THE SUPPLY OF STORAGE

By MICHAEL J. BRENNAN*

It is a familiar proposition that the amount of a commodity held in storage is determined by the equality of the marginal cost of storage and the temporal price spread. Why then do we observe stocks being carried from one period to the next when the price expected to prevail in the next period—reflected in the futures price quotation for delivery in that period—is below the current price.

In an attempt to explain "inverse carrying charges" in futures markets (futures prices below spot prices or prices of deferred futures below that of near futures) the concept of a convenience yield on stocks has been introduced.[1] Stocks of all goods provide a yield or compensation to the holder which must be deducted from storage costs proper in calculating *net* storage costs. In equilibrium the spread between a futures and a spot price is equal to the marginal expenditure on rent for storage space, interest, handling charges, etc., minus the marginal convenience yield of stocks. Since marginal convenience yield is a decreasing function of stocks held, the marginal convenience yield may exceed the marginal expenditure on physical storage when stocks are relatively small; hence the futures price will be below the spot price.

We shall attempt here to generalize this theory in terms of the demand for and the supply of stocks for storage. Our theory purports to provide an explanation of the holding of all stocks, including those for which there is not an active futures market. It will be shown that, on the supply side, in addition to the marginal expenditure on physical storage and the marginal convenience yield another variable, a risk premium, is required to explain the holding of stocks as a function of price spreads. In the empirical part of the study the theory will be applied to stocks of several agricultural commodities. The risk premium for each commodity will be measured residually under specified con-

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[1] N. Kaldor, "Speculation and Economic Stability," *Rev. Econ. Stud.*, Oct. 1939, VII, 1-27. H. Working, "The Theory of the Inverse Carrying Charge in Futures Markets," *Jour. Farm Econ.*, Feb. 1948, XXX, 1-28.

ditions by deducting from the price spread between two periods the other two components of the marginal cost of storage.

I. *Theory of Storage*

During any period there will be firms carrying stocks of a commodity from that period into the next. Producers, wholesalers, etc., carry finished inventories from the periods of seasonally high production to the periods of low production. Processors carry stocks of raw materials. Speculators possess title to stocks held in warehouses. These firms may be considered as supplying inventory stocks or, briefly, supplying storage. The supply of storage refers not to the supply of storage *space* but to the supply of *commodities* as inventories. In general, a supplier of storage is anyone who holds title to stocks with a view to their future sale, either in their present or in a modified form.

On the other hand there will be groups who want to have stocks carried for them from one period (in which they do not intend to consume them) to another period (in which they do intend to consume them). These consumers may be regarded as demanding storage. Since production is not stable for all commodities, consumers demand that the storage function be so performed that the flow of commodities for sale will be made relatively stable. We assume that there is no significant time lag between sales out of stocks by suppliers of storage and utilization of the commodity by households, for otherwise we could not distinguish suppliers from demanders of storage. For example, in the case of butter, storage is normally performed by the manufacturer—stocks are carried from the months of seasonally high production to the months of seasonally low production. Sales out of stocks by manufacturers to wholesalers (or retailers) are assumed equivalent to sales to consumers, and the relevant price entering storage decisions is the wholesale price. This is a simple analogy to the derived demand for a commodity: with no time lag between wholesale purchases and household consumption the demand for consumption can be expressed as a function of the wholesale price.

A. *The Demand for Storage*

The demand for storage of a commodity can be derived from the demand for its consumption. We assume that consumption during any period depends only upon the price in that period; all other variables affecting consumption are exogenous. Let P_t be the price in period t and let C_t be consumption during t. The demand function in period t can then be written:

$$P_t = f_t(C_t), \qquad \frac{\partial f_t}{\partial C_t} < 0. \tag{1}$$

The subscript indicates that demand may shift periodically.

Consumption in any period equals stocks carried into the period plus current production minus stocks carried out of the period. Consequently we may rewrite (1) as:

$$P_t = f_t(S_{t-1} + X_t - S_t), \tag{2}$$

where S_{t-1} is stocks at the end of period $t - 1$, X_t is production during t and S_t is stocks at the end of t. For convenience it is assumed that current production and subsequent levels of production and stocks are known. To derive the demand for storage of the commodity from period t to period $t + 1$, consider the effect of an increase in carryout from period t, *i.e.*, an increase in end-of-period stocks. Under the specified assumptions, if the price in t rises, less will be consumed. With stocks carried into the period known and current production given, the rise in price in t results in less of the commodity offered for sale in t and more carried out of t. Since future levels of production and stocks are given, more of the commodity is consumed in period $t + 1$, *i.e.*, price in $t + 1$ will fall. Conversely a reduction in carryout from period t will, under these assumptions, be associated with an increase of P_{t+1} relative to P_t. In general, price in the next period minus price in the current period may be expressed as a decreasing function of stocks carried out of the current period.

Symbolically the demand for storage from period t to period $t + 1$ can be represented as follows:

$$\begin{aligned} P_{t+1} - P_t &= f_{t+1}(C_{t+1}) - f_t(C_t) \\ &= f_{t+1}(S_t + X_{t+1} - S_{t+1}) - f_t(S_{t-1} + X_t - S_t). \end{aligned} \tag{3}$$

If we differentiate this expression with respect to S_t, we see that the partial derivative is negative. With S_{t-1} known and X_t, X_{t+1} and S_{t+1} exogenously determined, the price spread is a decreasing function of S_t. The price spread may be positive or negative. Figure 1 shows the demand curve for storage.

The assumption that demand may shift periodically is a realistic and, for our purposes, useful one. In general, the demand curve for storage of a commodity from period t to period $t + 1$ will shift upward (*e.g.*, to $D'D'$ in Figure 1) as a result of (1) an increase in production in t, (2) a decrease in production in $t + 1$ or (3) an increase in stocks carried out of $t + 1$. Opposite movements of these exogenous variables will produce a shift downward.

B. *The Supply of Storage*

The supply of storage is forthcoming from those firms holding title to stocks carried from one period to another. In a competitive industry in an uncertain world a firm seeking to maximize net revenue will hold an amount of stocks such that the net marginal cost of storage per unit of time equals the expected change in price per unit of time. We have seen that the net marginal cost of storage need not be positive. The net marginal cost of storage is defined as the marginal outlay on physical storage plus a marginal risk-aversion factor minus the marginal convenience yield on stocks.

