector 2. The transition path is illustrated in Figure 3. Upon impact, low-skill and high-skill workers (respectively) younger

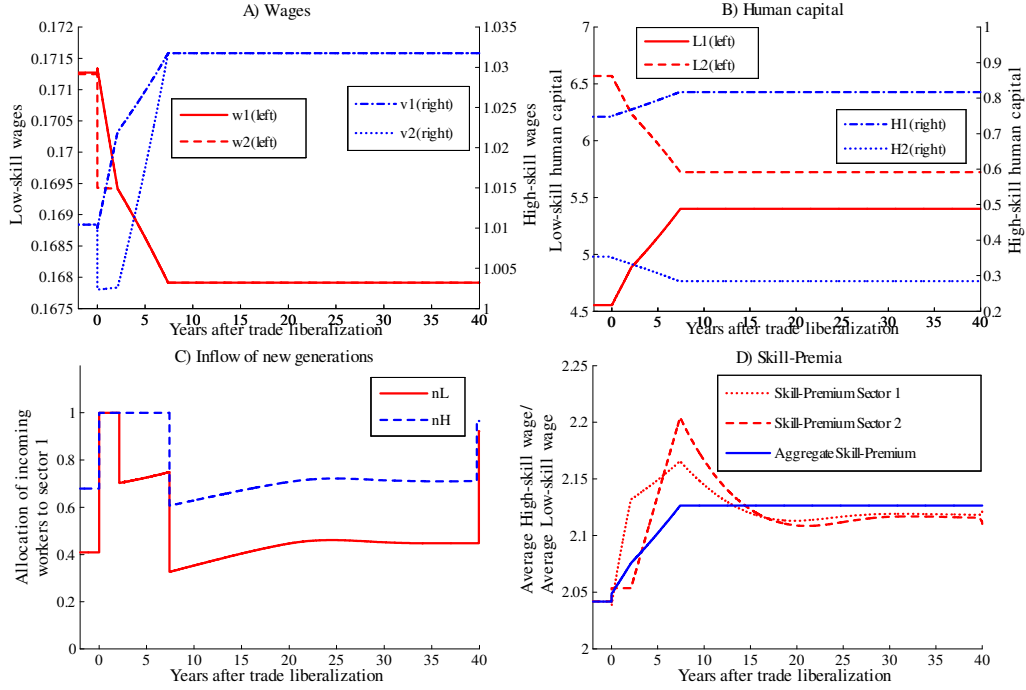


Figure 3: Simulated Transition After Trade Shock. Case with no capital, 1 percent price drop in sector 2.

than 0.041 and 0.27 years move from sector 2 to sector 1. Since only few workers switch and since these workers have hardly any sector-specific human capital, the initial loss of low-skill (respectively high-skill) human capital in sector 2 is only 0.1 (respectively 0.62) percent and the gain in sector 1 is only 0.14 (respectively 0.3) percent; the total loss in human capital of either type is even smaller (less than a hundredth of a percent). Because of this minuscule amount of immediate reallocation, wage changes are initially sector-dependent, as illustrated in Figure 3.A. New incoming generations will all enter sector 1 (as can be seen in Figure 3.C) and human capital in this sector grows as shown in Figure 3.B. Low-skill wages are equalized after 2.11 years and high-skill wages after 7.4 years. Eventually, the total stock of human capital in sector 1 will have increased by 18.6 and 9.3 percent for (respectively) low-skill and high-skill workers. As predicted by our approximation method, the initial adjustment's share of total labor reallocation for low-skill workers,  $\chi_L$ , is quite small: 0.74 percent and similarly  $\chi_H$  is equal to 3.16 percent, which, in return, explains why the transition is so protracted.

Figure 3.D studies the evolution of the skill premium, both within sector and at the aggregate level. The skill premium here is defined as the average pay of a low-skill worker divided by the average pay of a high-skill worker (not controlling for industry-tenure or age). The evolution at the aggregate level follows exactly the analysis in section 4.1: the wage premium increases on impact as the price shocks negatively affects the low-skill intensive sector and

then keeps increasing smoothly to its steady-state value. More surprising is the behavior of the within sector skill-premium. In sector 1, the skill premium increases quickly as initially all incoming generations enter sector 1 (so that the ratio of low-skill human capital over high-skill human capital increases). When low-skill workers start entering sector 2, the skill premium keeps increasing but slower (as the generations with a high ratio of low-skill over high-skill workers accumulate more human capital). The skill premium overshoots its steady-state value (which is equal to the aggregate value) and starts declining once high-skill workers enter both sectors. In sector 2, the skill premium initially jumps as more high-skill workers leave the sector than low-skill ones do, it stays essentially flat until  $t_1$ , and increases sharply thereafter, as only low-skill workers restart entering sector 2. It also overshoots its steady-state value. The figure demonstrates that the full impact on wage premia from changing producer prices is only reached after several years. This is consistent with the findings of Robertson (2004) and suggests that it might be difficult to find Stolper-Samuelson effects when using contemporaneous changes in wages and producer-prices.

We investigate numerically how the speed of human capital accumulation affects the transition by using the accumulation functions  $\tilde{x}_Z(a, \theta) = x_Z(a/2, \theta/2)$ , so that specific human capital is accumulated half as fast as in the baseline scenario. As previously discussed, we indeed find that the transition is slightly shorter: it takes 2 years for low-skill wages to equalize (instead of 2.11) and 7.24 years (instead of 7.4) for high-skill wages. More workers switch sectors on impact—namely, low-skill workers whose age is less than 0.053 (instead of 0.041) and high-skill workers whose age is less than 0.32 (instead of 0.27). The discount rate is sufficiently large to ensure that, in the long-run, the smaller cost of switching dominates the impact of a shorter period of wage differences on the number of workers switching sectors.

Next we turn to the welfare implications. Because the immediate wage impact is tied to sector of employment, the oldest workers in sector 1 gain from the drop in sector 2-prices; in contrast, the effect of a trade shock on the youngest workers will be dominated by standard HOS effects in the long run. Figure 4 shows the welfare gains from trade liberalization for low-skill workers for each generation alive at  $t = 0$ , in the aggregate and for each initial sector of employment, it also shows the welfare gains in the alternative case where human capital is general (so that welfare is the same regardless of the initial sector of employment). Welfare gains are expressed according to the equivalent variation measure in percentage gains in consumption (that is the figure displays for each type of worker the percentage change of consumption without the trade shock which yields the same welfare as the trade shock). The gradual reallocation of factors towards the skill-intensive sector means that the youngest low-skill workers lose the most from the trade liberalization. In addition, although all low-skill workers are better-off than they would have been in a fully flexible world, only the oldest low-skill workers in sector 1 benefit from the trade shock. A corresponding graph for high-skill

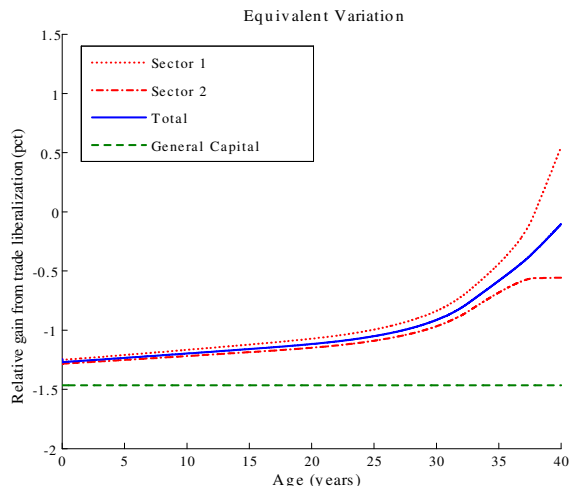


Figure 4: Welfare gains (in percentage of consumption) to low-skill workers, by age, who are alive at time 0. Case with no capital, 1 percent price drop in sector 2.

workers would demonstrate that, although all high-skill workers are worse-off than they would be in a world of complete capital mobility, only the oldest high-skill workers in sector 2 are hurt by the trade shock.

We examine the effects of a subsidy program (as in Section 4.3) that taxes high-skill workers to subsidize all workers who switch sector. We do indeed find that this program reduces the aggregate income of all low-skill workers alive at  $t = 0$ . For instance if the ratio  $s$  between the subsidy and the pay of a new worker is equal to 10%, the payment of the subsidy itself represents a direct transfer from high-skill to low-skill workers equivalent to 0.006 percent of the lifetime income of all low-skill workers (alive at  $t = 0$ ). Yet, the negative general equilibrium effect is sufficiently large, that overall the loss for low-skill workers alive at  $t = 0$  is equal to 0.028 percent of their lifetime income.

Finally, to illustrate the accuracy of our approximation technique, Figure 5.A shows the numerically computed values for  $t_1$ ,  $t_2$ ,  $a_L$ , and  $a_H$  and compares them to the values obtained with the Taylor approximations for different price changes. In all cases, an equilibrium of the type described in Proposition 1 exists. The approximation does quite well for  $a_L$ ,  $t_1$ , and  $a_H$ . For  $t_2$ , the fit worsens significantly as the price change increases and  $t_2$  becomes a relatively large number. As shown in Figure 5.B, the initial share of adjustment increases with the size of the price change, but all along, the adjustment largely occurs through the incoming generations. As already discussed, this version of the model predicts large labor reallocations even for small price changes, and the fit worsens significantly from 2.5 percent. The economy specializes quickly (it is specialized at 10 percent), and at that point an equilibrium of the type described in Proposition 1 ceases to exist.

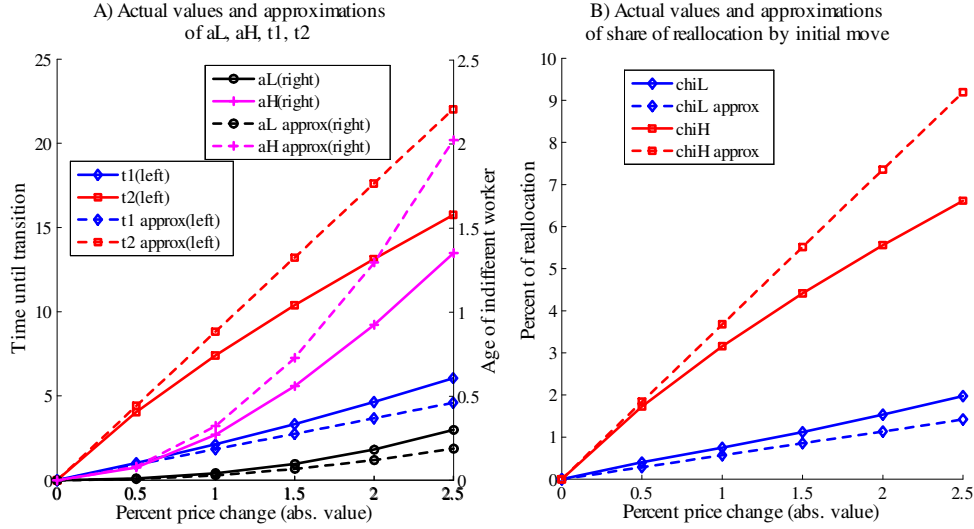


Figure 5: Times until wage equalization, ages of indifferent workers, and shares of reallocation on impact for different price changes (simulation results and approximation results). Case without physical capital.

### 6.3 Simulation results with sector-specific physical capital

We now turn to the case with sector-specific physical capital. With sector-specific physical capital, the model predicts much less reallocation so that with a 1 percent price shock, the times until wages are equalized are given by  $t_1 = 0.45$  and  $t_2 = 0.79$  while the ages of the indifferent workers are  $a_L = 0.01$  for low-skill and  $a_H = 0.017$  for high-skill. We focus on a 10 percent price drop in sector 2. An equilibrium of the type described by Proposition 1 exists. As described below, the qualitative features are similar to the baseline model, the major difference being that the Stolper-Samuelson theorem does not apply.

The transition path is illustrated in Figure 6. Figure 6.A looks similar to Figure 3.A (we omit the panels on the allocation of human capital, the allocation of entering generations and on the skill premium as they look similar<sup>26</sup>). We find that low-skill and high-skill workers (respectively) younger than 0.64 and 1.12 years move from sector 2 to sector 1. Low-skill wages are equalized after 4.01 years and high-skill wages after 6.21 years. Therefore, most of the adjustment happens through the entry of new generations since the initial adjustment's shares  $\chi_L$  and  $\chi_H$  are equal to 12.3 and 13.7 percents respectively.<sup>27</sup>

<sup>26</sup> Although in sector 1 the skill premium now decreases between  $t_1$  and  $t_2$  because the mass of low-skill workers entering sector 1 is quite low.

<sup>27</sup> One can show that for marginal price changes a slower accumulation of human capital leads to a slower transition when  $x_L$  and  $x_H$  are proportional and physical capital enters the production functions in a Cobb-Douglas way with the same share in both sectors. In our calibrations, the shares are quite close, and therefore we still find that if human capital is accumulated half as fast,  $t_1$  and  $t_2$  decrease to 3.72 and 5.95 years ( $a_L$  and  $a_H$  increase to 0.73 and 1.27).

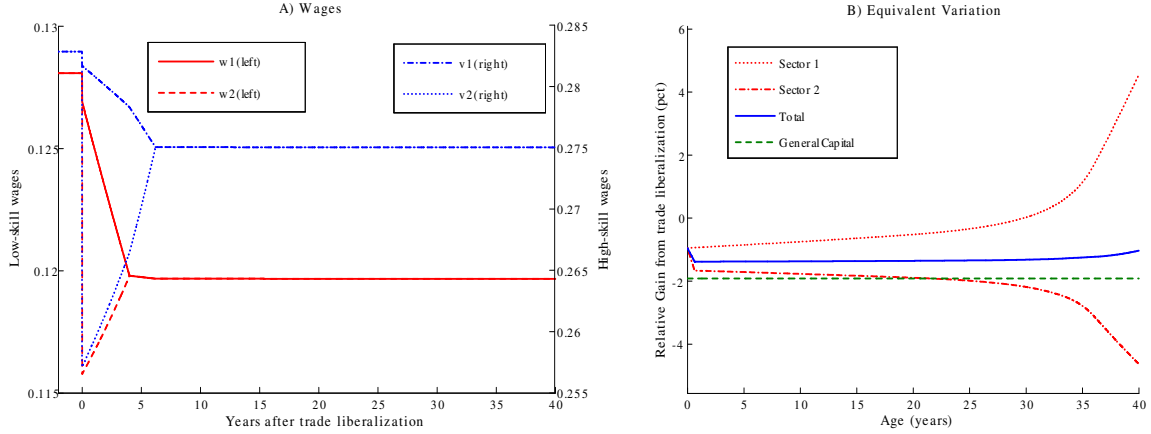


Figure 6: Simulated Transition After Trade Shock. Case with sector-specific physical capital, 10 percent price drop in sector 2.

