Learning the BEAST tendency

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Definition of the basic neurons

$\forall trial \in \{5(block-1), ..., 5block\}$ $STy_{i,trial} = \frac{\Sigma_{j=1}^{kapa_i} outcomeY_{i,trial,j}}{kapa_i}$
$Where:$

$$kapa_i = \begin{cases} 1 & \text{w.p } \frac{1}{3} \\ 2 & \text{w.p } \frac{1}{3} \\ 3 & \text{w.p } \frac{1}{3} \end{cases}$$

$$outcomeY_{i,trial,j} = \begin{cases} unbiased_{i,trial} = sample(valY_{unbiased}, distY_{unbiased}, Ry_{i,trial,j}) & \text{w.p } p_{unbiased,block} \\ uniform_{i,trial} = sample(valY_{uniform}, distY_{uniform}, Ry_{i,trial,j}) & \text{w.p } p_{uniform,block} \\ pessimism_{i,trial} = sample(valY_{pessimism}, distY_{pessimism}, Ry_{i,trial,j}) & \text{w.p } p_{pessimism,block} \\ sign_{i,trial} = sample(valY_{sign}, distY_{sign}, Ry_{i,trial,j}) & \text{w.p } p_{sign,block} \end{cases}$$

$valY_{unbiased}, distY_{unbiased} = valY, distY$
$valY_{uniform}, distY_{uniform} = valY, uniform(valY)$
$valY_{pessimism}, distY_{pessimism} = \begin{cases} minY, 1 & signMax > 0 \wedge ratioMin < \gamma \\ valY, uniform(valY) & otherwise \end{cases}$
$valY_{sign}, distY_{sign} = range * sign(valY), distY$

$$Ra_{i,trial,j} \sim U(0,1), Rb_{i,trial,j} = \begin{cases} Ra_{i,trial,j} & corr = 1 \text{ or } trial \leq 5 \\ 1 - Ra_{i,trial,j} & corr = -1 \text{ and } trial \geq 6 \\ \sim U(0,1) & otherwise \end{cases}$$

The empirical expectation of a block's prediction is computed by:

$\frac{1}{nsims}\Sigma_{i=1}^{nsims}\frac{1}{5}\Sigma_{trial=5(block-1)+1}^{5block} sigmoid(BEVb - BEVa + STb_{i,trial} - STa_{i,trial})$

The expectation of a block's prediction is

$E_{sta,stb}[sigmoid(BEVb - BEVa + STb_{i,trial} - STa_{i,trial})] =$

$\Sigma_{sta}\Sigma_{stb} sigmoid(BEVb - BEVa + stb - sta)p(sta, stb) =$
$\Sigma_{sta}\Sigma_{stb} sigmoid(BEVb - BEVa + stb - sta)\Sigma_{k=1}^{3}\frac{1}{3}p(sta, stb|kapa = k) =$

$$\Sigma_{k=1}^{3}\frac{1}{3}\Sigma_{sta}\Sigma_{stb}sigmoid(BEVb-BEVa+stb-sta)p(sta,stb|kapa=k) \tag{1}$$

For each kapa we can say that:

$\Sigma_{sta}\Sigma_{stb}sigmoid(BEVb-BEVa+stb-sta)p(sta,stb|kapa)=$

$\Sigma_{sta}\Sigma_{stb}sigmoid(BEVb-BEVa+stb-sta)$
$\Sigma_{T_1\in\{unbiased,uniform,pessimism,sign\}}...\Sigma_{T_{kapa}\in\{unbiased,uniform,pessimism,sign\}}$
$p_{T_1,block}...p_{T_{kapa},block}p(sta,stb|T_1,...,T_{kapa},kapa)=$

$$\Sigma_{T_1\in\{unbiased,uniform,pessimism,sign\}}...\Sigma_{T_{kapa}\in\{unbiased,uniform,pessimism,sign\}}p_{T_1,block}...p_{T_{kapa},block}$$
$$\Sigma_{sta}\Sigma_{stb}sigmoid(BEVb-BEVa+stb-sta)p(sta,stb|T_1,...,T_{kapa},kapa) \tag{2}$$