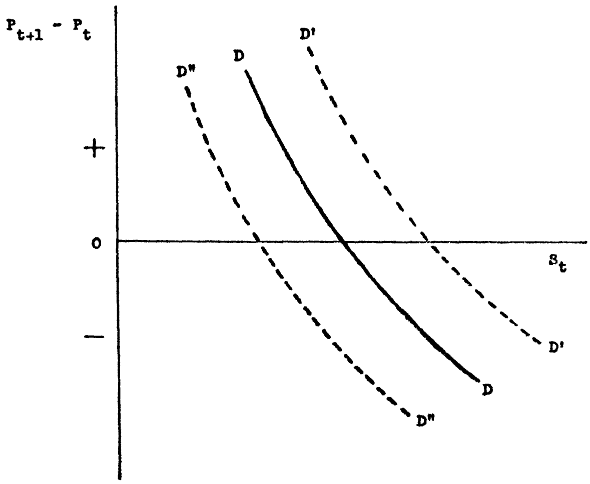

FIGURE 1

The total outlay on physical storage is the sum of rent for storage space, handling or in-and-out charges, interest, insurance, etc. As the quantity of stocks held by a firm increases, the total outlay increases. Although for any single firm this cost may increase at either a constant or an increasing rate, it seems reasonable to suppose that the marginal outlay is approximately constant until total warehouse capacity is almost fully utilized (each firm can store all it wishes without affecting the cost per unit of the commodity stored). Beyond this level marginal outlay will rise at an increasing rate.

Suppliers of storage are mostly engaged in production, processing or merchandising with storage as an adjunct. The costs of storage must be considered as charged against the business operation as a whole. Given day-to-day fluctuations in the market, a producing firm can meet a sudden and "unexpected" increase in demand by filling orders out of finished inventories or by adjusting its production schedule or by some combination of these. The convenience yield is attributed to the advantage (in terms of less delay and lower costs) of being able to keep regular customers satisfied or of being able to take advantage of a rise in demand and price without resorting to a revision of the production schedule. Similarly, for a processing firm the availability of stocks as raw materials permits variations in production without incurring the trouble, cost and perhaps delays of frequent spot purchases and deliveries. A wholesaler can vary his sales in response to an increased flow of orders only if he has sufficient stocks on hand.

The smaller the level of stocks on hand the greater will be the convenience yield of an additional unit. It is assumed that there is some quantity of stocks so large that the marginal convenience yield is zero. Distinction is sometimes made between "surplus" stocks which will not be carried to a future period without expectation of a monetary return and "pipeline" or "working" stocks. There is an implication that pipeline stocks are relatively small and fixed in quantity. Such a distinction has little functional meaning. Actually working stocks may vary through a considerable range, their upper limit being defined as the level at which the marginal convenience yield is zero.

The third component of the net marginal cost of storage is the marginal risk-aversion factor. We should expect total risk aversion to be an increasing function of stocks. If a comparatively small quantity of stocks is held, the risk involved in undertaking the investment in stocks is also small. An unexpected fall in the price at which stocks must be sold will result in a relatively small loss to the firm holding stocks for later sale. For firms holding a small quantity of stocks as raw materials for use in production, an unexpected fall in the price will involve a relatively small loss. However, given the total capital resources of the firm, the greater the quantity of stocks held, the greater will be the loss to the firm from the same unexpected fall in the future price. There is probably some critical level of stocks at which the loss would seriously endanger the firm's credit position, and as stocks increase up to this point the risk incurred in holding them will steadily increase also—the risk of loss will constitute a part of the cost of storage. The marginal risk-aversion factor may be assumed to be either constant or, more likely, an increasing function of stocks held.

Again let S_t denote the stocks carried out of period t. Let $o_t(S_t)$ be the total outlay on physical storage, $r_t(S_t)$ the total risk-aversion factor and $c_t(S_t)$ the total convenience yield. Then the net total cost of storage $m_t(S_t)$, is defined as:

$$m_t(S_t) = o_t(S_t) + r_t(S_t) - c_t(S_t). \tag{4}$$

o_t and r_t are increasing functions of S_t so that the marginal outlay and marginal risk aversion are either constant or are increasing functions of S_t, *i.e.*, $o_t' > 0$ and $o_t'' \geq 0$; $r_t' > 0$ and $r_t'' \geq 0$. c_t is also an increasing function of S_t, but the marginal convenience yield declines and reaches zero at some large level of stocks, *i.e.*, $c_t' \geq 0$ and $c_t'' \leq 0$. The net marginal cost of storage in period t may be written as:

$$m_t'(S_t) = o_t'(S_t) + r_t'(S_t) - c_t'(S_t). \tag{5}$$

The net marginal cost of storage need not be positive. When stocks are relatively small, c_t' will be large. If c_t' is large enough relative to o_t' plus r_t', the net marginal cost of storage will be negative. Figure 2 de-

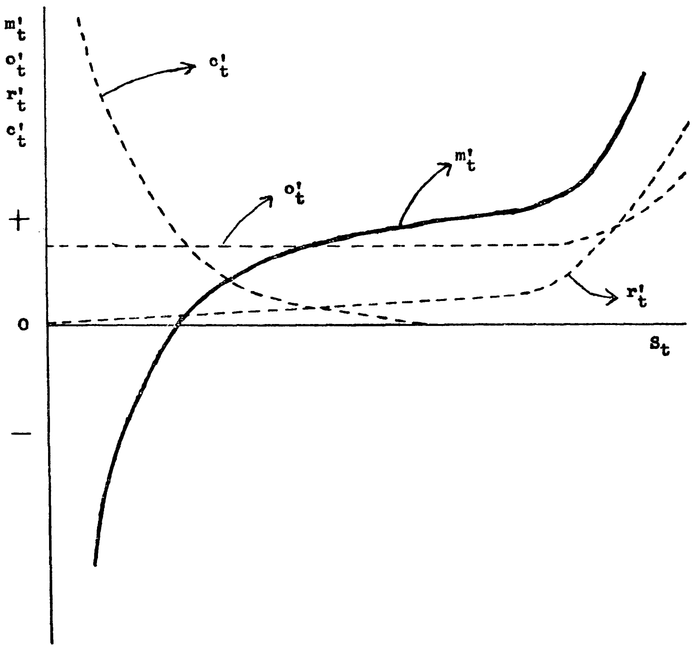

FIGURE 2

picts graphically the net marginal cost of storage and its three components for a typical firm.

