

# GOING FURTHER WITH PROVERIF

VeriCrypt 2021

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# Outline

- A. Additional modeling techniques
  - 1. Equational Theory vs Rewrite rules
  - 2. Equivalence properties
  - 3. Memory cell and locks for stateful protocols
- B. Dealing with « cannot be proved »
  - 1. Adding precision
  - 2. Trace with assumption
  - 3. Proof by induction
  - 4. Lemma
- C. Dealing with non-termination

# Equational theory vs Rewrite rules

## Strengths and weaknesses of rewrite rules

- + Verification efficient
- + Very expressive with **otherwise**

```
fun ifthenelse(bool,bitstring,bitstring):bitstring
  reduc
    forall x,y:bitstring; ifthenelse(true,x,y) = x
    otherwise forall b:bool,x,y:bitstring; ifthenelse(b,x,y) = y.
```



the term  
ifthenelse(true,m,decrypt(a,k))  
fails

```
fun lazy_ite(bool,bitstring,bitstring):bitstring
  reduc
    forall x:bitstring; y:bitstring or fail; lazy_ite(true,x,y) = x
    otherwise forall b:bool,x:bitstring or fail,y:bitstring; lazy_ite(b,x,y) = y.
```

# Equational theory vs Rewrite rules

Strengths and weaknesses of rewrite rules

- + Verification efficient
- + Very expressive with otherwise
- Cannot call itself

Algebraic properties that cannot be modeled with rewrite rules in ProVerif

$$\text{dec}(\text{enc}(x, y), y) = x \quad \text{with} \quad \text{enc}(\text{dec}(x, y), y) = x$$

$$\text{exp}(\text{exp}(g, x), y) = \text{exp}(\text{exp}(g, y), x)$$

Diffie-Hellman

# Equational theory vs Rewrite rules

## Strengths and weaknesses of equational theory

- + Extremely expressive
- Makes the verification slow
- Not all equational theory can be handled (may not terminate from the start)

```
fun enc(G, passwd): G.  
fun dec(G, passwd): G.  
equation forall x: G, y: passwd; dec(enc(x,y),y) = x.  
equation forall x: G, y: passwd; enc(dec(x,y),y) = x.
```

```
const g: G.  
fun exp(G, exponent): G.  
equation forall x: exponent, y: exponent; exp(exp(g, x), y) = exp(exp(g, y), x).
```

---

# Equivalence properties

## Type of security properties

Reachability

Bad event in one system



Authentication



Secrecy

# Equivalence properties

## Type of security properties

### Reachability

Bad event in one system



Authentication



Secrecy

### Equivalence

Privacy as indistinguishability



Anonymity



Vote privacy



Unlinkability

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# Equivalence properties

## Indistinguishability

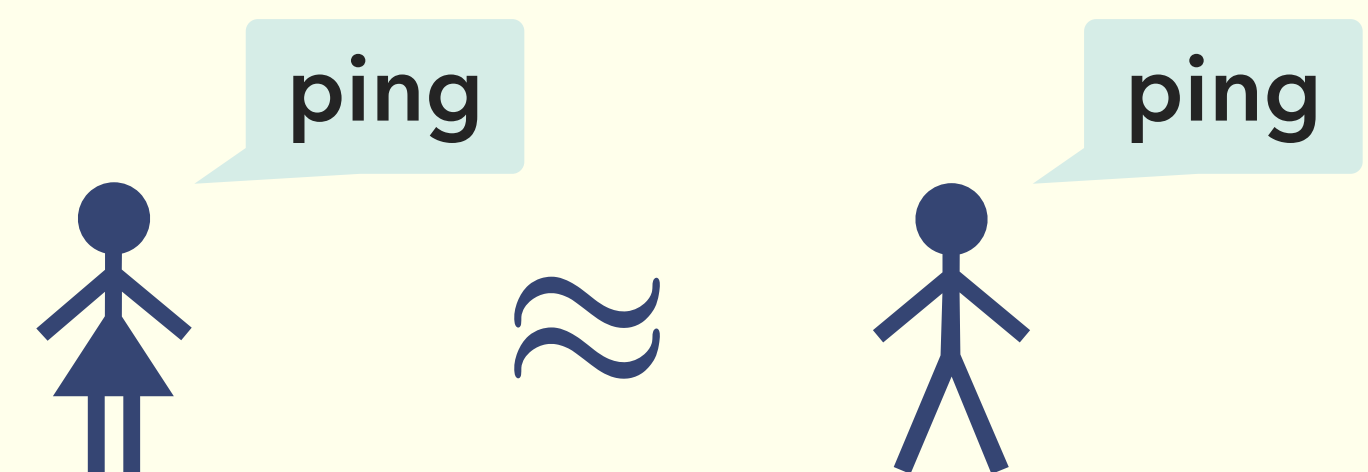
of two situations where the private attribute differs



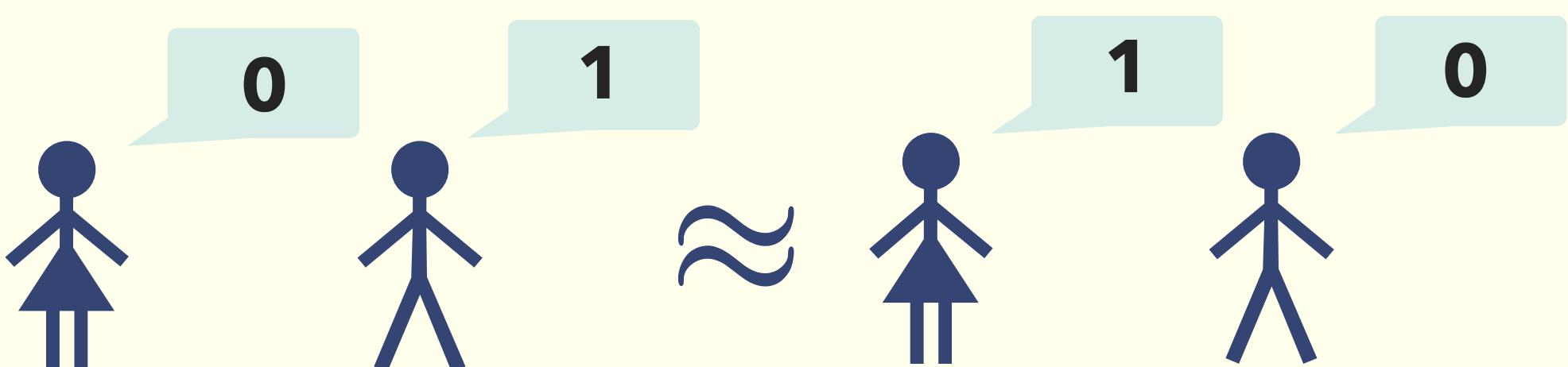
# Equivalence properties

## Indistinguishability

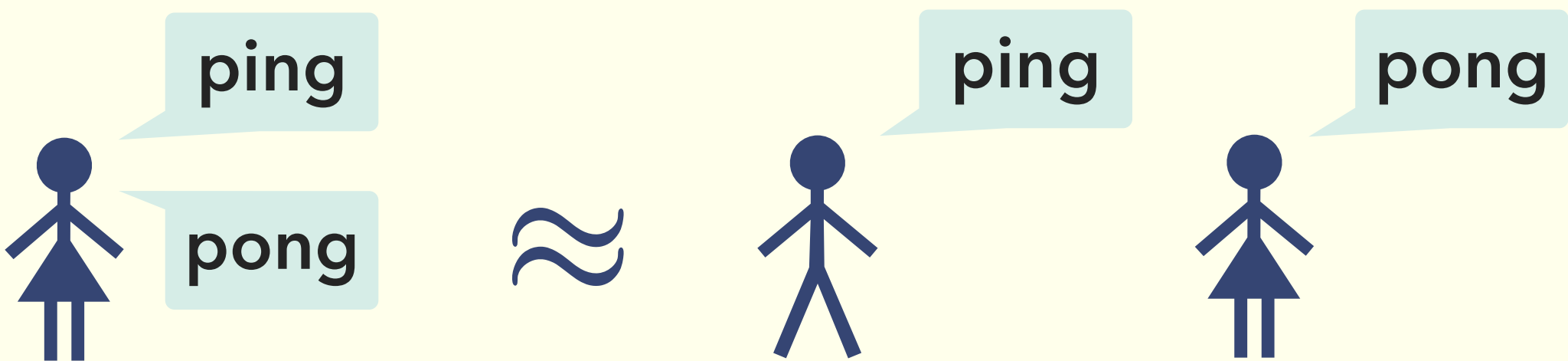
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Anonymity

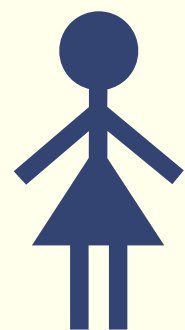
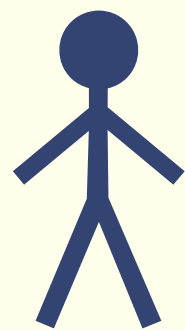


