

Learning the BEAST tendency

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Definition of the basic neurons

$$\forall trial \in \{5(block - 1), \dots, 5block\} STy_{i,trial} = \frac{\sum_{j=1}^{kapa_i} outcomeY_{i,trial,j}}{kapa_i}$$

Where :

$$kapa_i = \begin{cases} 1 & \text{w.p } \frac{1}{3} \\ 2 & \text{w.p } \frac{1}{3} \\ 3 & \text{w.p } \frac{1}{3} \end{cases}$$

$$outcomeY_{i,trial,j} = \begin{cases} unbiased_{i,trial} = sample(valY_{unbiased}, distY_{unbiased}, Ry_{i,trial,j}) & \text{w.p } p\phi_{unbiased,block} \\ uniform_{i,trial} = sample(valY_{uniform}, distY_{uniform}, Ry_{i,trial,j}) & \text{w.p } p\phi_{uniform,block} \\ pessimism_{i,trial} = sample(valY_{pessimism}, distY_{pessimism}, Ry_{i,trial,j}) & \text{w.p } p\phi_{pessimism,block} \\ sign_{i,trial} = sample(valY_{sign}, distY_{sign}, Ry_{i,trial,j}) & \text{w.p } p\phi_{sign,block} \end{cases}$$

$$valY_{unbiased}, distY_{unbiased} = valY, distY$$

$$valY_{uniform}, distY_{uniform} = valY, uniform(valY)$$

$$valY_{pessimism}, distY_{pessimism} = \begin{cases} minY, 1 & signMax > 0 \wedge ratioMin < \gamma \\ valY, uniform(valY) & otherwise \end{cases}$$

$$valY_{sign}, distY_{sign} = range * sign(valY), distY$$

$$Ra_{i,trial,j} \sim U(0, 1), Rb_{i,trial,j} = \begin{cases} Ra_{i,trial,j} & corr = 1 \text{ or } trial \leq 5 \\ 1 - Ra_{i,trial,j} & corr = -1 \text{ and } trial \geq 6 \\ \sim U(0, 1) & otherwise \end{cases}$$

The empirical expectation of a block's prediction is computed by:

$$\frac{1}{nsims} \sum_{i=1}^{nsims} \frac{1}{5} \sum_{trial=5(block-1)+1}^{5block} sigmoid(BEVb - BEVa + STb_{i,trial} - STa_{i,trial})$$

The expectation of a block's prediction is

$$E_{sta, stb} [sigmoid(BEVb - BEVa + STb_{i,trial} - STa_{i,trial})] =$$

$$\Sigma_{sta} \Sigma_{stb} sigmoid(BEVb - BEVa + stb - sta) p(sta, stb) =$$

$$\Sigma_{sta} \Sigma_{stb} sigmoid(BEVb - BEVa + stb - sta) \Sigma_{k=1}^3 \frac{1}{3} p(sta, stb | kapa = k) =$$

$$\Sigma_{k=1}^3 \frac{1}{3} \Sigma_{sta} \Sigma_{stb} \text{sigmoid}(BEVb - BEVa + stb - sta) p(sta, stb | kapa = k) \quad (1)$$

For each kapa we can say that:

$$\Sigma_{sta} \Sigma_{stb} \text{sigmoid}(BEVb - BEVa + stb - sta) p(sta, stb | kapa) =$$

$$\Sigma_{sta} \Sigma_{stb} \text{sigmoid}(BEVb - BEVa + stb - sta)$$

$$\Sigma_{T_1 \in \{\text{unbiased}, \text{uniform}, \text{pessimism}, \text{sign}\}} \dots \Sigma_{T_{kapa} \in \{\text{unbiased}, \text{uniform}, \text{pessimism}, \text{sign}\}} p\phi_{T_1, \text{block}} \dots p\phi_{T_{kapa}, \text{block}} p(sta, stb | T_1, \dots, T_{kapa}, kapa) =$$

$$\Sigma_{T_1 \in \{\text{unbiased}, \text{uniform}, \text{pessimism}, \text{sign}\}} \dots \Sigma_{T_{kapa} \in \{\text{unbiased}, \text{uniform}, \text{pessimism}, \text{sign}\}} p\phi_{T_1, \text{block}} \dots p\phi_{T_{kapa}, \text{block}} \Sigma_{sta} \Sigma_{stb} \text{sigmoid}(BEVb - BEVa + stb - sta) p(sta, stb | T_1, \dots, T_{kapa}, kapa) \quad (2)$$

