The General Sieve Kernel

or, The Algorithmic Ant and the Sandpile

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February 27, 2023

The Algorithmic Ant and the Sandpile

Once upon a time...

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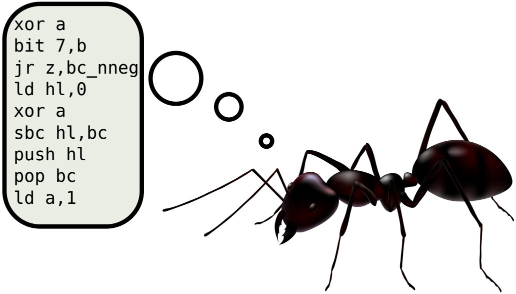

An algorithmic ant.[1]

[1] Thanks to Léo Ducas for the ants and images in the story!

The Queen of ants summoned the algorithmic ant,

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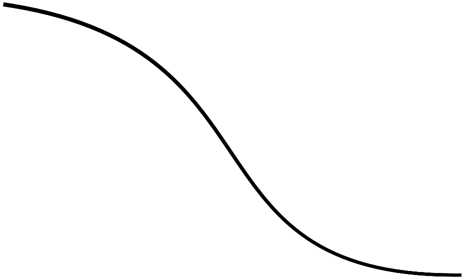
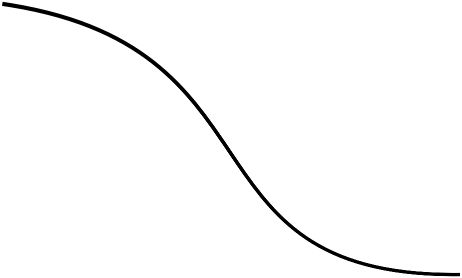
"See this sandpile?"





"I want it flat!"

Looking closer at the sandpile,

the algorithmic ant ponders.

“One grain at the time,

I shall pull the sand downhill.”

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(. . . so hoped the ant.)

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each excavation is a puzzle of its own.

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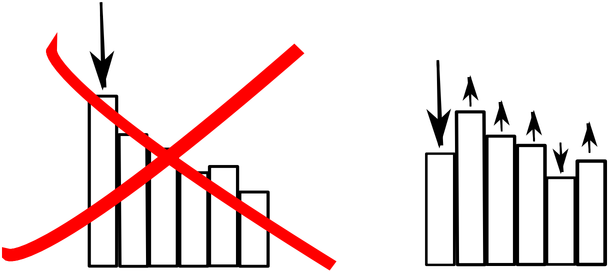

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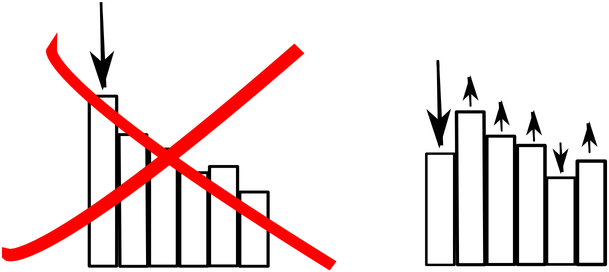
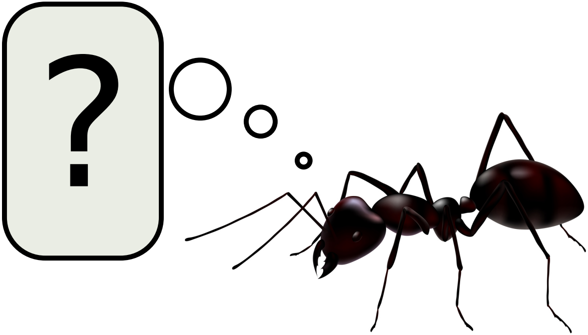


Columns are tied in mysterious ways;

to push one down, one must find

the right combination.

Unsure how to proceed,



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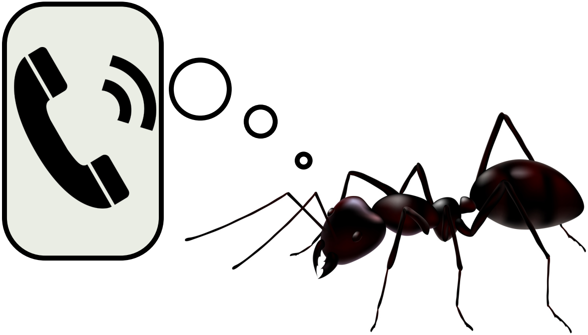


the ant calls a computer scientist.

"Let's start from the top!"

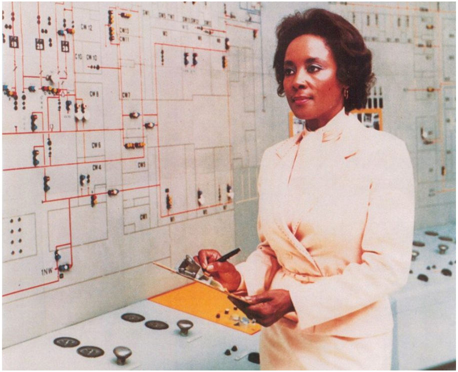
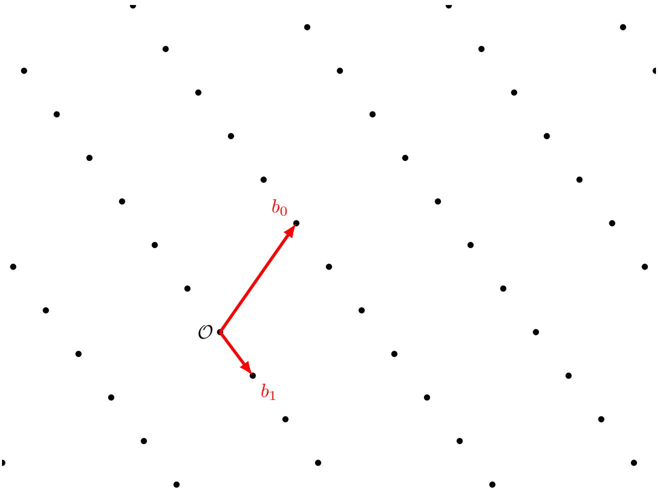