Vote privacy



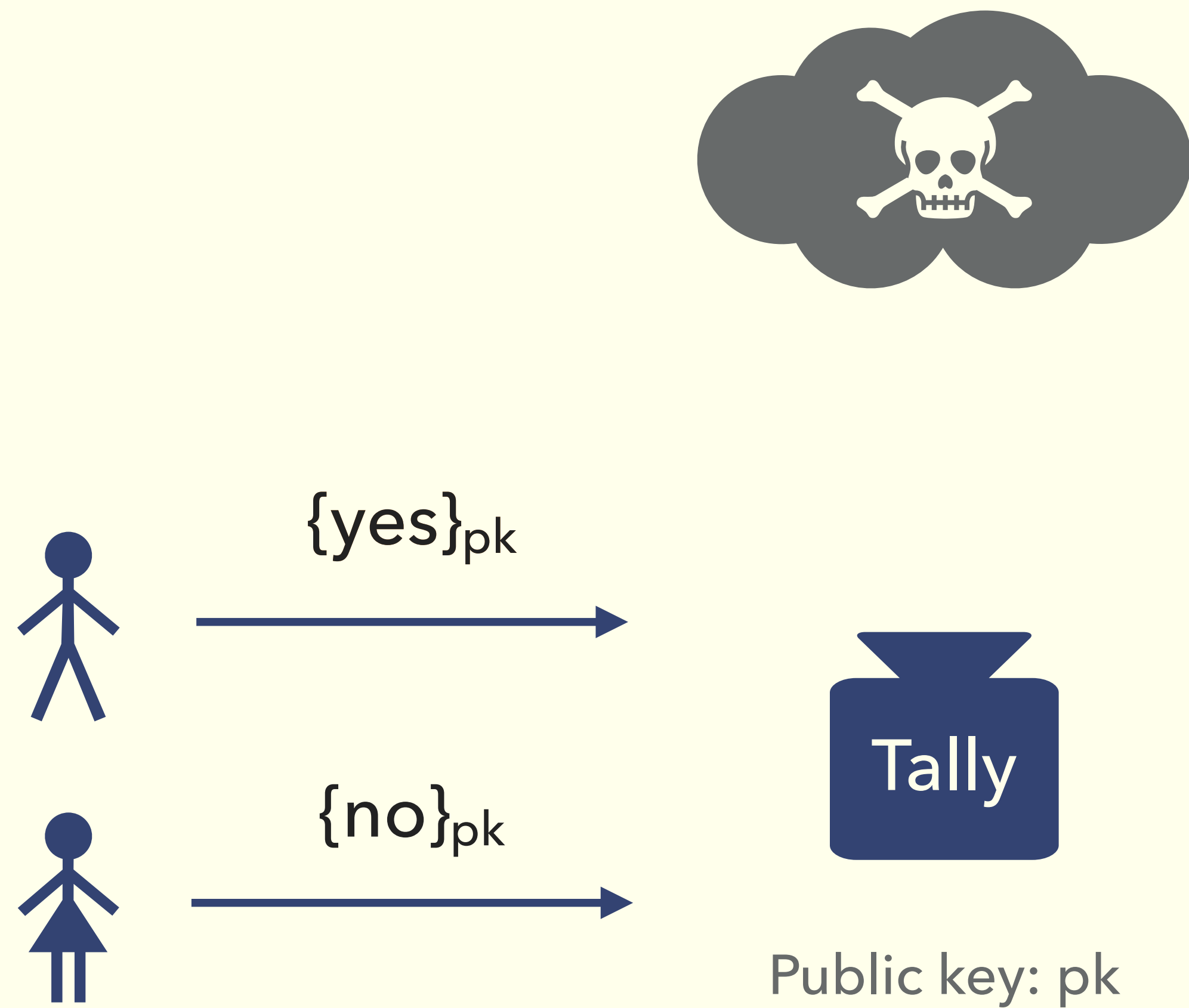
Unlinkability

# A simple e-voting protocol

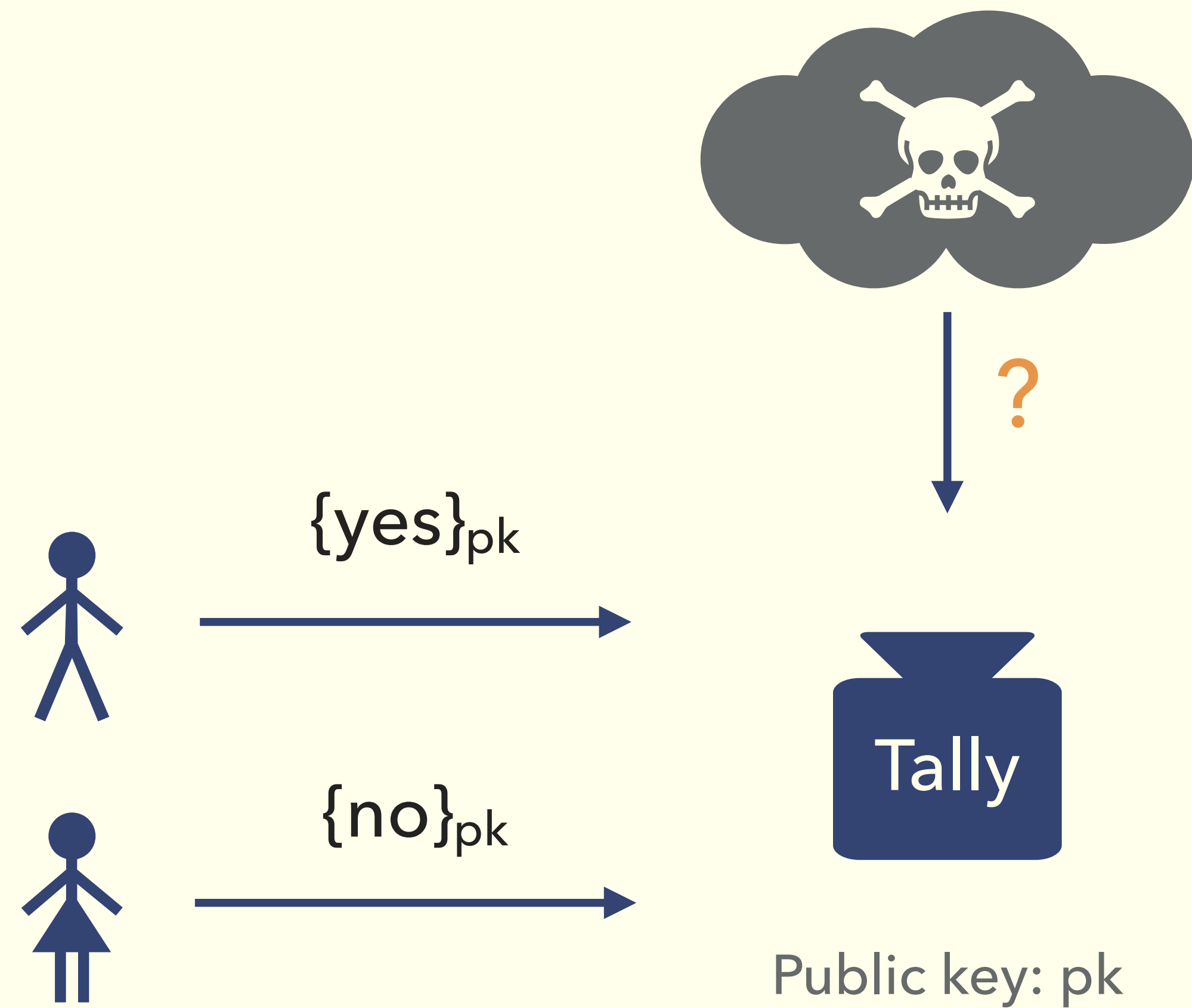


Public key:  $pk$

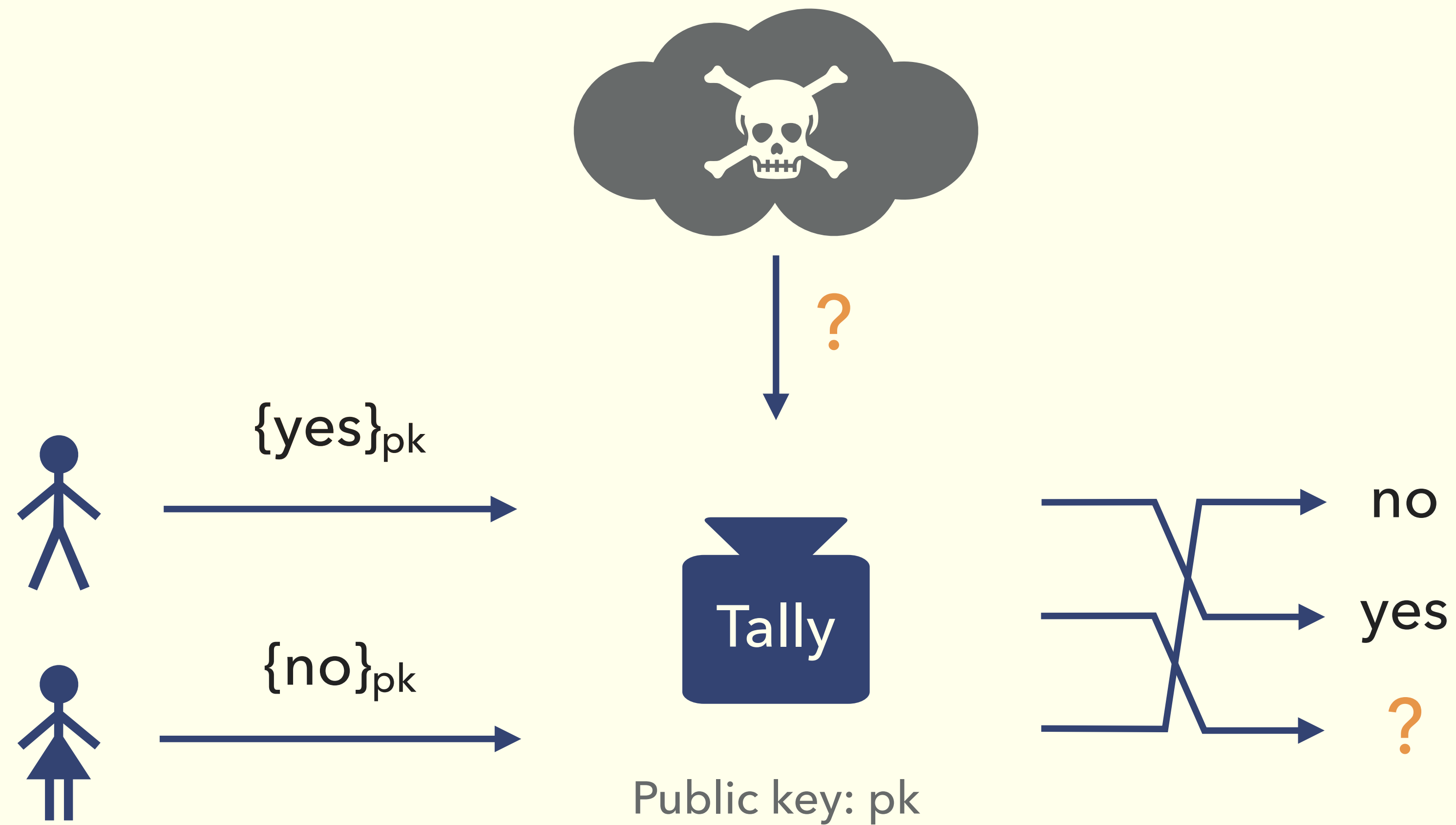
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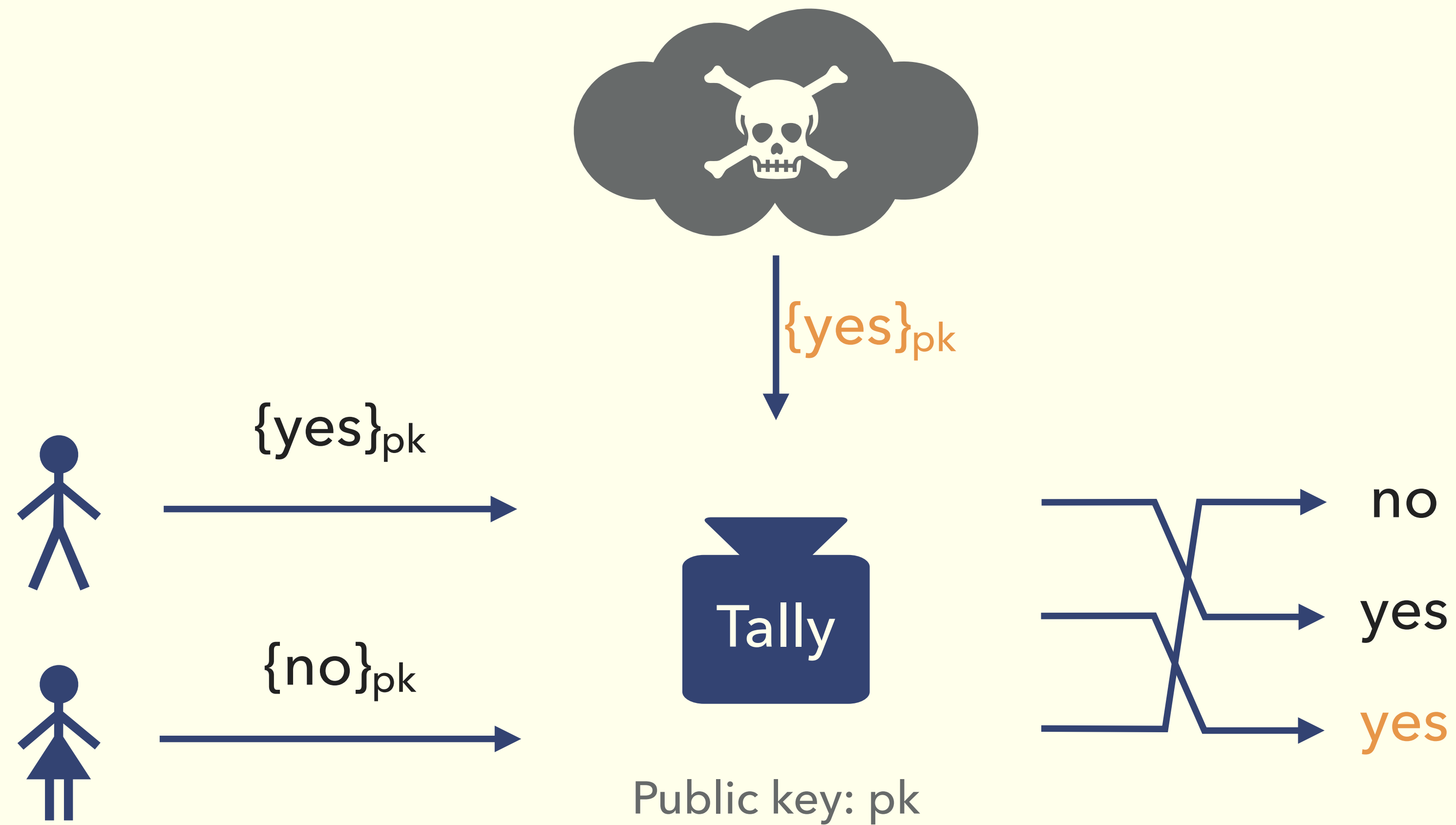
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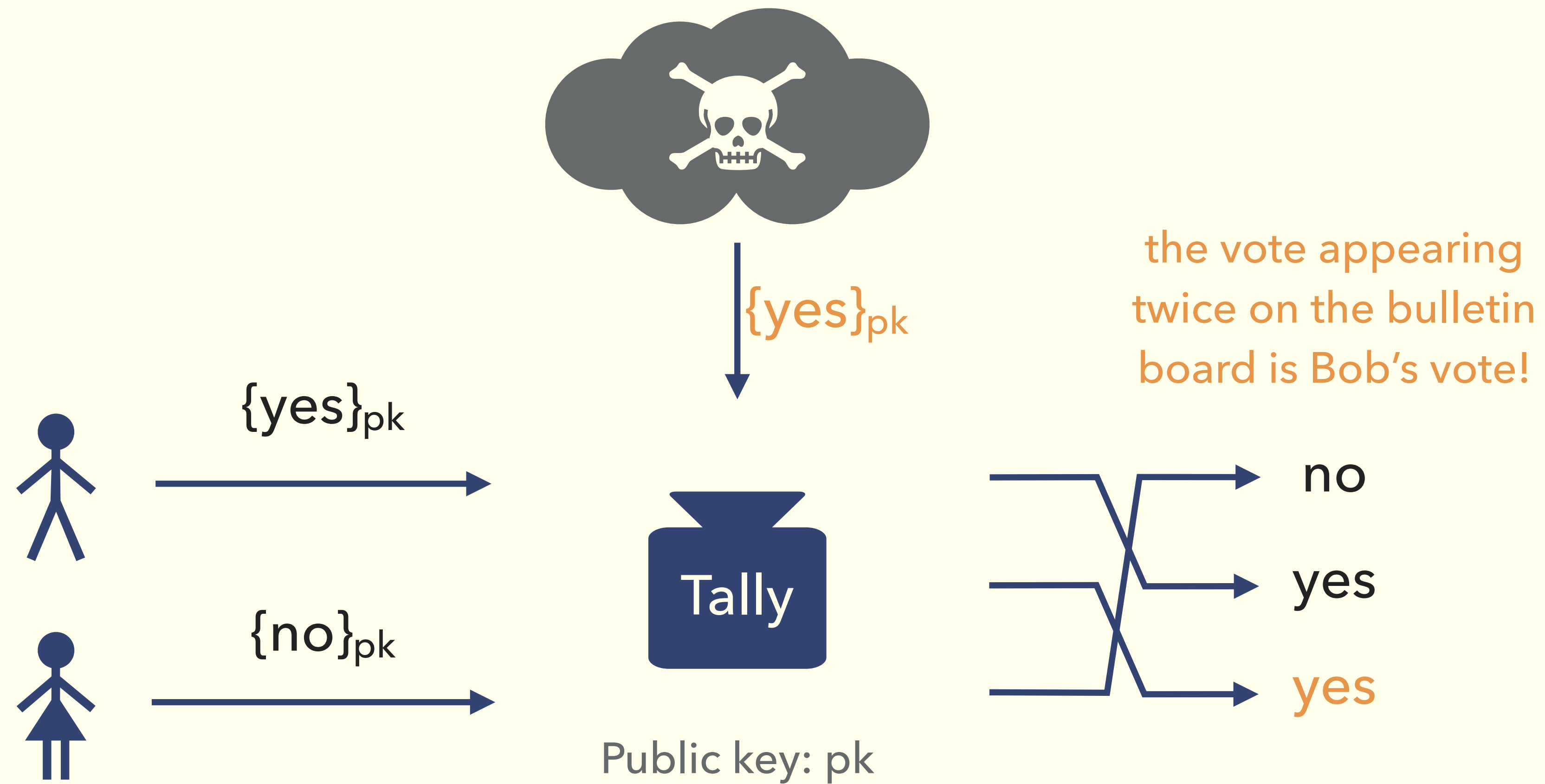
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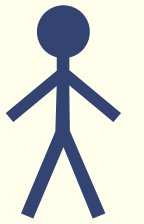
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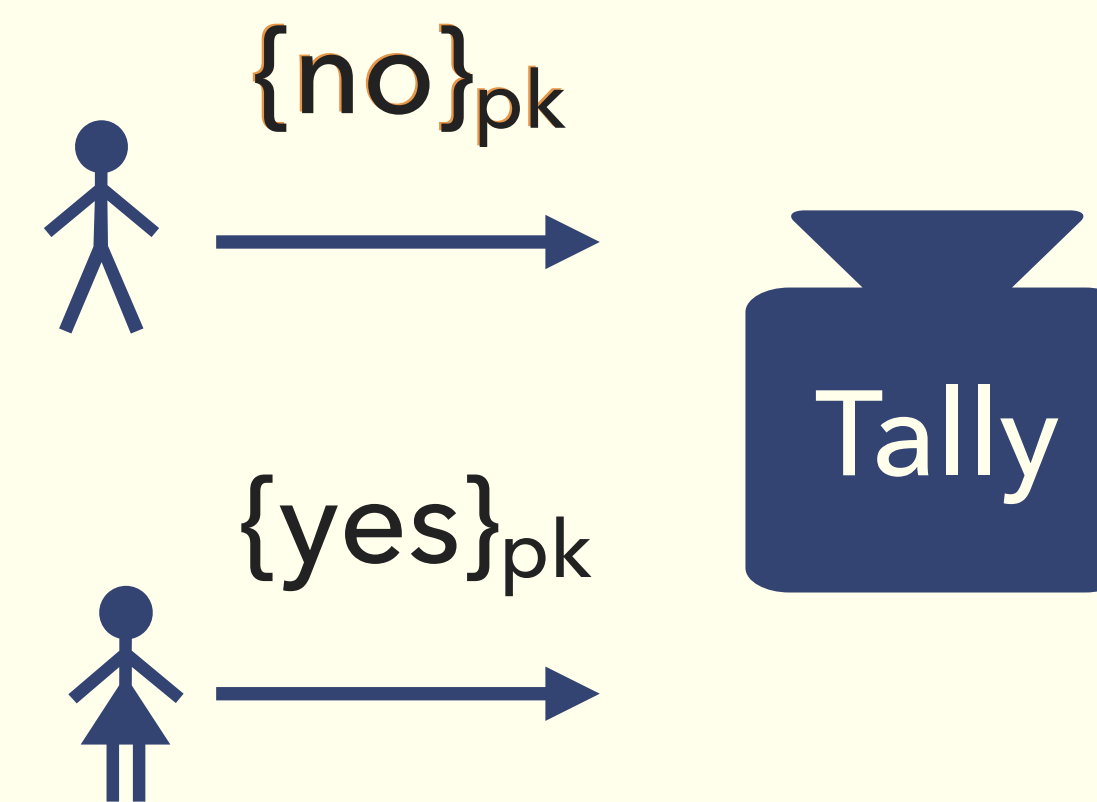
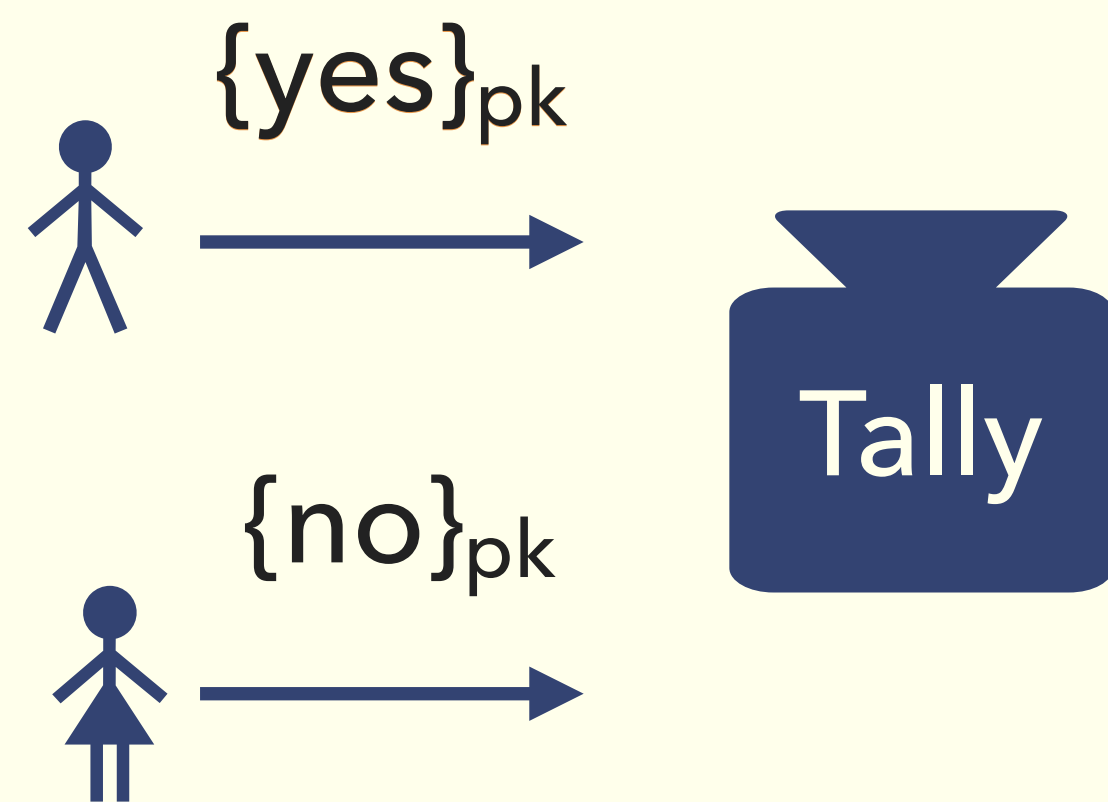


