String attractors of Rote sequences

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Outline

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Motivation: Unifying repetitiveness measures

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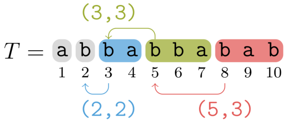
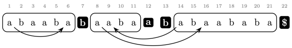
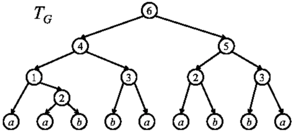
Dictionary compressors



Lempel-Ziv methods



Pointer macro scheme



Grammars

Motivation: Unifying repetitiveness measures

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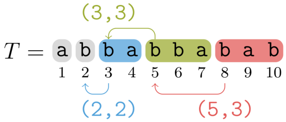

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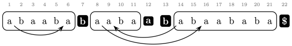

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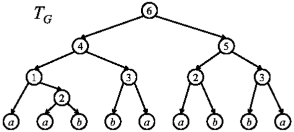


Grammars

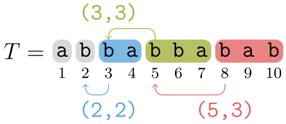
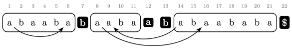
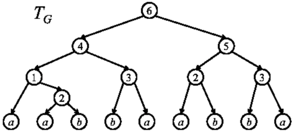
Induced repetitiveness measures

$\rightarrow z =$ size of the parsing

$\rightarrow b =$ size of the scheme

$\rightarrow g =$ size of the straight-line program

Motivation: Unifying repetitiveness measures

Dictionary compressors	Induced repetitiveness measures	Bounds via the smallest string attractors
 Lempel-Ziv methods	$\rightarrow z =$ size of the parsing	$z^* \in \mathcal{O}(\gamma^* \log^2(\frac{n}{\gamma^*}))$
 Pointer macro scheme	$\rightarrow b =$ size of the scheme	$b^* \in \mathcal{O}(\gamma^* \log(\frac{n}{\gamma^*}))$
 Grammars	$\rightarrow g =$ size of the straight-line program	$g^* \in \mathcal{O}(\gamma^* \log^2(\frac{n}{\gamma^*}))$ [Kempa & Prezza, 2018]

Motivation: Unifying repetitiveness measures

- Repetitiveness measures also upper bounds for the smallest string attractor

[Kempa & Prezza, STOC 2018]

Dictionary compressors can be interpreted as approximation algorithms for the smallest string attractor.

- Minimum size of an attractor gives us
  - lower and upper bounds for dictionary compression methods
  - direct stringological measure instead of the result of a specific compression method

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Dictionary compressors can be interpreted as approximation algorithms for the smallest string attractor.

- Minimum size of an attractor gives us
  - lower and upper bounds for dictionary compression methods
  - direct stringological measure instead of the result of a specific compression method
- Finding the smallest attractor size is NP-hard
  - $\rightarrow$ CoW approach:
    structural assumption (e.g., special classes of words) may make the computation tractable

Outline

String attractors: definition and example

Definition: Let $w = w_0 w_1 \dots w_n$ be a word, let $u = w_i w_{i+1} \dots w_j$ be its factor. Then $\{i, i+1, \dots, j\}$ is an *occurrence* of u in w.

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Definition of a string attractor [Prezza, ICTCS 2017]

Let $w = w_0 w_1 \ldots w_n$ be a finite word over alphabet $\mathcal{A}$. A *string attractor* of w is a set of positions $\Gamma \subseteq \{0, \ldots, n\}$ such that every substring of w has an occurrence containing an element of Γ.