Figure 1: Annie Easley (NASA/NACA)

- Lattices $\leftrightarrow$ Sandpiles
- Sieving for a grain of sand
- Beat the Oracle! G6K, a stateful machine

From Lattices to Sandpiles



$\Lambda = \mathrm{Span}_{\mathbb{Z}}(b_0, \ldots, b_{d-1})$, $B = (b_0, \ldots, b_{d-1}) \subset \mathbb{R}^d$ basis

Define the orthogonal projection of v on u,

$$\pi_u(v) = \frac{\langle v, u \rangle}{\langle u, u \rangle} u.$$

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$$\pi_u(v) = \frac{\langle v, u\rangle}{\langle u, u\rangle} u.$$

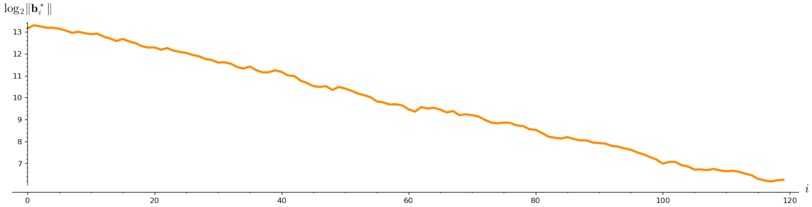
Given B we define the Gram–Schmidt vectors iteratively

$$\begin{aligned}
b_0^* &= \pi_0^\perp(b_0) & &= b_0 \\
b_1^* &= \pi_1^\perp(b_1) & &= b_1 - \pi_{b_0^*}(b_1) \\
b_2^* &= \pi_2^\perp(b_2) & &= b_2 - \pi_{b_0^*}(b_2) - \pi_{b_1^*}(b_2) \\
&\vdots \\
b_i^* &= \pi_i^\perp(b_i) & &= b_i - \sum_{j<i} \pi_{b_j^*}(b_i).
\end{aligned}$$

Let $B^* = (b_0^*, \ldots, b_{d-1}^*)$, the Gram–Schmidt basis of B.

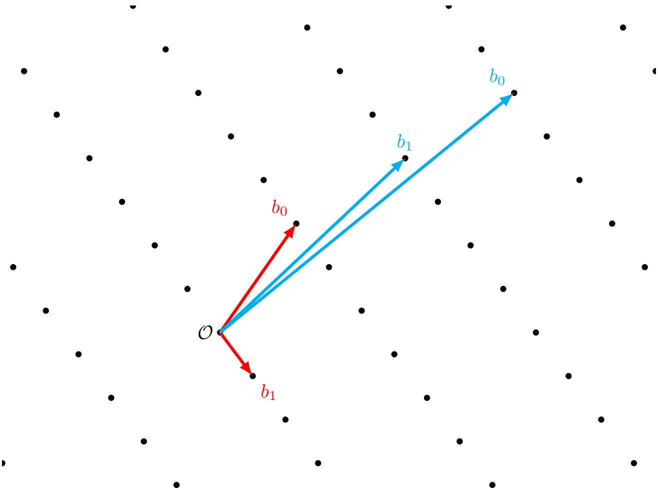
WARNING: $\mathsf{Span}_{\mathbb{Z}}(B^*) \neq \Lambda$!

We get our sandpile from B^*.[2]

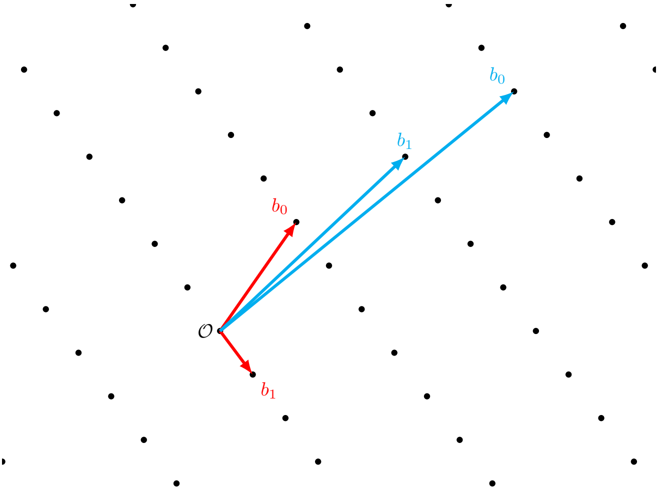


But how to flatten it, and why is it so whimsical?

[2] Thanks to Martin Albrecht for the LLL/BKZ experiments throughout!



Good basis B, bad basis B



B is also a basis of $\Lambda \iff \exists$ unimodular U such that $BU = B$

We have an invariant quantity

$$\mathrm{vol}(\Lambda) = \det(B) = \det(B) = \prod_{i=0}^{d-1} \|b_i^*\| = \prod_{i=0}^{d-1} \|b_i^*\|.$$

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So our desire to “flatten” is constrained by

- constant product of $\|b_i^*\|$
- strong dependence between b_i^*, b_j^*.

Lattice reduction is any procedure P

$$B \xrightarrow{P} B \text{ such that } B^* \text{ is flatter than } B^*$$

- Search for global U that minimises slope?

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BKZ!