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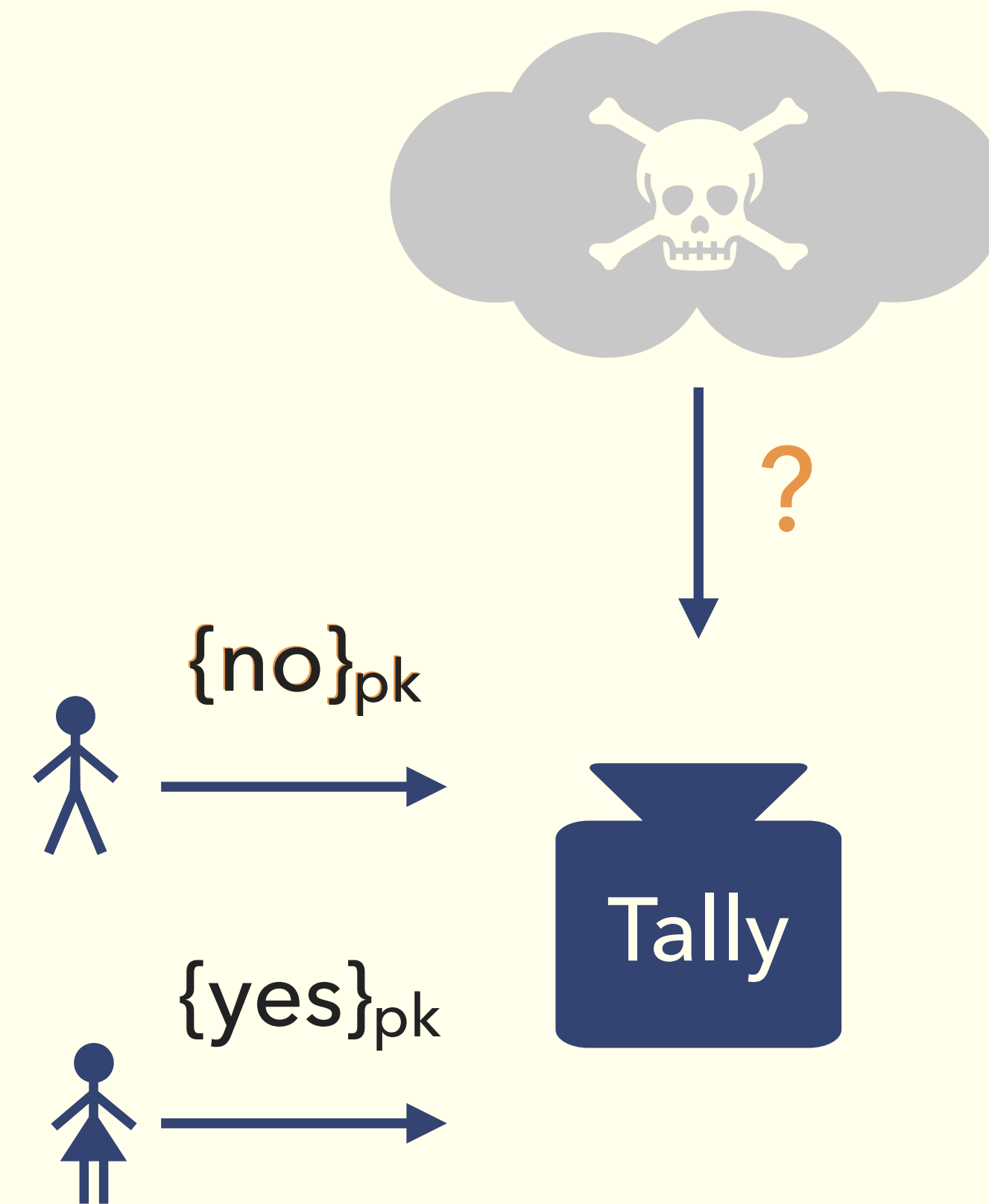
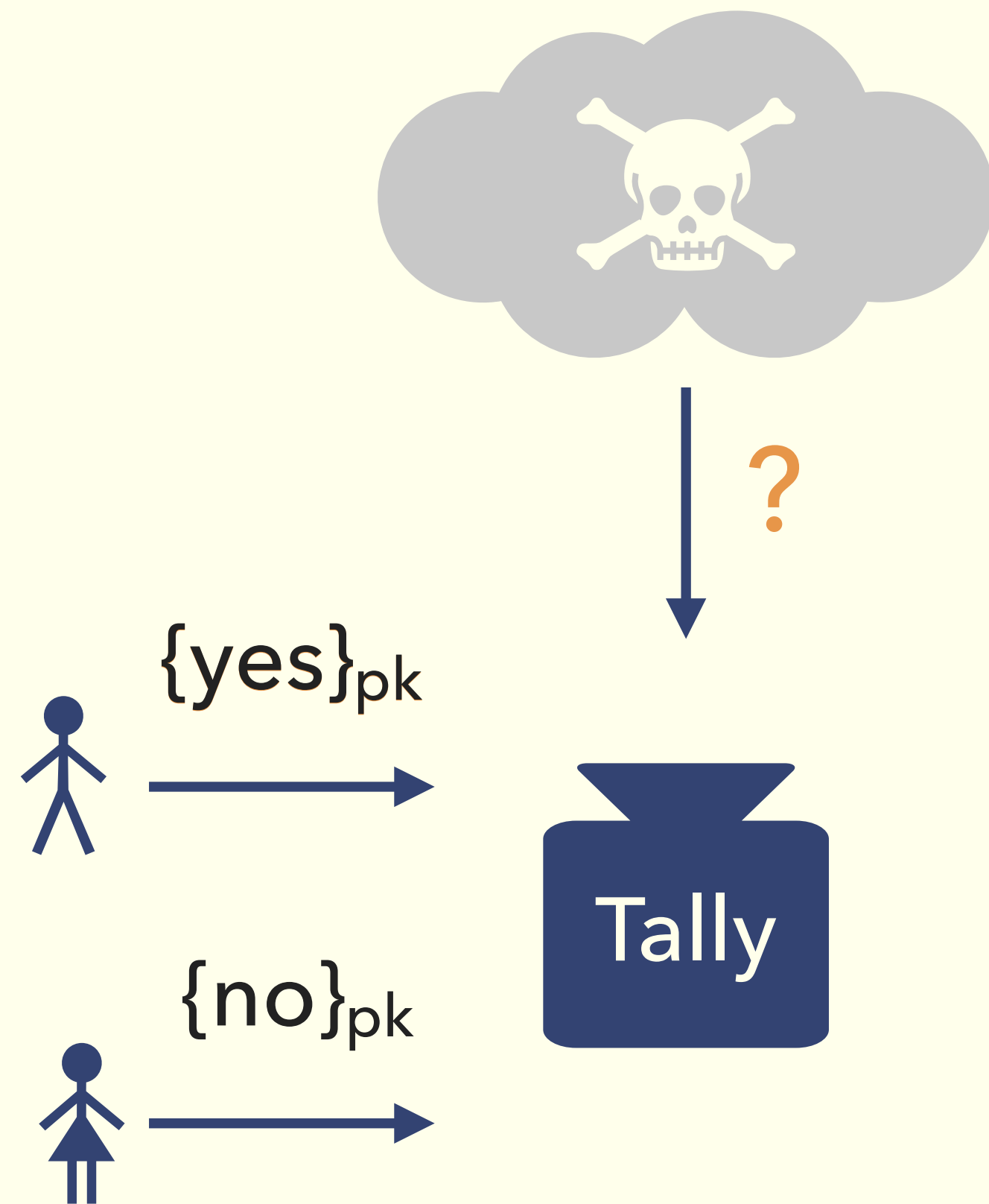