Let the expected marginal revenue of stocks carried out of period t be represented by $u_t'(S_t)$. In a competitive industry u_t' equals the expected change in price from period t to a future period, say $t+1$.[2] Total expected net revenue is equal to:

$$u_t(S_t) - m_t(S_t). \tag{6}$$

The quantity of stocks which maximizes expected net revenue is found by differentiating (6) with respect to S_t and setting the derivative equal

[2] For a firm operating under conditions of nonpure competition, u_t^1 is a decreasing function of stocks held.

to zero. This gives

$$u_t'(S_t) = m_t'(S_t), \tag{7}$$

which expresses the familiar condition that expected net revenue is maximized when expected marginal revenue equals net marginal cost. The conditions on the second derivatives of o_t, r_t and c_t insure that the solution is a maximum.

Under the assumptions of pure competition and no external economies or diseconomies in the storage industry the supply curve of storage is the horizontal sum of all individual net marginal cost functions. Thus the sum of equations like (7), when solved for S_t as a function of u_t', is the supply curve of storage. We denote the supply of storage by

$$u_t' = g_t(S_t). \tag{8}$$

C. *Equilibrium*

We can now use the demand for and supply of storage to determine the equilibrium quantity of stocks carried out of t as a function of the price spread. We suppose that prices expected to prevail in future periods are the same for each firm. The equilibrium quantity of stocks is determined by the equality of the demand for stocks and the supply of stocks:

$$u_t' = EP_{t+1} - P_t, \tag{9}$$

where EP_{t+1} is the price expected in period $t + 1$ and P_t is assumed known. Using (3) and (8) this can be written

$$g_t(S_t) = Ef_{t+1}(S_t + X_{t+1} - S_{t+1}) - f_t(S_{t-1} + X_t - S_t). \tag{10}$$

For a two-period model the equilibrium is illustrated in Figure 3. *DD*, *D'D'* and *D''D''* are demand-for-storage curves; *CC* is the supply curve of storage. If *DD* is the demand curve in *t*, the equilibrium price spread will be *OR* and the equilibrium quantity of stocks carried out of *t* will be *OL*. If production during *t* or the expected production and/or carryout in $t+1$ were to change, the demand curve would shift. Take, for example, the case in which production is reduced in period *t*, production in $t+1$ is expected to increase or stocks carried out of $t+1$ are expected to decrease. Then the demand curve will shift downward to *D''D''* so that the equilibrium expected price spread is negative, *OQ*, and the equilibrium quantity of stocks is *OK*.

II. *The Supply Curve of Storage of Selected Agricultural Commodities*

Suppose that the period under consideration is one month. Then as production and planned future inventories vary seasonally the demand curve will shift. Normally the cost per unit of storage space, handling charges per unit, the allowance for spoilage and negotiation charges will not vary from month to month. Interest and insurance charges per unit vary seasonally with the price. Since insurance is a very small proportion of the cost per unit stored and since the variation in price would be responsible for only a very small percentage variation in interest charges per unit stored, we may take the marginal outlay function as relatively stable. Furthermore, it is unlikely that the marginal convenience yield function and the marginal risk-aversion

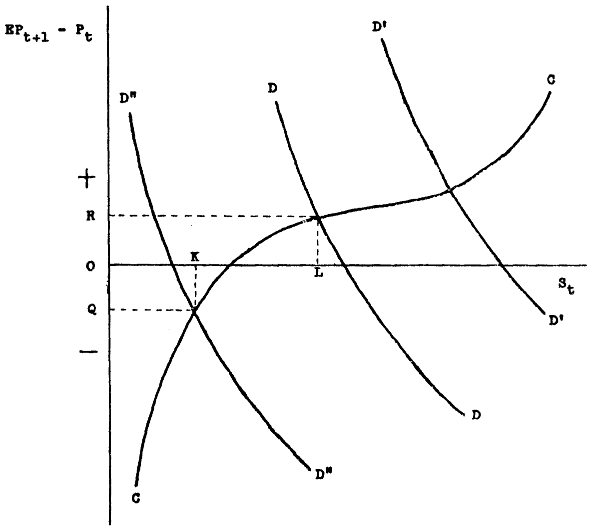

FIGURE 3

function will shift significantly from month to month. Consequently, we may reasonably assume that the supply curve of storage is relatively stable. If we allow demand to fluctuate over a stable supply function, a supply curve of storage can be generated empirically by measuring the relationship between $EP_{t+1} - P_t$ and S_t for $t = 1, 2, 3, \cdots, n$.

However, merely plotting appropriate price spreads against stocks would conceal the relative importance of the three components in the net marginal cost of storage. Indeed it would not tell us whether or not more than two components are required to explain end-of-period stocks. Consider the equilibrium equation $EP_{t+1} - P_t = o_t' + r_t' - c_t'$. The price in period t, P_t, and the marginal outlay, o_t', are in principle observable. Granted that a means can be obtained for estimating the expected price, EP_{t+1}, these three variables can be used to estimate the residual, $r_t' - c_t'$. If in conjunction with the marginal outlay the marginal convenience yield is sufficient to explain the level of stocks held, *i.e.*, if

$r_t'=0$, the residual should be a monotonically nondecreasing function with zero as a maximum. If, on the other hand, the residual rises above zero over a significant range of stocks, another variable is required to explain the holding of stocks as a function of price spreads. For several agricultural commodities the residual $EP_{t+1}-P_t-o_t'$ is labeled $r'-c'$ and is related to total stocks in Figures 4 to 8. Before proceeding to these empirical conclusions, however, let us examine the means used to derive estimates of the relevant variables.

A. *Expected Prices*

For stocks which are hedged on an active futures market the price spread relevant to a decision about storage levels is the difference between a futures and a spot price. Arbitrage between cash and futures markets will insure that the cash price expected to exist in a future period is accurately reflected in the current quotation of the futures price for delivery in that period. Some commodities have no futures market, yet they are stored in economically significant quantities. For others a futures market exists but the amount of hedging is negligible. Consequently we need a method for obtaining reasonable approximations to expectations of future prices.

In general it will be assumed that decision-makers include in their expectations of monthly prices notions of trend, seasonal normal, and transitory factors. In the absence of more specific information it seems reasonable to suppose that the sequence of prices expected to prevail within one planning interval, for example one production and storage year, depends in some way on past prices. Of course price expectations are shaped by a variety of influences so that the expression of expected price as a function of past prices is merely a convenient way to summarize these diverse influences.