As already mentioned, a major difference with Figure 3.A is that the Stolper-Samuelson theorem no longer applies. High-skill wages still increase relatively more from the price change than low-skill wages; but here, steady-state high-skill wages are lower in nominal terms after the price shock and nominal low-skill wages decrease less than the price of good 2. Low-skill wages in sector 2 drop on impact with the price shock. Then, up to  $t_1$ , incoming generations enter sector 1, and some low-skill and high-skill workers employed in sector 2 retire, as a result, low-skill wages increase in sector 2 as the ratio of physical capital per low-skill workers increases. Between  $t_1$  and  $t_2$ , low-skill workers start reentering sector 2, but not high-skill workers, the wage of low-skill workers drops but not enough to compensate the increase between  $t_1$  and  $t_2$ . After  $t_2$ , workers of both types enter both sectors and wages are roughly constant. This different dynamic for wages has important welfare consequences. As shown in Figure 6.B, older low-skill workers in sector 2 lose more than younger ones, and in fact they would be better off if human capital were general instead of sector-specific (however, aggregating sector 1 and sector 2, each generation of low-skill worker still loses relative to the general human capital case).<sup>28</sup>

As before, we compare the theoretical results with the numerical ones by studying price changes of 5, 10, 15 and 20 percent. Contrary to the cases with no or general physical capital, an equilibrium with the characteristics of Proposition 1 exists even for a 20 percent price change (as the new steady-state does not feature full specialization). Figure 7.A shows that the approximation does very well for a 5 percent price change, but the fit worsens for  $t_2$  and  $a_H$  for price changes above 15 percent. In Appendix B.5 we show that, in this case, extending

<sup>28</sup>For the parameter values considered, a subsidy as in Section 4.3, does not decrease the aggregate income of low-skill workers as the negative general equilibrium effect is not strong enough to overturn the positive transfer from high-skill to low-skill.



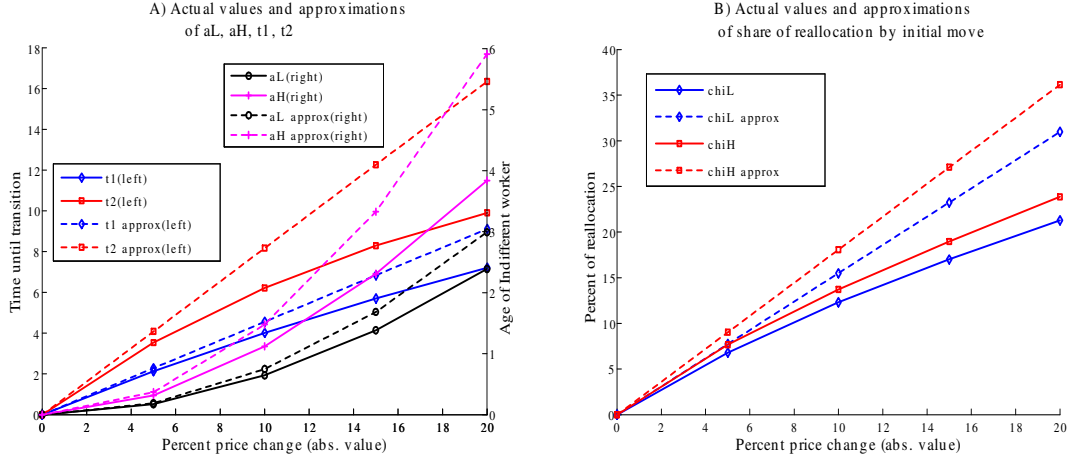


Figure 7: Times until wage equalization, ages of indifferent workers, and shares of reallocation on impact for different price changes (simulation results and approximation results). Case with sector-specific physical capital.

our approximation technique to include one additional order can significantly improve the precision of the analytical approximations. As before, the share of labor reallocation occurring on impact still increases with the size of the price change (equation 10 still holds). Nevertheless, Figure 7.B show that even at 20 percent, the adjustment mostly occurs through the incoming generations ( $\chi_L$  and  $\chi_H$  are equal to 21.3 and 23.9 percent respectively). Therefore, even though at large price changes, the gap between the values given by the approximation method and the actual values increases, the shape of the equilibrium is conserved, and so are the insights brought about by our analysis.

A drawback of this specification is that the allocation of new investment is exogenous and constant, even though the interest rates are permanently different in both sectors (we assumed that the capital stock was growing at the same rate as the population, which is why new investments take place). To address this concern, Appendix A.4 presents an extension of the model where in each sector a fraction of capital is fully sector specific (this may represent physical capital but also natural resources or workers with a very strong link to a particular sector) while the rest of capital is slowly transferable (installed capital is fixed but new investments are fully mobile). Quantitatively, the quality of the approximation for  $a_H$  and  $t_2$  worsens for large price changes as the share of slowly transferable capital increases. Yet, qualitatively the results are similar as long as the new steady-state does not feature full specialization and the population growth rate  $\eta$  is large enough.<sup>29</sup> The youngest workers are reallocated on impact,

<sup>29</sup> As specified in footnote 9, a sufficiently large  $\eta$  is necessary to ensure that at the time where the switchers die, the drop in the mass of workers in sector 1 is not too large. For a large price change, where the share of workers allocated to sector 1 is very large, a  $\eta > 0.02$  may be necessary. The equilibrium is more likely to break down if the new steady-state features full specialization (which can only occur asymptotically and when there

while the majority of the reallocation happens through incoming generations. After a period where wages are higher in sector 1 (with new workers all entering sector 1), wages get equalized and new workers start entering both sectors.

## 7 Conclusion

The mobility of factors is crucial for understanding the welfare effects of trade shocks. This paper adds sector-specific human capital to an otherwise classic dynamic HOS model. Our model replicates the standard HOS model in steady state but it differs during the transitional phase. In particular, our model endogenously generate (i) low levels of worker reallocation immediately after a trade shock and (ii) a protracted period of adjustment before wages reequilibrate. The model replicates previous empirical findings that mostly young people switch sectors, that most of the adjustment happens through the entry of new generations, and that wages after the shock are tied to sector and not to skill type. We also show that the model’s qualitative predictions are unaltered by the inclusion of either general human capital, physical capital or gross flows from nonpecuniary sectoral preferences.

Moreover, the model delivers some surprising results: a faster accumulation of human capital can make the transition *longer* and low-skill workers *benefit* from rigid labor markets when the low-skill-intensive sector is hit by a negative price shock. This last point is crucial for assessing the welfare effects of a subsidy for switching sectors. Although such a moving subsidy directly benefits the low-skill workers who receive it, the subsequent faster reallocation of resources to the high-skill-intensive sector hurts low-skill workers as a group. For a wide range of parameter values, this latter effect dominates. The intuitions of the model are illustrated with a calibration, which reveals that an equilibrium with the same structure still exists and that the approximation methods are accurate for discrete price changes. The calibration shows in particular that the initial adjustment’s share in total labor reallocation is small, which explains why the transition is protracted. The quantitative predictions of the approximation method do, however, worsen when the trade shocks leads to large labor reallocation in the long-run.

This paper employs the analytical approach of extending a classic trade model to analyze the interaction between labor market rigidities and international trade; it therefore complements the literature that studies similar questions using estimated and numerically solved models. The analytical approach has several virtues: it allows for greater generality, it provides linkages to well-understood models in trade, and it is easy to extend. These advantages open up several paths for future research. For instance, one could add firms and firm-specific human capital (or occupations and occupation-specific capital) to the model as a means for assessing the importance of the type of human capital specificity. Such models might build on

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is no fully sector-specific capital) as then the share of incoming workers allocated to sector 1 must be close to 1.

the literature that addresses firm heterogeneity and the intraindustry reallocations triggered by trade liberalization and perhaps could illuminate why little interindustry labor reallocation is observed in the short run despite substantial intraindustry reallocations.

## References

- Artuc, E., Chaudhuri, S., McLaren, J., 2010. Trade shocks and labor adjustment: A structural empirical approach. *The American Economic Review* 103 (3), 1008–1045.
- Autor, D., Dorn, D., Hanson, G., 2013. The China syndrome: Local labor market effects of import competition in the United States. *The American Economic Review* 103 (6), 2121–2168.
- Card, D., Lemieux, T., 2001. Can falling supply explain the rising return to college for younger men? A cohort-based analysis. *The Quarterly Journal of Economics* 116 (2), 705–746.
- Cosar, A. K., 2013. Adjusting to trade liberalization: Reallocation and labor market policies, mimeo.
- Davis, S., Haltiwanger, J., 1992. Gross job creation, gross job destruction and employment reallocation. *The Quarterly Journal of Economics* 107 (3), 819–863.
- Dix-Carneiro, R., 2014. Trade liberalization and labor market dynamics. *Econometrica* 82 (3), 825–885.
- Dix-Carneiro, R., Kovak, B., 2015. Trade reform and regional dynamics: Evidence from 25 years of brazilian matched employer-employee data, NBER Working Paper No 20908.
- Goldberg, P. K., Pavcnik, N., 2007. Distributional effects of globalization in developing countries. *Journal of Economic Literature* 45 (1), 39–82.
- Gonzaga, G., Menezes Filho, N., Terra, C., 2006. Trade liberalization and the evolution of skill earnings differentials in Brazil. *Journal of International Economics* 68 (2), 345–367.
- Haskel, J., Lawrence, R., Leamer, E., Slaughter, M., 2012. Globalization and U.S. wages: Modifying classic theory to explain recent facts. *Journal of Economic Perspectives* 26 (2), 119–140.
- Helpman, E., Itskhoki, O., Muendler, M.-A., Redding, S., 2012. Trade and inequality: From theory to estimation. NBER Working Paper No 17991.
- Helpman, E., Itskhoki, O., Redding, S., 2010. Inequality and unemployment in a global economy. *Econometrica* 78 (4), 1239–1283.
- Jones, R., 1971. A three-factor model in theory, trade, and history. In: *Trade, Balance of Payments, and Growth: Papers in International Economics in Honor of Charles P. Kindleberger*. North-Holland Pub. Co.

- Kambourov, G., 2009. Labour market regulations and the sectoral reallocation of workers: The case of trade reforms. *The Review of Economic Studies* 76 (4), 1321–1358.
- Kambourov, G., Manovskii, I., 2009. Occupational specificity of human capital. *International Economic Review* 50(1), 63–115.
- Kletzer, L. G., 2001. Job Loss from Imports: Measuring the Costs. Institute for International Economics, Washington, DC.
- Krugman, P. R., 2008. Trade and wages, reconsidered. *Brookings Papers on Economic Activity*, 103–154.
- Matsuyama, K., 1992. A simple model of sectoral adjustment. *The Review of Economic Studies* 59 (2), 375–387.
- Mayda, A. M., Rodrik, D., 2005. Why are some people (and countries) more protectionist than others? *European Economic Review* 49 (6), 1393–1430.
- Mayer, W., 1974. Short-run and long-run equilibrium for a small open economy. *The Journal of Political Economy* 82 (5), 955–967.
- Menezes Filho, N., Muendler, M., 2011. Labor reallocation in response to trade reform, NBER Working Paper No. 17372.
- Mussa, M., 1978. Dynamic adjustment in the Heckscher-Ohlin-Samuelson model. *The Journal of Political Economy* 86 (5), 775–791.
- Neal, D., 1995. Industry-specific human capital: Evidence from displaced workers. *Journal of Labor Economics* 13 (4), 653–677.
- Neary, J. P., 1978. Short-run capital specificity and the pure theory of international trade. *The Economic Journal* 88 (351), 488–510.
- Parent, D., 2000. Industry-specific capital and the wage profile: Evidence from the national longitudinal survey of youth and the panel study of income dynamics. *Journal of Labor Economics* 18 (2), 306–323.
- Revenge, A., 1992. Exporting jobs? The impact of import competition on employment and wages in U.S. manufacturing. *Quarterly Journal of Economics* 107 (1), 255 – 284.
- Robertson, R., 2004. Relative prices and wages inequality: Evidence from Mexico. *Journal of Economic Literature* 64 (2), 387–409.
- Schott, P., 2010. U.S. manufacturing exports and imports by SIC or NAICS category and partner country, 1972 to 2005, Technical Note.

- Slaughter, M., 2000. What are the results of product-price studies and what can we learn from their differences? In: Feenstra, R. C. (Ed.), *The Impact of International Trade on Wages*. University of Chicago Press, pp. 129–169.
- Timmer, M., O’Mahony, M., van Ark, B., March 2008. EU KLEMS database. In: *The EU KLEMS Growth and Productivity Accounts: An Overview*. University of Groningen & University of Birmingham, downloadable at [www.euklems.net](http://www.euklems.net).
- Topalova, P., 2010. Factor immobility and regional impacts of trade liberalization: Evidence on poverty from India. *American Economic Journal: Applied Economics* 2 (4), 1–41.
- Wacziarg, R., Wallack, J. S., 2004. Trade liberalization and intersectoral labor movements. *Journal of International Economics* 64 (2), 411–439.
- Wood, A., 1995. How trade hurt unskilled workers. *Journal of Economic Perspectives* 9 (3), 57–80.

# A Appendix

## A.1 Existence of the equilibrium

This section proves Propositions 1 and 2. First we derive the times until equalization of high-skill wages and of low-skill wages as well as the ages of the indifferent high-skill and low-skill worker ( $t_1$ ,  $t_2$ ,  $a_H$ , and  $a_L$ , respectively) in an equilibrium which has the structure described in Proposition 1. Second, we show that workers switch sectors only at time 0. In the online Appendix B, we show that for  $\eta$  sufficiently large, it is possible to keep wages equalized forever once they have been equalized once; and we show that the equilibrium maximizes the present value of production.

### A.1.1 Ages of the indifferent workers and times until wage equalization

Equations (5) and (6) pin down the indifferent workers. For a marginal price change, the difference between  $w_1$  and  $w_2$  is at most first order,<sup>30</sup> and workers whose age is nonmarginal will not switch sector (for these workers,  $w_1(\tau)x_L(\tau) < w_2(\tau)x_L(a + \tau)$ ). Hence only the workers whose age is at most first order in the price change may be willing to move. We can therefore take a first-order expansion of (5) with respect to  $(a_L, w_1(t), w_2(t))$  around  $a_L = 0$  and  $w_1(t) = w_2(t) = w^{ss}$  and then simplify to obtain

$$a_L = \frac{\int_0^{t_1} (dw_1(\tau) - dw_2(\tau)) x_L(\tau) e^{-\delta\tau} d\tau}{w^{ss} \int_0^T x_L'(\tau) e^{-\delta\tau} d\tau} + o(t_1 dp), \quad (\text{A.1})$$

where  $dw_i(t) \equiv w_i(t) - w^{ss}$  and analogous definitions are used for high-skill wages. Since  $t_1$  and  $dw_i(t)$  will both be of at most first order in  $dp$ , it follows that  $a_L$  is at most second order. An analogous expression holds for  $a_H$ .