$\forall T \in \{\text{unbiased}, \text{uniform}, \text{pessimism}, \text{sign}\}$, given T , kapa and ra_1, \dots, ra_{kapa} sta and stb are independent, therefore :

$$p(sta, stb | T_1, \dots, T_{kapa}, kapa) = \quad (3)$$

$$\int_{ra_1=0}^1 \dots \int_{ra_{kapa}=0}^1 p(sta | ra_1, \dots, ra_{kapa}, T) p(stb | ra_1, \dots, ra_{kapa}, T) dra_1 \dots dra_{kapa}$$

given ra_1, \dots, ra_{kapa} , we also can compute sta and stb probabilities:

$$p(sta | ra_1, \dots, ra_{kapa}, T_1, \dots, T_{kapa}, kapa) = \quad (4)$$

$$\begin{cases} 1 & \exists i_1, \dots, i_{kapa} : sta = \frac{\Sigma_{z=1}^{kapa} valA_{T_z z}[i_z]}{kapa} \wedge \Sigma_{k=0}^{i_z-1} distA_{T_z z}[k] < ra_z \leq \Sigma_{k=0}^{i_z} distA_{T_z z}[k] \quad \forall z \in \{1, \dots, kapa\} \\ 0 & otherwise \end{cases}$$

$$p(stb | ra_1, \dots, ra_{kapa}, T_1, \dots, T_{kapa}, kapa) = \quad (5)$$

if corr=1 or block=1:

$$\begin{cases} 1 & \exists j_1, \dots, j_{kapa} : stb = \frac{\Sigma_{z=1}^{kapa} valB_{T_z z}[j_z]}{kapa} \wedge \Sigma_{k=0}^{j_z-1} distB_{T_z z}[k] < ra_z \leq \Sigma_{k=0}^{j_z} distB_{T_z z}[k] \quad \forall z \in \{1, \dots, kapa\} \\ 0 & otherwise \end{cases}$$

if corr=-1 and block > 1 :

$$\begin{cases} 1 & \exists j_1, \dots, j_{kapa} : stb = \frac{\Sigma_{z=1}^{kapa} valB_{T_z z}[j_z]}{kapa} \wedge 1 - \Sigma_{k=0}^{j_z} distB_{T_z z}[k] \leq ra_z < 1 - \Sigma_{k=0}^{j_z-1} distB_{T_z z}[k] \quad \forall z \in \{1, \dots, kapa\} \\ 0 & otherwise \end{cases}$$

if corr=0 and block > 1 :

$$\int_{rb_1=0}^1 \dots \int_{rb_{kapa}=0}^1 p(stb | rb_1, \dots, rb_{kapa}, T) drb \text{ where } p(stb | rb_1, \dots, rb_{kapa}, T) =$$

$$\begin{cases} 1 & \exists j_1, \dots, j_{kapa} : stb = \frac{\sum_{z=1}^{kapa} valB_{T_z}[j_z]}{kapa} \wedge \sum_{k=0}^{j_z-1} distB_{T_z}[k] < rb_z \leq \sum_{k=0}^{j_z} distB_{T_z}[k] \quad \forall z \in \{1, \dots, kapa\} \\ 0 & otherwise \end{cases}$$

Combining (3),(4) and (5) we get :

$$\Sigma_{sta} \Sigma_{stb} sigmoid(BEVb - BEVa + stb - sta) p(sta, stb | T_1, \dots, T_{kapa}, kapa) = \quad (6)$$

if corr=1 or block=1 :