To estimate the trend in monthly price expectations a linear multiple regression of annual average price on past annual average prices is used. This yields a prediction of the expected annual average price in the planning year. Upon this estimated annual average price a seasonal price index is superimposed as an estimate of the "normal" sequence of future monthly prices.[3] This method gives a *typical* structure to

[3] Because it has yielded the smallest standard error of estimate for all commodities studied a linear multiple regression of annual average price on past annual average prices has been chosen as the means for estimating expected annual average price. For the period covered by the data the seasonal indexes were applied as follows: a new 5-year index was computed after each 3 years in order to take into account gradual changes in the seasonal price pattern. The 5 years used to compute the index were taken as the 5 years preceding the first year of the 3-year period in which it was applied so as to take into account the fact that future prices are unknown and to (partially) eliminate any transitory elements in the monthly price pattern of the immediately preceding year.

the sequence of expected monthly prices. Since there are a number of diverse transitory influences which could possibly be operative in any given planning interval (leading to a complex expectation function) and since those which are operative will result in shifts in the demand and/or supply schedules, we assume for simplicity that the transitory factors can be summarized in the price variable. Recent prices are a partial result of forces expected to continue to operate in the near future. It is highly probable that the more recent the past price the more that price expresses the operation of forces relevant to expectations. The transitory influences may not have been operative three or four months previously; if transitory factors were operative then, they will likely be reflected to more or less degree in the prices of more recent months.

Assuming a planning interval of n months, before adjustment for transitory factors we have a vector of n expected monthly prices which reflect trend and seasonal normal. Let the vector of $n-t$ adjusted expected prices formulated in month t be represented by

$$P^*(t)_{t+1}^{n} = (EP_{t+1}, EP_{t+2}, \cdots, EP_n). \tag{11}$$

There will also be a vector of $n-(t-1)$ adjusted expected prices which was formulated in month $t-1$. Let the vector of $n-t$ of these expected prices formulated in $t-1$ be represented by $P^*(t-1)_{t+1}^n$. That is, of the $n-(t-1)$ expected prices formulated in month $t-1$, $P^*(t-1)_{t+1}^n$ represents $n-t$ of them extending from period $t+1$ to period n. In this way both $P^*(t)_{t+1}^n$ and $P^*(t-1)_{t+1}^n$ contain the same number of expected prices covering the same future months. The adjustment for transitory factors in period t is:

$$\begin{aligned} P^*(t)_{t+1}^{n} - P^*(t-1)_{t+1}^{n} &= \alpha(P_t - EP_t) + \alpha(1-\alpha)(P_{t-1} - EP_{t-1}) \\ &\quad + \alpha(1-\alpha)^2(P_{t-2} - EP_{t-2}) + \cdots \end{aligned} \tag{12}$$

where α is a constant less than or equal to 1, EP_t is obtained from $P^*(t-1)_t^n$, EP_{t-1} is obtained from $P^*(t-2)_{t-1}^n$, etc.

Expression (12) states that the sequence of expected future prices is adjusted for transitory factors by a weighted moving average of the errors made in past predictions in which the weights decline toward zero as one goes back in time. Decision-makers in revising their expectations upward or downward take into account not only the error made in the preceding month but also those of several past months. In each month, as each new price becomes known, a new adjustment of all remaining future prices in the planning interval is made. The number of past errors to be included will depend upon the value of α. The assurance that the weights decline means that practically we can ignore errors in the very distant past. The closer is α to 1, the less importance distant past errors assume.[4] If we try a number of different values for α, we can find a value which will yield the least deviation between expected and actual prices (and for the methods used to estimate trend and seasonal normal the greatest correlation between expected and actual prices).[5] This iterative procedure has been applied to the 5 commodities studied and the smallest errors of estimate are consistently obtained by using 3 past errors for some commodities and 4 for others with rounded values of α varying between .8 and .7.

What might be said in favor of this "naive" expectation model? First, it is likely that decision-makers with first-hand information available to them in each period will form price expectations with deviations from actual prices at least as small as those obtained by this method. Expected price spreads in relation to stocks derived from the use of this model are not significantly different in general form from actual price spreads, *i.e.*, where $EP_t = P_t$. If the price expectations of decision-makers should lie between actual prices and our estimates of their expectations, then the substantive conclusions about the supply curve of storage would not be significantly different from the case in which we knew exactly and completely how each price expectation was formed. Secondly, attempts to construct expectation functions of a more sophisticated nature have not yielded results as accurate in predictability as those derived from this model. Finally, in the case of those commodities for which there are active futures markets the futures price has also been used as the expected price. When the fitted curve relating stocks to expected price spreads derived by the above method is superimposed on observations involving futures price spreads, the scatter around the curve is close enough to warrant confidence in this model of expected prices.

B. *Marginal Outlay*

There are no published data on storage charges per unit per month made by warehouses to owners of stocks; it was necessary to collect this information directly from a sample of cold-storage warehouses,

[4] The sum of the weights for m past errors is $1-(1-\alpha)^{m+1}$. The number of past errors to be included can be found from the formula

$$\left|1-(1-\alpha)^{m+1}\right| \leq 1$$

in which the sum of the weights is made as close to 1 as is practicable. In this way we allow the data to choose the number of past errors to be included subject to the constraint that the sum of the weights is less than or equal to unity.

[5] Under the assumptions made above it can be shown by maximization of the likelihood function for each chosen α that this iterative procedure yields maximum likelihood estimates.

grain elevators and other sources.[6] The total cost per unit is composed of rent for storage space, handling charges, interest, insurance and an allowance for negotiation charges and spoilage. The charge for storage space is relatively stable from 1924 to 1938 for eggs, butter and cheese and from 1924 to 1932 for wheat and oats. To determine the handling charge per month the charge, quoted in cents per unit, was divided by the average number of months that stocks were held in storage during the year minus one month. One month was deducted in order to give a maximal estimate of marginal outlay, and thereby the hypothesis being tested was placed at its greatest disadvantage.

From 1924 to 1930 interest was figured at 6 per cent per annum, the rate normally charged by warehouses on funds advanced to owners of stocks during this period.[7] The customers' rate charged by banks on commercial loans in 19 cities averaged a little less than 5 per cent for this period. Between 1930 and 1939 this bank rate fluctuated between 5 and 2½ per cent, averaging about 3½ per cent. Since warehouses report that they normally charge a rate slightly higher than the bank rate, interest has been imputed at 1 per cent above the bank rate. The monthly interest cost per unit was computed as a percentage of the weighted average monthly price during the months in which stocks were being accumulated seasonally. Each monthly price was weighted by the increase in stocks during that month.