If $U_{\ell,r} = \begin{pmatrix} I_\ell & & \\ & U'_{\ell,r} & \\ & & I_{d-r} \end{pmatrix}$, then $BU_{\ell,r}$ gives

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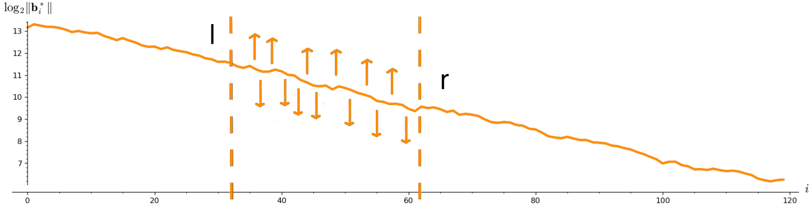

Even this is too hard, so instead find $U_{\ell,r}$ that minimises $\|b_\ell^*\|$

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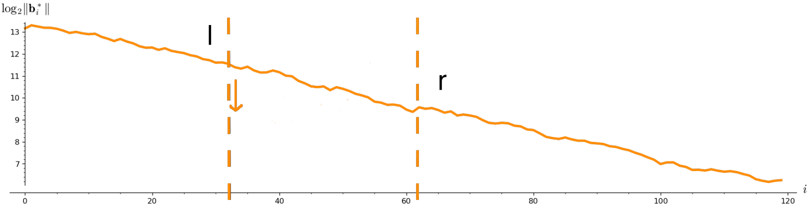

More specifically let

$$B_{[\ell:\,r]} = (\pi_\ell^\perp(b_\ell), \ldots, \pi_\ell^\perp(b_{r-1})),$$

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then $B_{[\ell:\,r]} = (b_\ell^*, \ldots)$, so let us solve SVP in $\Lambda_{[\ell:\,r]}$!

$$\left(\begin{array}{ccccccccc} \overbrace{b_0 \quad b_1 \quad b_2 \quad b_3 \quad b_4}^{\beta = 5} & b_5 & b_6 & b_7 & \dots \end{array}\right)$$

$$\begin{pmatrix} \underbrace{\overbrace{b_0 \quad b_1 \quad b_2 \quad b_3 \quad b_4}^{\beta = 5}} \quad b_5 \quad b_6 \quad b_7 \quad \dots \end{pmatrix}$$

$$\begin{pmatrix} \overbrace{\color{green}{b_0} \quad b_1 \quad b_2 \quad b_3 \quad b_4}^{\beta = 5} \quad b_5 \quad b_6 \quad b_7 \quad \ldots \end{pmatrix}$$

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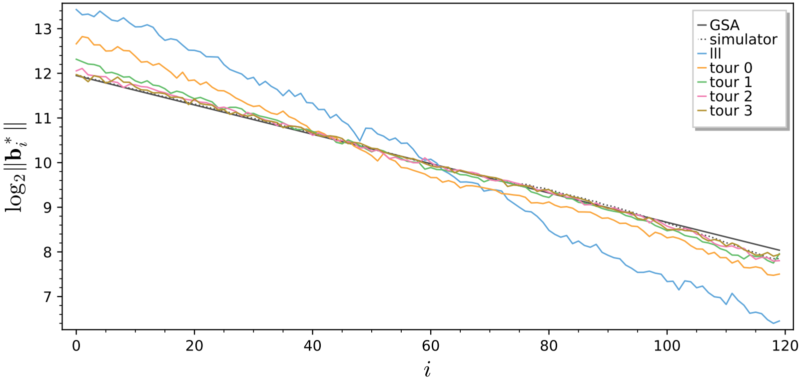
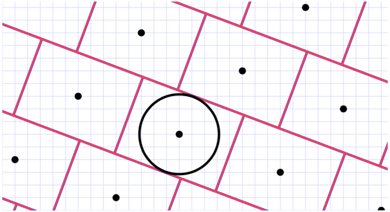


GSA
simulator
lll
tour 0
tour 1
tour 2
tour 3
$\log_2 \|\mathbf{b}_i^*\|$
i
13
12
11
10
9
8
7
0
20
40
60
80
100
120

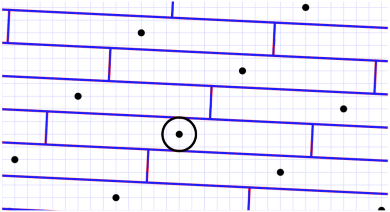
A flat basis helps with the

- *dual* and *primal* LWE attacks (practical)
- shortest independent vectors problem (fundamental)
- approximate closest vector problem (pertinent here!)

We solve the "approx CVP" with Babai's Nearest Plane.



Decoding radius with $\mathbf{B}^\star$



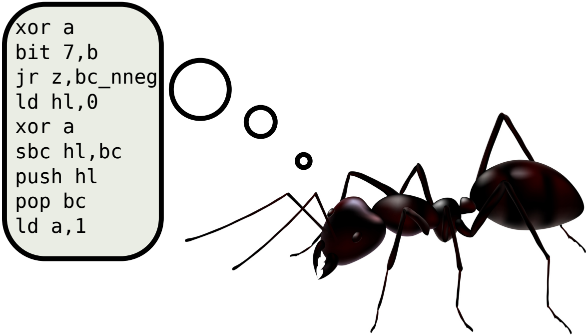
Decoding radius with $\mathbf{B}^\star$

- Worst case distance $\frac{1}{2}\sqrt{\sum \|b_i^*\|^2}$ (approx CVP)
- Correct decoding of $t = v + e$ where $v \in \Lambda$ if (BDD)

$$\|e\| \leq \frac{1}{2} \min \|b_i^*\|$$

Liberté, Égalité, Fraternité!

“Thanks for the lecture, but...



