

The equation (7) in "A Single-Step Localization Method in the Presence of Unknow Mutual Coupling" can be derived according to the following equation, where \odot denotes the Hadamard product, u_k and α_k ($k = 1, 2, \dots, m-1$).

$$\tilde{\mathbf{a}}(\mathbf{p}) = \mathbf{C}\mathbf{a}(\mathbf{p})$$

$$= \begin{pmatrix} 1 + c_1\beta(\mathbf{p}) + \dots + c_{m-1}\beta(\mathbf{p})^{m-1} \\ \vdots \\ c_k + c_{k-1}\beta(\mathbf{p}) + \dots + c_1\beta(\mathbf{p})^{k-2} + \beta(\mathbf{p})^{k-1} + c_1\beta(\mathbf{p})^k + \dots + c_{m-1}\beta(\mathbf{p})^{k+m-2} \\ \vdots \\ c_{m-1} + c_{m-2}\beta(\mathbf{p}) + \dots + c_1\beta(\mathbf{p})^{m-2} + \beta(\mathbf{p})^{m-1} + c_1\beta(\mathbf{p})^m + \dots + c_{m-1}\beta(\mathbf{p})^{2m-2} \\ \vdots \\ c_{m-1}\beta(\mathbf{p})^{M-2m-1} + \dots + c_1\beta(\mathbf{p})^{M-m-1} + \beta(\mathbf{p})^{M-m} + c_1\beta(\mathbf{p})^{M-m-1} + \dots + c_{m-1}\beta(\mathbf{p})^{M-1} \\ \vdots \\ c_{m-1}\beta(\mathbf{p})^{M-2m-1+k} + \dots + c_1\beta(\mathbf{p})^{M-m-1+k} + \beta(\mathbf{p})^{M-m+k} + c_1\beta(\mathbf{p})^{M-m+k-1} + \dots + c_{m-1}\beta(\mathbf{p})^{M-1} \\ \vdots \\ c_{m-1}\beta(\mathbf{p})^{M-m} + \dots + c_1\beta(\mathbf{p})^{M-2} + c_{m-1}\beta(\mathbf{p})^{M-1} \end{pmatrix} \quad (1)$$

$$= \begin{pmatrix} 1 + \sum_{j=1}^{m-1} c_j\beta(\mathbf{p})^j \\ \vdots \\ \sum_{j=1}^{k-1} c_j\beta(\mathbf{p})^{-j} + 1 + \sum_{j=1}^{m-1} c_j\beta(\mathbf{p})^j \\ \vdots \\ \sum_{j=1}^{m-1} c_j\beta(\mathbf{p})^{-j} + 1 + \sum_{j=1}^{m-1} c_j\beta(\mathbf{p})^j \\ \vdots \\ \sum_{j=1}^{m-1} c_j\beta(\mathbf{p})^{-j} + 1 + \sum_{j=1}^{m-1} c_j\beta(\mathbf{p})^j \\ \vdots \\ \sum_{j=1}^{k-1} c_j\beta(\mathbf{p})^{-j} + 1 + \sum_{j=1}^{m-k-1} c_j\beta(\mathbf{p})^j \\ \vdots \\ 1 + \sum_{j=1}^{m-1} c_j\beta(\mathbf{p})^{-j} \end{pmatrix} \odot \begin{pmatrix} 1 \\ \beta(\mathbf{p}) \\ \vdots \\ \beta(\mathbf{p})^{k-1} \\ \vdots \\ \beta(\mathbf{p})^{M-1} \end{pmatrix} = \left[1 + \sum_{j=1}^{m-1} c_j \left(\beta(\mathbf{p})^j + \beta(\mathbf{p})^{-j} \right) \right] \begin{pmatrix} u_1 \\ \vdots \\ u_{m-1} \\ 1 \\ \vdots \\ 1 \\ \alpha_1 \\ \vdots \\ \alpha_{m-1} \end{pmatrix} \odot \mathbf{a}(\mathbf{p})$$

$$\begin{aligned}
u_k &= \frac{1 + \sum_{j=1}^{k-1} c_j \beta(\mathbf{p})^{-j} + \sum_{j=1}^{m-1} c_j \beta(\mathbf{p})^j}{1 + \sum_{j=1}^{m-1} c_j \beta(\mathbf{p})^{-j} + \sum_{j=1}^{m-1} c_j \beta(\mathbf{p})^j} \\
\alpha_k &= \frac{1 + \sum_{j=1}^{m-1} c_j \beta(\mathbf{p})^{-j} + \sum_{j=1}^{m-1-k} c_j \beta(\mathbf{p})^j}{1 + \sum_{j=1}^{m-1} c_j \beta(\mathbf{p})^{-j} + \sum_{j=1}^{m-1} c_j \beta(\mathbf{p})^j}
\end{aligned} \tag{2}$$