$\forall T\in\{unbiased,uniform,pessimism,sign\}$, given T, kapa and $ra_1,...,ra_{kapa}$
sta and stb are independent,therefore :

$$p(sta,stb|T_1,...T_{kapa},kapa)= \tag{3}$$

$\int_{ra_1=0}^{1}...\int_{ra_{kapa}=0}^{1}p(sta|ra_1,...,ra_{kapa},T)p(stb|ra_1,...,ra_{kapa},T)dra_1...dra_{kapa}$

given $ra_1,...,ra_{kapa}$,we also can compute sta and stb probabilities:

$$p(sta|ra_1,...,ra_{kapa},T_1,...T_{kapa},kapa)= \tag{4}$$

$\begin{cases}1 & \exists i_1,...,i_{kapa}:sta=\frac{\Sigma_{z=1}^{kapa}valA_{T_z}z[i_z]}{kapa}\wedge\Sigma_{k=0}^{i_z-1}distA_{T_z}z[k]<ra_z\le\Sigma_{k=0}^{i_z}distA_{T_z}[k]\ \forall z\in\{1,...,kapa\}\\ 0 & otherwise\end{cases}$

$$p(stb|ra_1,...,ra_{kapa},T_1,...T_{kapa},kapa)= \tag{5}$$

if corr=1 or block=1:
$\begin{cases}1 & \exists j_1,...,j_{kapa}:stb=\frac{\Sigma_{z=1}^{kapa}valB_{T_z}[j_z]}{kapa}\wedge\Sigma_{k=0}^{j_z-1}distB_{T_z}[k]<ra_z\le\Sigma_{k=0}^{j_z}distB_{T_z}[k]\ \forall z\in\{1,...,kapa\}\\ 0 & otherwise\end{cases}$

if corr=-1 and block > 1 :
$\begin{cases}1 & \exists j_1,...,j_{kapa}:stb=\frac{\Sigma_{z=1}^{kapa}valB_{T_z}[j_z]}{kapa}\wedge 1-\Sigma_{k=0}^{j_z}distB_{T_z}[k]\le ra_z<1-\Sigma_{k=0}^{j_z-1}distB_{T_z}[k]\ \forall z\in\{1,...,kapa\\ 0 & otherwise\end{cases}$

if corr=0 and block > 1 :
$\int_{rb_1=0}^{1}...\int_{rb_{kapa}=0}^{1}p(stb|rb_1,,..,rb_{kapa},T)drb$ where $p(stb|rb_1,,..,rb_{kapa},T)=$

$$\begin{cases} 1 & \exists j_1, ..., j_{kapa} : stb = \frac{\Sigma_{z=1}^{kapa} valB_{T_z}[j_z]}{kapa} \wedge \Sigma_{k=0}^{j_z-1} distB_{T_z}[k] < rb_z \leq \Sigma_{k=0}^{j_z} distB_{T_z}[k] \ \forall z \in \{1, ..., kapa\} \\ 0 & otherwise \end{cases}$$

Combining (3),(4) and (5) we get :

$$\Sigma_{sta}\Sigma_{stb} sigmoid(BEVb - BEVa + stb - sta)p(sta, stb|T_1, ...T_{kapa}, kapa) = \tag{6}$$

if corr=1 or block=1 :

$$\Sigma_{i_1=0}^{len(valA_{T_1})}\Sigma_{j_1=0}^{len(valB_{T_1})}...\Sigma_{i_{kapa}=0}^{len(valA_{T_z})}\Sigma_{j_1=0}^{len(valB_{T_{kapa}})} sigmoid(BEVb - BEVa + \frac{\Sigma_{z=1}^{kapa} valB_{T_z}[j_z]}{kapa} - \frac{\Sigma_{z=1}^{kapa} valA_{T_z}[i_z]}{kapa})$$