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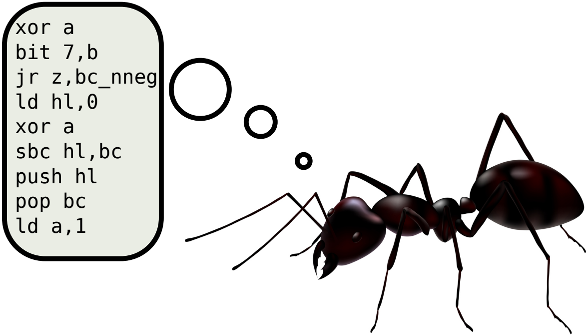
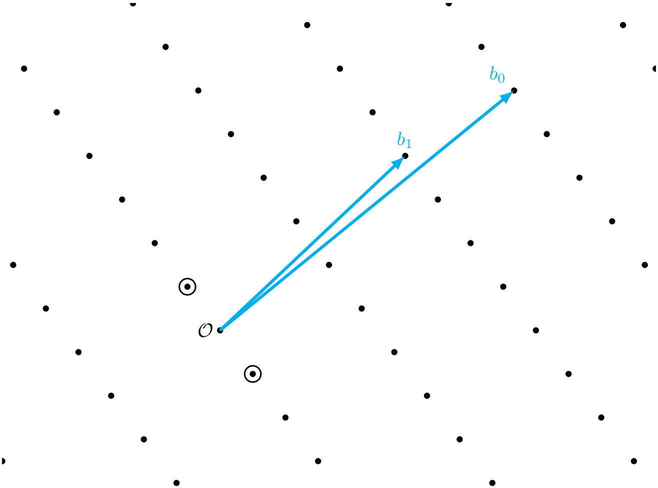
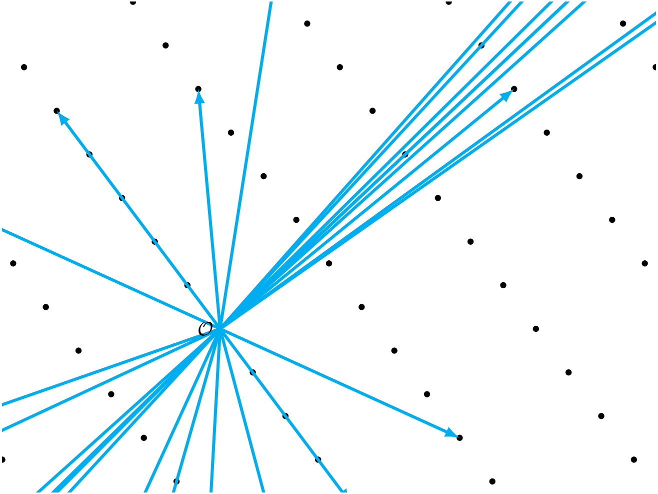


how should I solve these SVP puzzles?"

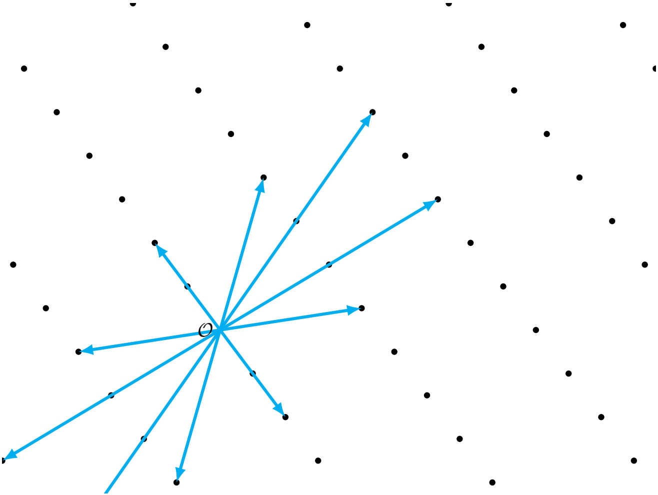
Sieving for a Grain of Sand



SVP: find $v \in \Lambda \setminus \{0\}$ such that $\|v\| \leq \|w\|$ for all $w \in \Lambda \setminus \{0\}$



Sieve: $v, w \in L \subset \Lambda$, if $\|v \pm w\| \leq \|v\|$, $v \leftarrow v \pm w$



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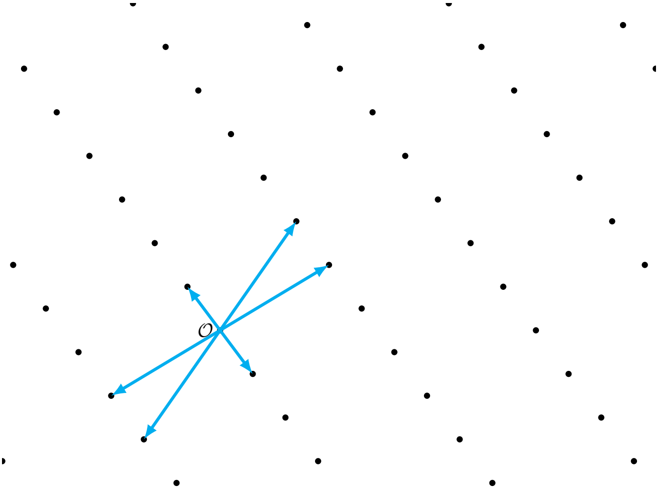

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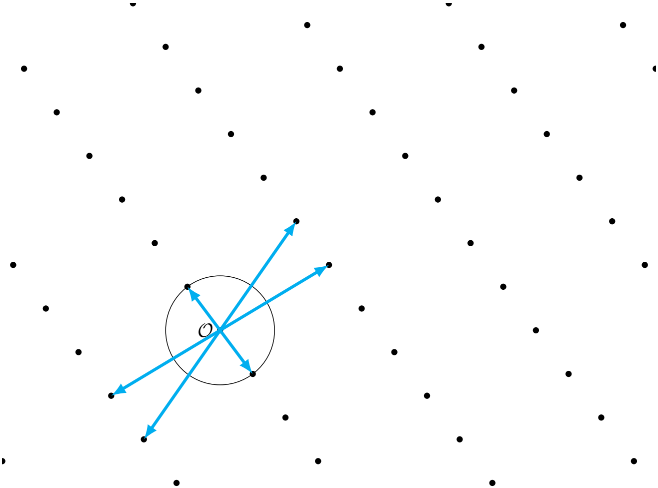

Sieve: $v, w \in L \subset \Lambda$, if $\|v \pm w\| \leq \|v\|$, $v \leftarrow v \pm w$

So $\mathrm{BKZ} = (\mathrm{Sieve}(\Lambda_{[0:\,\beta]}), \mathrm{Sieve}(\Lambda_{[1:\,\beta+1]}), \ldots)^{\mathrm{tours}}$.

So BKZ $= (\mathrm{Sieve}(\Lambda_{[0:\,\beta]}), \mathrm{Sieve}(\Lambda_{[1:\,\beta+1]}), \ldots)^{\mathrm{tours}}$.

Some sieving facts

- dimension d lattice (time, space) $= (\exp(\Theta(d)), \exp(\Theta(d)))$
- a Sieve outputs most of the shortest vectors in a lattice…

Define the Gaussian heuristic (expected length of SVP solution)

$$\mathrm{gh}(\Lambda) = \sqrt{\frac{d}{2\pi e}}\mathrm{vol}(\Lambda)^{1/d}.$$

[3] Can parameterise termination by ensuring some constant fraction of L.