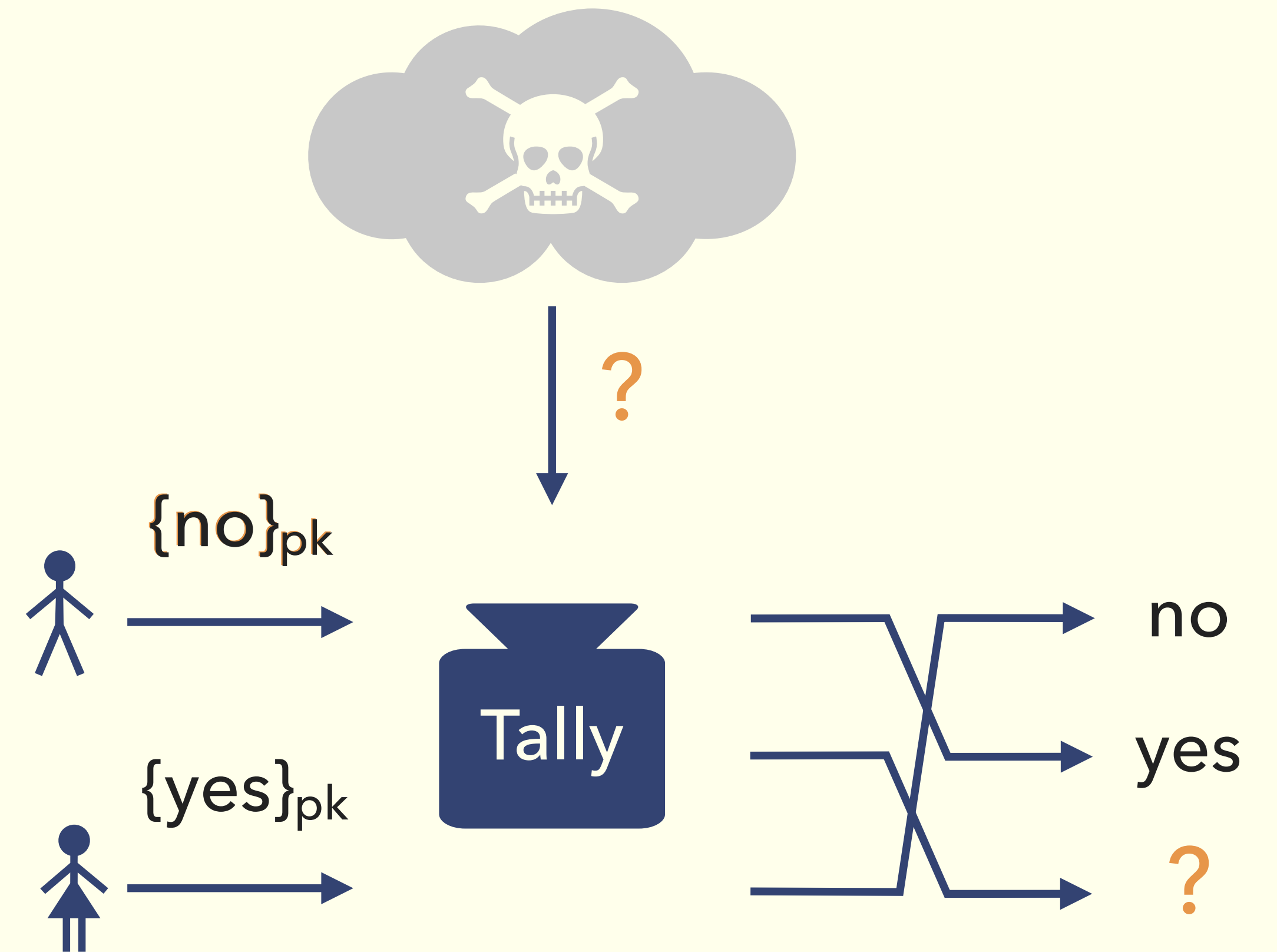
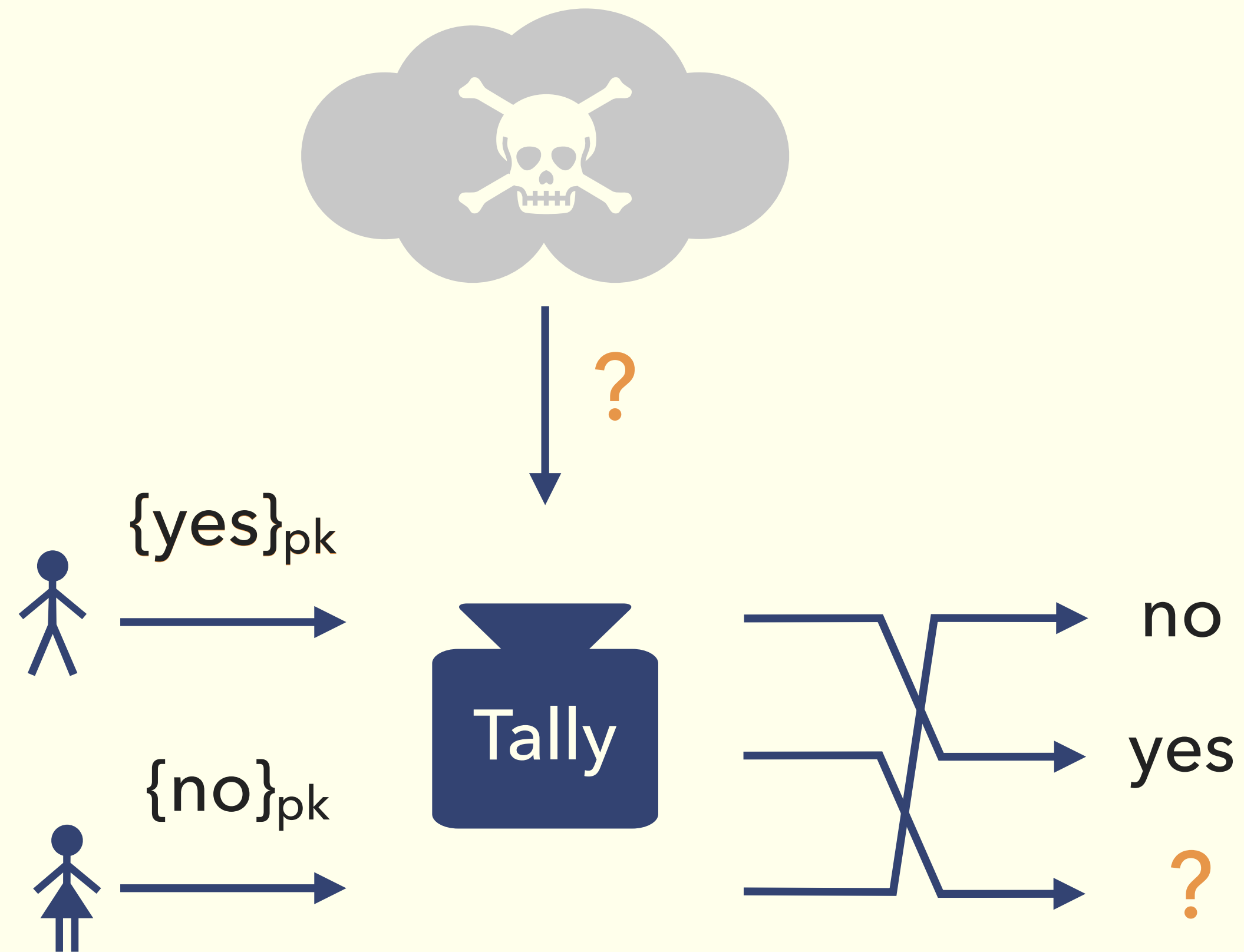
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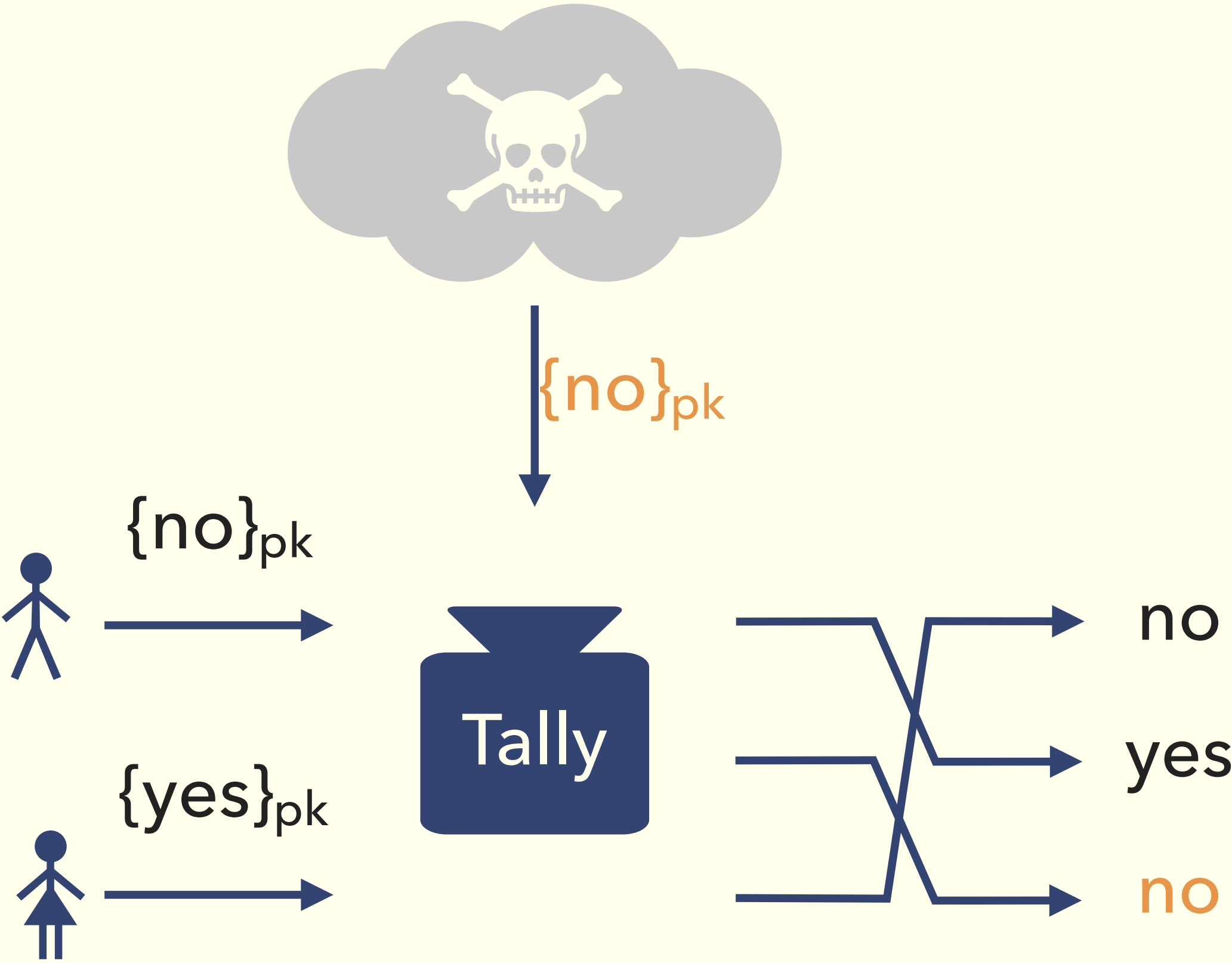
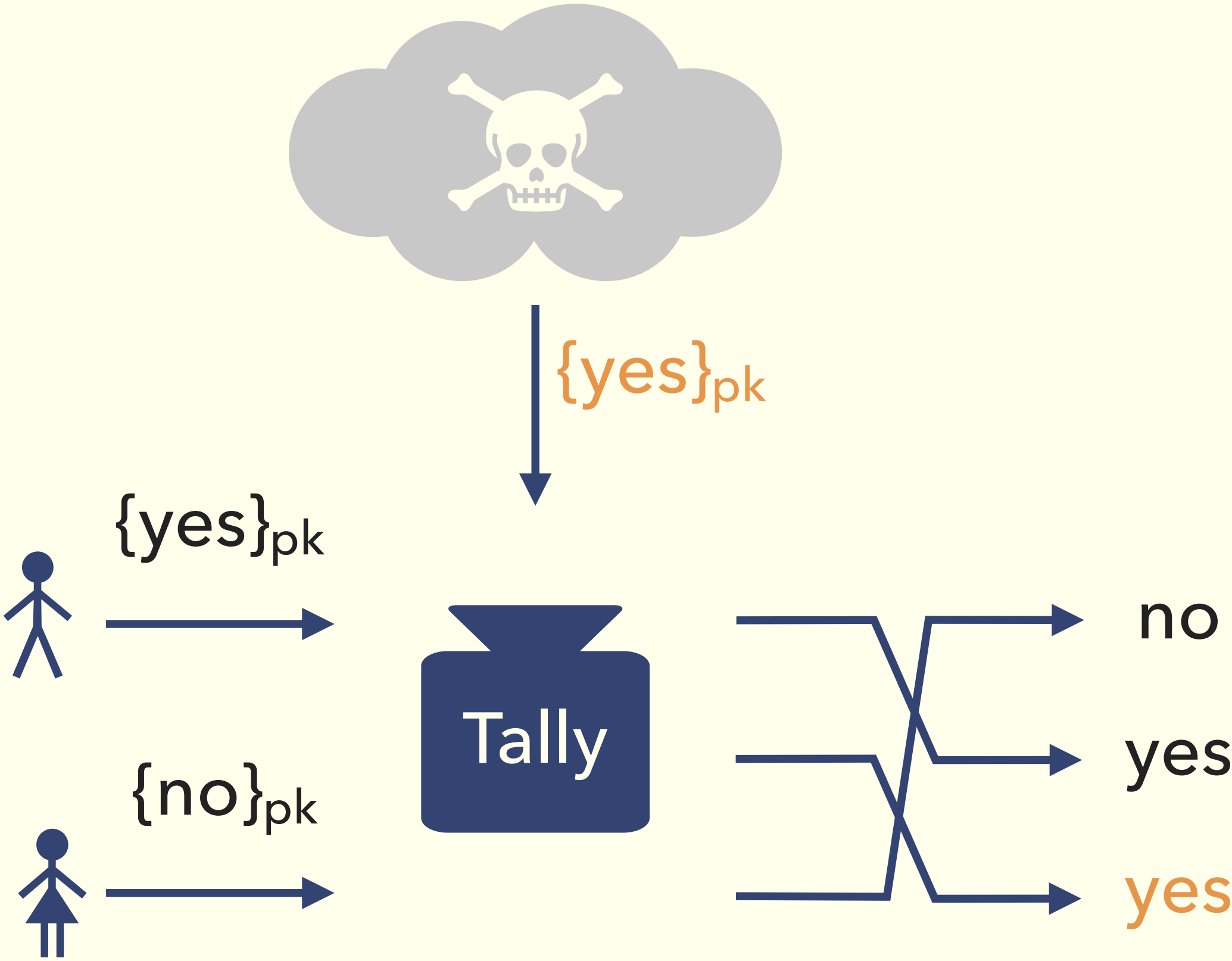
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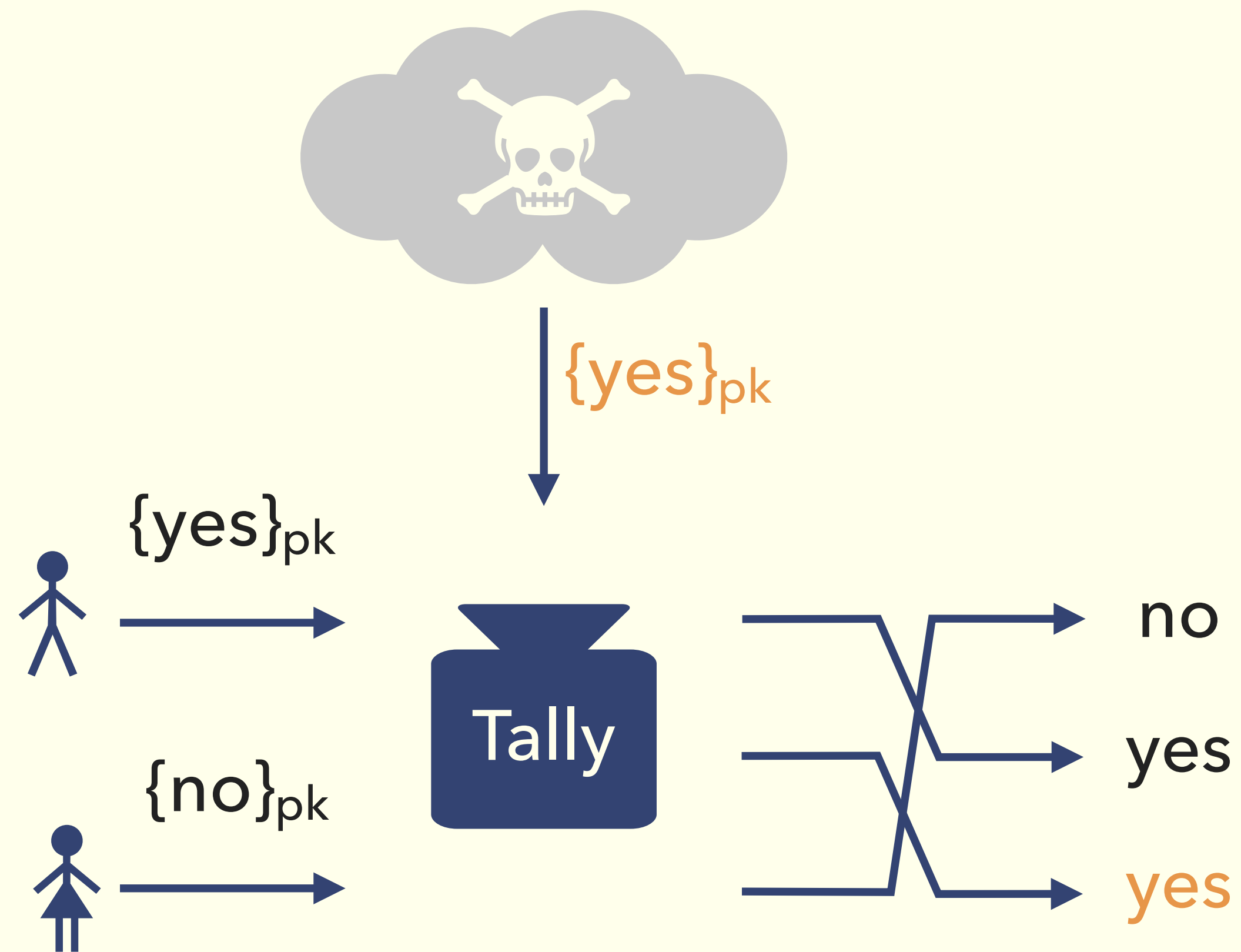
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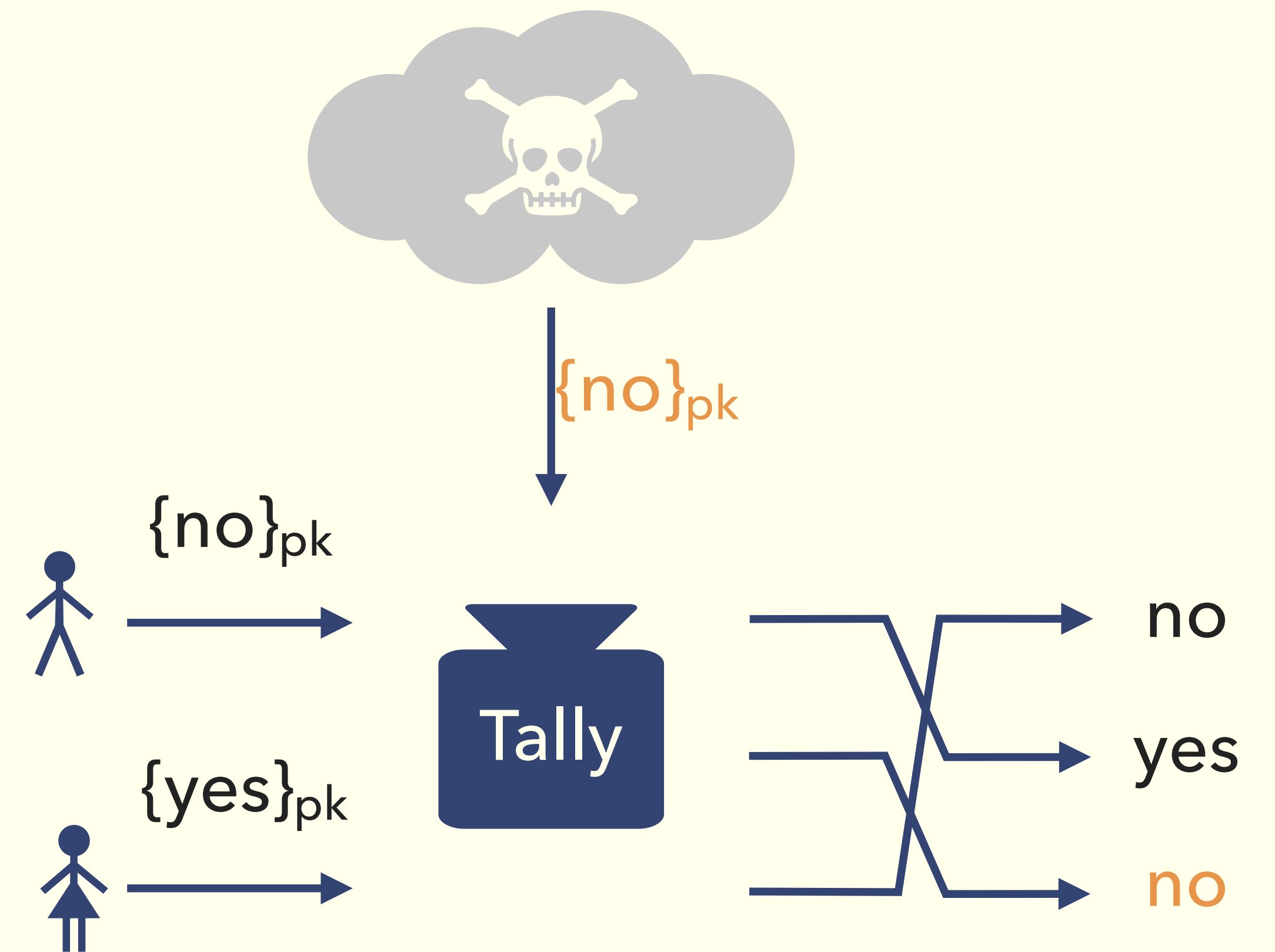
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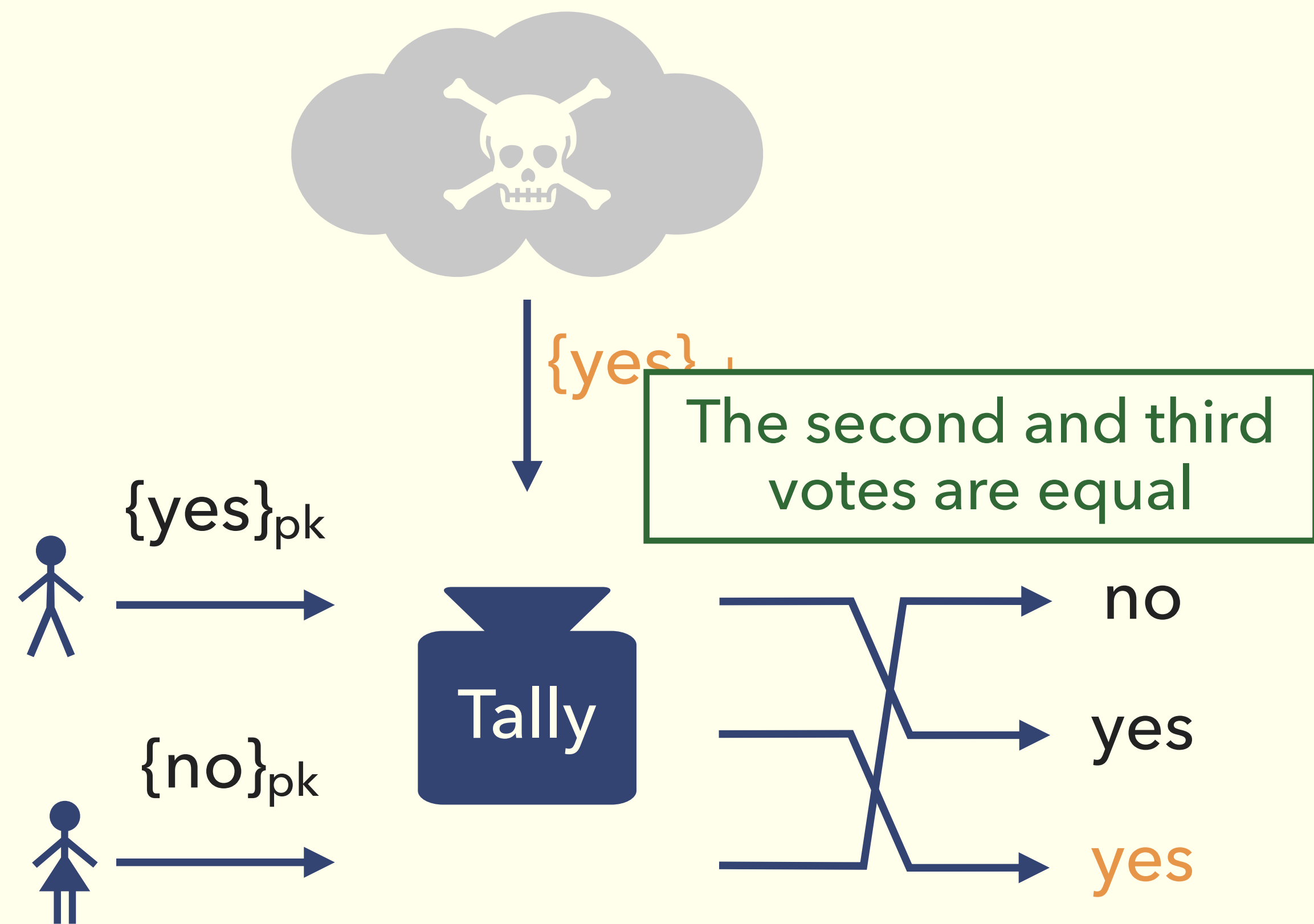


the vote appearing twice on the bulletin board is Bob's vote!