Now define  $g_H(t) \equiv dv_1(t) - dv_2(t)$  and  $g_L(t) \equiv dw_1(t) - dw_2(t)$  such that  $t_1$  (resp.  $t_2$ ) denotes the lowest  $t$  for which  $g_L(t_1) = 0$  (resp.  $g_H(t_2) = 0$ ). Define  $dl_i(t) = l_i(t) - l_{ss}$ . Given that  $a_L$  is at most second order, one can differentiate equation (7) and an analogous expression for  $l_2$  to obtain:

$$dl_1(t) = -dl_2(t) = (1 - n_L) x_L(0) t + o(dp) \text{ for } 0 \leq t \leq t_1, t_2; \quad (\text{A.2})$$

similarly,

$$dh_1(t) = -dh_2(t) = (1 - n_H) x_H(0) \bar{H} t + o(dp) \text{ for } 0 \leq t \leq t_1, t_2. \quad (\text{A.3})$$

Taking a first-order expansion of  $g_H(t)$  around  $dp = 0$  and then using equations (A.2) and (A.3), we find that

$$g_H(t) = t \left[ (v_{1L} + v_{2L}) (1 - n_L) x_L(0) + (v_{1H} + v_{2H}) (1 - n_H) x_H \bar{H}(0) \right] - \frac{v^{ss}}{p} dp + o(dp) \text{ for } 0 \leq t \leq t_1, t_2,$$

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<sup>30</sup> An economic variable that is "at most  $n^{th}$  order" in the price change is one that can be of order  $m^{th}$  for  $m \geq n$ .

where  $v_{iZ} = \frac{\partial v_i}{\partial Z}$  at the steady-state value (prior to the price change), and similarly that

$$g_L(t) = t \left[ (w_{1H} + w_{2H})(1 - n_H)x_H(0)\bar{H} + (w_{1L} + w_{2L})(1 - n_L)x_L(0) \right] - \frac{w^{ss}}{p} dp + o(dp) \text{ for } 0 \leq t \leq t_1, t_2,$$

where  $w_{iZ} = \frac{\partial w_i}{\partial Z}$ . Since both  $F_1$  and  $F_2$  are CRS, it follows that  $(v_{1L} + v_{2L})(1 - n_L)x_L(0) + (v_{1H} + v_{2H})(1 - n_H)x_H(0)\bar{H}$  or  $(w_{1H} + w_{2H})(1 - n_H)x_H(0)\bar{H} + (w_{1L} + w_{2L})(1 - n_L)x_L(0)$  (or both) must be strictly negative.<sup>31</sup> Consider the case where

$$\begin{aligned} & \frac{1}{v^{ss}} \left( (v_{1L} + v_{2L})(1 - n_L)x_L(0) + (v_{1H} + v_{2H})(1 - n_H)x_H(0)\bar{H} \right) \\ & > \frac{1}{w^{ss}} \left( (w_{1H} + w_{2H})(1 - n_H)x_H(0)\bar{H} + (w_{1L} + w_{2L})(1 - n_L)x_L(0) \right), \end{aligned} \quad (\text{A.4})$$

which implies  $(w_{1H} + w_{2H})(1 - n_H)x_H(0)\bar{H} + (w_{1L} + w_{2L})(1 - n_L)x_L(0) < 0$ .<sup>32</sup> Then  $g_L$  is positive but decreases over time until  $t_1 > 0$ ; over the same time period,  $g_H$  remains strictly positive (and may increase or decrease). Therefore, if (A.4) holds then wages are equalized for low-skill workers first:  $t_1 < t_2$ . The Appendix focuses on this case; if  $t_1 > t_2$  then symmetric expressions would obtain.

For  $t \in (t_1, t_2)$ , low-skill workers are allocated such that their wages are equalized across sectors but (A.3) still holds. At first order,  $dl_1 + dl_2 = 0 + o(dp)$  because the number of low-skill workers who switch is of second order at most; therefore,  $dw_1 = dw_2$  implies that

$$dl_1(t) = -\frac{w_{1H} + w_{2H}}{w_{1L} + w_{2L}} (1 - n_H)x_H(0)\bar{H}t + \frac{w^{ss}}{w_{1L} + w_{2L}} \frac{dp}{p} + o(dp) \text{ for } t_1 \leq t \leq t_2. \quad (\text{A.5})$$

Using this equation and (A.3), we can write  $g_H(t)$  as

$$g_H(t) = \left( v_{1H} + v_{2H} - \frac{(v_{1L} + v_{2L})(w_{1H} + w_{2H})}{w_{1L} + w_{2L}} \right) (1 - n_H)x_H(0)\bar{H}t + \left( \frac{v_{1L} + v_{2L}}{w_{1L} + w_{2L}} w^{ss} - v^{ss} \right) \frac{dp}{p} + o(dp),$$

for  $t_1 \leq t \leq t_2$ , which is decreasing in  $t$  (using the properties of CRS functions) and positive when (A.4) holds. Hence  $t_2$  is defined by

$$t_2 = \frac{v^{ss} - \frac{v_{1L} + v_{2L}}{w_{1L} + w_{2L}} w^{ss}}{(1 - n_H)x_H(0)\bar{H} \left( v_{1H} + v_{2H} - \frac{(v_{1L} + v_{2L})(w_{1H} + w_{2H})}{w_{1L} + w_{2L}} \right)} \frac{dp}{p} + o(dp). \quad (\text{A.6})$$

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<sup>31</sup> Assume that this is *not* the case then both

$$(F_{1LH} + pF_{2LH})(1 - n_L)x_L(0) \geq -(F_{1HH} + pF_{2HH})(1 - n_H)x_H(0)\bar{H}$$

and

$$(F_{1LH} + pF_{2H})(1 - n_H)x_H(0)\bar{H} \geq -(F_{1LL} + pF_{2LL})(1 - n_L)x_L(0),$$

with strict inequality for at least one of the two expressions. This implies, since  $F_1$  and  $F_2$  are CRS, that  $\frac{F_{1HH}F_{2LL} + F_{2HH}F_{1LL}}{F_{1LH}F_{2LH}} < 2$ . However the properties of CRS functions dictate that  $\frac{F_{1HH}F_{2LL} + F_{2HH}F_{1LL}}{F_{1LH}F_{2LH}} = \frac{H_2}{L_2} \frac{L_1}{H_1} + \frac{L_2}{H_2} \frac{H_1}{L_1}$ , which is strictly greater than 2 if  $F_1$  and  $F_2$  have different factor intensities.

<sup>32</sup> We rule out the case where (A.4) holds with equality. The same logic would apply, but then  $t_1$  and  $t_2$  would differ only at second order.



Using (A.2), (A.3), and that  $dw_1 = dw_2$  for  $t > t_1$ , we can rewrite (A.1) as (9). Since  $dw_1 > dw_2$  on  $(0, t_1)$  and  $dw_1 = dw_2$  from  $t_1$ , the only low-skill workers who switch from sector 2 are those who are younger than  $a_L$ . In an analogous manner we can use (A.2), (A.3), (A.5), and the counterpart of (A.1) for high-skill workers to solve for  $a_H$  as follows:

$$a_H = -\frac{x_H(0)}{2 \int_0^T x'_H(\tau) e^{-\delta\tau} d\tau} \frac{dp}{p} \left( t_2 - \frac{w^{ss}(v_{1L} + v_{2L})}{v^{ss}(w_{1L} + w_{2L})} (t_2 - t_1) \right). \quad (\text{A.7})$$

Similarly, the only high-skill workers who switch from sector 2 are those who are younger than  $a_H$ .

Note that, in the opposite case where wages of high-skill workers are equalized first ( $t_1 > t_2$ ), one can analogously derive the following expressions:

$$\begin{aligned} t_1 &= \frac{w^{ss} - \frac{w_{1H} + w_{2H}}{v_{1H} + v_{2H}} v^{ss}}{(1 - n_L) x_L(0) \left( w_{1L} + w_{2L} - (w_{1H} + w_{2H}) \frac{v_{1L} + v_{2L}}{v_{1H} + v_{2H}} \right)} \frac{dp}{p} + o(dp), \\ t_2 &= \frac{v^{ss}}{(1 - n_L) x_L(0) [v_{1L} + v_{2L}] + (1 - n_H) x_H(0) \bar{H} [v_{1H} + v_{2H}]} \frac{dp}{p} + o(dp); \\ a_L &= -\frac{x_L(0)}{2 \int_0^T x'_L(\tau) e^{-\delta\tau} d\tau} \frac{dp}{p} \left( t_1 - \frac{v^{ss}(w_{1H} + w_{2H})}{w^{ss}(v_{1H} + v_{2H})} (t_1 - t_2) \right) + o(dp^2), \\ a_H &= -\frac{x_H(0) t_2}{2 \int_0^T e^{-\delta\tau} x'_H(\tau) d\tau} \frac{dp}{p} + o(dp^2). \end{aligned}$$

To establish that Proposition 1 describes an equilibrium, we must still show that (i) workers will switch sectors only once and only time  $t = 0$ , and (ii) after  $t_1$  (resp.  $t_2$ ) it is always possible to adjust the flow of entrants such that wages of low-skill (resp. high-skill) workers remain equalized. We focus on low-skill workers below in what follows but the same reasoning applies as well to high-skill workers.

### A.1.2 Workers switch only once

We begin by noting that low-skill workers who enter sector 1 will never switch because this sector always has wages that are weakly higher than those of sector 2. Furthermore, workers will not switch after time  $t_1$  because then wages are equalized; workers will always remain in the sector where they have accumulated the most experience until time  $t_1$ . Therefore, the only workers who may switch are those born before  $t = 0$  who entered sector 2, and they may switch only during the time period  $[0, t_1]$ . Let us consider such a worker. We denote her age by  $a$ , the time she spends in sector 1 during the time period  $[0, t_1]$  by  $\mu_1$ , and the time spent in sector 2 during that same time period by  $\mu_2 = t_1 - \mu_1$ . We seek to show that if such a worker were to stick to the same sector during the time period  $(0, t_1]$ , she would be better-off.

First consider the case where, at time  $t_1$ , the total experience accumulated in sector 1 is weakly greater than the total experience accumulated in sector 2, that is  $\mu_1 \geq \mu_2 + a$  (which

implies that  $a$  is at most of first order since  $t_1$  is first order). Therefore from time  $t_1$  onward, this worker would (weakly) prefer working in sector 1. We compare the welfare of this worker under this strategy to her welfare under the alternative strategy where she switches to sector 1 at time 0 (when the trade shock hits). During the time interval  $[0, t_1]$ , the worker benefits from a higher wage under the alternative strategy for periods where she works in sector 2 in the original strategy, but she suffers from a lower level of human capital. The loss in human capital is bounded above by  $x_L(\mu_2 + a) - x_L(0)$ , and it is suffered during a time period of length  $\mu_2$ ; hence this loss is at most of the same order as  $(\mu_2 + a)\mu_2$ . For periods where she works in sector 1 in the original strategy, she benefits from a higher level of human capital under the alternative strategy. The gain is equal to  $x_L(\tau) - x_L(\tau - \mu_2)$ . This gain is endured for a nonnegligible period of time and so is of the same order as  $\mu_2$ . Hence the gain is of a higher order than the lower, and this worker would be better-off switching to sector 1 upon impact.

Now consider the opposite case where  $\mu_1 < \mu_2 + a$  (at time  $t_1$  the total experience accumulated in sector 1 is smaller than the total experience accumulated in sector 2) and the alternative strategy where the worker stays in sector 2 forever. During the time interval  $[0, t_1]$ , when the worker is employed in sector 1 under the original strategy, she suffers from a lower wage in the alternative strategy; the resulting welfare loss is at most of the same order as  $\mu_1 dp$ . For time periods where she works in sector 2 in the original strategy, she benefits from a higher level of human capital under the alternative strategy. In particular, from period  $t_1$  onward, her human capital is higher by  $x_L(\tau + a) - x_L(\tau + a - \mu_1)$ . This gain lasts a nonnegligible period of time, so that the welfare gain is of the same order as  $\mu_1$ . In this case, then, the gains are larger than the losses and so the worker is better-off under the alternative strategy, staying in sector 2 all along. This establishes point (i).

In Appendix B.1.1, we show that  $n_L(t)$  and  $n_H(t)$  are in  $(0, 1)$  for  $\eta$  sufficiently large, which achieves the proof of existence of the equilibrium.

## A.2 Proof of Proposition 3

The argument is most easily made with reference to Figure 1. Along the transition path, wages in sector 1 must remain weakly higher than wages in sector 2 ( $w_1(t) \geq w_2(t)$  and  $v_1(t) \geq v_2(t)$ ). From the figure it follows that  $w_1(t), w_2(t) \geq w^{ss'}$  and  $v_2(t), v_1(t) \leq v^{ss'}$ ; therefore, any low-skill worker who does not switch industries (and so does not lose any human capital) will benefit from the rigidity engendered by sector-specific human capital.

Consider, moreover, a low-skill worker of  $\hat{t} \leq a_L$  who switches from sector 2 to sector 1. The lifetime income of this worker obeys

$$\int_0^{T-\hat{t}} w_1(\tau) x_L(\tau) e^{-\delta\tau} d\tau \geq \int_0^{T-\hat{t}} w_2(\tau) x_L(\hat{t} + \tau) e^{-\delta\tau} d\tau \geq \int_0^{T-\hat{t}} w^{ss'} x_L(\hat{t} + \tau) e^{-\delta\tau} d\tau.$$

Here the first inequality follows from equation (5) and the second from  $w_1(t) \geq w^{ss'}$ . Since lifetime income is higher under the rigid regime yet prices are the same, even those workers who switch are better-off. An analogous argument demonstrates that all high-skill workers would be better-off if human capital were *not* sector-specific.

### A.3 Calibration details

In this appendix we provide some details on our calibration: the list of industries in each group, how we parameterize the accumulation function from Neal (1995), and how we derive the parameters  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_1$ ,  $\beta_2$ ,  $A_2$ ,  $x_L(0,0)$ , and  $x_H(0,0)$ .

As described in the main text, we split the 2-digit industries from EU KLEMS for 2000 into a high-skill group and a low-skill group of equal value-added size. “High skill” is defined as college graduate or above, and “low skill” is defined as some college or below (i.e., the sum of low and medium skills in the EU KLEMS’s US classification). The high-skill industries—in decreasing order of their wage bill devoted to high-skill workers—are: office, accounting and computing machinery; medical, precision, and optical instruments; chemicals and chemical products; transport equipment; printing, publishing and reproduction, electrical engineering; and coke, refined petroleum, and nuclear fuel. The low-skill industries (in decreasing order of the wage bill share of high-skill workers) are tobacco; manufacturing not otherwise classified; food and beverage; pulp and paper; machinery not otherwise classified; textiles; rubber and plastics; nonmetallic minerals; basic metals; fabricated metal; wood; and leather and footwear. The high-skill wage bill share for the cutoff industries are 40.2% and 34.4%.