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$$\mathrm{gh}(\Lambda) = \sqrt{\frac{d}{2\pi e}} \mathrm{vol}(\Lambda)^{1/d}.$$

The output of Sieve is most[3] of

$$L = \mathrm{Sieve}(\Lambda) = \left\{ v \in \Lambda \text{ s.t. } \|v\| \leq \sqrt{\frac{4}{3}}\, \mathrm{gh}(\Lambda) \right\}.$$

[3]Can parameterise termination by ensuring some constant fraction of L.

Can we use more of L than simply the shortest vector?

---

[4] Thijs Laarhoven and Artur Mariano, Progressive lattice sieving, Post-Quantum Cryptography – 9th International Conference, PQCrypto 2018 (Tanja Lange and Rainer Steinwandt, eds.), Springer, Heidelberg, 2018, pp. 292–311.

[5] L. Babai, On lovász' lattice reduction and the nearest lattice point problem, Combinatorica 6 (1986), no. 1, 1–13.

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Can we use more of L than simply the shortest vector? Yes!

- sieve in sublattices to “seed” higher dimensional sieves[4]
- sieve in (projected) sublattices and lift[5] to the full lattice[6]

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Imagine we want to sieve in $\Lambda = \mathsf{Span}_{\mathbb{Z}}(b_0, \ldots, b_{d-1})$.

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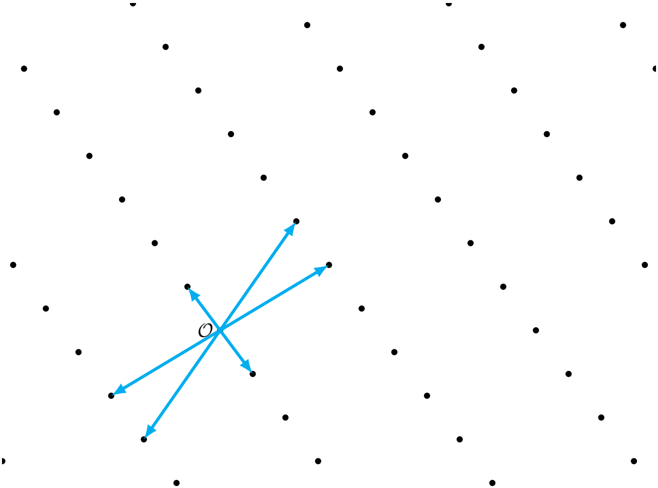
function Progressive Sieve($b_0, \ldots, b_{d-1}$)
 $L \leftarrow \text{Sieve}((b_0, b_1), \emptyset)$
 for $i \in \{2, \ldots, d-1\}$ **do**
  $L \leftarrow \text{Sieve}((b_0, \ldots, b_i), L)$
 end for
 Return L
end function



Some $v \in \mathsf{Span}_{\mathbb{Z}}(b_0, b_1, b_2)$ will shorten quicker!

Let $s \in \Lambda$ be a shortest vector. Rather than (eventually)

$$s \in L = \text{Sieve}(b_0, \ldots, b_{d-1}),$$

Let $s \in \Lambda$ be a shortest vector. Rather than (eventually)

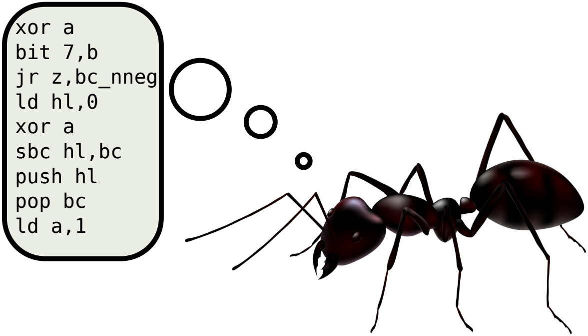
$$s \in L = \mathrm{Sieve}(b_0, \ldots, b_{d-1}),$$

hope that

$$\pi_f^{\perp}(s) \in L = \mathrm{Sieve}(\Lambda_{[f\,:\,d]}), \text{ and } s \in \mathrm{Lift}(L).$$

Both conditions are determined by how flat the sandpile is!

“All that work for a single grain of sand!



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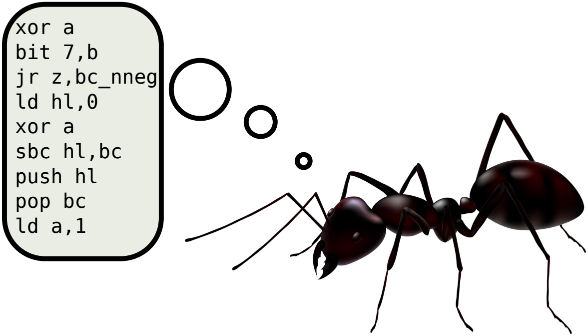
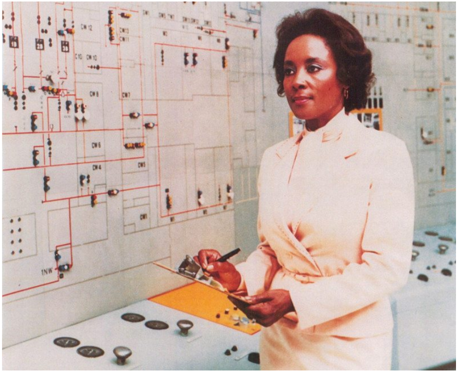


Must I repeat it all for each grain?"

“Hum. Let me think.



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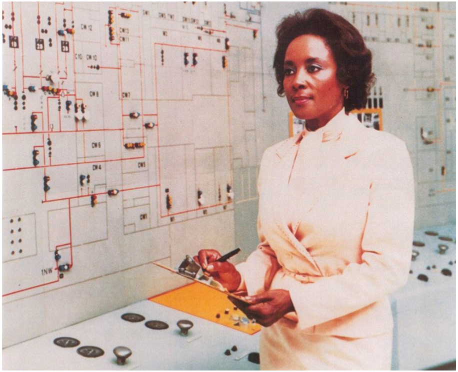

Maybe we don’t need to repeat all of it...”