The median insurance charge appears to be about .08 per cent of the value per month and was estimated at .10 per cent of the weighted average monthly price. An allowance of .05 per cent of the weighted average monthly price was attributed to butter, eggs and cheese for spoilage and negotiation charges. An allowance of .03 per cent was attributed to wheat and oats. In the light of comments made by warehouse operators these allowances are probably slight overestimates. The estimate of storage costs is the average of the charges made in the important storage areas. Geographical differences in storage rates and value of stocks are not sufficient to alter the empirical conclusions. The variance of o_t' relative to the residual $r'-c'$ is small enough to dismiss the possibility that the size of $r'-c'$ is significantly dependent upon the size of the variance of o_t'.

The charge per unit made by warehouses to owners of stocks does

[6] Sources in the survey from which replies were received for butter, cheese and eggs are 18 cold storage warehouses in 8 geographically dispersed cities; 2 meat-packers with national distribution; 2 middle-western cooperatives; the Chicago Mercantile Exchange and the U. S. Department of Agriculture. Sources for wheat and oats are 14 grain elevators in 7 dispersed cities; the Chicago and Kansas City Boards of Trade; the Grain Exchange Institute, Inc., Chicago and the U. S. Department of Agriculture.

[7] E. A. Duddy and D. A. Revzan, "Profits and Losses in the Storage of Butter," *Jour. Bus. Univ. Chicago,* Oct. 1933, VI, 293-317.

not increase as stocks increase. This, together with the fact that interest, insurance and handling charges per unit are not rising functions of stocks, suggests that the marginal outlay is constant and equal to the average outlay. It is still possible, however, that marginal outlay rises as stocks increase over the relevant range because at or near the peak storage months warehouses in the important storage areas are filled to capacity and additional stocks must be diverted to warehouses located at a greater distance from the point of production or sale. Then the marginal cost would rise with the distance from the normal storage area and with the additional amount of handling required. The facts do not support this supposition. In the years 1924 to 1938, for the economy as a whole the peak occupancy period for public and private cold storage warehouses was around October, at which time about 70 to 80 per cent of capacity was utilized. The peak seasonal storage period for butter, cheese and eggs occurred in the summer months when 60 to 70 per cent of capacity was utilized. In the survey of warehouses, firms in the most important storage areas replied unanimously that capacity was never utilized during this period to the extent that stocks had to be diverted to other areas. The same reply was received from grain elevators with respect to wheat and oats. Except for a few geographically and temporally isolated instances, such as 1929 in Chicago, there was never exceptional congestion of elevators during the period 1924 to 1932. As a consequence the average outlay has been used as an approximation to the marginal outlay assumed constant over the range of observed levels of stocks.

C. *Risk and Convenience Yield*

For several agricultural commodities end-of-month stocks are plotted against the residuals $EP_{t+1}-P_t-o_t'$, where t is equal to one month, P_t is the average monthly price and EP_{t+1} is calculated by the method outlined previously. The relations of the residuals, $r'-c'$, with stocks are shown in Figures 4 to 8.[8] Three semiperishable and two durable farm commodities were selected for study.The commodities are further differentiated by the importance of futures markets. For shell eggs, cheese and butter the data cover the period 1924 to 1938; for wheat and oats, 1924 to 1932. Later dates were omitted because of the influence of government price-support and storage programs. Prices were deflated by the wholesale price index of all commodities (1926=100). Stocks

[8] Data for these figures were derived from the following sources: (1) cash prices: *Yearbook of Agriculture, Agricultural Statistics;* (2) futures prices: *Grain Futures Statistics,* Stat. Bull. 131, USDA, July 1953; (3) stocks: *Agricultural Statistics; Cold Storage Report,* USDA monthly; *Dairy Statistics and Related Series,* Stat. Bull. 134, USDA, Oct. 1953; *Grain and Feed Statistics,* Stat. Bull. 159, USDA, Mar. 1955 and information made available by the U. S. Department of Agriculture.

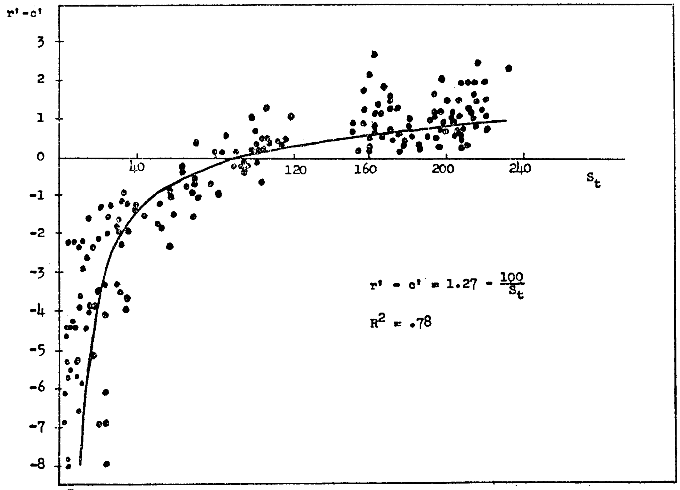

FIGURE 4.—Shell Eggs: Average wholesale price spreads per dozen, Chicago, by months, 1924-1938 minus marginal outlay per month related to end-of-month total cold-storage holdings.

are expressed as percentages of a 13-month moving average in order to facilitate monthly comparisons allowing for long-term growth in production and stocks. A number of curves that conform generally to the shape of the scatter diagram were fitted by least squares, and the curve giving the best fit was used.

Although most of the diagrams are self-explanatory, Figures 5, 7 and 8b call for some comment. The special circumstances surrounding the aging of cheese in storage account in large part for the relatively small percentage seasonal fluctuations in stocks. Because certain cheeses are undergoing a relatively long aging process the amount of

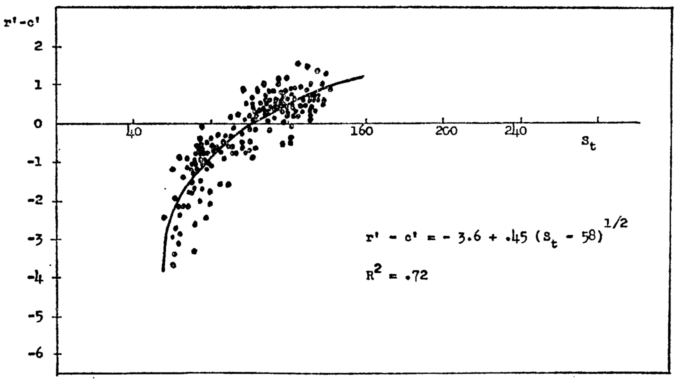

FIGURE 5.—Cheese: Fresh single daisies, average wholesale price spreads per pound, Chicago, by months, 1924-1938 minus marginal outlay per month related to end-of-month total cold storage holdings.