We base our estimate for the human capital accumulation function on columns 2 and 3 of Table 4 in Neal (1995). Neal regresses the log wage of displaced workers on experience (pre-displacement), experience squared, tenure (in the firm prior to the displacement), and tenure squared (plus a constant and some control variables that include education). He runs this regression separately for displaced workers who switch 2-digit industries and for displaced workers who stay in the same 2-digit industry. We reproduce the coefficients of interest from his table below (specifying notation in parenthesis).

The Relationship between Wages and Job Tenure: Men (Neal, 1995)

	Log Postdisplacement Wage	
	Switchers	Stayers
Experience (predisplacement)	0.016 ( $\gamma_e^a$ )	0.027 ( $\gamma_e^s$ )
Experience <sup>2</sup>	-0.0003 ( $\gamma_{e^2}^a$ )	-0.0004 ( $\gamma_{e^2}^s$ )
Tenure (predisplacement)	0.011 ( $\gamma_t^a$ )	0.030 ( $\gamma_t^s$ )
Tenure <sup>2</sup>	-0.0004 ( $\gamma_{t^2}^a$ )	-0.0010 ( $\gamma_{t^2}^s$ )

Since our model does not distinguish between experience in a sector and experience in a specific firm in a given sector, we add up the coefficients for experience and tenure. Then we identify the coefficient for switchers as the impact of general human capital on wages, and identify the

difference between the coefficients for stayers and switchers as the effect of sector-specific human capital on wages. Following the specification of this regression, we posit capital accumulation functions of the form given by (17). We obtain

$$\begin{aligned}
\varphi_a &\equiv \gamma_e^s + \gamma_t^s - (\gamma_e^a + \gamma_t^a) = 0.03, \\
\varphi_{a^2} &\equiv \gamma_{e^2}^s + \gamma_{t^2}^s - (\gamma_{e^2}^a + \gamma_{t^2}^a) = -0.0007, \\
\varphi_\theta &\equiv \gamma_e^a + \gamma_t^a = 0.027, \\
\varphi_{\theta^2} &\equiv \gamma_{e^2}^a + \gamma_{t^2}^a = -0.0007, \\
m_a &= 21.42, \\
m_\theta &= 19.28.
\end{aligned}$$

In this specification, nearly half of total human capital is sector specific.

We find that the consumption share for sector 1 is given by  $\nu = 0.5087$  and that  $\bar{H} = 0.286$ . The capital shares are empirically given by  $\hat{\beta}_1 = 0.358$  in sector 1 and  $\hat{\beta}_2 = 0.329$  in sector. The empirical estimate for the wage bill in sector 1 is given by  $\hat{\alpha}_1 = 0.487$  and in sector 2 by  $\hat{\alpha}_2 = 0.242$ . The estimates for output—where one unit of good in each sector corresponds to \$1 trillion of output in the data—are  $\hat{Y}_1 = 2.275$  and  $\hat{Y}_2 = 2.194$  in sectors 1 and 2, respectively. The estimates of the share of high-skill and low-skill workers are  $\hat{n}_H = 0.618$  and  $\hat{n}_L = 0.341$ .

Let  $F_L \equiv \int_0^{40} e^{-\eta t} \frac{x_L(t,t)}{x_{L0}} dt$  and  $F_H \equiv \int_0^{40} e^{-\eta t} \frac{x_H(t,t)}{x_{H0}} dt$  (therefore  $F_L = F_H$ ). Then we can express steady-state (that is, time 0) output in the model as:

$$\begin{aligned}
Y_1 &= \left( \alpha_1 (n_H x_{H0} \bar{H} F_H)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_1) (n_L x_{L0} F_L)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}(1-\beta_1)}, \\
Y_2 &= \tilde{A}_2 \left( \alpha_2 ((1 - n_H) x_{H0} \bar{H} F_H)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_2) ((1 - n_L) x_{L0} F_L)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}(1-\beta_2)},
\end{aligned}$$

where  $n_H$  and  $n_L$  denote the endogenous steady-state allocations of workers of each type in sector 1 and  $\tilde{A}_2 = A_2 (K_2(0))^{\beta_2}$ . Moreover, wage equalization in both sectors imposes that

$$\begin{aligned}
&C_H \\
&\equiv \frac{\alpha_1 (1 - \beta_1) (n_H x_{H0} \bar{H} F_H)^{-\frac{1}{\sigma}} \left( \alpha_1 (n_H x_{H0} \bar{H} F_H)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_1) (n_L x_{L0} F_L)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma(1-\beta_1)}{\sigma-1} - 1}}{\alpha_2 (1 - \beta_2) \tilde{A}_2 ((1 - n_H) x_{H0} \bar{H} F_H)^{-\frac{1}{\sigma}} \left( \alpha_2 ((1 - n_H) x_{H0} \bar{H} F_H)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_2) ((1 - n_L) x_{L0} F_L)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{(1-\beta_2)\sigma}{\sigma-1} - 1}} - 1 \\
&= 0,
\end{aligned}$$

$$\begin{aligned}
& C_L \\
& \equiv \frac{(1 - \alpha_1)(1 - \beta_1)(n_L x_{L0} F_L)^{-\frac{1}{\sigma}} \left( \alpha_1 (n_H x_{H0} \bar{H} F_H)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_1)(n_L x_{L0} F_L)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma(1-\beta_1)}{\sigma-1} - 1}}{(1 - \alpha_2)(1 - \beta_2) \tilde{A}_2 ((1 - n_L) x_{L0} F_L)^{-\frac{1}{\sigma}} \left( \alpha_2 ((1 - n_H) x_{H0} \bar{H} F_H)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_2)((1 - n_L) x_{L0} F_L)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{(1-\beta_2)\sigma}{\sigma-1} - 1}} - 1 \\
& = 0.
\end{aligned}$$

For the version with no capital, we impose  $\beta_1 = \beta_2 = 0$  and we pin down the parameters  $\alpha_1, \alpha_2, x_{L0}, x_{H0}$ , and  $A_2 = \tilde{A}_2$  by solving for

$$\min_{\alpha_1, \alpha_2, x_{L0}, x_{H0}, n_L, n_H, A_2} M_0(\alpha_1, \alpha_2, x_{L0}, x_{H0}, n_L, n_H, A_2),$$

where  $M_0$  is the following distance function:

$$M_0 \equiv (\alpha_1 - \widehat{\alpha_1})^2 + (\alpha_2 - \widehat{\alpha_2})^2 + (n_H - \widehat{n_H})^2 + (n_L - \widehat{n_L})^2 + \left( \frac{Y_1}{\widehat{Y_1}} - 1 \right)^2 + \left( \frac{Y_2}{\widehat{Y_2}} - 1 \right)^2 + C_H^2 + C_L^2.$$

This results in:

$$\alpha_1 = 0.705, \alpha_2 = 0.5779, x_{L0} = 0.2537, x_{H0} = 0.0878 \text{ and } A_2 = 0.7089.$$

With these parameters, the model predicts a steady-state allocation of  $n_H = 0.68$  and  $n_L = 0.41$

In the presence of sector-specific capital, we pin down the parameters  $\alpha_1, \alpha_2, \beta_1, \beta_2, x_{L0}, x_{H0}$ , and  $\tilde{A}_2$  by solving for

$$\min_{\alpha_1, \alpha_2, \beta_1, \beta_2, x_{L0}, x_{H0}, n_L, n_H, A_2} M(\alpha_1, \alpha_2, \beta_1, \beta_2, x_{L0}, x_{H0}, n_L, n_H, A_2),$$

where  $M$  is the distance function:

$$M \equiv M_0 + \left( \beta_1 - \widehat{\beta_1} \right)^2 + \left( \beta_2 - \widehat{\beta_2} \right)^2$$

We thus obtain the following parameters:

$$\alpha_1 = 0.5952, \alpha_2 = 0.4567, x_{L0} = 0.3262, x_{H0} = 0.2985, \tilde{A}_2 = 0.7991, \beta_1 = 0.3678 \text{ and } \beta_2 = 0.3212$$

With these parameters, the model predicts a steady-state allocation of  $n_H = 0.66$  and  $n_L = 0.39$ . We then use the value of  $\tilde{A}_2$ , the normalization  $A_1 K_1(0)^{\beta_1} = 1$ , the assumption that the total capital stock at  $t = 0$  is equal to 2 and the equality in the rental rate of capital to derive  $A_1 = 0.9705$  and  $A_2 = 0.8222$ .

Note that since the system is overidentified, one cannot pinpoint exactly which moment determines which parameter. Yet, the capital shares in the data are directly linked to the

$\beta_i$  coefficients, the high-skill cost shares depend on the coefficient  $\alpha_i$  as well as the ratio of low-skill to high-skill human capital in the sector (that is  $n_H x_{H0} \bar{H} / n_L x_{L0}$  in sector 1 and  $(1 - n_H) x_{H0} \bar{H} / (1 - n_L) x_{L0}$  in sector 2). The output levels depend on the low-skill and high-skill steady-state allocations  $(n_L, n_H)$ , as well as on the levels of human capital  $x_{H0}$  and  $x_{L0}$ , the capital shares  $\beta_i$  and the productivity coefficient  $\tilde{A}_2$ . The constraints that wages must be equalized and that the steady-state allocations must be close to the ones observed in the data further constrain the values for  $\alpha_1, \alpha_2, \beta_1, \beta_2, x_{L0}, x_{H0}$  and  $\tilde{A}_2$ .

## A.4 Slowly transferable capital

### A.4.1 Analytical results

We now assume that while old capital cannot move, new investments can be freely allocated between the two sectors. We denote by  $\Delta$  the depreciation rate of capital, and since we assume a constant capital - labor ratio (at the level of the economy), the exogenous total investment flow must be given by  $I(t) = (\eta + \Delta) e^{\eta t} K_0$  (with  $K_0 \equiv K(0)$ ). As a result the normalized stock of capital in sector 1 obeys

$$\dot{k}_1(t) = (\eta + \Delta) (n_I(t) K_0 - k_1(t)),$$

where  $n_I$  is the share of investment going to sector 1. Similarly, the normalized stock of capital in sector 2 obeys

$$\dot{k}_2(t) = (\eta + \Delta) ((1 - n_I(t)) K_0 - k_2(t)).$$

We assume that at time  $t = 0$ , the economy is in a steady-state so that the rental rate of capital is equalized in the two sectors. As the price of good 2 drops, the interest rate in sector 2 drops below that in sector 1. Old capital cannot be reallocated, but new investments get reallocated to sector 1. Over time, the gap in the interest rates narrows and just as for wages, there exists a time  $t_K$  from which, interest rates are equalized and new investments get allocated to both sectors (this is only the case if the long-run steady-state does not feature full specialization, otherwise, new investments never get allocated to the shrinking sector). Except for this, the structure of the equilibrium is similar to the baseline case.

Using the same type of analysis as in Appendix A.1, we can derive first-order approximations for the time at which low-skill wages ( $t_1$ ), high-skill wages ( $t_2$ ) and capital are equalized and second order approximations for the ages of the low-skill ( $a_L$ ) and high-skill ( $a_H$ ) workers who are indifferent between switching sectors or not at the time of the shock. More specifically, if  $t_1 < t_2, t_K$ , we can derive

$$t_1 = \frac{w^{ss} \frac{dp}{p} + o(dp)}{(w_{1H} + w_{2H}) (1 - n_H) x_H(0) \bar{H} + (w_{1L} + w_{2L}) (1 - n_L) x_L(0) + (w_{1K} + w_{2K}) (1 - n_I) (\eta + \Delta) K_0}.$$

Using this expression, it is easy to show that in this case,  $t_1$  is between its value when capital is completely fixed and when capital is fully flexible. In this case, the age of the indifferent

low-skill worker  $a_L$  is given by (9), and we have that the mass of low-skill workers switching sectors on impact is greater than in the case of sector-specific capital but smaller than in the case where capital can immediately be transferred across sectors.

When  $t_1 < t_2 < t_K$ , we can further derive the following expressions:

$$t_2 = \frac{\left( v^{ss} - \frac{(v_{1L}+v_{2L})w^{ss}}{(w_{1L}+w_{2L})} \right) \frac{dp}{p} + o(dp)}{\left[ \left( v_{1H} + v_{2H} - \frac{(v_{1L}+v_{2L})(w_{1H}+w_{2H})}{w_{1L}+w_{2L}} \right) (1 - n_H) x_H(0) \bar{H} \right. \\ \left. + \left( v_{1K} + v_{2K} - \frac{(v_{1L}+v_{2L})(w_{1K}+w_{2K})}{w_{1L}+w_{2L}} \right) (1 - n_I) (\eta + \Delta) K_0 \right]},$$

$$t_K = \frac{\det \begin{pmatrix} w_{1L} + w_{2L} & w_{1H} + w_{2H} & w^{ss} \\ v_{1L} + v_{2L} & v_{1H} + v_{2H} & v^{ss} \\ r_{1L} + r_{2L} & r_{1H} + r_{2H} & r^{ss} \end{pmatrix} \frac{dp}{p} + o(dp)}{(1 - n_I) (\eta + \Delta) K_0 \det(Hessian)}.$$

where *Hessian* is the Hessian matrix of  $F_1 + pF_2$ :

$$Hessian = \begin{pmatrix} w_{1L} + w_{2L} & w_{1H} + w_{2H} & w_{1K} + w_{2K} \\ v_{1L} + v_{2L} & v_{1H} + v_{2H} & v_{1K} + v_{2K} \\ r_{1L} + r_{2L} & r_{1H} + r_{2H} & r_{1K} + r_{2K} \end{pmatrix}.$$

The age of the indifferent high-skill worker  $a_H$  is still given by (A.7).