The Generalised Sieve Kernel[7]
(G6K, pronounced /ζe.si.ka/)

[7] Albrecht M. R., Ducas L., Herold G., Kirshanova E., Postlethwaite E. W., Stevens M. (2019) The General Sieve Kernel and New Records in Lattice Reduction. In: Ishai Y., Rijmen V. (eds) Advances in Cryptology – EUROCRYPT 2019. EUROCRYPT 2019. Lecture Notes in Computer Science, vol 11477. Springer, Cham

Idea: Recycle vectors between overlapping SVP instances.

Rather than an SVP oracle, Sieve is a stateful machine!

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Rather than an SVP oracle, Sieve is a stateful machine!

An algorithmic ant on a sandpile,
carrying a bag of vectors on its back.

$$
\begin{array}{ccccccccc}
\Lambda = & \Lambda_{[0:d]} & \supset & \Lambda_{[0:d-1]} & \supset & \cdots & \Lambda_{[0:2]} & \supset & \Lambda_{[0:1]} \\
& \downarrow \pi_1^{\perp} & & \downarrow \pi_1^{\perp} & & & \downarrow \pi_1^{\perp} & & \\
& \Lambda_{[1:d]} & \supset & \Lambda_{[1:d-1]} & \supset & \cdots & \Lambda_{[1:2]} & & \\
& \vdots & & \vdots & & \iddots & & & \\
& \Lambda_{[d-2:d]} & \supset & \Lambda_{[d-1:d]} & \iddots & & & & \\
& \downarrow \pi_{d-1}^{\perp} & & & & & & & \\
& \Lambda_{[d-1:d]} & & & & & & &
\end{array}
$$

The $\pi_i^{\perp}$ are “inverted” via Nearest Plane.

If $L = \text{Sieve}\left(\Lambda_{[\ell:r]}\right)$ then we take our vectors with us.

$$\begin{tikzcd}
\Lambda_{[\ell-1:r]} \arrow[d, shift left, "\pi_\ell^\perp"] & \supset & \Lambda_{[\ell-1:r-1]} \\
\Lambda_{[\ell:r]} \arrow[u, shift left, "\left(\pi_\ell^\perp\right)^{-1}"]
\end{tikzcd}$$

Extend right: $\subset$ (Inclusion – ER)
Shrink left: $\pi_\ell^\perp$ (Project – SL)
Extend left: $\left(\pi_\ell^\perp\right)^{-1}$ (Lift – EL)

$$
\begin{pmatrix}
 & & | & & & & | & & \\
b_0 & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 & \dots \\
 & & | & & & & | & &
\end{pmatrix}
$$

$$\left(b_0 \quad b_1 \quad \overbrace{\underbrace{b_2 \quad b_3 \quad b_4 \quad b_5 \quad b_6}}^{\beta = 5} \quad b_7 \quad \dots \right)$$

$$\begin{array}{c} \begin{matrix} i_0 & i_1 & i_2 & & & & & & \end{matrix} \\ \begin{pmatrix} & & \vert & & & & \vert & & \\ b_0 & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 & \dots \\ & & \vert & & & & \vert & & \end{pmatrix} \end{array}$$

$$
\begin{array}{c}
\begin{array}{ccccccccc} i_0 & i_1 & i_2 & & & & & & \end{array} \\
\begin{pmatrix}
 & & & \vert & & & & \vert & \\
b_0 & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 & \ldots \\
 & & & \vert & & & & \vert &
\end{pmatrix}
\end{array}
$$

$$\begin{pmatrix} i_0 & i_1 & i_2 & \overbrace{\hphantom{b_3 \quad b_4 \quad b_5 \quad b_6 \quad b_7}}^{\beta = 5} & \\ b_0 & b_1 & b_2 & \underbrace{b_3 \quad b_4 \quad b_5 \quad b_6 \quad b_7} & \dots \end{pmatrix}$$

$$
\begin{array}{c}
\begin{array}{cccccccc} i_0 & i_1 & i_2 & i_3 & & & & \end{array} \\
\begin{pmatrix}
 & & & | & & & & | & \\
b_0 & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 & \dots \\
 & & & | & & & & | &
\end{pmatrix}
\end{array}
$$

The Full Abstract Machine

State:

- Basis B,
- Indices $0 \leq \kappa \leq \ell \leq r \leq d$, $[\kappa : r]$ "lifting context", $[\ell : r]$ "sieving context",
- Database, L, of (short) vectors in $\Lambda_{[\ell:r]}$,
- Insertion candidates $i_\kappa, \ldots, i_\ell$, $i_j \in \Lambda_{[j:r]}$, or $i_j = \perp$.

Instructions:

- Reset (R): empty L, set (κ, ℓ, r),
- Sieve (S): sieve in $\Lambda_{[\ell:r]}$, shorten $v \in L$, better i_j,
- $\{$EL, ER, SL$\}$: change $[\ell : r]$ and apply to L,
- Insert (I): update B and L, $[\ell : r] \to [\ell + 1 : r]$.

A Pump

$$\text{Pump}_{f,r}\text{: Reset}_{r-1,r}\ \overbrace{(\text{EL S})^{r-f-1}}^{\text{pump-up}}\ \overbrace{(\text{I S})^{r-f-1}}^{\text{pump-down}}$$

Key: sieve, lift, c_i = insert

A Pump

$\text{Pump}_{f,r}$: $\text{Reset}_{r-1,r}\ \overbrace{(\text{EL S})^{r-f-1}}^{\text{pump-up}}\ \overbrace{(\text{I S})^{r-f-1}}^{\text{pump-down}}$

$$(b_0, b_1, \ldots, b_{f-1}, b_f, b_{f+1}, \ldots, b_{d-2}, b_{d-1})$$

Reset!

Key: sieve, lift, c_i = insert

A Pump

$$\text{Pump}_{f,r}\text{: Reset}_{r-1,r}\ \overbrace{(\text{EL S})^{r-f-1}}^{\text{pump-up}}\ \overbrace{(\text{I S})^{r-f-1}}^{\text{pump-down}}$$

$$(b_0, b_1, \ldots, b_{f-1}, b_f, b_{f+1}, \ldots, \color{red}{b_{d-2}}, \color{red}{b_{d-1}})$$

Extend Left!