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Example:

$$w = 012300123012$$

$$\Gamma = \{2, 3, 4, 8, 10\} \leftrightarrow w = 01\colorbox{lime}{230}012\colorbox{lime}{3}0\colorbox{lime}{1}2$$

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$$\Gamma^* = \{2, 3, 4, 10\} \leftrightarrow w = 01\underline{230}01230\underline{1}2$$

$$\Gamma^* = \{3, 5, 7, 10\} \leftrightarrow w = 012\underline{3}0\underline{0}1\underline{2}30\underline{1}2$$

Γ^* = some attractor with the minimum length

Overview of attractors in CoW

In CoW, minimal attractors have been determined for

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- particular prefixes
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- for factors
  - of episturmian sequences by Dvořáková, 2022
  - of the Thue-Morse sequence by Dolce, 2023

Overview of attractors in CoW

- Schaeffer & Shallit, 2021: study of attractors in linearly recurrent and in automatic sequences
- Restivo, Romana, Sciortino, 2022: combinatorial properties of attractors (relation to factor complexity, recurrence function, etc.), study of attractors in fixed points of morphisms
- Romana: String Attractor: a Combinatorial Object from Data Compression, 2022
- Gheeraert, Romana, Stipulanti, 2023: study of attractors in fixed points of k-bonacci-like morphisms

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Way to generate Sturmian sequences: Palindromic closures

Palindromes:
Word u is a *palindrome* if it reads the same forward and backward.

e.g. $1001, 11011, 10101$

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e.g. $100 \to 1001$, $1011 \to 101101$

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[X. Droubay, J. Justin, G. Pirillo, 2001]

Algorithm for generating Sturmian sequences

- Take any binary sequence (= directive sequence)
- Add letters from directive sequence one by one to generated word
- After each letter addition, make a palindromic closure

Example: Fibonacci word

Directive sequence $\Delta = (01)^\omega = 0\,1\,0\,1\,0\,1\ldots$

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$$\text{Directive sequence } \Delta = (01)^\omega = 0\ 1\ 0\ 1\ 0\ 1 \ldots$$

$$u_1 = 0$$

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$$u_2 = 01$$

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$$u_2 = 010$$
$$u_3 = 010010$$
$$u_4 = 0100101001 0$$

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$$u_2 = 010$$
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$$\vdots$$

Highlights mark the *longest palindromic prefixes* followed by 0 and 1

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Highlights mark the *longest palindromic prefixes* followed by 0 and 1

Longest palindromic prefixes followed by distinct letters mark attractors for all palindromic prefixes of standard Sturmian (and episturmian) words [L. Dvořáková, 2022]

Attractors of episturmian sequences via palindromic closures

$\rightarrow$ generalizing for any alphabet size

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Directive sequence $\Delta = (012)^{\omega} = 0\,1\,2\,0\,1\,2\,0\,1\,2\ldots$

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$u_1 = 0$

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$$\text{Directive sequence } \Delta = (012)^{\omega} = \boxed{0}\boxed{1}\boxed{2}\boxed{0}\boxed{1}\boxed{2}\boxed{0}\boxed{1}\boxed{2}\ldots$$

$$u_1 = \boxed{0}$$

$$u_2 = \boxed{0}\boxed{1}0$$

$$u_3 = \boxed{0}\boxed{1}0\boxed{2}010$$

$$u_4 = 0\boxed{1}0\boxed{2}010\boxed{0}102010$$

$$u_5 = 010\boxed{2}010\boxed{0}102010\boxed{1}020100102010$$

$$\vdots$$

The *longest palindromic prefixes* followed by $\boxed{0}$, $\boxed{1}$, and $\boxed{2}$ create attractors for these words. [L. Dvořáková, 2022]

Outline

1 Motivation

2 String attractors overview

3 Palindromic closures and Sturmian sequences

4 Pseudopalindromic closures and Rote sequences

5 Open questions

Our interest: Rote sequences

Definition

Complementary-symmetric (CS) Rote sequences are binary sequences having complexity $2n$ and such that their language is closed under letter exchange.