When  $t_1 < t_K < t_2$ , we instead get (we present both cases because in our simulations we encountered both situations):

$$t_K = \frac{\left( r^{ss} - \frac{(r_{1L}+r_{2L})w^{ss}}{(w_{1L}+w_{2L})} \right) \frac{dp}{p} + o(dp)}{\left( r_{1H} + r_{2H} - \frac{(r_{1L}+r_{2L})(w_{1H}+w_{2H})}{w_{1L}+w_{2L}} \right) (1 - n_H) x_H(0) \bar{H} \\ + \left( r_{1K} + r_{2K} - \frac{(r_{1L}+r_{2L})(w_{1K}+w_{2K})}{w_{1L}+w_{2L}} \right) (1 - n_I) (\eta + \Delta) K_0}$$

$$t_2 = \frac{\det \begin{pmatrix} w_{1L} + w_{2L} & w^{ss} & w_{1K} + w_{2K} \\ v_{1L} + v_{2L} & v^{ss} & v_{1K} + v_{2K} \\ r_{1L} + r_{2L} & r^{ss} & r_{1K} + r_{2K} \end{pmatrix} \frac{dp}{p} + o(dp)}{(1 - n_H) x_H(0) \bar{H} \det(Hessian)},$$

and the age of the indifferent high-skill worker obeys:

$$a_H = \frac{-x_H(0) \frac{dp}{p}}{2 \int_0^T x'_H(\tau) e^{-\delta\tau} d\tau} \\ \times \left( + \frac{t_2 - (t_K - t_1) \frac{(v_{1L}+v_{2L})w^{ss}}{(w_{1L}+w_{2L})v^{ss}}}{\frac{w^{ss}((v_{1L}+v_{2L})(r_{1K}+r_{2K}) - (v_{1K}+v_{2K})(r_{1L}+r_{2L})) + r^{ss}((w_{1L}+w_{2L})(v_{1K}+v_{2K}) - (w_{1K}+w_{2K})(v_{1L}+v_{2L}))}{((r_{1L}+r_{2L})(w_{1K}+w_{2K}) - (r_{1K}+r_{2K})(w_{1L}+w_{2L}))v^{ss}}} (t_2 - t_K) \right)$$

This analysis straightforwardly extends to the case where the production functions depend both on a stock of slowly transferable capital and a stock of fully sector-specific capital (which grows at rate  $\eta$ ): all expressions are identical (with the derivatives taken with respect to the transferable capital).

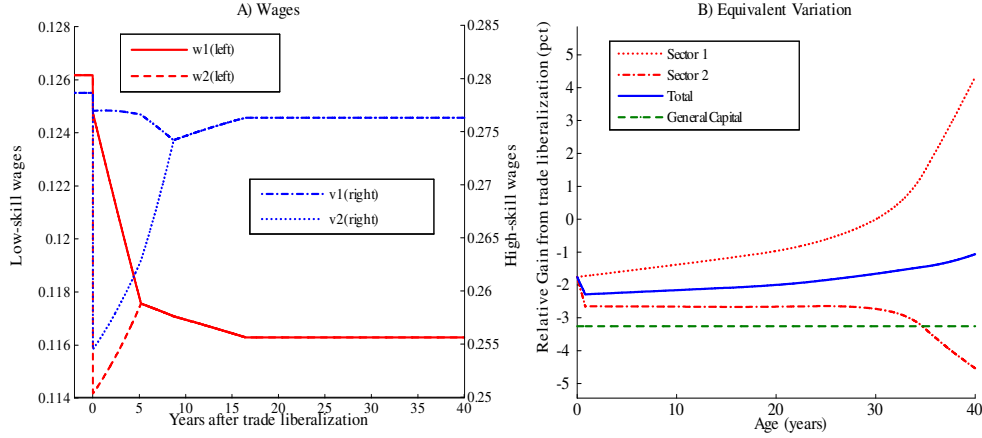


Figure A.1: Simulated Transition After Trade Shock. Case where half of the physical capital stock is slowly transferable and half is fixed. 10 percent price drop in sector 2.

#### A.4.2 Simulations

The calibration is done with the same parameters as in the case where capital is entirely fixed, but we assume that half of the capital stock in each sector is fully fixed while the other half is modeled as above. The depreciation rate is null ( $\Delta = 0$ ) and we focus on a 10% price change.

For price changes larger than 12%, an equilibrium akin to that described in Proposition 1 ceases to exist for the reason described in footnote 9 (however such an equilibrium would exist for a higher level of the population growth rate  $\eta$ ). Similarly as the share of fully sector-specific capital shrinks, more labor must reallocate in the long-run. This shrinks the range of price changes and population growth rates  $\eta$  for which an equilibrium as in Proposition 1 exists.

Figure A.1 reproduces Figure 6 for this scenario. Initially all factor prices are higher in sector 1 than in sector 2. As a result, all new workers and all investments are allocated to this sector until the relevant factor prices are equalized. Low-skill wages are equalized at  $t_1 = 5.18$ , then high-skill wages are equalized at  $t_2 = 8.68$  and finally the rental rates of transferable capital are equalized at  $t_K = 16.5$ . Low-skill workers younger than  $a_L = 0.82$  switch sectors on impact, while high-skill workers younger than  $a_H = 1.59$  do so. As expected from the theoretical analysis, the reallocation takes longer and more worker switch on impact relative to the case where capital is fully fixed. The share of reallocation which happens on impact for low-skill workers is given by  $\chi_L = 14.98\%$ , while for high-skill workers it is given by  $\chi_H = 20.17\%$ . Figure A.1.B shows that the welfare consequences are similar to the case where all capital is fixed (when the share of fully fixed capital shrinks, the welfare consequences look more like that in the case with no capital—Figure 4—with low-skill workers in sector 2 gaining relative to the case with general human capital).

Figure A.2 reproduces Figure 7.A albeit for price changes of 0.1 to 10%. Quantitatively,



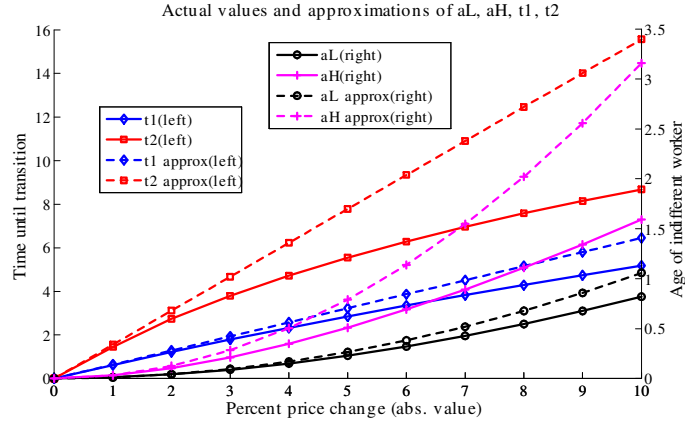


Figure A.2: Times until wage equalization and ages of indifferent workers for different price changes (simulation results and approximation results). Case where half of the physical capital stock is slowly transferable and half is fixed. 10 percent price drop in sector 2.

the quality of the approximation is in between the case where capital is entirely fixed and that where there is no capital. In particular the approximations for  $t_1$  and  $t_2$  are not too far off, but the approximations for  $a_L$  and  $a_H$  are less good for price changes over 5%.

## B Online Appendix

### B.1 Rest of the proof of Proposition 1

#### B.1.1 $n_L(t)$ and $n_H(t)$ are in $(0, 1)$ for $\eta$ sufficiently large

We now seek to show that, once wages have been equalized, it is possible to adjust the flow of entrants  $n_L(t)$ ,  $n_H(t)$  in sector 1 so as to keep wages equalized for  $\eta$  sufficiently large ( $\eta > \frac{1}{T} \ln \left( \frac{x_L(T)}{x_L(0)} \right)$  is a sufficient condition). We denote by  $n_Z(t)$  the mass of workers of type  $Z = L, H$  entering sector 1; to avoid confusion, we rewrite the original steady-state value as  $n_Z^{ss}$  instead of  $n_Z$ . Once wages have been equalized, the normalized mass of low-skill workers in sector 1 can be written as

$$\begin{aligned} l_1(t) &= n_L^{ss} \int_{\min(t, T)}^T e^{-\eta\tau} x_L(\tau) d\tau + (1 - n_L^{ss}) e^{-\eta t} x_L(t) \int_0^{\min(a_L, |T-t|)} e^{-\eta\tau} d\tau \\ &+ \int_{\min(t-t_1, T)}^{\min(t, T)} e^{-\eta\tau} x_L(\tau) d\tau + \int_0^{\min(t-t_1, T)} e^{\eta(-\tau)} n_L(t-\tau) x_L(\tau) d\tau \text{ for } t \geq t_1. \end{aligned} \quad (\text{B.1})$$

Compared with equation (7), there is a new term representing the entrants who have arrived since the equalization of wages (the first three terms are generalized to the case where  $t$  is sufficiently large that workers who switched start to die off). Similarly, the normalized mass of low-skill workers in sector 2—and of high-skill workers in sectors 1 and 2—can be written as follows:

$$l_2(t) = (1 - n_L^{ss}) \int_{\min(t+a_L, T)}^T e^{-\eta\tau} x_L(\tau) d\tau + \int_0^{\min(t-t_1, T)} e^{\eta(-\tau)} (1 - n_L(t-\tau)) x_L(\tau) d\tau \text{ for } t \geq t_1, \quad (\text{B.2})$$

$$\begin{aligned} \frac{h_1(t)}{\overline{H}} &= n_H^{ss} \int_{\min(t, T)}^T e^{-\eta\tau} x_H(\tau) d\tau + e^{-\eta t} x_L(t) (1 - n_H^{ss}) \int_0^{\min(a_H, |T-t|)} e^{-\eta\tau} d\tau \\ &+ \int_{\min(t-t_2, T)}^{\min(t, T)} e^{-\eta\tau} x_H(\tau) d\tau + \int_0^{\min(t-t_2, T)} e^{\eta(-\tau)} n_H(t-\tau) x_H(\tau) d\tau \text{ for } t \geq t_2, \end{aligned} \quad (\text{B.3})$$

$$\frac{h_2(t)}{\overline{H}} = (1 - n_H^{ss}) \int_{\min(t+a_H, T)}^T e^{-\eta\tau} x_L(\tau) d\tau + \int_0^{\min(t-t_2, T)} e^{\eta(-\tau)} (1 - n_H(t-\tau)) x_H(\tau) d\tau \text{ for } t \geq t_2. \quad (\text{B.4})$$

The functions  $n_L(t)$  and  $n_H(t)$  must satisfy  $v_1(t) = v_2(t)$  for  $t \geq t_2$  and  $w_1(t) = w_2(t)$  for  $t \geq t_1$ , and our goal is to show that the solutions to these equations fall in the range  $(0, 1)$ . Summing (B.1) and (B.2) and then summing (B.3) and (B.4), we obtain:

$$\begin{aligned} l(t) &= l_1(t) + l_2(t) = l^{eff} - (1 - n_L^{ss}) e^{-\eta t} \int_0^{\min(a_L, |T-t|)} e^{-\eta\tau} (x_L(\tau+t) - x_L(t)) d\tau = l^{eff} + o(dp^4), \\ h(t) &= h_1(t) + h_2(t) = h^{eff} - \overline{H} (1 - n_H^{ss}) e^{-\eta t} \int_0^{\min(a_H, |T-t|)} e^{-\eta\tau} (x_H(\tau+t) - x_H(t)) d\tau = h^{eff} + o(dp^4); \end{aligned}$$

these equalities show that, at forth order, the masses of normalized factors are constant.

First, we consider the case where  $t \in (t_1, t_2)$ , so that only wages of low-skill workers are equalized and  $t$  is small. Equation (A.3) still holds, and differentiating (B.1) and (B.2) for small price changes and small  $t$  yields

$$dl_1(t) = -dl_2(t) = ((t - t_1)(n_L(t) - n_L^{ss}) + t_1(1 - n_L^{ss}))x_L(0) + o(dp).$$

Plugging these expressions into  $dw_1 = dw_2 + o(dp)$  and using (8), the result is

$$(t - t_1)((w_{1H} + w_{2H})(1 - n_H^{ss})x_H(0)\bar{H} + (w_{1L} + w_{2L})(n_L(t) - n_L^{ss})x_L(0)) = o(dp);$$

therefore,

$$n_L(t) - n_L^{ss} = -\frac{(w_{1H} + w_{2H})(1 - n_H)x_H(0)\bar{H}}{(w_{1L} + w_{2L})x_L(0)} + o(1).$$

Since  $(w_{1H} + w_{2H})(1 - n_H)x_H(0)\bar{H} + (w_{1L} + w_{2L})(1 - n_H)x_L(0) < 0$  if  $t_1 < t_2$ , it follows that  $n_L(t)$  is in  $(0, 1)$  for small price changes when  $t \in (t_1, t_2)$ .

For  $t > t_2$ , wages are equalized in both sectors. This implies, absent any factor intensity reversal, that wages must be at the new steady-state value:  $w_2(t) = w_1(t) = w^{ss'}$  and  $v_2(t) = v_1(t) = v^{ss'}$ . Hence factor intensity in each sector must likewise be at the new steady state values, so we must have

$$l_1(t) = \frac{l(t) - \frac{l_2^{ss'}}{l_2^{ss'}}h(t)}{\left(1 - \frac{l_2^{ss'}}{h_2^{ss'}}\frac{h_1^{ss'}}{l_1^{ss'}}\right)}$$

(as well as similar expressions for  $h_2(t)$ ,  $l_1(t)$ , and  $l_2(t)$ ). Furthermore, observe that  $\frac{dl(t)}{dt} = O(dp^2)$ , where  $O(dp^2)$  is a function of time and  $dp$ , which satisfies  $|O(dp^2)| < Mdp^2$  for some positive real  $M$ ; similarly,  $\frac{dh(t)}{dt} = O(dp^2)$ . Therefore  $\frac{dl_1(t)}{dt} = O(dp^2)$  and similarly for  $\frac{dl_2(t)}{dt}$ ,  $\frac{dh_1(t)}{dt}$ , and  $\frac{dh_2(t)}{dt}$ .

More specifically, assume that  $t_2 < t < T - \max(a_H, a_L)$ . Then

$$\begin{aligned} \frac{dh_1(t)}{dt} \frac{1}{\bar{H}} &= -e^{-\eta t} n_H^{ss} x_H(t) + (1 - n_H^{ss}) \frac{d}{dt} (e^{-\eta t} x_H(t)) \int_0^{a_H} e^{-\eta \tau} d\tau + e^{-\eta t} x_H(t) \\ &\quad - e^{\eta(t-t_2)} x_H(t-t_2) + n_H(t) x_H(0) + \int_0^{t-t_2} n_H(t-\tau) \frac{d(e^{-\eta \tau} x_H(\tau))}{d\tau} d\tau, \end{aligned}$$

$$\frac{dh_2(t)}{dt} \frac{1}{\bar{H}} = -e^{-\eta(t+a_H)} (1 - n_H^{ss}) x_H(t+a_H) + (1 - n_H(t)) x_H(0) + \int_0^{t-t_2} (1 - n_H(t-\tau)) \frac{d(e^{-\eta \tau} x_H(\tau))}{d\tau} d\tau.$$

Similar expressions can be derived for low-skill workers:

$$\begin{aligned} \frac{dl_1(t)}{dt} &= -e^{-\eta t} n_L^{ss} x_L(t) + (1 - n_L^{ss}) \frac{d}{dt} (e^{-\eta t} x_L(t)) \int_0^{a_L} e^{-\eta \tau} d\tau + e^{-\eta t} x_L(t) \\ &\quad - e^{-\eta(t-t_1)} x_L(t-t_1) + n_L(t) x_L(0) + \int_0^{t-t_1} n_L(t-\tau) \frac{d(e^{-\eta \tau} x_L(\tau))}{d\tau} d\tau, \end{aligned}$$

$$\frac{dl_2(t)}{dt} = -e^{-\eta(t+a_L)}(1-n_L^{ss})x_L(t+a_L) + (1-n_L(t))x_L(0) + \int_0^{t-t_1} (1-n_L(t-\tau)) \frac{d(e^{-\eta\tau}x_L(\tau))}{d\tau} d\tau.$$

Since  $\frac{dl_1(t)}{dt} = O(dp^2)$ , we have

$$-e^{-\eta t}n_L^{ss}x_L(t) + n_L(t)x_L(0) + \int_0^{t-t_1} n_L(t-\tau) \frac{d(e^{-\eta\tau}x_L(\tau))}{d\tau} d\tau = O(dp).$$

Because  $t_1$  and  $t_2$  are first order, this equation can be rewritten as

$$(n_L(t) - n_L^{ss})x_L(0) + \int_0^{t-t_2} (n_L(t-\tau) - n_L^{ss}) \frac{d(e^{-\eta\tau}x_L(\tau))}{d\tau} d\tau = O(dp). \quad (\text{B.5})$$

Note that  $n_L(t) = n_L^{ss}$  for  $t \in (t_2, T - \max(a_H, a_L))$  is a solution to this functional equation when the right-hand side is exactly equal to 0. Hence there must be a solution of (B.5) that can be written as  $n_L(t) = n_L^{ss} + O(dp)$ , where  $O(dp)$  is a function of time and  $dp$ , which satisfies  $|O(dp)| < Ndp$  for some number  $N$  and all  $t$ . Then, for  $dp$  sufficiently small,  $n_L(t)$  will be close to  $n_L^{ss}$ ; in particular,  $n_L(t)$  belongs to  $(0, 1)$ . The same reasoning holds for  $n_H(t)$ .