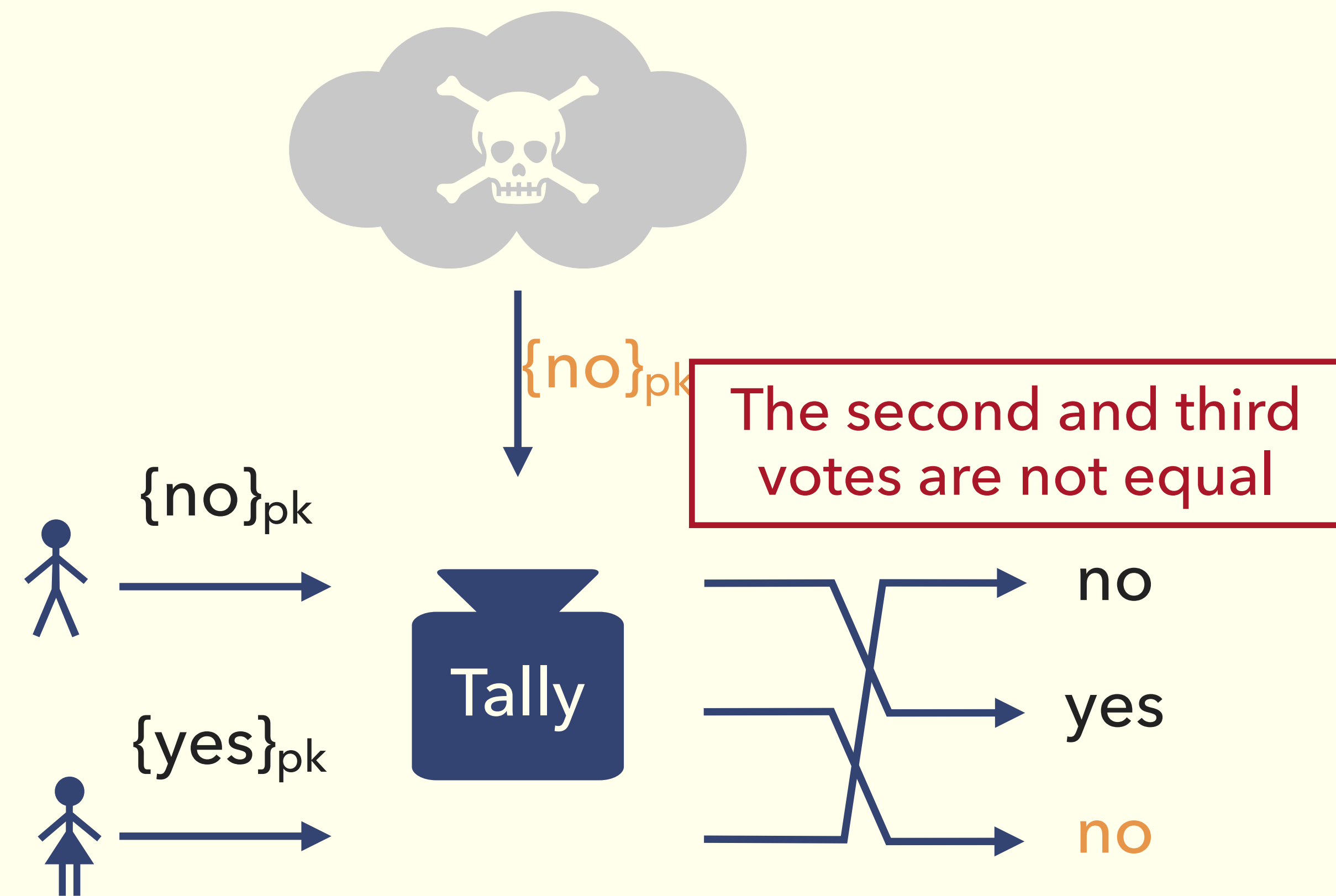


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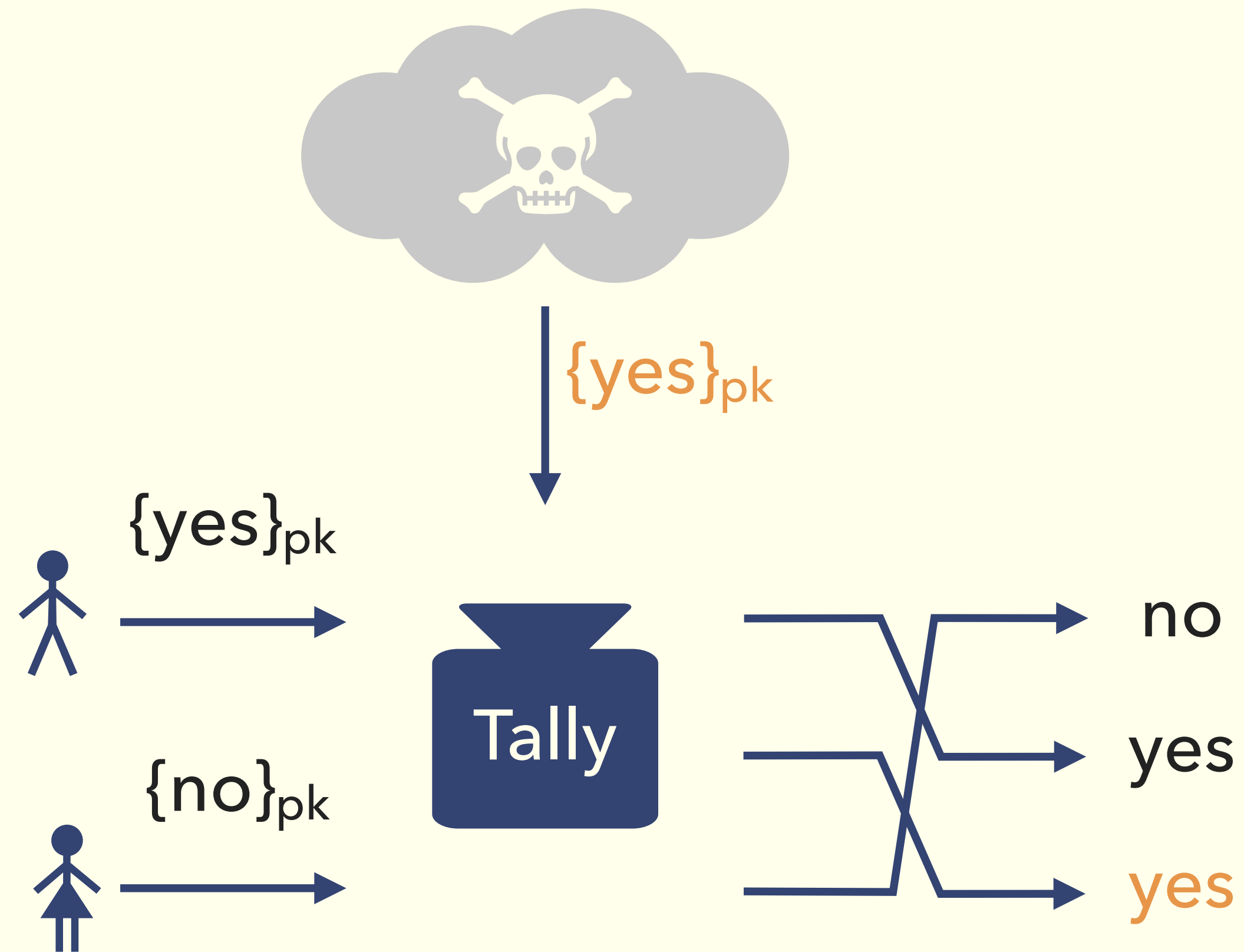


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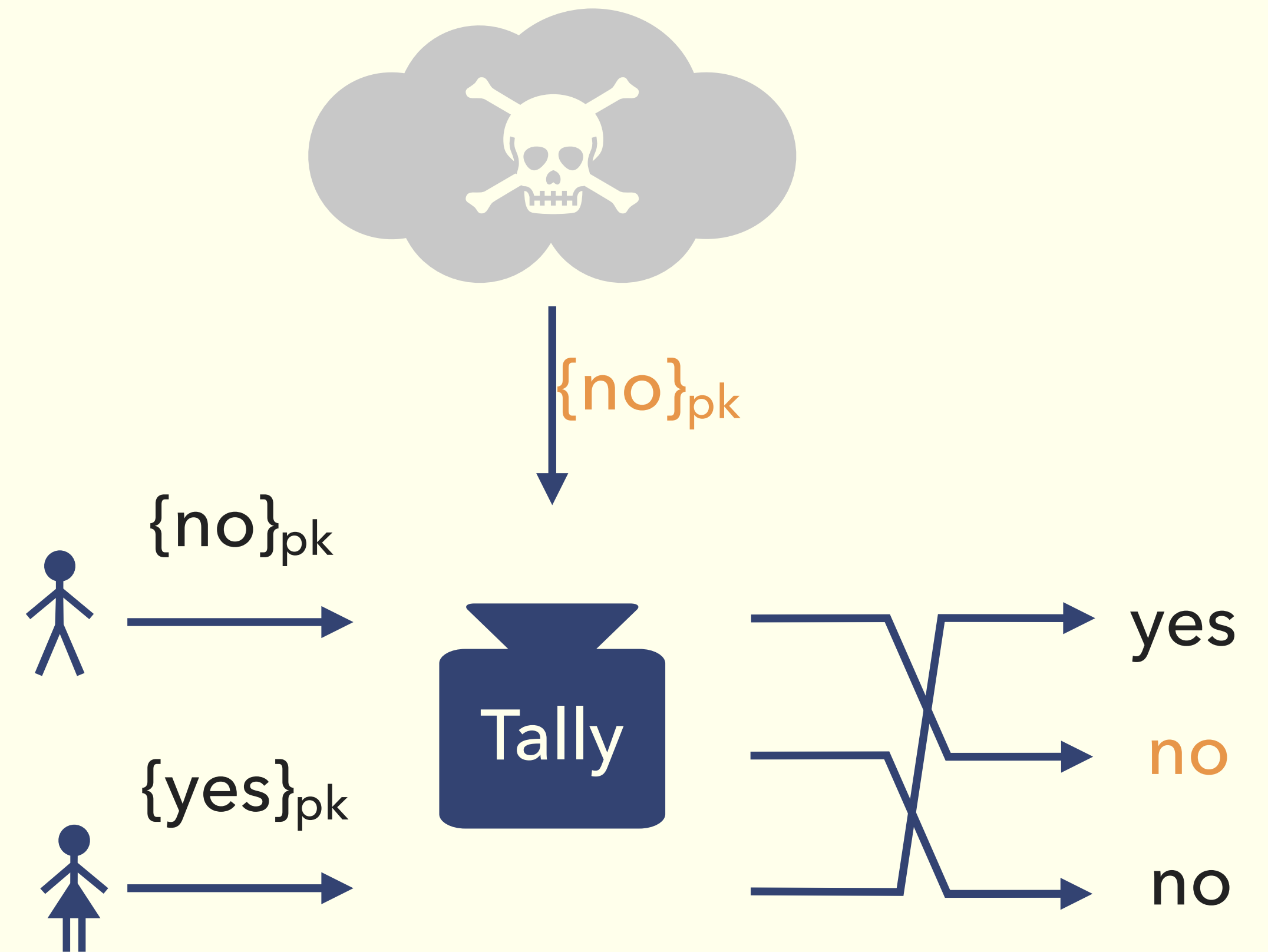


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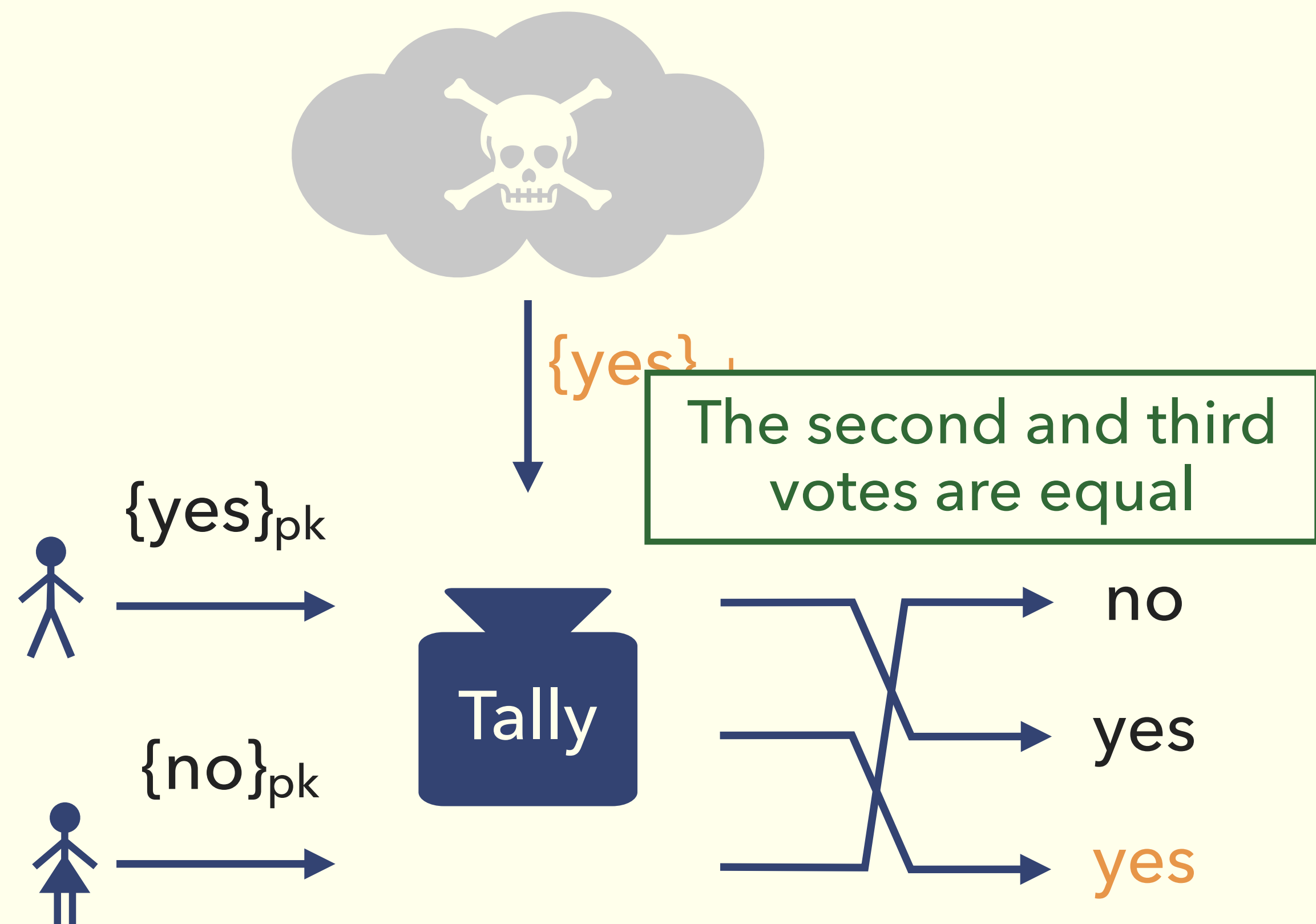


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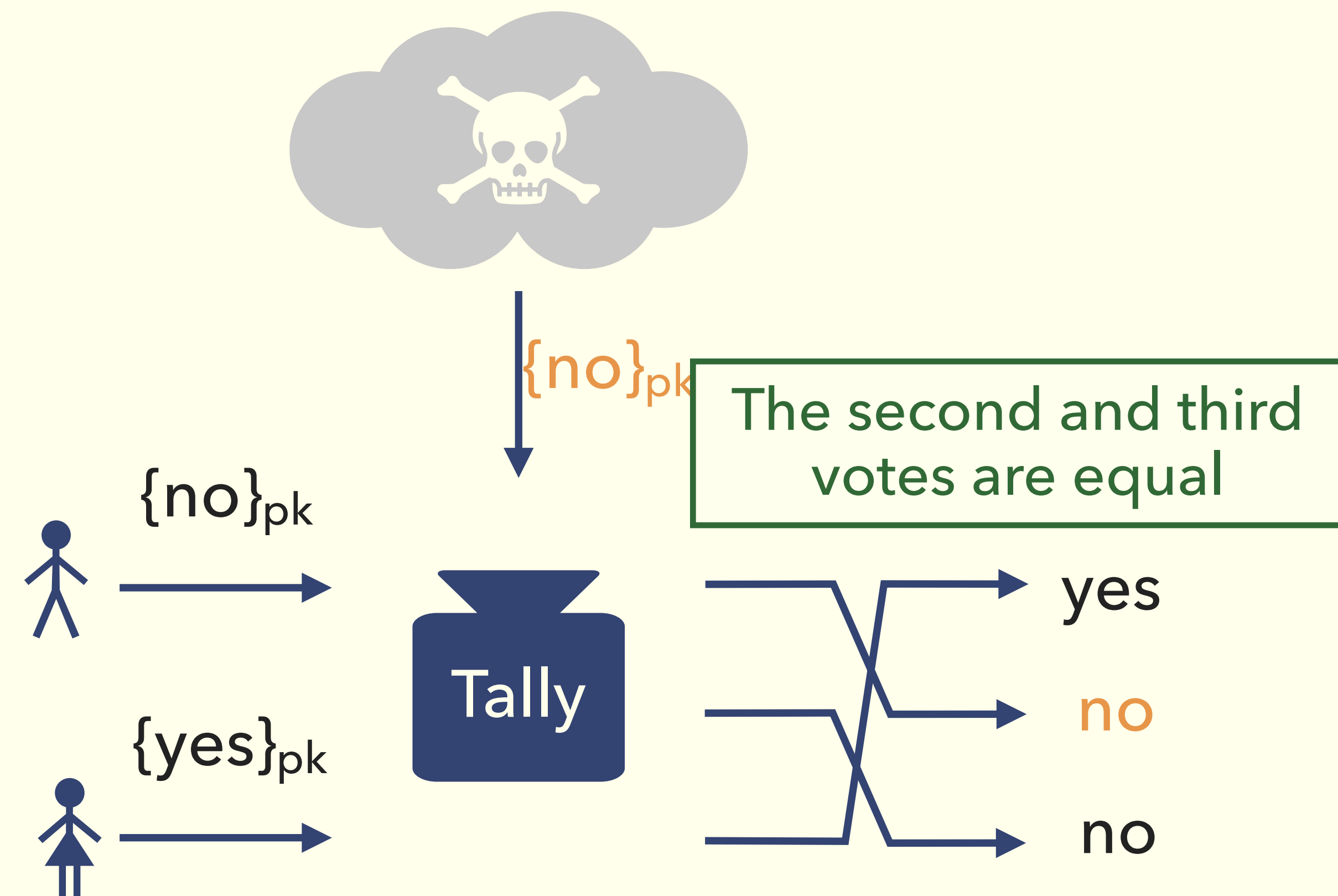