Closely connected to Sturmian sequences by words' sum:

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Definition: Let $w = w_0 \dots w_n$ be a binary word. Its *sum* is defined as $S(w) = u = u_0 \dots u_{n-1}$, where $u_i = w_i + w_{i+1} \mod 2$.

$$w = 0011100$$
$$S(w) = 010010$$

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$$\begin{array}{rcll} w & = & 0011100 & \text{Rote} \\ & & \bigvee\bigvee\bigvee\bigvee\bigvee\bigvee & \\ S(w) & = & 010010 & \text{Sturmian} \end{array}$$

Structural theorem [G. Rote, 1994]

A binary sequence w is a CS Rote sequence if and only if the sequence $S(w)$ is a Sturmian sequence.

How can we obtain Rote attractors from Sturmian ones?

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Example:

Rote: $w = 0011\underline{\mathbf{10}}0011$ - unique factor underlined

Sturmian: $u = 0100\underline{\mathbf{1}}0010$ - attractor should contain this position

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Example:

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Sturmian: $u = 0100\underline{\mathbf{1}}0010$ - attractor should contain this position

Currently known Sturmian attractors:

$u = 0\boxed{1}0010\boxed{0}10$ $\qquad$ $u = 010010\boxed{01}0$

No straightforward way how to obtain the necessary position from these.

Back to closures: Generalized pseudostandard sequences

Antipalindromes (on binary alphabet):
Word w is an *antipalindrome* if it reads forward and backward the same, only with letter exchange ($\overline{1} = 0$, $\overline{0} = 1$).

e.g. $1010, 110100, 10110010$

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e.g. $100 \to 100110$, $101 \to 1010$

Algorithm for generating generalized pseudostandard sequences

- Take any binary bisequence (= directive bisequence) specifying letters $\{0, 1\}$ and closures $\{R, E\}$
- Add letters from directive bisequence one by one to generated word
- After each letter addition, make an (anti)palindromic closure

Rote sequences are subset of generalized pseudostandard sequences

Theorem [Blondin-Massé A. et al., 2013]

Let (Δ, Θ) be a directive bisequence. Then w generated by this bisequence is a standard CS Rote sequence if and only if w is aperiodic and no factor of the directive bisequence is in the following sets:
$\{(ab, EE) : a, b \in \{0,1\}\}$,
$\{(aa, RR) : a \in \{0,1\}\}$,
$\{(aa, RE) : a \in \{0,1\}\}$.

Omitting these pairs in the bisequence, we can generate Rote sequences using pseudopalindromic closures!

Rote sequences via closures

CS Rote sequences generated by pseudopalindromic closures omitting the following patterns:
$\{(ab, EE) : a, b \in \{0,1\}\} \cup \{(a\bar{a}, RR) : a \in \{0,1\}\} \cup$
$\{(aa, RE) : a \in \{0,1\}\}$

Example:

$\Delta = \boxed{0}\ \boxed{0}\ \boxed{1}\ \boxed{1}\ \boxed{0}\ \boxed{0}\ \mathbf{....}$
$\Theta = \boxed{\mathrm{R}}\ \boxed{\mathrm{R}}\ \boxed{\mathrm{E}}\ \boxed{\mathrm{R}}\ \boxed{\mathrm{E}}\ \boxed{\mathrm{R}}\ \mathbf{....}$

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Example:

$\Delta = 0\ 0\ 1\ 1\ 0\ 0 \ \ldots.$

$\Theta = R\ R\ E\ R\ E\ R \ \ldots.$

$w_1 = 0$

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$w_2 = 00$

$w_3 = 0011$

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$\Theta = \boxed{R}\ \boxed{R}\ \boxed{E}\ \boxed{R}\ \boxed{E}\ \boxed{R}$ ....

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$w_2 = 00$

$w_3 = 0011$

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Example:

$\Delta =$ 0 0 1 1 0 0 ....
$\Theta =$ R R E R E R ....

