Evolution Strategies

- Particularities
- General structure
- Recombination
- Mutation
- Selection
- Adaptive and self-adaptive variants

Particularities

Evolution strategies: evolutionary techniques used in solving continuous optimization problems

History: the first strategy has been developed in 1964 by Bienert, Rechenberg si Schwefel (students at the Technical University of Berlin) in order to design a flexible pipe

Main ideas [Beyer &Schwefel – ES: A Comprehensive Introduction, 2002]:

- Use one candidate (containing several variables) which is iteratively evolved
- Change all variables at a time, mostly slightly and at random.
- If the new set of variables does not diminish the goodness of the device, keep it, otherwise return to the old status.

Particularities

Data encoding: real (the individuals are vectors of float values belonging to the definition domain of the objective function)

Main operator: mutation (based on parameterized random perturbation)

Secondary operator: recombination

Particularity: self adaptation of the mutation control parameters

General structure

Problem (minimization):

Find x^* in D (subset of R^n) such that

$f(x^*)<f(x)$ for all x in D

The population consists of elements from D (vectors with real components)

Rmk. A configuration is better if the value of f is smaller.

Structure of the algorithm

Population initialization

Population evaluation

REPEAT

construct offspring by recombination

change the offspring by mutation

offspring evaluation

survivors selection

UNTIL <stopping condition>

Resource related criteria (e.g.: generations number, nfe)

Criteria related to the convergence (e.g.: value of f)

Recombination

Aim: construct an offspring starting from a set of parents

$$y = \sum_{i=1}^{\rho} c_i x^i, \quad 0 < c_i < 1, \ \sum_{i=1}^{\rho} c_i = 1$$

Intermediate (convex): the offspring is a linear (convex) combination of the parents

$$y_j = \begin{cases} x_j^1 & \text{with probability } p_1 \\ x_j^2 & \text{with probability } p_2 \\ \vdots & \\ x_j^\rho & \text{with probability } p_\rho \end{cases},$$

$$0 < p_i < 1, \ \sum_{i=1}^{\rho} p_i = 1$$

Discrete: the offspring consists of components randomly taken from the parents

Recombination

Geometrical recombination:

$$y_j = (x_j^1)^{c_1}(x_j^2)^{c_2}\ldots(x_j^\rho)^{c_\rho}, \quad 0 < c_i < 1, \ \sum_{i=1}^{\rho} c_i = 1$$

Remark: introduced by Z. Michalewicz for solving constrained optimization problems with constraints involving the product of components (e.g. $x_1 x_2 \ldots x_n > c$)

Heuristic recombination:

$y = x^i + u(x^i - x^k)$ with x^i an element at least as good as x^k

u – random value from (0,1)

Recombination

Simulated Binary Crossover (SBX)

- It is a recombination variant which simulates the behavior of one cut point crossover used in the case of binary encoding
- It produces two children c1 and c2 starting from two parents p1 and p2

$$c_1 = \overline{p} - \frac{\beta}{2}(p_2 - p_1)$$

$$c_2 = \overline{p} + \frac{\beta}{2}(p_2 - p_1)$$

$$\overline{p} = (p_1 + p_2)/2$$

Rmk: β is a random value generated according to the distribution given by:

$$prob(\beta) = \begin{cases} 0.5(k+1)\beta^k & \beta \le 1 \\ 0.5(k+1)\dfrac{1}{\beta^{k+2}} & \beta > 1 \end{cases}$$

Rmk: k can be any natural value; high values of k lead to children which are close to the parents