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# A simple e-voting protocol



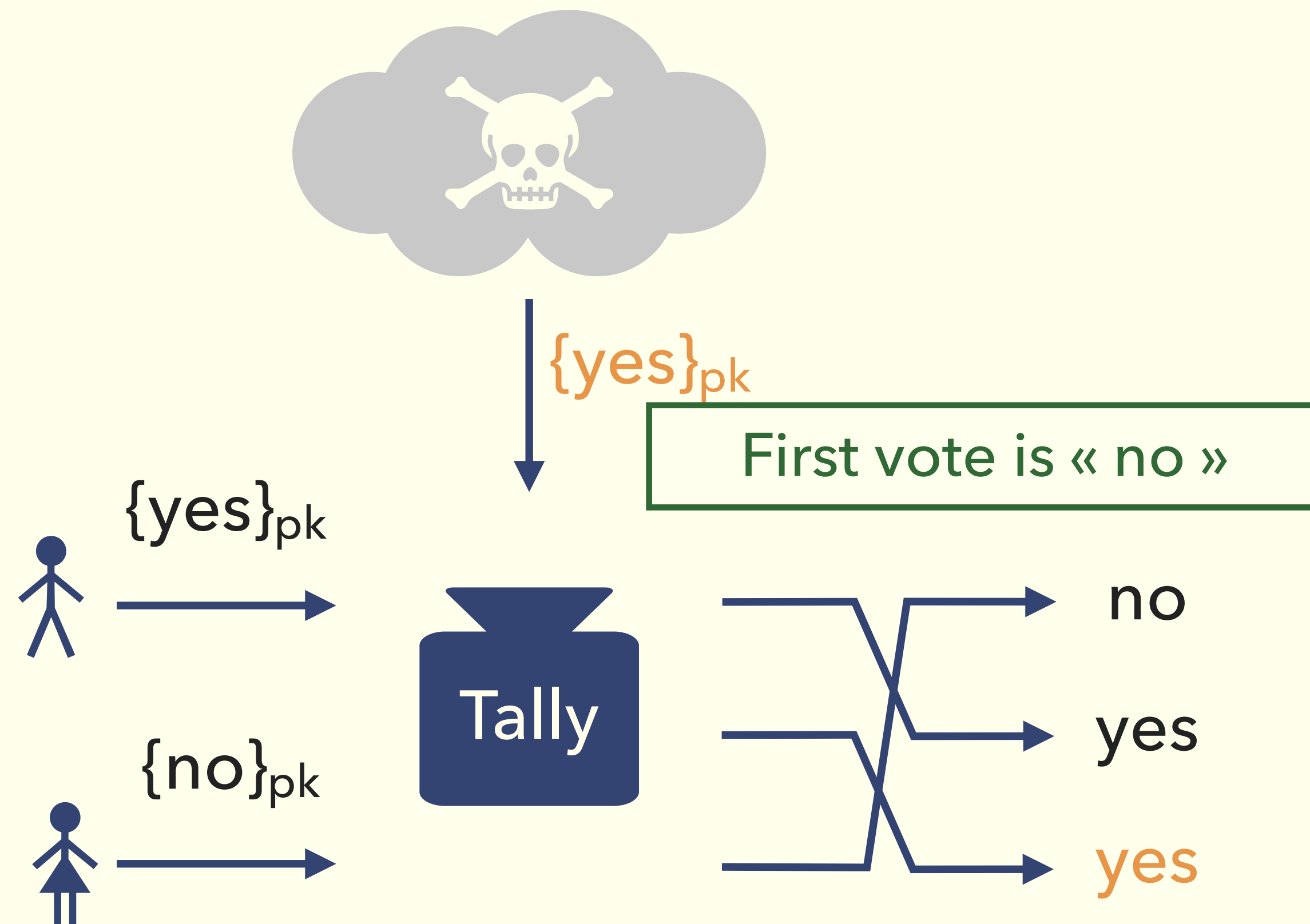
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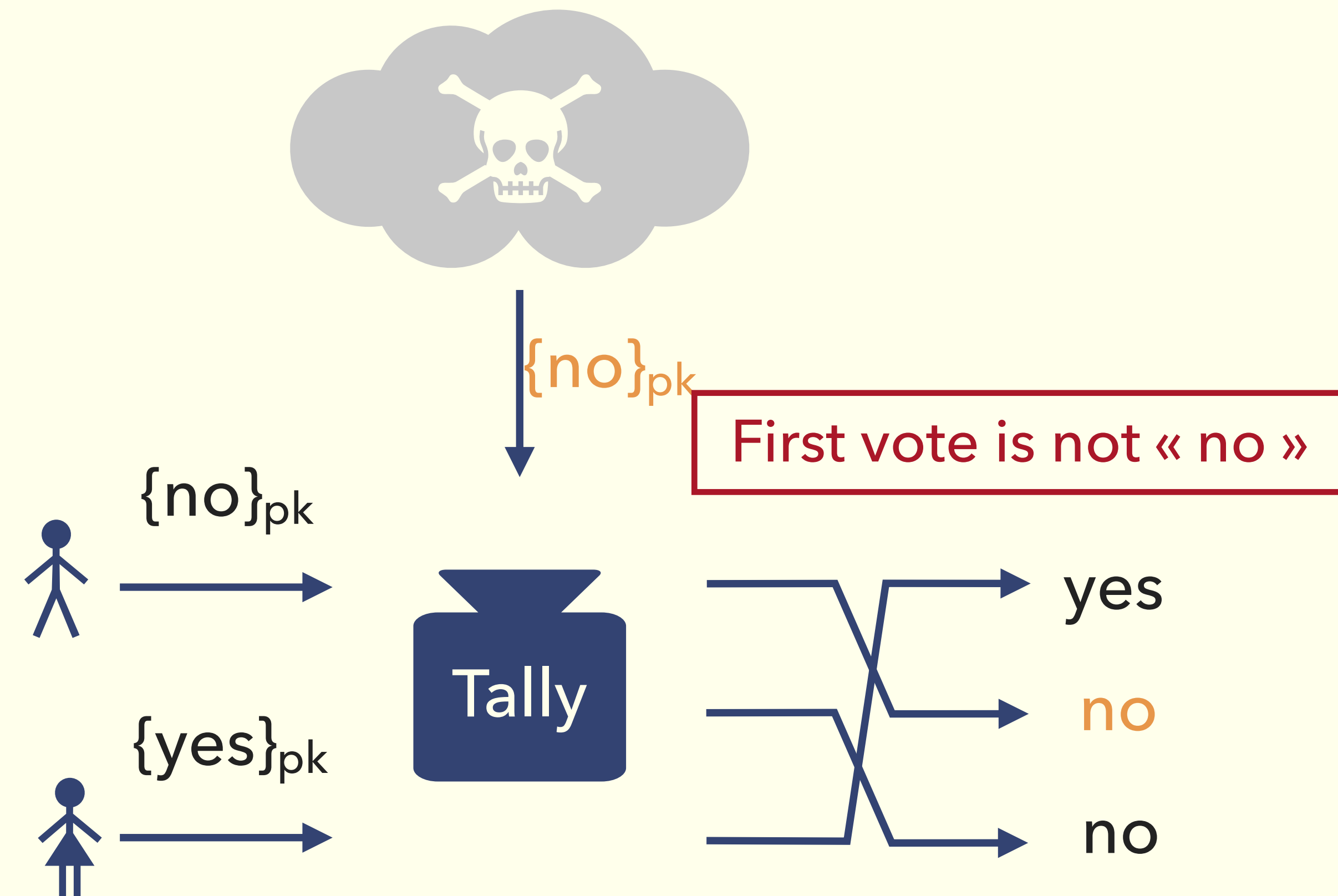
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# A simple e-voting protocol



the vote appearing twice on the bulletin board is Bob's vote!



the vote appearing twice on the bulletin board is Bob's vote!

# Equivalence of processes in ProVerif

```
let system1 = setup | voter(skA,v1) | voter(skB,v2).  
let system2 = setup | voter(skA,v2) | voter(skB,v1).  
equivalence system1 system2
```

Equivalence between two  
processes

```
let system(vA,vB) = setup | voter(skA,vA) | voter(skB,vB).  
process system(choice[v1,v2],choice[v2,v1])
```

Equivalence as a biprocess

# Equivalence of processes in ProVerif

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let system1 = setup | voter(skA,v1) | voter(skB,v2).  
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```

```
let system(vA,vB) = setup | voter(skA,vA) | voter(skB,vB).  
process system(choice[v1,v2],choice[v2,v1])
```

Equivalence between two  
processes

Internally



Equivalence as a biprocess

---

# Equivalence of processes in ProVerif

## Equivalence between two processes

- + Easier to model,
- + No need to know « how to match » the processes
- Can be slow
- Difficult to « fix » when not working

## Equivalence as a biprocess

- + Also easy to model,
- + Works better with other features (e.g. lemmas, axioms)
- + More efficient
- Need to have a good idea why processes are equivalent

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# Memory cell

ProVerif's calculus is stateless

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## Memory cell

ProVerif's calculus is stateless ... but we have private channels

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# Memory cell

ProVerif's calculus is stateless ... but we have private channels

A Ocaml like version

`let x = ref 0`

Initialisation

`!x`

Reading

`x := n`

Writing

# Memory cell

ProVerif's calculus is stateless ... but we have private channels

A Ocaml like version

let x = ref 0

Initialisation

```
free cell:channel [private]  
let init = out(cell,0).
```

!x

Reading

x := n

Writing



# Memory cell

ProVerif's calculus is stateless ... but we have private channels

A Ocaml like version

let x = ref 0

Initialisation

```
free cell:channel [private]
let init = out(cell,0).
```

!x

Reading

```
let P =
  ...
  in(cell,x:nat); out(cell,x);
  ...
```

x := n

Writing

# Memory cell

ProVerif's calculus is stateless ... but we have private channels

A Ocaml like version

let x = ref 0

Initialisation

```
free cell:channel [private]
let init = out(cell,0).
```

!x

Reading

```
let P =
  ...
  in(cell,x:nat); out(cell,x);
  ...
```

x := n

Writing

```
let Q =
  ...
  in(cell,x:nat); out(cell,n);
  ...
```

# Memory cell

Initialisation

```
free cell:channel [private]  
let init = out(cell,0).
```

Reading

```
let P =  
  ...  
  in(cell,x:nat); out(cell,x);  
  ...
```

Writing

```
let Q =  
  ...  
  in(cell,x:nat); out(cell,n);  
  ...
```

Reading/Writing both consist of inputting the « current value » of the cell and outputting the « new value »

Communication are synchronous on private channels: always one single output available at all time.

The system

```
process  
  init | P | Q | !in(cell,x:nat);out(cell,x)
```

Avoids « blocking » an agent

# Locking memory cell

Initialisation

```
free cell:channel [private]  
let init = out(cell,0).
```

Reading

```
let P =  
  ...  
  in(cell,x:nat);  
  event B;  
  out(cell,x);  
  ...
```

Writing

```
let Q =  
  ...  
  in(cell,x:nat);  
  event A;  
  event C;  
  out(cell,n);  
  ...
```

Communication are synchronous  
on private channels:  
If no output available, all processes  
trying to input are « blocked »

The sequence of events A, B, C is not possible

# Locking memory cell

Initialisation

```
free cell:channel [private]  
let init = out(cell,0).
```

Lock and read

```
let P =  
  ...  
  in(cell,x:nat);  
  event B;  
  out(cell,x);  
  ...
```

Write and unlock

```
let Q =  
  ...  
  in(cell,x:nat);  
  event A;  
  event C;  
  out(cell,n);  
  ...
```

Communication are synchronous  
on private channels:  
If no output available, all processes  
trying to input are « blocked »

The sequence of events A, B, C is not possible

# Simplified Yubikey protocol

P only accepts increasing sequence of natural numbers.

Q emits sequentially all natural numbers encrypted with k

```
free k:key [private].
free cellP,cellQ:channel [private]

let P =
  in(c,x:bitstring);
  in(cellP,i:nat);
  let j = sdec(x,k) in
  if j > i
  then
    event Accept(j);
    out(cellP,j)
  else
    out(cellP,i).

let Q =
  in(cellQ,i:nat);
  out(c,senc(I,k));
  out(cellQ,i+1).

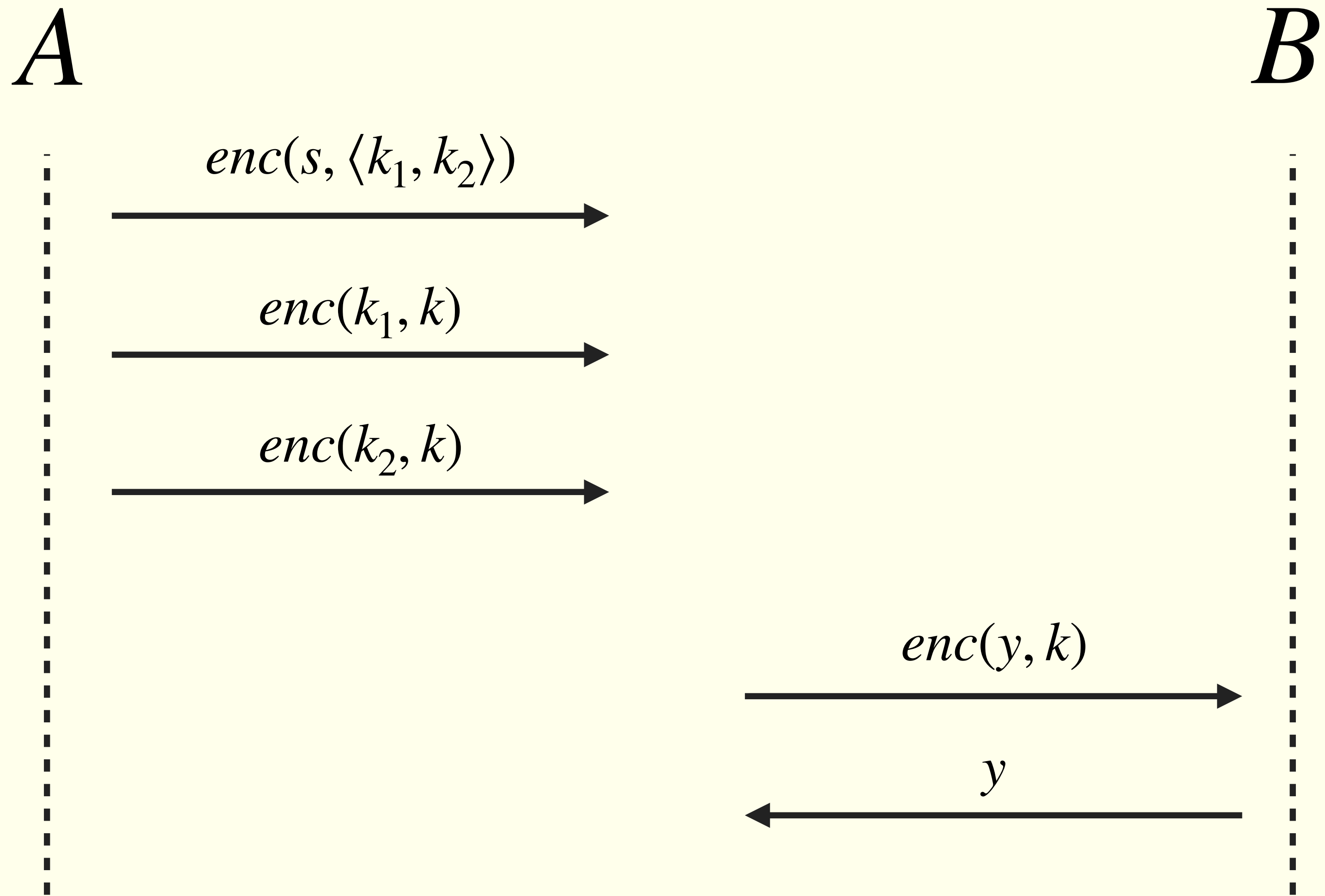
process out(cellP,0) | out(cellQ,0) | !P | !Q
```

---

# Outline

- A. Additional modeling techniques
  - 1. Equational Theory vs Rewrite rules
  - 2. Equivalence properties
  - 3. Memory cell and locks for stateful protocols
- B. Dealing with « cannot be proved »
  - 1. Adding precision
  - 2. Trace with assumption
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- C. Dealing with non-termination