$w_1 = 0$

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$w_5 = 00111000011$

$w_6 = 00111000011000011100$

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Example:

$\Delta =$ 0 0 1 1 0 0 ....
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$w_2 = 00$

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$w_4 = 0011100$

$w_5 = 00111000011$

$w_6 = 00111000011000011100$

Can we use the longest pseudopalindromic prefixes followed by distinct letters to obtain attractors of pseudopalindromic prefixes of Rote sequences?

Result: Attractors of Rote sequences

Theorem [Dvořáková L., Hendrychová V., 2023]

Assume (Δ, Θ) is the directive bisequence of a standard CS Rote sequence w, and w_n contains both letters. Then

1. If w_n is antipalindromic, w_n has an attractor $\Gamma = \{|w_i|, |w_{n-1}|\}$, where w_i is the longest antipalindromic prefix followed by $\overline{\Delta[n]}$ in w.

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Theorem [Dvořáková L., Hendrychová V., 2023]

Assume (Δ, Θ) is the directive bisequence of a standard CS Rote sequence w, and w_n contains both letters. Then

1. If w_n is antipalindromic, w_n has an attractor $\Gamma = \{|w_i|, |w_{n-1}|\}$, where w_i is the longest antipalindromic prefix followed by $\overline{\Delta[n]}$ in w.
2. If w_n is palindromic and w_{n-1} is antipalindromic, w_n has an attractor $\Gamma = \{|w_j|, |w_{n-1}|\}$, where w_j is the longest palindromic prefix followed by $\overline{\Delta[n]}$ in w.

Result: Attractors of Rote sequences

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3. If w_n is palindromic and w_{n-1} is palindromic, w_n has the same attractor as w_{n-1}.

$\rightarrow$ The form of attractor depends not only on the current closure, but also on the preceding one.

Example: Attractor of Rote sequence

"LPPn" = longest palindromic prefix followed by n
"LAPn" = longest antipalindromic prefix followed by n

Example:

$\Delta =$ 0 0 1 1 0 0 0 ....

$\Theta =$ R R E R E R R ....

w_i	attractor
$w_1 = 0$	-
$w_2 = 0\,0$	-
$w_3 = \underline{\mathbf{0}}\,0\,\underline{\mathbf{1}}\,1$	$\lvert \text{LAP } 0 \rvert$, $\lvert w_2 \rvert$

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$\Delta = 0\ 0\ 1\ 1\ 0\ 0\ 0 \ldots.$
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w_i		attractor
w_1	$=0$	-
w_2	$=00$	-
w_3	$=\underline{\mathbf{0}}0\underline{\mathbf{1}}1$	$\lvert$LAP 0$\rvert$, $\lvert w_2\rvert$
w_4	$=0\underline{\mathbf{0}}11\underline{\mathbf{1}}00$	$\lvert$LPP 0$\rvert$, $\lvert w_3\rvert$

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w_3	$= \underline{\mathbf{0}}0\underline{\mathbf{1}}1$	$\lvert \text{LAP } 0\rvert$, $\lvert w_2\rvert$
w_4	$= 0\underline{\mathbf{0}}11\underline{\mathbf{1}}00$	$\lvert \text{LPP } 0\rvert$, $\lvert w_3\rvert$
w_5	$= 0011\underline{\mathbf{1}}00\underline{\mathbf{0}}11$	$\lvert \text{LAP } 1\rvert$, $\lvert w_4\rvert$
w_6	$= 00\underline{\mathbf{1}}110001 1\underline{\mathbf{0}}0011100$	$\lvert \text{LPP } 1\rvert$, $\lvert w_5\rvert$
w_7	$= 00\underline{\mathbf{1}}1100011\underline{\mathbf{0}}00111000110001 1100$	same as previous

Outline

Open problems

- What are the attractors of prefixes of **generalized pseudostandard sequences**?
  - i.e. what if we don't omit any (anti)palindromic combinations in the generating bisequence?

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- What about attractors of (generalized) pseudostandard sequences over **larger alphabets**?
- How does the minimum attractor size affect the **form of examined words compressed** by dictionary compressors? Do they also remain constant?

Thank you for your attention!