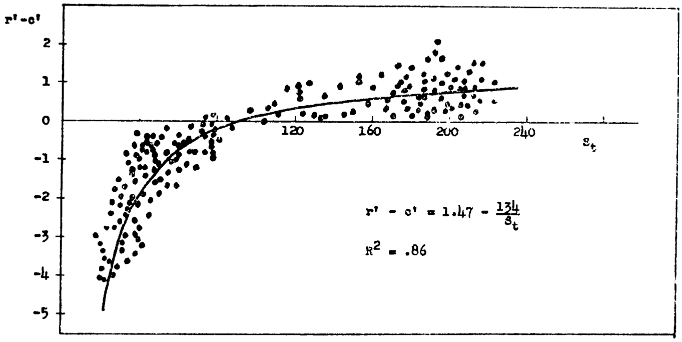

FIGURE 6.—Creamery Butter: Ninety-two score, average wholesale price spreads per pound, Chicago, by months, 1924-1938 minus marginal outlay per month related to end-of-month total cold-storage holdings.

cheese held in storage does not respond readily to changes in monthly price spreads. The effect of this is to raise the absolute level of the seasonal trough for total cheese holdings. The absolute quantity of stocks held during the peak months may be larger than stocks of commodities for which processing and storing are not a single operation;

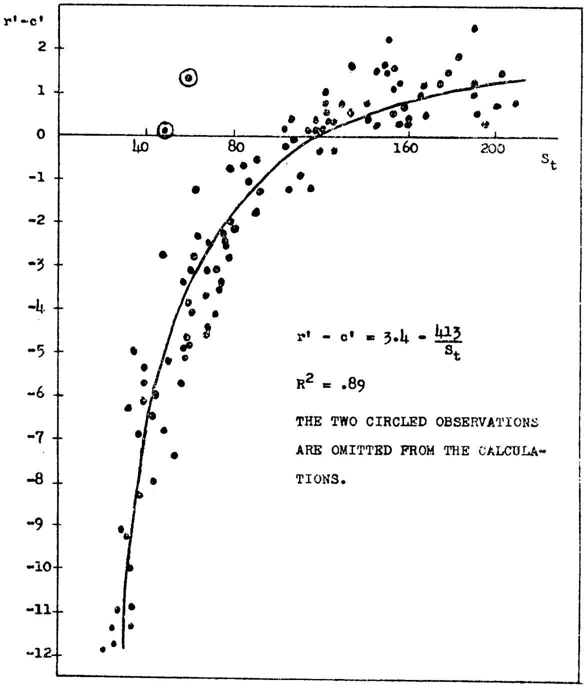

FIGURE 7a.—Wheat: No. 2 hard winter, average price spreads per bushel, Kansas City, by months, 1924-1932 minus marginal outlay per month related to total end-of-month wheat stocks.

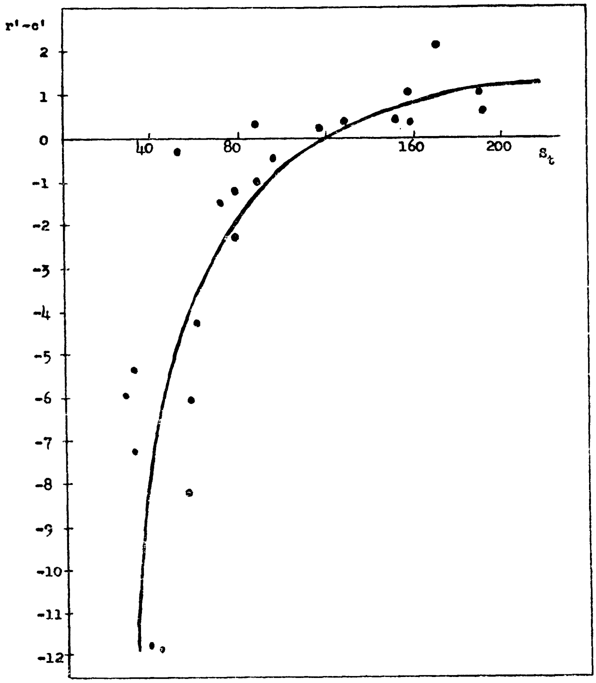

FIGURE 7b.—Wheat: Futures price spreads,* end-of-month closing prices of principal futures, Chicago Board of Trade, 1924-1932 minus marginal outlay related to total end-of-month wheat stocks.

* See footnote 9.

nevertheless, the higher annual average level of stocks per month will result in a relatively smaller seasonal peak when expressed as a percentage of trend. To enable comparison of expected-price calculations Figures 7b and 8b show the curves derived in Figures 7a and 8a superimposed on the scatter of points yielded by futures price spreads minus marginal outlay.[9] Though futures price quotations provide fewer ob-

[9] Rather than compare a current cash price with the current quotation of a distant futures price, the current quotation of the futures price for delivery in a distant month was subtracted from the current quotation of the futures price for delivery in the current month or a near month. This was done to insure that the prices being used referred to identical commodities. The cash wheat price in July may differ from the July futures wheat price quoted in July by the inclusion of wheat "on track" in the cash price. The futures price spreads used in Figures 7b and 8b are the following: (1) September futures minus July futures at the end of July 1924 to 1932 exclusive of 1928 and 1931; (2) September futures minus July futures at the end of June 1924 to 1932 exclusive of 1928 and 1931; (3) May futures minus March futures at the end of March 1929 to 1932 exclusive of 1931; (4) December futures minus September futures at the end of September 1925 to 1932 exclusive of 1928 and 1931. Years in which corners, squeezes or exceptional circumstances affected the price spreads are omitted in order to depict the normal relationship more clearly. March quotations are not available prior to 1929. The exceptional price spreads in 1929 are due in part to unusual congestion of grain elevators in Chicago.