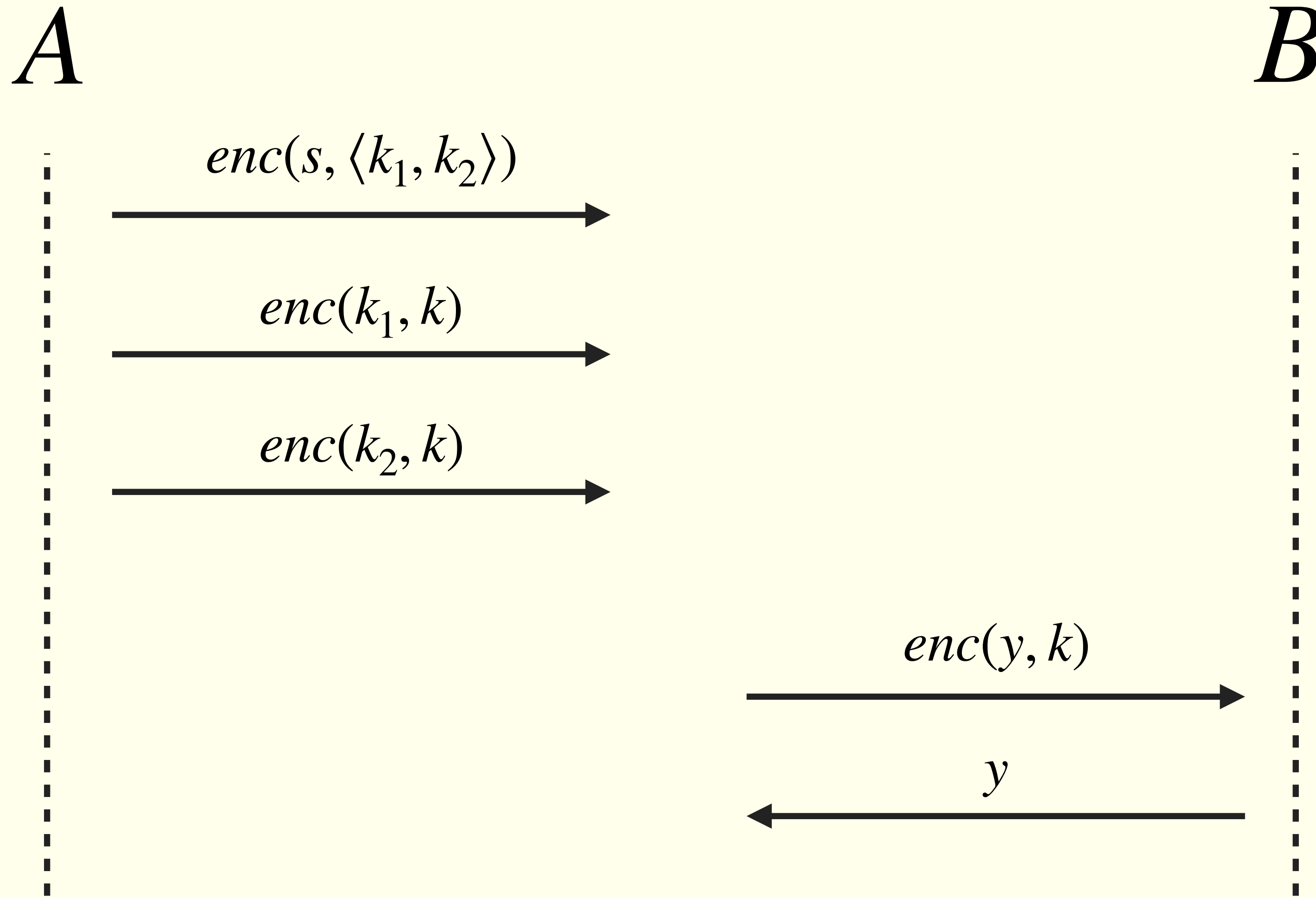
# Toy-example





## Toy-example

$B$  acts as an oracle for decryption with the key  $k$   
but only one time !



# Why does it fail ?

Transform process  
in Horn clauses

```
free s,k1,k2,k:bitstring [private].  
  
let A =  
  out(c,senc(s,(k1,k2)));  
  out(c,senc(k1,k));  
  out(c,senc(k2,k)).  
  
let B =  
  in(c,x);  
  out(c,dec(x,k)).  
  
process A | B
```

$\rightarrow att(enc(s, \langle k_1, k_2 \rangle))$

$\rightarrow att(enc(k_1, k))$

$\rightarrow att(enc(k_2, k))$

$att(enc(y, k)) \rightarrow att(y)$

Horn clauses for  
the attacker

$att(x) \wedge att(y) \rightarrow att(enc(x, y))$

$att(enc(x, y)) \wedge att(y) \rightarrow att(x)$

$att(x) \wedge att(y) \rightarrow att(\langle x, y \rangle)$

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Secrecy of  $s$  is preserved if  $att(s)$  is not logically  
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Horn clauses can be applied an  
arbitrary number of times for  
arbitrary instantiations

Secrecy of  $s$  is preserved if  $att(s)$  is not logically  
decucible from the set of Horn clauses

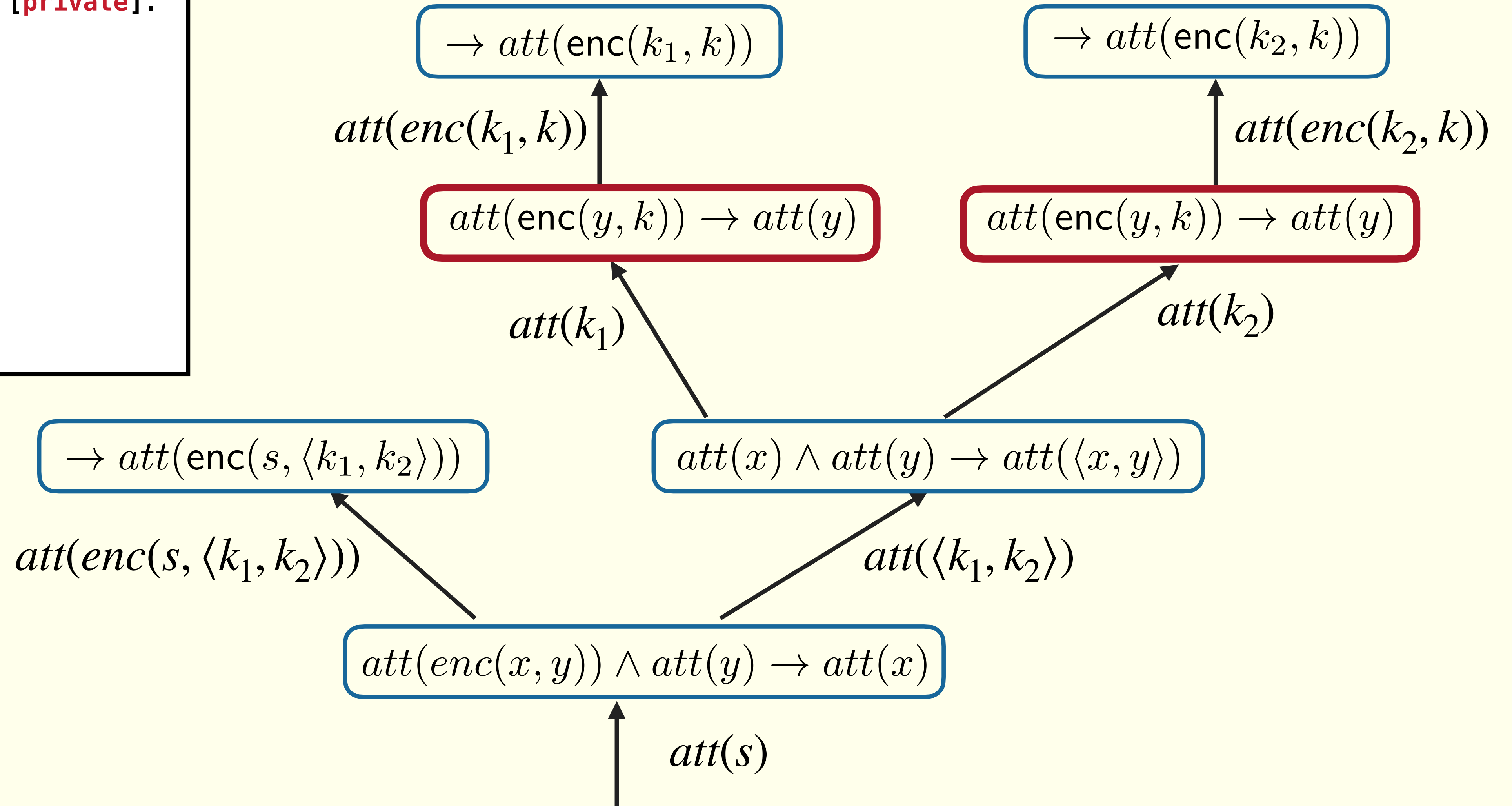
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```
let A =  
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  out(c,senc(k2,k)).
```

```
let B =  
  in(c,x);  
  out(c,dec(x,k)).
```

```
process A | B
```



# What to do ?

Add a [precise] option to the problematic input !

```
free s,k1,k2,k:bitstring [private].  
  
let A =  
  out(c,senc(s,(k1,k2)));  
  out(c,senc(k1,k));  
  out(c,senc(k2,k)).  
  
let B =  
  in(c,x) [precise];  
  out(c,dec(x,k)).  
  
process A | B
```

```
-- Query not attacker(s[]) in process 0.  
Translating the process into Horn clauses...  
Completing...  
Starting query not attacker(s[])  
  
RESULT not attacker(s[]) is true.
```

-----  
Verification summary:

Query not attacker(s[]) is true.  
-----

Global setting

```
set preciseActions = true.
```



Adding [precise] options may increase the verification time or lead to non-termination

# How to know where to put precise ?

Going through  
the derivation !

Find two different  
messages received  
by the same input {n}

Check on your  
process if it should  
be possible

Derivation:

1. The message `enc(k2[],k[])` may be sent to the attacker at output {5}.  
`attacker(enc(k2[],k[]))`.
2. The message `enc(k2[],k[])` that the attacker may have by 1 may be received at input {7}.  
So the message `k2[]` may be sent to the attacker at output {8}.  
`attacker(k2[])`.
3. The message `enc(k1[],k[])` may be sent to the attacker at output {4}.  
`attacker(enc(k1[],k[]))`.
4. The message `enc(k1[],k[])` that the attacker may have by 3 may be received at input {7}.  
So the message `k1[]` may be sent to the attacker at output {8}.  
`attacker(k1[])`.
5. By 4, the attacker may know `k1[]`.  
By 2, the attacker may know `k2[]`.  
Using the function 2-tuple the attacker may obtain `(k1[],k2[])`.  
`attacker((k1[],k2[]))`.
6. The message `enc(s[],(k1[],k2[]))` may be sent to the attacker at output {6}.  
`attacker(enc(s[],(k1[],k2[])))`.
7. By 6, the attacker may know `enc(s[],(k1[],k2[]))`.  
By 5, the attacker may know `(k1[],k2[])`.  
Using the function `dec` the attacker may obtain `s[]`.  
`attacker(s[])`.
8. By 7, `attacker(s[])`.  
The goal is reached, represented in the following fact:  
`attacker(s[])`.



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**4. The message `enc(k1[],k[])` that the attacker may have by 3 may be received at input {7}.**  
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8. By 7, `attacker(s[])`.  
The goal is reached, represented in the following fact:  
`attacker(s[])`.



# Two strange situations !

## Simplified Yubikey

```
free k:key [private].
free cellP,cellQ:channel [private]

let P =
  in(c,x:bitstring);
  in(cellP,i:nat);
  let j = sdec(x,k) in
  if j > i
  then
    event Accept(j);
    out(cellP,j)
  else
    out(cellP,i).

let Q =
  in(cellQ,i:nat);
  out(c,senc(I,k));
  out(cellQ,i+1).

process out(cellP,0) | out(cellQ,0) | !P | !Q
```

## Can't disprove the sanity check...

```
query i:nat; event(Accept(i)).
```

-- Query not event(Accept(i\_2)) in process 0.

Translating the process into Horn clauses...

mess(cellQ[],i\_2) -> mess(cellQ[],i\_2 + 1)

select mess(cellQ[],i\_2)/-5000

Completing...

Starting query not event(Accept(i\_2))

goal reachable:  $i_2 \geq 1$  && mess(cellQ[],i\_2) -> end(Accept(i\_2))

Derivation:

1. We assume as hypothesis that  
mess(cellQ[],i\_2).

2. The message i\_2 that may be sent on channel cellQ[] by 1 may be received  
at input {12}.

So the message senc(i\_2,k[]) may be sent to the attacker at output {13}.

attacker(senc(i\_2,k[])).

...

Could not find a trace corresponding to this derivation.

# Two strange situations !

```
-- Query not attacker(S(kAminus[!1 = v],x_1)) in process 1.
select member(*x_1,y)/-5000
select memberid(*x_1,y)/-5000
Translating the process into Horn clauses...
Completing...

...

A more detailed output of the traces is available with
  set traceDisplay = long.

new exponent: channel creating exponent_3 at {1}
new honestC: channel creating honestC_3 at {8}
new kAminus: skey creating kAminus_3 at {10} in copy a
...
...

event enddosi(Pk(kAminus_3),NI_3) at {37} in copy a, a_4, a_8
event mess3(Pk(kAminus_3)...)) at {46} in copy a, a_4, a_8
out(c, cons3(~M_9,...)) at {47} in copy a, a_4, a_8

The attacker has the message 3-proj-3-tuple(D(H(...)).
A trace has been found, assuming the following hypothesis:
memberid(Pk(a_12[]),a_5[])
Stopping attack reconstruction attempts. To try more traces,
modify the setting reconstructTrace.
RESULT not attacker(S(kAminus[!1 = v],x_1)) cannot be proved.
```

A trace is found... but ProVerif  
assume that the attacker has  
magically a term

# A closer look

```
-- Query not attacker(S(kAminus[!1 = v],x_1)) in process 1.  
select member(*x_1,y)/-5000  
select memberid(*x_1,y)/-5000  
Translating the process into Horn clauses...  
Completing...  
  
...  
A more detailed output of the traces is available with  
  set traceDisplay = long.  
  
new exponent: channel creating exponent_3 at {1}  
new honestC: channel creating honestC_3 at {8}  
new kAminus: skey creating kAminus_3 at {10} in copy a  
...  
...  
event enddosi(Pk(kAminus_3),NI_3) at {37} in copy a, a_4, a_8  
event mess3(Pk(kAminus_3)...) at {46} in copy a, a_4, a_8  
out(c, cons3(~M_9,...)) at {47} in copy a, a_4, a_8  
  
The attacker has the message 3-proj-3-tuple(D(H(...)).  
A trace has been found, assuming the following hypothesis:  
memberid(Pk(a_12[]),a_5[])  
Stopping attack reconstruction attempts. To try more traces,  
modify the setting reconstructTrace.  
RESULT not attacker(S(kAminus[!1 = v],x_1)) cannot be proved.
```

```
-- Query not event(Accept(i_2)) in process 0.  
Translating the process into Horn clauses...  
mess(cellQ[],i_2) -> mess(cellQ[],i_2 + 1)  
select mess(cellQ[],i_2)/-5000  
Completing...  
Starting query not event(Accept(i_2))  
  
goal reachable: i_2 ≥ 1 && mess(cellQ[],i_2) ->  
end(Accept(i_2))  
  
Derivation:  
  
1. We assume as hypothesis that  
mess(cellQ[],i_2).  
  
2. The message i_2 that may be sent on channel cellQ[] by 1 may  
be received at input {12}.  
So the message senc(i_2,k[]) may be sent to the attacker at  
output {13}.  
attacker(senc(i_2,k[])).  
  
...  
  
Could not find a trace corresponding to this derivation.
```

ProVerif decided to prevent  
resolution on some facts

# Why ProVerif prevent resolution ?

## Simplified Yubikey

```
free k:key [private].
free cellP,cellQ:channel [private]

let P =
  in(c,x:bitstring);
  in(cellP,i:nat);
  let j = sdec(x,k) in
  if j > i
  then
    event Accept(j);
    out(cellP,j)
  else
    out(cellP,i).

let Q =
  in(cellQ,i:nat);
  out(c,senc(I,k));
  out(cellQ,i+1).

process out(cellP,0) | out(cellQ,0) | !P | !Q
```

```
select mess(cellQ[],i_2)/-5000
```

Clauses generated from the process Q

$$\begin{aligned} & \text{mess}(\text{cellQ}, i) \rightarrow \text{mess}(\text{cellQ}, i + 1) \\ & \rightarrow \text{mess}(\text{cellQ}, 0) \end{aligned}$$

If  $\text{mess}(\text{cellQ}, i)$  was selected then by resolution:

$$\rightarrow \text{mess}(\text{cellQ}, 1)$$
$$\rightarrow \text{mess}(\text{cellQ}, 2)$$
$$\vdots$$

# What to do to solve the problem ?

Use a new setting

```
set nounifIgnoreAFewTimes = auto.
```

When solving the query, ProVerif will ignore a « few times » the prevention of resolution.

By default, only one time but it can be parametrized

```
set nounifIgnoreNtimes = 3.
```



The bigger the number, the slower the verification will be



Useful for proofs and finding attacks



Not always enough !

# Proof of queries by induction

## Simplified Yubikey

```
free k:key [private].
free cellP,cellQ:channel [private]

query i:nat; mess(cellQ,i) ==> is_nat(i).

let P =
  in(c,x:bitstring);
  in(cellP,i:nat);
  let j = sdec(x,k) in
  if j > i
  then
    event Accept(j);
    out(cellP,j)
  else
    out(cellP,i).

let Q =
  in(cellQ,i:nat);
  out(c,senc(I,k));
  out(cellQ,i+1).

process out(cellP,0) | out(cellQ,0) | !P | !Q
```

## Even with

```
set nounifIgnoreAFewTimes = auto.
set nounifIgnoreNtimes = 10.
```

## With obtain

```
goal reachable: is_not_nat(i_2 + 10) && mess(cellQ[],i_2) ->
mess(cellQ[],i_2 + 10)

...