Key: sieve, lift, c_i = insert

A Pump

$$\text{Pump}_{f,r}\text{: Reset}_{r-1,r}\ \overbrace{(\text{EL S})^{r-f-1}}^{\text{pump-up}}\ \overbrace{(\text{I S})^{r-f-1}}^{\text{pump-down}}$$

$$(b_0, b_1, \ldots, b_{f-1}, b_f, b_{f+1}, \ldots, b_{d-2}, b_{d-1})$$

Sieve!

Key: sieve, lift, c_i = insert

A Pump

$$\text{Pump}_{f,r}\text{: Reset}_{r-1,r}\ \overbrace{(\text{EL S})^{r-f-1}}^{\text{pump-up}}\ \overbrace{(\text{I S})^{r-f-1}}^{\text{pump-down}}$$

$$({\color{blue}b_0}, {\color{blue}b_1}, \ldots, {\color{blue}b_{f-1}}, {\color{blue}b_f}, {\color{red}b_{f+1}}, \ldots, {\color{red}b_{d-2}}, {\color{red}b_{d-1}})$$

Extend Left!

Key: sieve, lift, c_i = insert

A Pump

$$\text{Pump}_{f,r}\text{: Reset}_{r-1,r}\ \overbrace{(\text{EL S})^{r-f-1}}^{\text{pump-up}}\ \overbrace{(\text{I S})^{r-f-1}}^{\text{pump-down}}$$

$$(b_0, b_1, \ldots, b_{f-1}, b_f, b_{f+1}, \ldots, b_{d-2}, b_{d-1})$$

Sieve!

Key: sieve, lift, c_i = insert

A Pump

$$\text{Pump}_{f,r}\text{: Reset}_{r-1,r}\ \overbrace{(\text{EL S})^{r-f-1}}^{\text{pump-up}}\ \overbrace{(\text{I S})^{r-f-1}}^{\text{pump-down}}$$

$$(b_0, b_1, \ldots, b_{f-1}, b_f, b_{f+1}, \ldots, b_{d-2}, b_{d-1})$$

Extend Left!

Key: sieve, lift, c_i = insert

A Pump

$$\text{Pump}_{f,r}\text{: Reset}_{r-1,r}\ \overbrace{(\text{EL S})^{r-f-1}}^{\text{pump-up}}\ \overbrace{(\text{I S})^{r-f-1}}^{\text{pump-down}}$$

$$({\color{blue}b_0}, {\color{blue}b_1}, \ldots, {\color{blue}b_{f-1}}, {\color{red}b_f}, {\color{red}b_{f+1}}, \ldots, {\color{red}b_{d-2}}, {\color{red}b_{d-1}})$$

Sieve!

Key: sieve, lift, c_i = insert

A Pump

$$\text{Pump}_{f,r}\text{: Reset}_{r-1,r}\ \overbrace{(\text{EL S})^{r-f-1}}^{\text{pump-up}}\ \overbrace{(\text{I S})^{r-f-1}}^{\text{pump-down}}$$

$$({c_0}, {\color{blue}b_1}, \ldots, {\color{blue}b_{f-1}}, b_f, {\color{red}b_{f+1}}, \ldots, {\color{red}b_{d-2}}, {\color{red}b_{d-1}})$$

Insert!

Key: sieve, lift, c_i = insert

A Pump

$$\text{Pump}_{f,r}\text{: Reset}_{r-1,r}\ \overbrace{(\text{EL S})^{r-f-1}}^{\text{pump-up}}\ \overbrace{(\text{I S})^{r-f-1}}^{\text{pump-down}}$$

$$(b_0, b_1, \ldots, b_{f-1}, b_f, b_{f+1}, \ldots, b_{d-2}, b_{d-1})$$

Sieve!

Key: sieve, lift, c_i = insert

Pump$_{f,r}$: Reset$_{r-1,r}$ $\overbrace{(\text{EL S})^{r-f-1}}^{\text{pump-up}}\ \overbrace{(\text{I S})^{r-f-1}}^{\text{pump-down}}$

$$(b_0, c_1, \ldots, b_{f-1}, b_f, b_{f+1}, \ldots, b_{d-2}, b_{d-1})$$

Insert!

Key: sieve, lift, c_i = insert

A Pump

$$\text{Pump}_{f,r}\text{: Reset}_{r-1,r}\ \overbrace{(\text{EL S})^{r-f-1}}^{\text{pump-up}}\ \overbrace{(\text{I S})^{r-f-1}}^{\text{pump-down}}$$

$$(b_0, b_1, \ldots, b_{f-1}, b_f, b_{f+1}, \ldots, b_{d-2}, b_{d-1})$$

Sieve!

Key: sieve, lift, c_i = insert

A Pump

$$\mathrm{Pump}_{f,r}\text{: } \mathrm{Reset}_{r-1,r}\ \overbrace{(\mathrm{EL\ S})^{r-f-1}}^{\text{pump-up}}\ \overbrace{(\mathrm{I\ S})^{r-f-1}}^{\text{pump-down}}$$

$$(c_0, c_1, \dots, c_{f-1}, c_f, c_{f+1}, \dots, c_{d-2}, b_{d-1})$$

Insert!