$$\int_{ra_1=max(\Sigma_{k=0}^{i_1-1} distA_{T_1}[k], \Sigma_{k=0}^{j_1-1} distB_{T_1}[k])}^{min(\Sigma_{k=0}^{i_1} distA_{T_1}[k], \Sigma_{k=0}^{j_1} distB_{T_1}[k])} ... \int_{ra_{kapa}=max(\Sigma_{k=0}^{i_{kapa}-1} distA_{T_{kapa}}[k], \Sigma_{k=0}^{j_{kapa}-1} distB_{T_{kapa}}[k])}^{min(\Sigma_{k=0}^{i_{kapa}} distA_{T_{kapa}}[k], \Sigma_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k])} 1 dra_1...dra_{kapa} =$$

$$\Sigma_{i_1=0}^{len(valA_{T_1})}\Sigma_{j_1=0}^{len(valB_{T_1})}...\Sigma_{i_{kapa}=0}^{len(valA_{T_{kapa}})}\Sigma_{j_{kapa}=0}^{len(valB_{T_{kapa}})} sigmoid(BEVb - BEVa + \frac{\Sigma_{z=1}^{kapa} valB_{T_z}[j_z]}{kapa} - \frac{\Sigma_{z=1}^{kapa} valA_{T_z}[i_z]}{kapa})$$

$$I_{min(\Sigma_{k=0}^{i_1} distA_{T_1}[k], \Sigma_{k=0}^{j_1} distB_{T_1}[k]) > max(\Sigma_{k=0}^{i_1-1} distA_{T_1}[k], \Sigma_{k=0}^{j_1-1} distB_{T_1}[k])}$$
$$(min(\Sigma_{k=0}^{i_1} distA_{T_1}[k], \Sigma_{k=0}^{j_1} distB_{T_1}[k]) - max(\Sigma_{k=0}^{i_1-1} distA_{T_1}[k], \Sigma_{k=0}^{j_1-1} distB_{T_1}[k]))$$
...
$$I_{min(\Sigma_{k=0}^{i_{kapa}} distA_{T_{kapa}}[k], \Sigma_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k]) > max(\Sigma_{k=0}^{i_{kapa}-1} distA_{T_{kapa}}[k], \Sigma_{k=0}^{j_{kapa}-1} distB_{T_{kapa}}[k])}$$
$$(min(\Sigma_{k=0}^{i_{kapa}} distA_{T_{kapa}}[k], \Sigma_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k]) - max(\Sigma_{k=0}^{i_{kapa}-1} distA_{T_{kapa}}[k], \Sigma_{k=0}^{j_{kapa}-1} distB_{T_{kapa}}[k]))$$

if corr=-1 and block > 1 :

$$\Sigma_{i_1=0}^{len(valA_{T_1})}\Sigma_{j_1=0}^{len(valB_{T_1})}...\Sigma_{i_{kapa}=0}^{len(valA_{T_{kapa}})}\Sigma_{j_{kapa}=0}^{len(valB_{T_{kapa}})} sigmoid(BEVb - BEVa + \frac{\Sigma_{z=1}^{kapa} valB_{T_z}[j_z]}{kapa} - \frac{\Sigma_{z=1}^{kapa} valA_{T_z}[i_z]}{kapa})$$

$$\int_{ra_1=max(\Sigma_{k=0}^{i_1-1} distA_{T_1}[k], 1-\Sigma_{k=0}^{j_1} distB_{T_1}[k])}^{min(\Sigma_{k=0}^{i_1} distA_{T_1}[k], 1-\Sigma_{k=0}^{j_1-1} distB_{T_1}[k])} ... \int_{ra_{kapa}=max(\Sigma_{k=0}^{i_{kapa}-1} distA_{T_{kapa}}[k], 1-\Sigma_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k])}^{min(\Sigma_{k=0}^{i_{kapa}} distA_{T_{kapa}}[k], 1-\Sigma_{k=0}^{j_{kapa}-1} distB_{T_{kapa}}[k])} 1 dra_1...dra_{kapa} =$$

$$\Sigma_{i_1=0}^{len(valA_{T_1})}\Sigma_{j_1=0}^{len(valB_{T_1})}...\Sigma_{i_{kapa}=0}^{len(valA_{T_{kapa}})}\Sigma_{j_1=0}^{len(valB_{T_{kapa}})} sigmoid(BEVb - BEVa + \frac{\Sigma_{z=1}^{kapa} valB_{T_z}[j_z]}{kapa} - \frac{\Sigma_{z=1}^{kapa} valA_{T_z}[i_z]}{kapa})$$