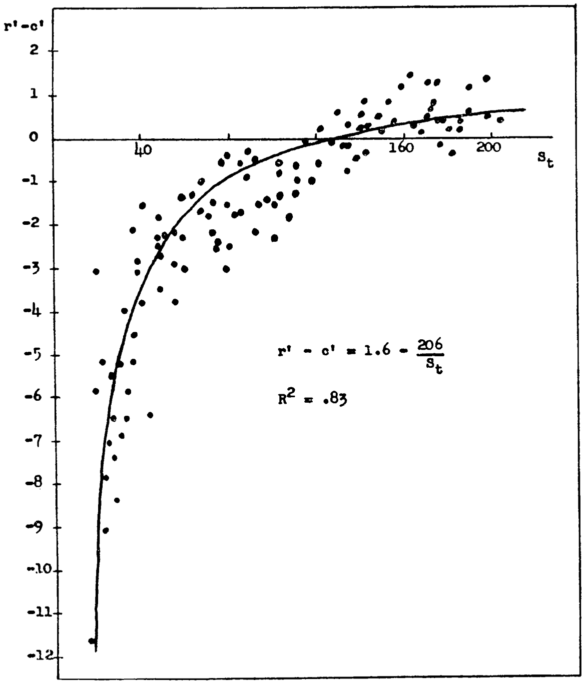

FIGURE 8a.—Oats: No. 3 white, average price spreads per bushel, Chicago, by months, 1924-1932 minus marginal outlay per month related to total end-of-month oats stocks.

servations, there is no significant difference between this curve and one fitted to the 23 observations by least squares. Similar comparisons were not made for the semiperishables because futures trading is often too light to afford good quotations.

If one is willing to allow a few "heroic" assumptions, it is possible to obtain a plausible quantitative estimate of the risk premium involved in storage. First, the marginal convenience yield is approximately zero in the peak storage months. Second, the marginal risk aversion is a linear increasing function of stocks; for the most plausible supposition about the marginal risk aversion is that it does increase with the level of stocks. If the marginal risk aversion were constant, this would imply that, given their capital resources, firms' decisions about additions to stocks would be independent of the amount already invested in stocks. This seems improbable. With c' equal to zero at the maximum observed level of monthly stocks, the residual is r', and the linear function passes through this point and the origin.

Since marginal outlay is equal to average outlay, the average monetary return or "profit" from storage can be obtained from knowledge

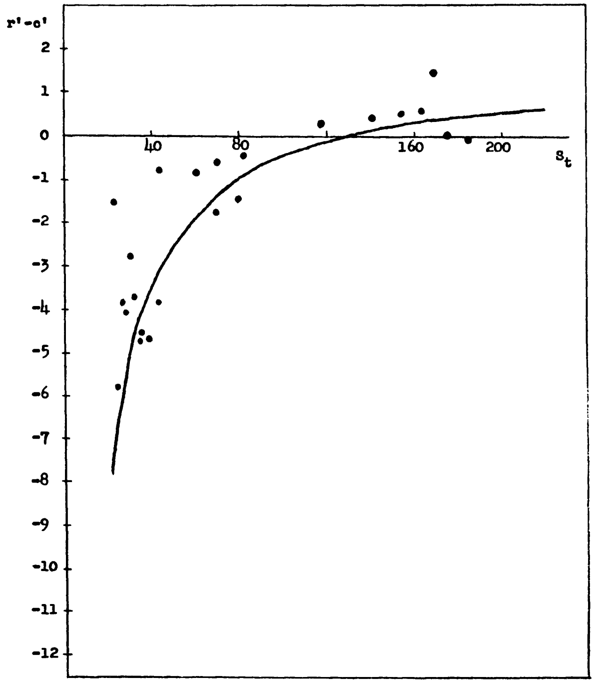

FIGURE 8b.—Oats: Futures price spreads,* end-of-month closing prices of principal futures, Chicago Board of Trade, 1924-1932 minus marginal outlay related to total end-of-month oats stocks.

* See footnote 9.

of this linear function. When stocks are relatively small, for each unit of the commodity put into storage a small risk premium will be required. The larger are stocks already held, the larger will be the risk premium of each unit put into storage in that month. If the average of these monthly returns is taken, an approximation to the return per month is obtained. Suppliers of storage move temporally along the risk function from the month of seasonal minimum stocks to the month of seasonal maximum and back again to the seasonal minimum. The average of these monthly values expressed as a percentage of price gives the average percentage return per month.

This has been done for each of the commodities studied and the estimated returns *per annum* are as follows: wheat 6.6 per cent, oats 6.8 per cent, butter 7.9 per cent, eggs 8.5 per cent and cheese 9.5 per cent. The relative difference in annual returns can be explained primarily by perishability. In addition to deterioration there is a quality factor which affects storage of and speculation in the semiperishables which does not exist to nearly the same degree in the case of more durable commodities. A speculator in grain can be comparatively certain that the commodity coming out of storage is the same as the commodity that went into storage. Because of a possible taste differential between fresh and stored semiperishables this is not true to the same extent for these commodities. Consequently this lack of knowledge on the part of speculators would require that they obtain a greater return for undertaking the risk of investment in semiperishable stocks. Shell eggs appear to be the commodity most affected by this uncertainty with respect to quality. There is another reason for the relatively higher return on cheese. The absolutely larger seasonal minimum level of stocks, shown in Figure 5 and explained earlier, means that the annual average investment in stocks is larger. As a result we should expect a risk premium in proportion to the larger annual average quantity of stocks held.

One interesting feature of the functions in Figures 4 to 8 is that they may be used to explain why such small proportions of semiperishable stocks have been hedged on the futures markets. By far the most important holders of semiperishable stocks are producers, wholesalers and dealers. These stocks are carried from the months of seasonally high production to the months of low production. A maximal estimate is that about one-third of total annual butter stocks were hedged prior to 1940. Of those owners of butter stocks who did engage in hedging—those who held the largest quantities of stocks—most hedged only a part of their total holdings.[10] A similar situation exists for eggs and cheese. Opinions expressed by officials of the various mercantile exchanges indicate that they have been unsuccessful in attempting to establish active futures trading on a large scale for these semiperishables.

When stocks are relatively small, the marginal convenience yield is relatively large; stocks will be held unhedged if the owner can expect to receive a marginal risk premium reflected in the expected seasonal price structure. There is always the possibility that the demand for consumption will increase *within* any one month, and the marginal convenience yield expresses the allowance made for this by owners of stocks. If this increase in demand is considered to be permanent, expected prices will rise in each future month within the planning interval. The immediate effect of the rise in demand is to raise the current cash price. As we have seen in the theory of the demand for storage, if future levels of production and stocks were not to adjust to the

[10] H. S. Irwin, *Impressions of Trading in Butter and Egg Futures,* Comm. Exch. Admin., USDA (Washington, D.C., Feb. 1940).

permanently higher expected prices, this would mean a decrease in demand for stocks to be carried out of the current month and a decrease in the price spread. But when sufficient time is allowed for the production schedule to be adjusted, then production in the current month and perhaps production and storage in future months will increase.