Key: sieve, lift, c_i = insert

A Workout

$\text{Workout}_{f,f^+,r}$: $\text{Pump}_{0,r-f^+,r}$ $\text{Pump}_{0,r-2f^+,r}$ $\ldots\text{Pump}_{0,r-f,r}$

A Workout

$\text{Workout}_{f,f^+,r}$: $\text{Pump}_{0,r-f^+,r}$ $\text{Pump}_{0,r-2f^+,r}$ ...$\text{Pump}_{0,r-f,r}$

$$(\qquad\qquad\qquad | \qquad\qquad\qquad)$$

A Workout

$\mathrm{Workout}_{f,f^+,r}$: $\mathrm{Pump}_{0,r-f^+,r}$ $\mathrm{Pump}_{0,r-2f^+,r}$ ...$\mathrm{Pump}_{0,r-f,r}$

(|)

A Workout

$\text{Workout}_{f,f^+,r}$: $\text{Pump}_{0,r-f^+,r}$ $\text{Pump}_{0,r-2f^+,r}$ ...$\text{Pump}_{0,r-f,r}$

$$(\qquad\qquad | \qquad\qquad\qquad\qquad\qquad\qquad)$$

Workout$_{f,f^+,r}$: Pump$_{0,r-f^+,r}$ Pump$_{0,r-2f^+,r}$ ...Pump$_{0,r-f,r}$

$$(\qquad \mid \qquad\qquad\qquad)$$

$$(\quad \mid \qquad\qquad\qquad\quad)$$

A `Workout`

$\text{Workout}_{f,f^+,r}$: $\text{Pump}_{0,r-f^+,r}$ $\text{Pump}_{0,r-2f^+,r}$ ...$\text{Pump}_{0,r-f,r}$

$$(\quad | \quad)$$

$$(\quad | \quad)$$

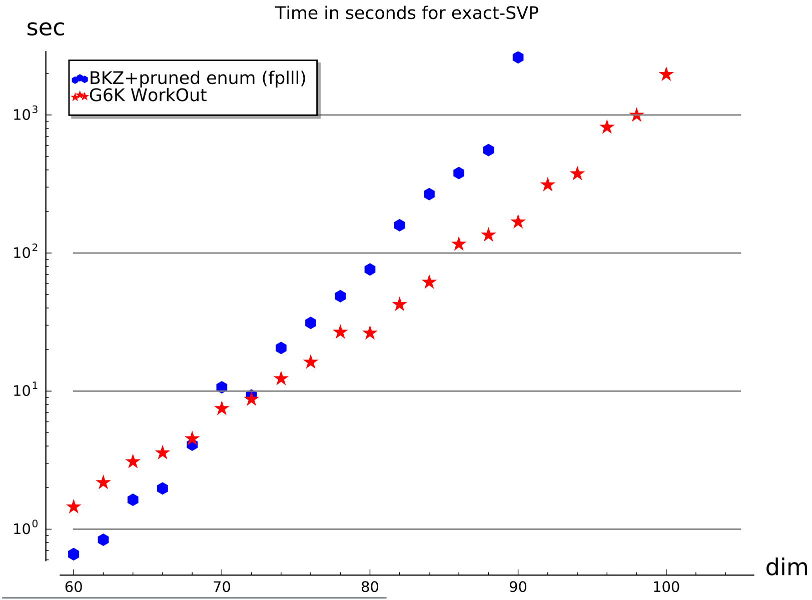
In short, more, smaller `Pumps` still faster than one larger `Pump`.

G6K has three high level design principles, to

- recycle (short) vectors between lattices,
- lift vectors, on the fly, to higher dimensional lattices,
- decide the insertion position only *after* sieving.

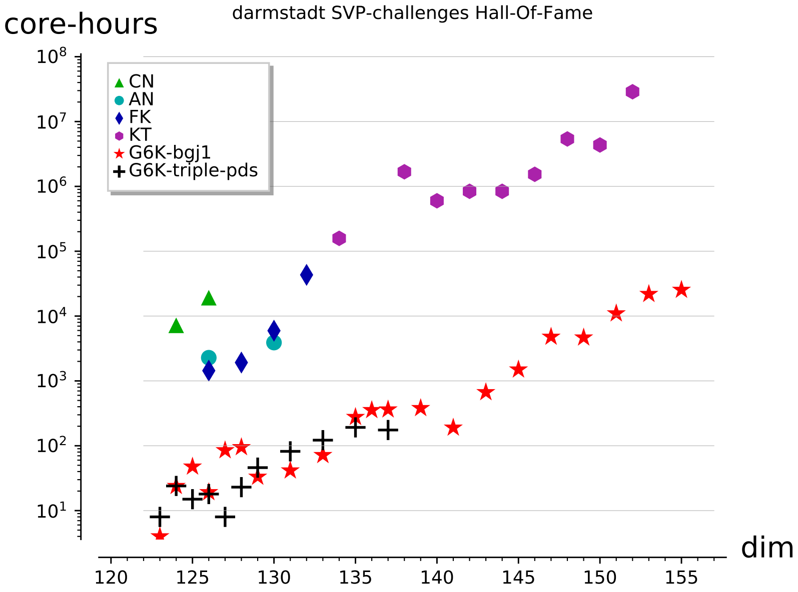
Records

Exact SVP, `Workout` vs. Enumeration (FPLLL)[8]

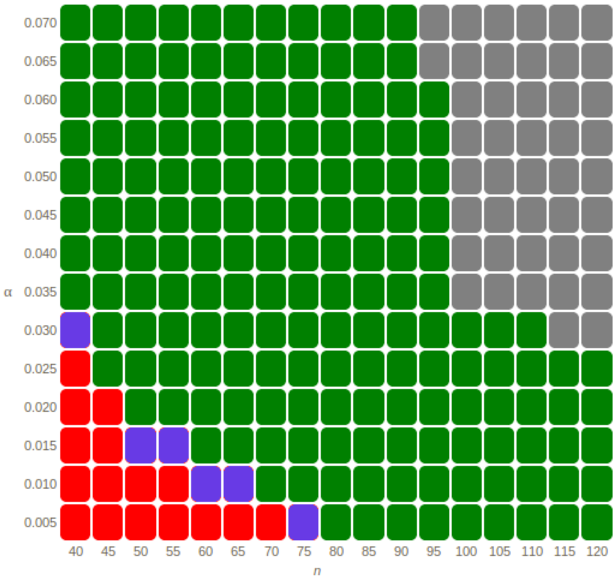


[8] The FPLLL development team, fplll, a lattice reduction library, 2019, available at https://github.com/fplll/fplll

Hermite SVP (Darmstadt Challenges), with `Workout`



LWE (Darmstadt Challenges), with Pump&JumpBKZ



Please hack around with the open source implementation!

`https://www.github.com/fplll/g6k`