$$I_{min(\Sigma_{k=0}^{i_1} distA_{T_1}[k], 1-\Sigma_{k=0}^{j_1-1} distB_{T_1}[k]) > max(\Sigma_{k=0}^{i_1-1} distA_{T_1}[k], 1-\Sigma_{k=0}^{j_1} distB_{T_1}[k])}$$
$$(min(\Sigma_{k=0}^{i_1} distA_{T_1}[k], 1 - \Sigma_{k=0}^{j_1-1} distB_{T_1}[k]) - max(\Sigma_{k=0}^{i_1-1} distA_{T_1}[k], 1 - \Sigma_{k=0}^{j_1} distB_{T_1}[k]))$$

...
$I_{min(\Sigma_{k=0}^{i_{kapa}} distA_{T_{kapa}}[k], 1-\Sigma_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k]) > max(\Sigma_{k=0}^{i_{kapa}-1} distA_{T_{kapa}}[k], 1-\Sigma_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k])}$
$(min(\Sigma_{k=0}^{i_{kapa}} distA_{T_{kapa}}[k], 1-\Sigma_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k]) - max(\Sigma_{k=0}^{i_{kapa}-1} distA_{T_{kapa}}[k], 1-\Sigma_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k]))$

if corr=0 and block > 1 :

$\Sigma_{i_1=0}^{len(valA_{T_1})} \Sigma_{j_1=0}^{len(valB_{T_1})} ... \Sigma_{i_{kapa}=0}^{len(valA_{T_{kapa}})} \Sigma_{j_{kapa}=0}^{len(valB_{T_{kapa}})} sigmoid(BEVb - BEVa + \frac{\Sigma_{z=1}^{kapa} valB_{T_z}[j_z]}{kapa} - \frac{\Sigma_{z=1}^{kapa} valA_{T_z}[i_z]}{kapa})$

$\int_{ra_1=\Sigma_{k=0}^{i_1-1} distA_{T_1}[k]}^{\Sigma_{k=0}^{i_1} distA_{T_1}[k]} \int_{rb_1=\Sigma_{k=0}^{j_1-1} distB_{T_1}[k]}^{\Sigma_{k=0}^{j_1} distB_{T_1}[k]} ... \int_{ra_{kapa}=\Sigma_{k=0}^{i_1 kapa-1} distA_{T_{kapa}}[k]}^{\Sigma_{k=0}^{i_{kapa}} distA_{T_{kapa}}[k]} \int_{rb_{kapa}=\Sigma_{k=0}^{j_{kapa}-1} distB_{T_{kapa}}[k]}^{\Sigma_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k]} 1 dra_1 drb_1 ... dra_{kapa}$

$\Sigma_{i_1=0}^{len(valA_{T_1})} \Sigma_{j_1=0}^{len(valB_{T_1})} ... \Sigma_{i_{kapa}=0}^{len(valA_{T_{kapa}})} \Sigma_{j_{kapa}=0}^{len(valB_{T_{kapa}})} sigmoid(BEVb - BEVa + \frac{\Sigma_{z=1}^{kapa} valB_{T_z}[j_z]}{kapa} - \frac{\Sigma_{z=1}^{kapa} valA_{T_z}[i_z]}{kapa})$

$(\Sigma_{k=0}^{i_1} distA_{T_1}[k] - \Sigma_{k=0}^{i_1-1} distA_{T_1}[k])(\Sigma_{k=0}^{j_1} distB_{T_1}[k] - \Sigma_{k=0}^{j_1-1} distB_{T_1}[k])$
...
$(\Sigma_{k=0}^{i_{kapa}} distA_{T_{kapa}}[k] - \Sigma_{k=0}^{i_{kapa}-1} distA_{T_{kapa}}[k])(\Sigma_{k=0}^{j_{kapa}} distB_{T_{kapa}}[k] - \Sigma_{k=0}^{j_{kapa}-1} distB_{T_{kapa}}[k])$

where $valY_{T_{kapa}}[-1] = 0 \ \forall Y \in \{A, B\}$

We can now place (6) in equation (2) and place (2) in (1)