Without loss of generality, we assume that  $a_H > a_L$  and examine the case where  $T - a_L \leq t < T - a_H$ . The expressions for the derivatives of  $h_1(t)$  and  $h_2(t)$  remain identical, but those for the derivatives of  $l_1(t)$  and  $l_2(t)$  now become

$$\begin{aligned} \frac{dl_1(t)}{dt} &= -e^{-\eta t}n_L^{ss}x_L(t) + (1-n_L^{ss}) \int_0^{T-t} e^{-\eta\tau} d\tau \frac{d}{dt} (e^{-\eta t}x_L(t)) - (1-n_L^{ss})x_L(t)e^{-\eta T} \\ &\quad + e^{-\eta t}x_L(t) - e^{\eta(t-t_1)}x_L(t-t_1) + n_L(t)x_L(0) + \int_0^{t-t_1} n_L(t-\tau) \frac{d(e^{-\eta\tau}x_L(\tau))}{d\tau} d\tau, \\ \frac{dl_2(t)}{dt} &= (1-n_L(t))x_L(0) + \int_0^{t-t_1} (1-n_L(t-\tau)) \frac{d(e^{-\eta\tau}x_L(\tau))}{d\tau} d\tau. \end{aligned} \quad (\text{B.6})$$

Using a similar argument as the one for (B.5) and given that  $T-t$  is second order for this time interval, we obtain

$$-(1-n_L^{ss})e^{-\eta T}x_L(T) + (n_L(t) - n_L^{ss})x_L(0) + \int_0^{t-t_2} (n_L(t-\tau) - n_L^{ss}) \frac{d(e^{-\eta\tau}x_L(\tau))}{d\tau} d\tau = O(dp).$$

This equation implies, since  $n_L(t) = n_L^{ss} + O(dp)$  (except for  $t \leq t_2$ , that is for a time period of duration  $O(dp)$ ), that

$$n_L(t) = n_L^{ss} + (1-n_L^{ss})e^{-\eta T} \frac{x_L(T)}{x_L(0)} + O(dp). \quad (\text{B.7})$$

Therefore,  $n_L(t) \in (0, 1)$  for  $dp$  sufficiently small provided

$$\eta > \frac{1}{T} \ln \left( \frac{x_L(T)}{x_L(0)} \right).$$

The same reasoning holds when  $T - a_H \leq t < T$  but now it applies to both high-skill and low-skill workers. Suppose now that  $T \leq t < T + t_1$ . Then

$$\begin{aligned}\frac{dl_1(t)}{dt} &= -e^{\eta(t-t_1)} x_L(t-t_1) + n_L(t) x_L(0) + \int_0^{t-t_1} n_L(t-\tau) \frac{d(e^{-\eta\tau} x_L(\tau))}{d\tau} d\tau, \\ \frac{dh_1(t)}{dt} \frac{1}{H} &= -e^{\eta(t-t_2)} x_H(t-t_2) + n_H(t) x_H(0) + \int_0^{t-t_2} n_H(t-\tau) \frac{d(e^{-\eta\tau} x_H(\tau))}{d\tau} d\tau;\end{aligned}$$

at the same time, (B.6) and the equivalent equation for  $\frac{dh_2}{dt}$  hold. In this case,  $\frac{dh(t)}{dt} = \frac{dl(t)}{dt} = 0$  and so

$$-(1 - n_L^{ss}) e^{\eta(t-t_1)} x_L(t-t_2) + (n_L(t) - n_L^{ss}) x_L(0) + \int_0^{t-t_1} (n_L(t-\tau) - n_L^{ss}) \frac{d(e^{-\eta\tau} x_L(\tau))}{d\tau} d\tau = 0;$$

therefore (B.7) still holds. Hence for  $\eta > \frac{1}{T} \ln\left(\frac{x_L(T)}{x_L(0)}\right)$  and sufficiently small  $dp$ , we have  $n_L(t) \in (0, 1)$ . The same reasoning applies to high-skill workers and extends to low-skill workers when  $T + t_1 < t < T + t_2$ .

Finally if  $T + t_1 \leq t$  then the derivatives for high-skill workers can be written as

$$\begin{aligned}\frac{dh_1}{dt} \frac{1}{H} &= -\frac{dh_2}{dt} \frac{1}{H} = -(n_H(t-T) - n_H^{ss}) x_H(T) e^{-\eta T} + (n_H(t) - n_H^{ss}) x_H(0) \\ &\quad + \int_0^T (n_H(t-\tau) - n_H^{ss}) \frac{d(e^{-\eta\tau} x_H(\tau))}{d\tau} d\tau;\end{aligned}$$

as a result, the counterpart of (B.5) is now

$$(n_H(t) - n_H^{ss}) x_H(0) = (n_H(t-T) - n_H^{ss}) x_H(T) e^{-\eta T} + \int_0^T (n_H(t-\tau) - n_H^{ss}) \frac{d(e^{-\eta\tau} x_H(\tau))}{d\tau} d\tau.$$

From this expression it follows directly that a solution exists where  $n_H(t)$  belongs to  $(0, 1)$  when  $x_H(T) e^{-\eta T} / x_H(0) < 1$ . Moreover,  $n_H^{ss}(t)$  is close to  $n_H^{ss}$  except for a set of measure  $O(dp)$ . For  $T + t_2 \leq t$ , the reasoning extends to low-skilled workers. This completes the proof of existence

### B.1.2 Production efficiency

We now prove that this equilibrium (when it exists) maximizes the present value of production (if it is finite) or the present value of production up to some date  $t \geq T$ . At any instant  $t > T$ , the competitive equilibrium described reproduces the outcome of a general human capital economy and therefore it does indeed maximize production. Therefore, we only have to show that the competitive equilibrium maximizes the discounted value of production from 0 to  $T$ .

First, we compare the present value of production in the competitive equilibrium described above with the present value of production in the (unique) competitive equilibrium obtained

in a model with general human capital. Note that with general human capital, the competitive equilibrium maximizes the present value of production. Relative to the general human capital allocation, the allocation of the sector-specific human capital model corresponds to a misallocation of factors and a decrease in the endowments. The misallocation of factors is first order in the price change. By the envelope theorem, it has a second-order effect on the value of production at a given time. This misallocation of factors, however, only last for a period of time which is first order in the price change (until wages are equalized). Therefore, it has a third order effect on the present value of production. The loss of effective units of human capital occurs as workers switch sectors. Since only a second order mass of workers switch, and since these workers have accumulated only a second order amount of human capital, this loss only has a forth order impact. Therefore, the difference between the maximal present value in the general human capital case and the present value obtained in the competitive equilibrium with sector specific-human capital is third order in the price change. As the maximal present value of production with general human capital must be weakly greater than its counterpart with sector-specific human capital, it follows that their difference must be at most third order in the price change.

Since the steady-state allocation maximizes the value of output before the trade shock, the allocation of factors which maximizes the present value of production differs from the steady-state allocation at most at first order.<sup>33</sup> Therefore the marginal product of factors across sector differs at most at first order. Moreover, they may only differ at first order for a first order period of time (otherwise the difference in the present value of production relative to the case with general human capital will be more than third order).

Let us then consider a (low-skill or high-skill) worker born after the trade shock. Denote by  $\mu_1$  the time he spends in sector 1 and by  $\mu_2$  the time he spends in sector 2 in the present value maximizing allocation. Without loss of generality, assume that  $\mu_1 \geq \mu_2$ . Let us consider an alternative allocation where he would spend all of his time in sector 1 instead. This worker would then provide a higher level of human capital which translates into a gain in the present value of production of the same order as  $\mu_2$ . The potential loss arises from a (possible) lower value of the marginal product of his type of human capital in sector 1 than in the sector 2. This loss is at most of second order (since marginal products are identical at first order except potentially for a first order period of time - where they may differ at first order). Therefore, this allocation of the worker's time can only be part of the social optimum if  $\mu_2$  is second order. The potential loss is then at most of the same order as  $\mu_2 dp$ , and must be smaller than the benefit. As a result, in the optimal allocation, newborn workers must remain in the same sector. Similarly workers who were already born at the time of the shock may only switch at the time of the shock and their accumulated experience at that time can only be second order.

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<sup>33</sup>Technically this only holds almost everywhere, that is everywhere except for a time period of mass 0. All identities in the following are similarly holding "almost everywhere".

Given that the optimal number of workers switch at the time of the trade shock, no workers would subsequently switch sectors, the problem of finding the optimal allocation at  $t > 0$  is simply one of finding the share of workers entering each sector (the endowments in effective units of human capital are fixed). Maximizing the present value of production requires to have workers being allocated to the sector with the largest marginal product of their human capital up until there are no more new workers to allocate or the marginal products are equalized across sectors. Since the initial switch is second order, marginal factor products are initially higher in sector 1 and both types of workers are initially allocated to sector 1. Our analysis of the competitive equilibrium shows that once marginal products have been equalized, it is possible to ensure that they remain equalized all along (provided that  $\eta$  is sufficiently large and the price change is small). Therefore marginal products in sector 1 must remain higher than in sector 2, and the initial switch only concerns workers of sector 2.

As a result, the optimal allocation takes the same form as the competitive allocation described above, with the share of entrants being allocated to the different sectors solving the same equations, but the mass of workers switching sectors at time 0 may still be different. To find this mass, we solve:

$$\max_{a_L, a_H} \int_0^T e^{-(\delta-\eta)t} (y_1(t) + p y_2(t)),$$

where  $y_1$  and  $y_2$  are the normalized amount of output, with the following constraints:

$$y_1(t) = F_1(l_1(t), h_1(t)) \text{ and } y_2(t) = F_2(l_2(t), h_2(t)),$$

$$\begin{aligned} l_1(t) = & n_L^{ss} \int_t^T e^{-\eta\tau} x_L(\tau) d\tau + (1 - n_L^{ss}) e^{-\eta t} x_L(t) \int_0^{\min(a_L, T-t)} e^{-\eta\tau} d\tau \\ & + \int_{\max(0, t-t_1)}^t e^{-\eta\tau} x_L(\tau) d\tau + \int_0^{\max(0, t-t_1)} e^{-\eta\tau} n_L(t-\tau) x_L(\tau) d\tau. \end{aligned} \quad (\text{B.8})$$

(this expression comes from (7) and (B.1)),

$$l_2(t) = (1 - n_L^{ss}) \int_{t+a_L}^T e^{-\eta\tau} x_L(\tau) d\tau + \int_0^{\max(0, t-t_1)} e^{-\eta\tau} (1 - n_L(t-\tau)) x_L(\tau) d\tau, \quad (\text{B.9})$$

with similar constraints for  $h_1(t)$  and  $h_2(t)$ , and where  $t_1, t_2, n_L(t), n_H(t)$  are defined as in the competitive case.

The first order equation with respect to  $a_L$  leads to:

$$\int_0^T e^{-(\delta-\eta)t} \left( w_1(t) \frac{\partial}{\partial a_L} l_1(t) + w_2(t) \frac{\partial}{\partial a_L} l_2(t) \right) = 0,$$

where  $w_1(t)$  is the marginal product of low-skill labor in sector 1 (and similarly  $w_2(t)$  in sector 2). Computing the integral separately over the intervals  $(0, t_1)$ ,  $(t_1, T - a_L)$  and  $(T - a_L, T)$

and using that  $w_1(t) = w_2(t) = w(t)$  after  $t_1$  gives:

$$\begin{aligned} & \int_0^{t_1} e^{-(\delta-\eta)t} \left( w_1(t) e^{-\eta t} x_L(t) e^{-\eta a_L} - w_2(t) e^{-\eta(t+a_L)} x_L(t+a_L) \right) d\tau \\ & + \int_{t_1}^{T-a_L} e^{-(\delta-\eta)t} w(t) e^{-\eta(t+a_L)} (x_L(t) - x_L(t+a_L)) d\tau \\ & = 0, \end{aligned}$$

which, in turn, can be rewritten as (5). The same holds for  $a_H$ . Therefore the equilibrium allocation maximizes the present value of production.