$$\begin{aligned} & \sum_{i_1=0}^{len(valA_{T_1})} \sum_{j_1=0}^{len(valB_{T_1})} \dots \sum_{i_{kapa}=0}^{len(valA_{T_{kapa}})} \sum_{j_1=0}^{len(valB_{T_{kapa}})} sigmoid(BEVb - BEVa + \frac{\sum_{z=1}^{kapa} valB_{T_z}[j_z]}{kapa} - \\ & \frac{\sum_{z=1}^{kapa} valA_{T_z}[i_z]}{kapa}) \\ & \int_{ra_1=max(\sum_{k=0}^{i_1-1} distA_{T_1}[k], \sum_{k=0}^{j_1-1} distB_{T_1}[k])}^{\min(\sum_{k=0}^{i_1} distA_{T_1}[k], \sum_{k=0}^{j_1} distB_{T_1}[k])} \dots \int_{ra_{kapa}=max(\sum_{k=0}^{i_{kapa}-1} distA_{T_{kapa}}[k], \sum_{k=0}^{j_{kapa}-1} distB_{T_{kapa}}[k])}^{\min(\sum_{k=0}^{i_{kapa}} distA_{T_{kapa}}[k], \sum_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k])} 1dra_1 \dots dra_{kapa} = \\ & \sum_{i_1=0}^{len(valA_{T_1})} \sum_{j_1=0}^{len(valB_{T_1})} \dots \sum_{i_{kapa}=0}^{len(valA_{T_{kapa}})} \sum_{j_{kapa}=0}^{len(valB_{T_{kapa}})} sigmoid(BEVb - BEVa + \frac{\sum_{z=1}^{kapa} valB_{T_z}[j_z]}{kapa} - \\ & \frac{\sum_{z=1}^{kapa} valA_{T_z}[i_z]}{kapa}) \\ & I_{\min(\sum_{k=0}^{i_1} distA_{T_1}[k], \sum_{k=0}^{j_1} distB_{T_1}[k]) > \max(\sum_{k=0}^{i_1-1} distA_{T_1}[k], \sum_{k=0}^{j_1-1} distB_{T_1}[k])} \\ & \dots \\ & I_{\min(\sum_{k=0}^{i_{kapa}} distA_{T_{kapa}}[k], \sum_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k]) > \max(\sum_{k=0}^{i_{kapa}-1} distA_{T_{kapa}}[k], \sum_{k=0}^{j_{kapa}-1} distB_{T_{kapa}}[k])} \\ & (\min(\sum_{k=0}^{i_1} distA_{T_1}[k], \sum_{k=0}^{j_1} distB_{T_1}[k]) - \max(\sum_{k=0}^{i_1-1} distA_{T_1}[k], \sum_{k=0}^{j_1-1} distB_{T_1}[k])) \\ & \dots \\ & (\min(\sum_{k=0}^{i_{kapa}} distA_{T_{kapa}}[k], \sum_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k]) - \max(\sum_{k=0}^{i_{kapa}-1} distA_{T_{kapa}}[k], \sum_{k=0}^{j_{kapa}-1} distB_{T_{kapa}}[k])) \end{aligned}$$

if corr=-1 and block > 1 :

$$\begin{aligned} & \sum_{i_1=0}^{len(valA_{T_1})} \sum_{j_1=0}^{len(valB_{T_1})} \dots \sum_{i_{kapa}=0}^{len(valA_{T_{kapa}})} \sum_{j_{kapa}=0}^{len(valB_{T_{kapa}})} sigmoid(BEVb - BEVa + \frac{\sum_{z=1}^{kapa} valB_{T_z}[j_z]}{kapa} - \\ & \frac{\sum_{z=1}^{kapa} valA_{T_z}[i_z]}{kapa}) \\ & \int_{ra_1=max(\sum_{k=0}^{i_1-1} distA_{T_1}[k], 1 - \sum_{k=0}^{j_1-1} distB_{T_1}[k])}^{\min(\sum_{k=0}^{i_1} distA_{T_1}[k], 1 - \sum_{k=0}^{j_1} distB_{T_1}[k])} \dots \int_{ra_{kapa}=max(\sum_{k=0}^{i_{kapa}-1} distA_{T_{kapa}}[k], 1 - \sum_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k])}^{\min(\sum_{k=0}^{i_{kapa}} distA_{T_{kapa}}[k], 1 - \sum_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k])} 1dra_1 \dots dra_{kapa} = \\ & \sum_{i_1=0}^{len(valA_{T_1})} \sum_{j_1=0}^{len(valB_{T_1})} \dots \sum_{i_{kapa}=0}^{len(valA_{T_{kapa}})} \sum_{j_1=0}^{len(valB_{T_{kapa}})} sigmoid(BEVb - BEVa + \frac{\sum_{z=1}^{kapa} valB_{T_z}[j_z]}{kapa} - \\ & \frac{\sum_{z=1}^{kapa} valA_{T_z}[i_z]}{kapa}) \\ & I_{\min(\sum_{k=0}^{i_1} distA_{T_1}[k], 1 - \sum_{k=0}^{j_1-1} distB_{T_1}[k]) > \max(\sum_{k=0}^{i_1-1} distA_{T_1}[k], 1 - \sum_{k=0}^{j_1} distB_{T_1}[k])} \\ & \dots \end{aligned}$$