The increase in demand for consumption will take the form of an expanded flow of orders, and the availability of stocks to meet the increased flow provides the convenience yield. Suppose that the cash price on August 10 is 50 cents and the December futures price is 60 cents (the net marginal cost of storage is 10 cents). If the cash price on August 11 were to rise to 55 cents, arbitrage between the cash and futures markets would insure a higher futures price also, given that the change in demand is considered permanent. Let us suppose that it rises to 65 cents, reflecting a continued higher expected price level. Production and stocks can be adjusted to reach the levels consistent with the ten-cent price spread. Consider a firm which has on hand unhedged stocks. It can meet the increase in demand for consumption and cash price by filling orders out of stocks. This temporarily reduces stocks below the level which maximizes net revenue, but these stocks can be replaced from later production in such a way that the new total level of stocks held maximizes the expected net revenue reflected in the new ten-cent price spread. Now consider the owner of stocks who has hedged. Though the stocks have been committed to future sale at a futures price quoted at the time of the hedge, *e.g.*, 60 cents, the stocks could still be used to take advantage of the increase in the cash price; however, the futures price quotation will have risen also. Then the purchase of a futures contract to cover the short sale would (partially) eliminate the gain from sales out of stocks.

It might be argued that it would be unnecessary to purchase a futures contract. One could still meet the short sale out of later production. But this later production would have to be sold at the futures price originally contracted when the hedge was effected, and this would now be lower than the current futures price quotation. There is a return foregone. As long as there is a relatively high marginal convenience yield and the firm expects a risk premium for carrying stocks into a future period, it will be to the firm's advantage to hold some stocks unhedged in order to be prepared for a possible unforeseen contingency.

As stocks accumulate the marginal convenience yield will decline. Since the owner of stocks has on hand unhedged stocks out of which he can fill increased orders, he can now hedge any additional stocks and shift the risk to speculators. The reason why he may decide to shift the risk is that *total* risk aversion is approaching the critical level —critical in terms of the firm's capital resources and credit position as explained earlier. Since there are a potentially infinite number of speculators, the risk will be spread over a greater number of individuals in society and borne at a smaller cost than that required in the case of any limited number of firms. When the marginal risk premium required by any individual firm exceeds that required by speculators, stocks will be hedged.

Under the specified assumptions it has been shown, on the basis of the evidence presented in Figures 4 to 8, that speculators in grains require a smaller marginal risk premium than speculators in semi-perishables. Consequently we should expect that, given the capital resources and convenience yields of storage firms, a greater proportion of grain stocks would be hedged relative to semiperishable stocks. If the increasing marginal risk functions of a holder of butter and a holder of grains are approximately the same, the risk premium will exceed that of speculators at a smaller level of holdings for the storer of grains. Since the risk premium required by speculators in butter is larger than that required by speculators in grains, the butter storer must accumulate relatively greater stocks before his risk premium will exceed that of speculators. When stocks of any commodity are large enough to require hedging, the risk premium for the market as a whole is the risk premium of the speculators in that commodity. We may interpret the evidence on price spreads, risk premiums, convenience yields and hedgings for the economy as a whole as the combination of forces operating simultaneously to determine the level of stocks held as a function of price spreads and the proportion of these stocks hedged on the futures markets.

III. *Conclusion*

When referring to the motives for holding money, the convenience motive is ordinarily distinguished from the speculative motive. The exact counterpart of this convenience motive for holding stocks would apply strictly only to those held in order to avoid the nuisance and cost of (1) frequent deliveries for processing and (2) frequent revisions of the production schedule to effect increased sales. The definition presented in this paper is somewhat broader. To include in the definition of convenience yield the benefit of being in a position to take advantage of a possible price rise on short notice would seem to place these holdings in the category of speculative stocks.

In such questions of definition usefulness is generally accepted as the criterion for choice. The line of distinction between convenience and speculative stocks is a thin one. If a firm is presumed to hold stocks for the purpose of convenient handling of an expanded flow of orders, how can this increase in orders occur unless the total market demand for consumption increases? If it is not a mere shift in orders from one firm to another while market demand remains unchanged, the current price can remain constant only if supply is perfectly elastic in the short run; there is nothing a priori to guarantee that this is the case. On the contrary, once stocks have been put into storage with the intention of carrying them to a future period, price in the current period would have to be bid up by those who intend to "consume" them in order that some of the stocks be diverted to current consumption.

Holbrook Working has pointed out two shortcomings of the price-of-storage theory which expresses net marginal cost of storage as marginal outlay minus marginal convenience yield (defined in the narrow sense).[11] Much storage is supplied by firms which do not hedge (completely), and many hedged stocks earn a gross monetary return per unit which is not exactly equal to the marginal outlay. The virtue of our broader definition is that, by means of this concept together with the marginal risk premium, it is possible to explain the degree of hedging as well as the difference between the gross return and the marginal outlay. It emphasizes the importance of risk for seasonal accumulation and provides an explanation of the "inverse carrying charge."

The notion of risk has entered previous discussion of storage. Several British writers have viewed the marginal risk premium as an increasing function of stocks; however, the risk premium was used primarily in an attempt to explain inverse carrying charge by the phenomenon of discounting the future.[12] For hedgers who are forward sellers (the majority of suppliers of storage) the futures price must fall short of the expected price by the amount of the marginal risk premium; hence, if the current price is expected to remain unchanged, the futures price must be below the spot price. For hedgers who are forward buyers the futures price is above the expected price by the amount of the marginal risk premium. Although this summary is somewhat skeletal, it illustrates the different role played by the risk premium. Holbrook Working has effectively demonstrated that the attempt to use risk in this way to explain inverse carrying charge hinges on a concept of substantial independence of cash and futures markets.[13] Continuous effectiveness of arbitrage between cash and futures prices makes it necessary to regard the two sets of prices as determined in a single market, *i.e.*, expected price equals futures price. On the basis

[11] H. Working, "The Theory of Price of Storage," *Am. Econ. Rev.*, Dec. 1949, XXXIX, 1254-62.

[12] J. M. Keynes, *Treatise on Money* (New York, 1930), II, p. 143.

[13] Working, "Theory of Inverse Carrying Charge," *loc. cit.*, p. 2.

of statistical evidence inverse carrying charge is better explained by a theory that does not involve explicit consideration of any supposed discounting of the future.

The theory presented in this paper provides a general hypothesis to explain the degree of hedging as well as intra- and interyear storage behavior. Two of the components which determine the supply curve of storage are not directly observable, yet the hypothesis can also be used to obtain a measure of the relative risk premiums involved in the storage of different commodities.