## B.2 Speed of human capital accumulation and $t_2$

In this appendix, we show formally that a faster accumulation function delays  $t_2$ . We use  $\hat{\cdot}$  to denote variables under the alternative accumulation functions  $\hat{x}_L, \hat{x}_H$ . Define  $\zeta \equiv \left( \int_0^T \hat{x}_L(\tau) e^{-\delta\tau} d\tau \right) / \left( \int_0^T x_L(\tau) e^{-\delta\tau} d\tau \right) > 1$ , we then get  $\hat{l}^{\max} = \zeta l^{\max}$ , and since  $\hat{x}_H = \gamma \hat{x}_L$  and  $x_H = \gamma x_L$  we also have  $\hat{h}^{\max} = \zeta h^{\max}$ . The production functions are homogeneous of degree one. As a result, their first derivatives are homogeneous of degree 0, hence  $F_{1L}(n_L \hat{l}^{\max}, n_H \hat{h}^{\max}) = F_{1L}(n_L l^{\max}, n_H h^{\max})$ , and similarly for sector 2 and the derivatives with respect to  $H$ . Therefore the allocation shares  $n_L$  and  $n_H$  are identical in steady-state in both scenarios, and we have  $\hat{v}^{ss} = v^{ss}$  and  $\hat{w}^{ss} = w^{ss}$ . The second derivatives are homogeneous of degree -1, so that  $\hat{v}_{iZ} = \zeta^{-1} v_{iZ}$  for  $i \in \{1, 2\}$  and  $Z \in \{L, H\}$  and similarly for  $w$ . Applying (A.6), we then get:

$$\begin{aligned} \hat{t}_2 &= \frac{\hat{v}^{ss} - \frac{\hat{v}_{1L} + \hat{v}_{2L}}{\hat{w}_{1L} + \hat{w}_{2L}} \hat{w}^{ss}}{(1 - n_H) \hat{x}_H(0) \bar{H} \left( \hat{v}_{1H} + \hat{v}_{2H} - \frac{(\hat{v}_{1L} + \hat{v}_{2L})(\hat{w}_{1H} + \hat{w}_{2H})}{\hat{w}_{1L} + \hat{w}_{2L}} \right)} \frac{dp}{p} + o(dp) \\ &= \frac{v^{ss} - \frac{v_{1L} + v_{2L}}{w_{1L} + w_{2L}} w^{ss}}{(1 - n_H) x_H(0) \bar{H} \zeta^{-1} \left( v_{1H} + v_{2H} - \frac{(v_{1L} + v_{2L})(w_{1H} + w_{2H})}{w_{1L} + w_{2L}} \right)} \frac{dp}{p} + o(dp) \\ &= \zeta t_2 + o(dp) > t_2 + o(dp). \end{aligned}$$

## B.3 Proof of Proposition 4

This section provides details about the adjustment program. We restrict attention to the case where  $x_H = \gamma x_L$ , and the subsidy is financed lump-sum by high-skill workers. Assume that workers who switch sectors receive, over their lifetime, a sum with present value  $S_Z$ ,  $Z \in \{L, H\}$ , regardless of their age.  $S_Z$  is first order in the price change and small enough that full adjustment is not reached. The indifferent workers are now characterized by

$$\int_0^{T-a_L} w_1(\tau) x_L(\tau) e^{-\delta\tau} d\tau + S_L = \int_0^{T-a_L} w_2(\tau) x_L(a_L + \tau) e^{-\delta\tau} d\tau, \quad (\text{B.10})$$



$$\int_0^{T-a_H} v_1(\tau) x_H(\tau) e^{-\delta\tau} d\tau + S_H = \int_0^{T-a_H} v_2(\tau) x_H(a_H + \tau) e^{-\delta\tau} d\tau \quad (\text{B.11})$$

instead of by (5) and (6). Even for a marginal price change, an extremely old worker may be willing to switch sectors in order to claim the subsidy before dying even if  $w_1(\tau) x_L(\tau) < w_2(\tau) x_L(a_L + \tau)$  (and  $v_1(\tau) x_H(\tau) < v_2(\tau) x_H(a_H + \tau)$ ) throughout his lifetime. We therefore assume that the subsidy is distributed over a time period of nonnegligible duration so that very old workers do not switch. We assume that the subsidy is proportional to entrants' steady-state wages:  $S_L = s w^{ss} x_L(0)$  and  $S_H = s v^{ss} x_H(0)$ . The number of workers who switch is now first order, and taking into account that  $x_H = \gamma x_L$ , the age of the indifferent worker of skill-type  $Z$  must be given by (12). Note that we still have  $w_1(0) > w_2(0)$  and  $v_1(0) > v_2(0)$ , and that the structure of the equilibrium stays the same.

For  $t < t_1, t_2$ , (A.2) and (A.3) become:

$$dl_1(t) = -dl_2(t) = (1 - n_L) x_L(0) (\xi s + t),$$

$$dh_1(t) = -dh_2(t) = (1 - n_H) \bar{H} x_H(0) (\xi s + t),$$

so that low-skill wages are equalized first when (A.4) is satisfied.

Assume that it is indeed satisfied, then, following the same strategy as in Appendix (A.1), we get that

$$t_1 = t_1^0 - \xi s,$$

where  $t_1^0 \equiv \frac{w^{ss}}{p} dp / ((w_{1H} + w_{2H})(1 - n_H) x_H(0) \bar{H} + (w_{1L} + w_{2L})(1 - n_L) x_L(0))$  is the first-order approximation of  $t_1$  when there is no subsidy assuming that  $t_1 < t_2$ . For  $t < t_1$ , wages for low-skill workers are given by:

$$dw_1(t) = dw_1^0(\xi s + t) + o(dp) \text{ and } dw_2(t) = dw_2^0(\xi s + t) + o(dp), \quad (\text{B.12})$$

where  $dw_1^0$  and  $dw_2^0$  represent the changes in wages when there are no subsidy. Similarly

$$t_2 = t_2^0 - \xi s + o(dp), \quad (\text{B.13})$$

with  $t_2^0 \equiv \left( v^{ss} - \frac{v_{1L} + v_{2L}}{w_{1L} + w_{2L}} w^{ss} \right) \frac{dp}{p} / \left( (1 - n_H) x_H(0) \bar{H} \left( v_{1H} + v_{2H} - \frac{(v_{1L} + v_{2L})(w_{1H} + w_{2H})}{w_{1L} + w_{2L}} \right) \right)$  defined similarly as  $t_1^0$ . For  $t \in (t_1, t_2)$ , we get

$$dw_1(t) = dw_2(t) = dw^0(\xi s + t).$$

Therefore the subsidy simply shifts the transition. Since  $w_1(t), w_2(t) \geq w^{ss'}$  all along, and since (at first-order) discounting can be ignored, we get that all low-skill workers who do not receive the subsidy are worse-off with it (except, possibly, workers who are sufficiently old and retire during the transition period). Workers who do receive the subsidy get a first-order benefit from it. The direct impact of the subsidy on the aggregate life-time income of low-skill workers can be written as  $a_L(1 - n_L) S_L$  and is proportional to  $s^2$ .

Yet, the change in aggregate income relative to steady-state for all low-skill workers alive at  $t = 0$  can be written as

$$dW = -s \frac{\left( (1 - n_L) \left( w_{2L} \frac{v_{1H} + v_{2H}}{v_{1L} + v_{2L}} - w_{2H} \right) + n_L \left( w_{1H} - w_{1L} \frac{v_{1H} + v_{2H}}{v_{1L} + v_{2L}} \right) (v_{1L} + v_{2L}) w^{ss} \right)}{(w_{1L} + w_{2L}) (v_{1H} + v_{2H}) - (v_{1L} + v_{2L})^2} \frac{dp}{p} \\ \times \xi \left( \int_0^T x_L(\theta) e^{-\eta\theta} d\theta \right) + K_0 + K_1 s^2 + o(dp^2),$$

where  $K_0$  and  $K_1$  are independent of  $s$ . Knowing that  $w_{1H} - w_{1L} \frac{v_{1H} + v_{2H}}{v_{1L} + v_{2L}} < 0$ ,  $w_{2L} \frac{v_{1H} + v_{2H}}{v_{1L} + v_{2L}} - w_{2H} < 0$ ,  $w_{1H} - w_{1L} \frac{v_{1H} + v_{2H}}{v_{1L} + v_{2L}} < 0$  and  $(w_{1L} + w_{2L}) (v_{1H} + v_{2H}) - (v_{1L} + v_{2L})^2 > 0$ , the impact of the subsidy on the change in aggregate income  $dW$  is negative for  $s$  small enough.

## B.4 Gross flows model

In this appendix we solve the extended model with gross flows due to nonpecuniary sector preferences (as first presented in Section 5). First, we derive the mass of entrants in each state in a steady-state where the share of workers in a biased state is the same across all ages and then compute the gross flows. Second, we derive the system of partial derivatives equations that the utilities of workers must satisfy in equilibrium, and solve it in steady-state. Third, we derive the equilibrium following a small price shock.

### B.4.1 Step 1: Number of entrants and gross flows

Denote by  $g_i(t)$  the mass of high-skill workers in state  $i = 0, 1, 2$  at time  $t$  (that is workers with no sectoral preference, workers biased towards sector 1 and workers biased towards sector 2), and denote by  $\tilde{g}_i(t)$  the flow of entrants in state  $i$  at time  $t$ . Workers die with a Poisson rate  $\delta$ , switch from state 0 to each of the biased state with a Poisson rate  $\lambda/2$ , and switch from each of the biased state to state 0 with a Poisson rate  $1/2$ . We then have the following system of differential equations:

$$\begin{aligned} \frac{dg_0}{dt}(t) &= -(\lambda + \delta) g_0(t) + \frac{1}{2} g_1(t) + \frac{1}{2} g_2(t) + \tilde{g}_0(t), \\ \frac{dg_1}{dt}(t) &= \frac{\lambda}{2} g_0(t) - \left( \frac{1}{2} + \delta \right) g_1(t) + \tilde{g}_1(t), \\ \frac{dg_2}{dt}(t) &= \frac{\lambda}{2} g_0(t) - \left( \frac{1}{2} + \delta \right) g_2(t) + \tilde{g}_2(t). \end{aligned}$$

In steady state, all  $g_i(t)$  and  $\tilde{g}_i(t)$  are constant. Furthermore, if the share of biased individuals is the same across ages then  $g_i(t) = \frac{1}{\delta} \tilde{g}_i(t)$ . (Note that, for a death rate  $\delta$ , the total mass of high-skill workers alive is given by  $\delta^{-1} \bar{H}$ , where  $\bar{H}$  is the size of an incoming generation.) We

can now solve the system to obtain that, in steady state,

$$\tilde{g}_0 = \frac{1}{1+2\lambda}, \tilde{g}_1 = \tilde{g}_2 = \frac{\lambda}{1+2\lambda}.$$

The workers who switch sectors between  $t$  and  $t + dt$  are the normal workers who become biased toward working in a different sector than the one in which they are employed in at time  $t$ . Therefore, a share  $\frac{\lambda}{2} \frac{1}{1+2\lambda}$  (per unit of time) of workers switches sectors.

#### B.4.2 Step 2: System satisfied by the value functions of workers

Recall that our assumption that nonpecuniary benefits are sufficiently large that a biased worker in state  $i$  always works in sector  $i$ . Denote by  $W_0^j(t, a)$  the value function of a low-skill worker of experience  $a$  at time  $t$  in the *normal* state in sector  $j$ , and denote by  $W_i(t, a)$  the value function of a low-skill worker of age  $a$  at time  $t$  in the *biased* state  $i \in \{1, 2\}$  (and so, by assumption, in sector  $i$ ). The set of value functions must then satisfy

$$W_1(t, a) = (w_1(t) x_L(a) + b) dt + \left(1 - \frac{1}{2}dt - \delta dt\right) W_1(t + dt, a + dt) + \frac{1}{2}dt W_0^1(t + dt, a + dt). \quad (\text{B.14})$$

Intuitively, biased workers in state 1 work in sector 1, receive wages  $w_1(t) x_L(a)$  and enjoy the nonpecuniary benefit  $b$ , die with probability  $\delta dt$ , and revert back to the normal state with probability  $\frac{1}{2}dt$  (without switching sectors). Similarly, we have

$$W_2(t, a) = (w_2(t) x_L(a) + b) dt + \left(1 - \frac{1}{2}dt - \delta dt\right) W_2(t + dt, a + dt) + \frac{1}{2}dt W_0^2(t + dt, a + dt). \quad (\text{B.15})$$

Workers in the normal state can become biased with probability  $\frac{\lambda}{2}dt$  for each biased state. If they remain unbiased, they decide rationally whether or not switch sectors. Irrespective of the reason, a worker who switches sectors loses all her accumulated experience. Hence the value function for unbiased workers in sector 1 is

$$\begin{aligned} W_0^1(t, a) &= w_1(t) x_L(a) dt + (1 - \lambda dt - \delta dt) \max(W_0^1(t + dt, a + dt), W_0^2(t + dt, 0)) \\ &\quad + \frac{\lambda}{2}dt W_1(t + dt, a + dt) + \frac{\lambda}{2}dt W_2(t + dt, 0), \end{aligned} \quad (\text{B.16})$$

and similarly for sector 2:

$$\begin{aligned} W_0^2(t, a) &= w_2(t) x_L(a) dt + (1 - \lambda dt - \delta dt) \max(W_0^1(t + dt, 0), W_0^2(t + dt, a + dt)) \\ &\quad + \frac{\lambda}{2}dt W_1(t + dt, 0) + \frac{\lambda}{2}dt W_2(t + dt, a + dt). \end{aligned} \quad (\text{B.17})$$

In steady state,  $w_1(t) = w_2(t) = w$ , the problem is stationary and normal workers never switch sectors. Define  $\vec{W} \equiv (W_1 \ W_0^1 \ W_2 \ W_0^2)$ , and add a superscript  $^{ss}$  for the steady-state values ( $\vec{W}^{ss}$  depends only on experience, not on time). The problem then simplifies to

$$\frac{d}{da} \vec{W}^{ss}(a) = B \vec{W}^{ss}(a) - \begin{pmatrix} wx_L(a) + b & \frac{\lambda}{2} W_2^{ss}(0) + wx_L(a) & wx_L(a) + b & \frac{\lambda}{2} W_1^{ss}(0) + wx_L(a) \end{pmatrix}^T,$$

where

$$B = \begin{pmatrix} \frac{1}{2} + \delta & -\frac{1}{2} & 0 & 0 \\ -\frac{\lambda}{2} & \lambda + \delta & 0 & 0 \\ 0 & 0 & \frac{1}{2} + \delta & -\frac{1}{2} \\ 0 & 0 & -\frac{\lambda}{2} & \lambda + \delta \end{pmatrix}.$$

This matrix is diagonalizable with eigenvalues given by  $\lambda_1$  and  $\lambda_2$ , where  $\lambda_1 \equiv \frac{1}{4} + \delta + \frac{1}{2}\lambda - \frac{1}{4}(1 + 4\lambda^2)^{\frac{1}{2}}$  and  $\lambda_2 \equiv \frac{1}{4} + \delta + \frac{1}{2}\lambda + \frac{1}{4}(1 + 4\lambda^2)^{\frac{1}{2}}$  are both positive. The solution to such a linear system of differential equations can be written as

$$\vec{W}^{ss}(a) = e^{Ba} \vec{W}^{ss}(0) - w e^{Ba} \int_0^a x_L(s) e^{-Bs} \vec{1} ds + B^{-1} (I - e^{Ba}) \begin{pmatrix} b & \frac{\lambda}{2} W_2^{ss}(0) & b & \frac{\lambda}{2} W_1^{ss}(0) \end{pmatrix}^T,$$

where  $\vec{1} = (1 \ 1 \ 1 \ 1)^T$ . We assume that the accumulation function  $x_L$  is such that  $\lim_{s \rightarrow \infty} \|x_L(s) e^{-Bs}\| = 0$ , and we define  $\vec{X}_L(a) \equiv - \int_a^\infty x_L(s) e^{-Bs} \vec{1} ds$ . Since  $\vec{W}^{ss}$  must remain bounded and  $e^{Ba}$  is unbounded, we get the following condition:

$$\vec{W}^{ss}(0) = -w \vec{X}_L(0) + B^{-1} \begin{pmatrix} b & \frac{\lambda}{2} W_2^{ss}(0) & b & \frac{\lambda}{2} W_1^{ss}(0) \end{pmatrix}^T. \quad (\text{B.18})$$