$$\begin{aligned}
& I_{\min(\sum_{k=0}^{i_{kapa}} distA_{T_{kapa}}[k], 1 - \sum_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k]) > \max(\sum_{k=0}^{i_{kapa}-1} distA_{T_{kapa}}[k], 1 - \sum_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k])} \\
& (\min(\sum_{k=0}^{i_1} distA_{T_1}[k], 1 - \sum_{k=0}^{j_1-1} distB_{T_1}[k]) - \max(\sum_{k=0}^{i_1-1} distA_{T_1}[k], 1 - \sum_{k=0}^{j_1} distB_{T_1}[k])) \\
& \dots \\
& (\min(\sum_{k=0}^{i_{kapa}} distA_{T_{kapa}}[k], 1 - \sum_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k]) - \max(\sum_{k=0}^{i_{kapa}-1} distA_{T_{kapa}}[k], 1 - \sum_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k]))
\end{aligned}$$

if corr=0 and block > 1 :

$$\begin{aligned}
& \sum_{i_1=0}^{len(valA_{T_1})} \sum_{j_1=0}^{len(valB_{T_1})} \dots \sum_{i_{kapa}=0}^{len(valA_{T_{kapa}})} \sum_{j_{kapa}=0}^{len(valB_{T_{kapa}})} sigmoid(BEVb - BEVa + \frac{\sum_{z=1}^{kapa} valB_{T_z}[j_z] - \sum_{z=1}^{kapa} valA_{T_z}[i_z]}{kapa} - \\
& \int_{ra_1=\sum_{k=0}^{i_1-1} distA_{T_1}[k]}^{\sum_{k=0}^{i_1} distA_{T_1}[k]} \int_{rb_1=\sum_{k=0}^{j_1-1} distB_{T_1}[k]}^{\sum_{k=0}^{j_1} distB_{T_1}[k]} \dots \int_{ra_{kapa}=\sum_{k=0}^{i_{kapa}-1} distA_{T_{kapa}}[k]}^{\sum_{k=0}^{i_{kapa}} distA_{T_{kapa}}[k]} \int_{rb_{kapa}=\sum_{k=0}^{j_{kapa}-1} distB_{T_{kapa}}[k]}^{\sum_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k]} 1 dr a_1 dr b_1 \dots dr a_{kapa} \\
& \sum_{i_1=0}^{len(valA_{T_1})} \sum_{j_1=0}^{len(valB_{T_1})} \dots \sum_{i_{kapa}=0}^{len(valA_{T_{kapa}})} \sum_{j_{kapa}=0}^{len(valB_{T_{kapa}})} sigmoid(BEVb - BEVa + \frac{\sum_{z=1}^{kapa} valB_{T_z}[j_z] - \sum_{z=1}^{kapa} valA_{T_z}[i_z]}{kapa} - \\
& (\sum_{k=0}^{i_1} distA_{T_1}[k] - \sum_{k=0}^{i_1-1} distA_{T_1}[k]) (\sum_{k=0}^{j_1} distB_{T_1}[k] - \sum_{k=0}^{j_1-1} distB_{T_1}[k]) \\
& \dots \\
& (\sum_{k=0}^{i_{kapa}} distA_{T_{kapa}}[k] - \sum_{k=0}^{i_{kapa}-1} distA_{T_{kapa}}[k]) (\sum_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k] - \sum_{k=0}^{j_{kapa}-1} distB_{T_{kapa}}[k])
\end{aligned}$$

where $valY_{T_{kapa}}[-1] = 0 \ \forall Y \in \{A, B\}$

We can now place (6) in equation (2) and place (2) in (1)