Solving for this system defines  $\vec{W}^{ss}(0)$ , after which we can write

$$\vec{W}^{ss}(a) = -w e^{Ba} \vec{X}_L(a) + B^{-1} \begin{pmatrix} b & \frac{\lambda}{2} W_2^{ss}(0) & b & \frac{\lambda}{2} W_1^{ss}(0) \end{pmatrix}^T. \quad (\text{B.19})$$

### B.4.3 Step 3: Equilibrium following a small price shock

We assume the existence of an equilibrium with a structure similar to the equilibrium described in Proposition 1—that is, in which normal workers switch only upon impact and the wages of high-skill (resp. low-skill) workers are equalized after a time  $t_2$  (resp.  $t_1$ ). A continuous version of the system of equations (B.14), (B.15), (B.16), and (B.17) exists and can be written as

$$\begin{aligned} & \frac{\partial}{\partial a} \vec{W}(t, a) + \frac{\partial}{\partial t} \vec{W}(t, a) \\ &= B \vec{W}(t, a) - B \vec{W}(t, a) \\ & \quad - \begin{pmatrix} w_1(t) x_L(a) + b & \frac{\lambda}{2} W_2(t, 0) + w_1(t) x_L(a) & w_2(t) x_L(a) + b & \frac{\lambda}{2} W_1(t, 0) + w_2(t) x_L(a) \end{pmatrix}^T. \end{aligned} \quad (\text{B.20})$$

The worker from sector 2 who is indifferent between staying in that sector or switching will have an experience level given by  $a_L$ , which satisfies

$$W_0^2(0, a_L) = W_0^1(0, 0).$$

As before, after  $\max(t_1, t_2)$  all wages are constant at their steady-state value; hence  $\vec{W}$  is also constant over time at its new steady-state value. Without loss of generality, we assume that  $t_1 < t_2$ . Then

$$\begin{aligned} W_0^1(t_2, t_2) - W_0^1(0, 0) &= \int_0^{t_2} \left( \frac{\partial W_1}{\partial t} + \frac{\partial W_1}{\partial a} \right) d\tau \\ &= \int_0^{t_2} \left( (\lambda + \delta) W_0^1(\tau, \tau) - \frac{\lambda}{2} W_1(\tau, \tau) - \frac{\lambda}{2} W_2(\tau, 0) - w_1(\tau) x_L(\tau) \right) d\tau, \end{aligned}$$

where we have used (B.20). A first-order Taylor expansion of  $W_0^1(\tau, \tau)$ ,  $W_1(\tau, \tau)$ , and  $W_2(\tau, 0)$  of order 1 around  $(t_2, t_2)$ ,  $(t_2, t_2)$ , and  $(t_2, 0)$ , respectively, gives

$$\begin{aligned} &W_0^1(t_2, t_2) - W_0^1(0, 0) \\ &= t_2 \left( (\lambda + \delta) W_0^1(t_2, t_2) - \frac{\lambda}{2} W_1(t_2, t_2) - \frac{\lambda}{2} W_2(t_2, 0) \right) \\ &\quad - \left( (\lambda + \delta) \left( \frac{\partial W_0^1}{\partial t}(t_2, t_2) + \frac{\partial W_0^1}{\partial a}(t_2, t_2) \right) - \frac{\lambda}{2} \left( \frac{\partial W_1}{\partial t}(t_2, t_2) + \frac{\partial W_1}{\partial a}(t_2, t_2) \right) - \frac{\lambda}{2} \left( \frac{\partial W_2}{\partial t}(t_2, 0) \right) \right) \frac{t_2^2}{2} \\ &\quad - \int_0^{t_2} w_1(\tau) x_L(\tau) d\tau + o(t_2^2). \end{aligned}$$

After  $t_2$ ,  $\vec{W}$  is independent of time:  $\frac{\partial W_0^1}{\partial t}(t_2, t_2) = \frac{\partial W_1}{\partial t}(t_2, t_2) = \frac{\partial W_2}{\partial t}(t_2, 0) = 0$ . Using superscript  $ss'$  to denote the new steady state, we can now rewrite the previous equation as

$$\begin{aligned} W_1^0(t_2, t_2) - W_1(0, 0) &= t_2 \left( (\lambda + \delta) W_0^{1ss'}(t_2) - \frac{\lambda}{2} W_1^{ss'}(t_2) - \frac{\lambda}{2} W_2^{ss'}(0) \right) \\ &\quad - \left( (\lambda + \delta) \frac{dW_0^{1ss'}}{da}(t_2) - \frac{\lambda}{2} \frac{dW_1^{ss'}}{da}(t_2) \right) \frac{t_2^2}{2} - \int_0^{t_2} w_1(\tau) x_L(\tau) d\tau + o(t_2^2). \end{aligned}$$

Similarly, we have

$$\begin{aligned} &W_0^2(t_2, t_2 + a) - W_0^2(0, a) \\ &= t_2 \left( (\lambda + \delta) W_0^{2ss'}(t_2 + a) - \frac{\lambda}{2} W_2^{ss'}(t_2 + a) - \frac{\lambda}{2} W_1^{ss'}(0) \right) \\ &\quad - \left( (\lambda + \delta) \frac{dW_0^{2ss'}}{da}(t_2 + a) - \frac{\lambda}{2} \frac{dW_2^{ss'}}{da}(t_2 + a) \right) \frac{t_2^2}{2} - \int_0^{t_2} w_2(\tau) x_L(\tau + a) d\tau + o(t_2^2). \end{aligned}$$

Next we take the difference between the two equations and use that  $W_0^{1ss'} = W_0^{2ss'}$  and  $W_1^{ss'} = W_2^{ss'}$  to obtain

$$\begin{aligned} &W_0^1(t_2, t_2) - W_1(0, 0) - (W_0^2(t_2, t_2 + a) - W_0^2(0, a)) \\ &= t_2 \left( (\lambda + \delta) (W_0^{1ss'}(t_2) - W_0^{1ss'}(t_2 + a)) - \frac{\lambda}{2} (W_1^{ss'}(t_2) - W_1^{ss'}(t_2 + a)) \right) \\ &\quad + \left( \left( (\lambda + \delta) \left( \frac{dW_0^{1ss'}}{da}(t_2 + a) - \frac{dW_0^{1ss'}}{da}(t_2) \right) - \frac{\lambda}{2} \left( \frac{dW_1^{ss'}}{da}(t_2 + a) - \frac{dW_1^{ss'}}{da}(t_2) \right) \right) \right) \frac{t_2^2}{2} \\ &\quad + \int_0^{t_2} (w_2(\tau) x_L(\tau + a) - w_1(\tau) x_L(\tau)) d\tau + o(t_2^2). \end{aligned}$$

Consider the indifferent worker. Since  $a_L$  must be at most first order and since  $W_1(0,0) = W_0^2(0,a_L)$  by definition, it follows that

$$\begin{aligned} W_0^{1ss'}(t_2) - W_0^{1ss'}(t_2 + a_L) &= -t_2 a_L \left( (\lambda + \delta) \frac{dW_0^{1ss'}}{da}(t_2) - \frac{\lambda}{2} \frac{dW_1^{ss'}}{da}(t_2) \right) \\ &\quad + \int_0^{t_2} (w_2(\tau) x_L(\tau + a_L) - w_1(\tau) x_L(\tau)) d\tau + o(t_2^2) + o(a_L t_2). \end{aligned}$$

Now if  $a_L$  were first order, then the LHS would have the same order as  $a_L$  while the RHS would be of order 2 at most (because  $w_2(\tau) x_L(\tau + a) - w_1(\tau) x_L(\tau)$  is of first order in the price change). Since this is impossible,  $a_L$  must be of second order. As a result, the previous equation simplifies to

$$W_0^{1ss'}(t_2) - W_0^{1ss'}(t_2 + a_H) = \int_0^{t_2} (w_2(\tau) x_L(\tau + a_L) - w_1(\tau) x_L(\tau)) d\tau + o(dp^2).$$

Now using that  $t_2$  is at most first order—so that  $t_2 \left( \frac{dW_0^{1ss'}}{da}(0) - \frac{dW_0^{1ss'}(a_L)}{da} \right)$  would be of third order and wages would be equalized after  $t_1$ —we have

$$W_0^{1ss'}(0) - W_0^{1ss'}(a_L) = x_L(0) \int_0^{t_1} (w_2(\tau) - w_1(\tau)) d\tau + o(dp^2). \quad (\text{B.21})$$

Equation (B.19) yields

$$\begin{aligned} \overrightarrow{W^{ss'}}(a_L) - \overrightarrow{W^{ss'}}(0) &= w^{ss'} \left( \int_0^\infty (x_L(s + a_L) - x_L(s)) e^{-Bs} \vec{1} ds \right) \\ &= a_L w^{ss'} \left( \int_0^\infty x'_L(s) e^{-Bs} \vec{1} ds \right) + o(dp^2) \end{aligned} \quad (\text{B.22})$$

Matrix  $B$  can be decomposed as  $B = PDP^{-1}$  for

$$P \equiv \begin{pmatrix} 1 & 1 & 0 & 0 \\ \frac{1}{2} - \lambda + \left(\frac{1}{4} + \lambda^2\right)^{\frac{1}{2}} & \frac{1}{2} - \lambda - \left(\frac{1}{4} + \lambda^2\right)^{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & \frac{1}{2} - \lambda + \left(\frac{1}{4} + \lambda^2\right)^{\frac{1}{2}} & \frac{1}{2} - \lambda - \left(\frac{1}{4} + \lambda^2\right)^{\frac{1}{2}} \end{pmatrix}$$

and

$$D \equiv \text{Diag}(\lambda_1, \lambda_2, \lambda_1, \lambda_2),$$

so (B.22) leads to

$$\begin{aligned} &W_0^{1ss'}(a_L) - W_0^{1ss'}(0) \\ &= a_L w^{ss} \int_0^\infty x'_L(s) \frac{1}{2} \left( \left(1 + (1 + 4\lambda^2)^{-\frac{1}{2}}\right) e^{-\lambda_1 s} + \left(1 - (1 + 4\lambda^2)^{-\frac{1}{2}}\right) e^{-\lambda_2 s} \right) ds + o(dp^2). \end{aligned} \quad (\text{B.23})$$

As in Section A.1, we can establish the first-order development of changes in the effective mass of low-skill workers in each sector to obtain

$$dl_1(t) = -dl_2(t) = (1 - n_L) \frac{1}{1 + 2\lambda} x_L(0) t + o(dp) \text{ for } 0 \leq t \leq t_1, t_2. \quad (\text{B.24})$$

The difference between (B.24) and (A.2) arises because only a fraction  $(1 + 2\lambda)^{-1}$  of the new-born generation can be allocated to either sectors (the other workers are in biased states). A similar expression holds for  $dh_1$  and  $dh_2$ . This directly leads to (15), and (B.21), (B.23), and (B.24) together imply (16). Following the same steps as in Section A.1, we can then derive

$$t_2 = \frac{\left(v^{ss} - \frac{v_{1L} + v_{2L}}{w_{1L} + w_{2L}} w^{ss}\right) (1 + 2\lambda)}{(1 - n_H) x_H(0) \bar{H} \left(v_{1H} + v_{2H} - \frac{(v_{1L} + v_{2L})(w_{1H} + w_{2H})}{w_{1L} + w_{2L}}\right)} \frac{dp}{p} + o(dp),$$

$$a_H = - \frac{x_H(0) \left(t_2 - \frac{w^{ss}(v_{1L} + v_{2L})}{v^{ss}(w_{1L} + w_{2L})} (t_2 - t_1)\right)}{\int_0^\infty x'_H(s) \left(\left(1 + (1 + 4\lambda^2)^{-\frac{1}{2}}\right) e^{-\lambda_1 s} + \left(1 - (1 + 4\lambda^2)^{-\frac{1}{2}}\right) e^{-\lambda_2 s}\right) ds} \frac{dp}{p} + o(dp^2).$$

Finally, the same reasoning as in Section A.1 shows that this is indeed an equilibrium.

## B.5 Taylor expansion to one additional order

In Figure B.1, the left panel copies Figure 7.A, while the right panel compares the numerical values for  $t_1$ ,  $t_2$ ,  $a_L$  and  $a_H$  with analytical values obtained from approximation techniques when we include one additional order. Including this additional term improves the fit significantly (except for  $a_H$  for price changes beyond 15%).

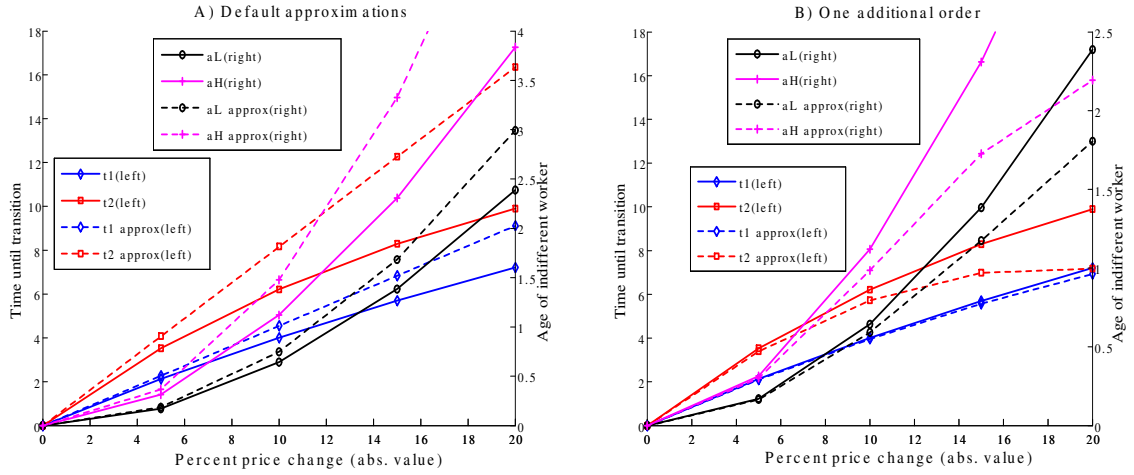


Figure B.1: Times until wage equalization and ages of indifferent workers, simulation results and approximation results. Panel A computes the approximation at first order for  $t_1$  and  $t_2$  and second order for  $a_L$  and  $a_H$ . Panel B computes them at second and third order respectively. Case with sector-specific physical capital.