Could not find a trace corresponding to this derivation.
RESULT mess(cellQ[],i_2) ==> is_nat(i_2) cannot be proved.
```



The attacker is untyped !



# Proof of queries by induction

## Simplified Yubikey

```
free k:key [private].
free cellP,cellQ:channel [private]

query i:nat; mess(cellQ,i) ==> is_nat(i).

let P =
  in(c,x:bitstring);
  in(cellP,i:nat);
  let j = sdec(x,k) in
  if j > i
  then
    event Accept(j);
    out(cellP,j)
  else
    out(cellP,i).

let Q =
  in(cellQ,i:nat);
  out(c,senc(I,k));
  out(cellQ,i+1).

process out(cellP,0) | out(cellQ,0) | !P | !Q
```

```
goal reachable: is_not_nat(i_2 + 10) && mess(cellQ[],i_2) ->
mess(cellQ[],i_2 + 10)
```

The fact `mess(cellQ[],i_2)` occurred strictly before `mess(cellQ[],i_2)` in the trace.

Induction on the size of the trace !

```
query i:nat; mess(cellQ,i) ==> is_nat(i) [induction].
```

# Proof of queries by induction

It also works for a group of queries !

Proof by mutual induction

```
query i:nat,...;  
  mess(cellQ,i) ==> is_nat(i);  
  mess(cellP,i) ==> is_nat(i);  
query_3;  
...  
query_n [induction].
```



As usual it, it may slow down the verification or lead to non-termination



Does not work for injective correspondence



# Lemmas, axioms, restrictions

Restrictions « restrict » the traces considered in axioms, lemmas and queries.

`query attacker(s).` holds if no trace satisfying `phi_1, ..., phi_n` reveals `s`

```
restriction phi_1.  
...  
restriction phi_n.  
  
axiom aphi_1.  
...  
axiom aphi_m.  
  
lemma lphi_1.  
...  
lemma lphi_k.  
  
query attacker(s).
```

**1** Proverif assumes that the axioms `aphi_1, ..., aphi_n` hold.

**2** Proverif tries to prove in order the lemmas `lphi_1, ..., lphi_k` reusing all axioms and previously proved lemmas

**3** Proverif tries to prove the query `query attacker(s).` reusing all axioms and all lemmas.

# The precise option under the hood

Option [precise] for inputs, table lookup and predicate testing is coded as an axiom internally .

```
free s,k1,k2,k:bitstring [private].
let A =
  out(c,senc(s,(k1,k2)));
  out(c,senc(k1,k));
  out(c,senc(k2,k)).
let B =
  in(c,x) [precise];
  out(c,dec(x,k)).
process A | B
```

Encoded as



```
type occurrence.
free s,k1,k2,k:bitstring [private].
event Precise(occurrence,bitstring).
axiom occ:occurrence,x1,x2:bitstring;
  event(Precise(occ,x1)) && event(Precise(occ,x2)) ==> x1 = x2.
let A =
  out(c,senc(s,(k1,k2)));
  out(c,senc(k1,k));
  out(c,senc(k2,k)).
let B =
  in(c,x);
  new occ[]:occurrence;
  event Precise(occ,x);
  out(c,dec(x,k)).
process A | B
```

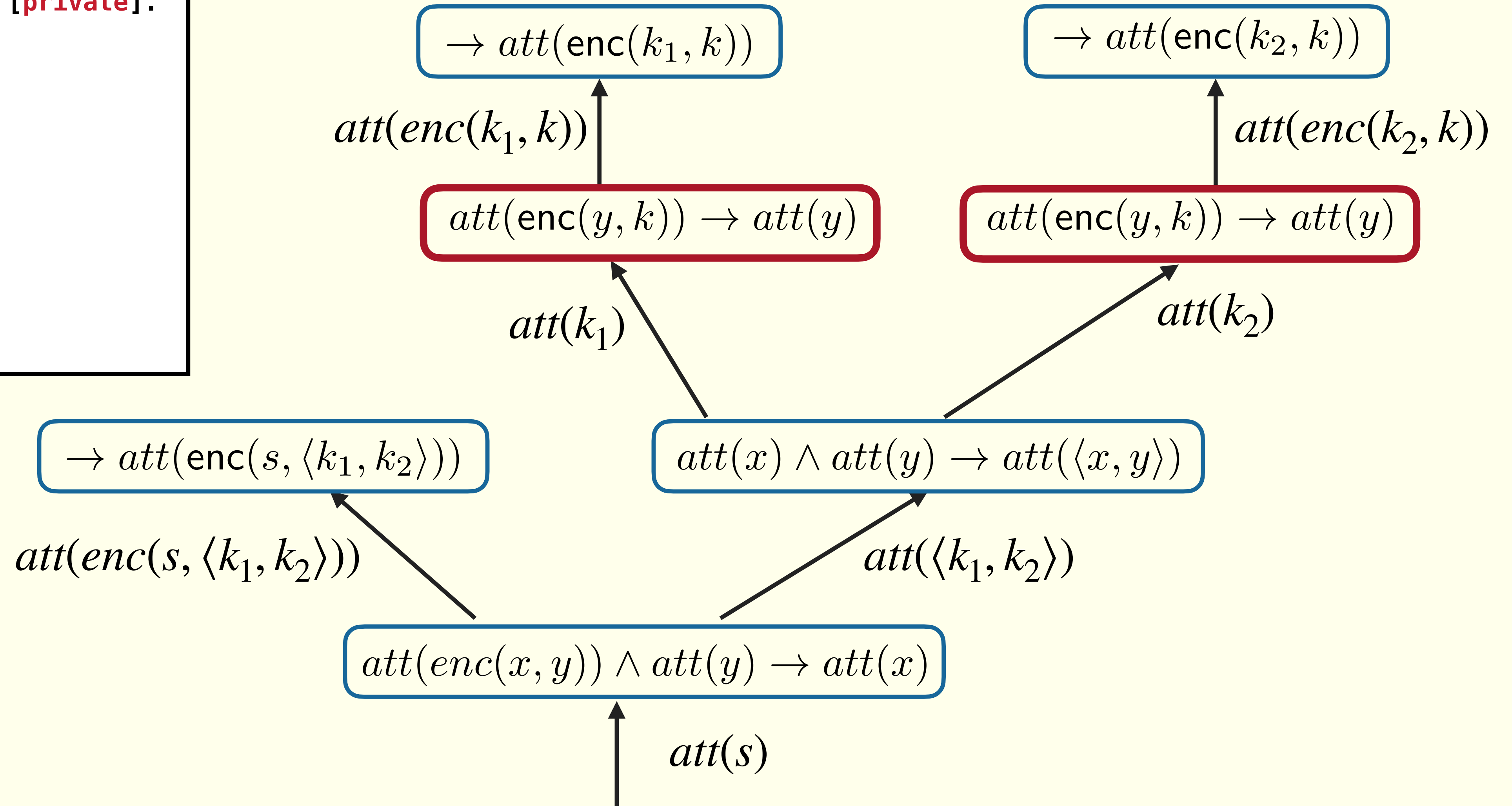
# On the derivation

```
free s,k1,k2,k:bitstring [private].
```

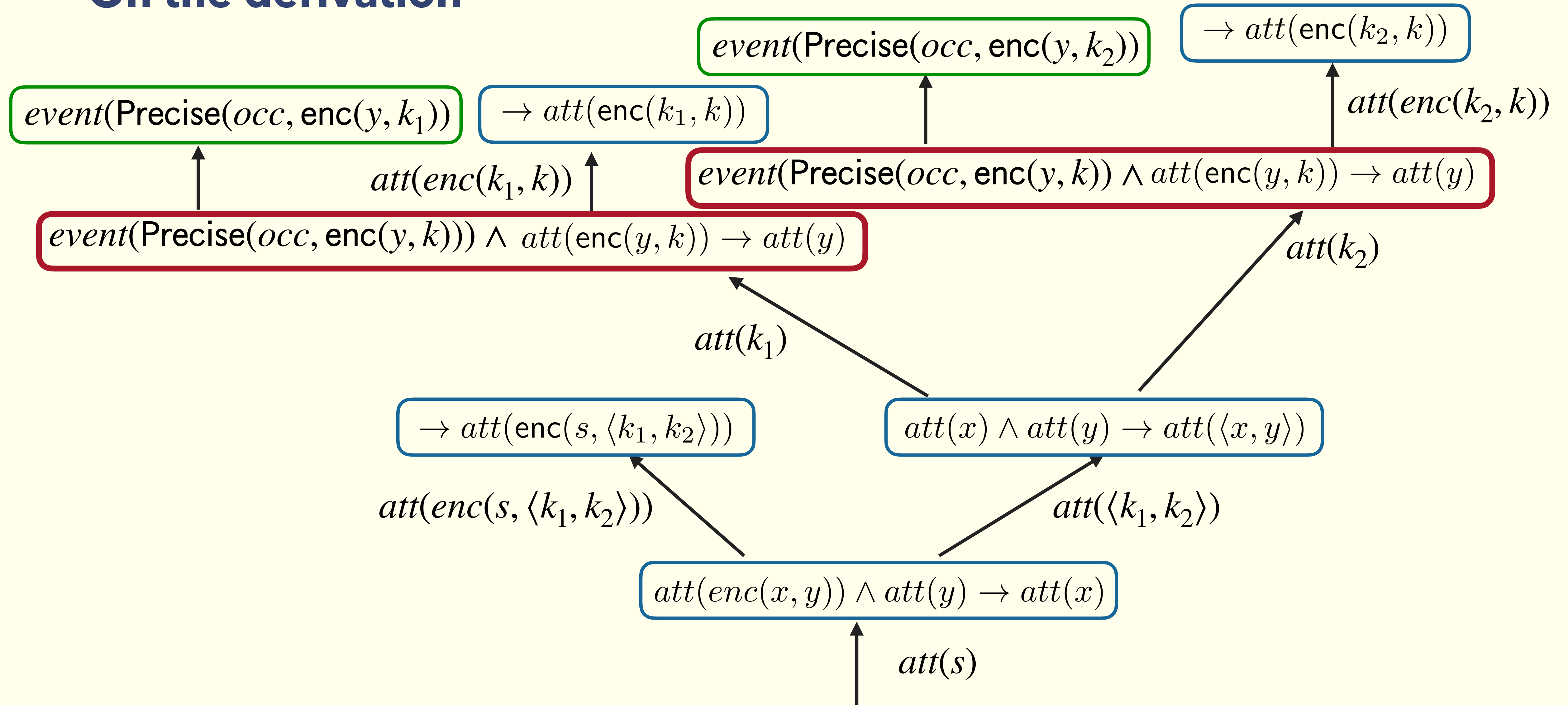
```
let A =  
  out(c,senc(s,(k1,k2)));  
  out(c,senc(k1,k));  
  out(c,senc(k2,k)).
```

```
let B =  
  in(c,x);  
  out(c,dec(x,k)).
```

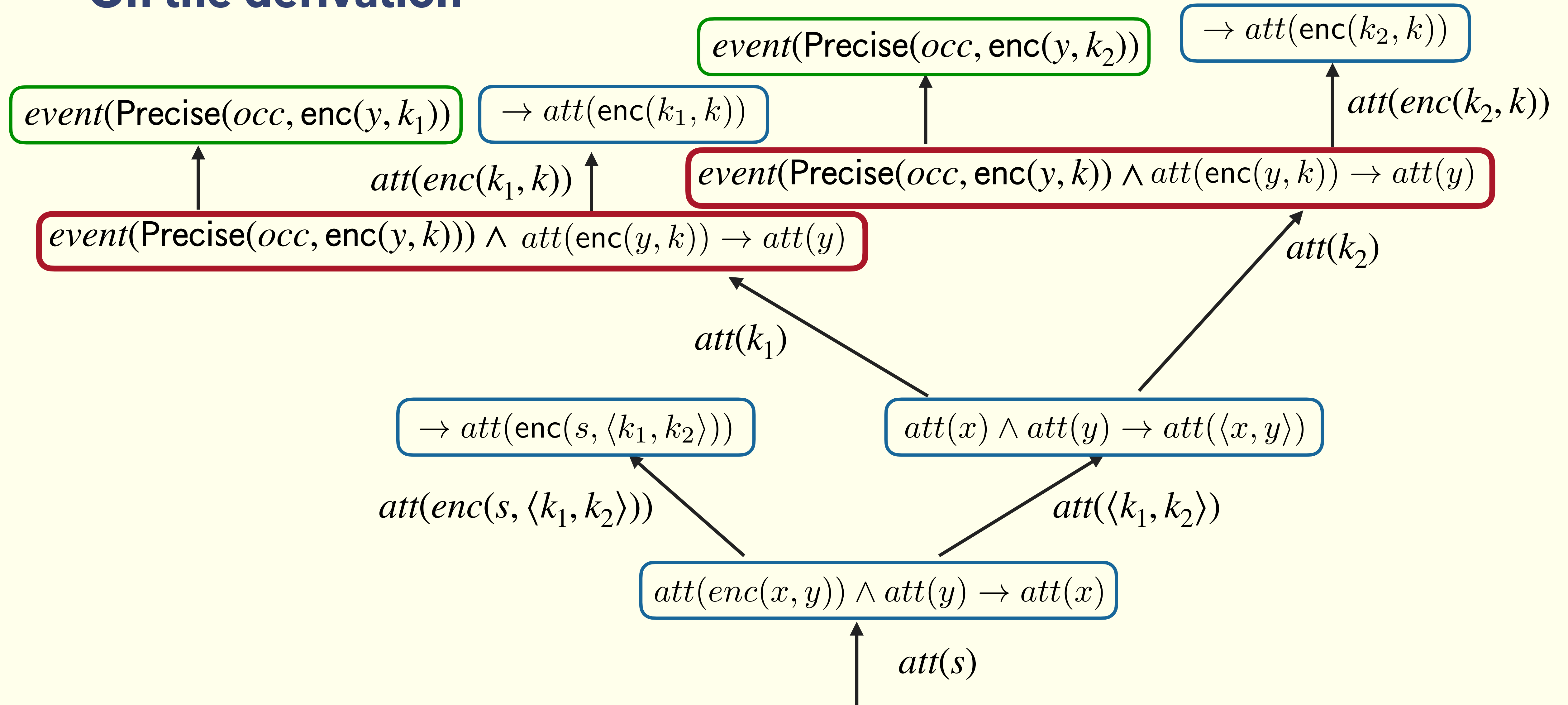
```
process A | B
```



## On the derivation



## On the derivation



# When to use Lemmas, Axioms and restrictions ?

## Restriction

To avoid heavy encoding in the calculus

Ex : To model that a process does not accept twice the same message through multiple session

```
restriction
occ1,occ2:occurrence,x:bitstring;
event(Unique(occ1,x)) &&
event(Unique(occ2,x)) ==> occ1 = occ2.

let P =
in(c,x);
new occ[:occurrence;
event Unique(occ,x);
...
```

## Lemma

When you the property can help proving the main query.

## Axiom

Ideally, always use lemma. Use axiom when you can prove by hand (or with another tool) that your property holds ... and ProVerif cannot.



# Does it work ?

## Published protocols

Protocol	Q	O	#	N
PCV Otway-Rees	eq	✗	1	✓
PCV Needham-Schreder	inj	✗	6	✓
			3	⚡
PCV Denning-Sacco	inj	✗	1	⚡
JFK	cor	✗	2	⚡
	inj		2	✓
Arinc823	cor	✗	6	⚡
Helios-norevote	eq	✗	4	✓
Signal	cor	✗	2	⚡
TLS12-TLS13-draft18	cor	✗	1	⚡

## Unpublished protocols

Protocol	Q	O	#	N
QBC_4qbits	cor	✗	1	✓
			1	⚡
Voting-draft	eq	✗	1	✓
LAK-simplified	cor	🕒	1	✓
PACE_v3-sequence	cor	✗	1	✓
			3	⚡
DP-3T-simpl-draft	cor	🕒	1	✓
			2	⚡
student1	cor	🕒	2	✓
	inj		1	⚡
student2	inj	✗	1	✓
student3	cor	🕒	1	⚡
student4	cor	✗	2	⚡
student5	cor	🕒	1	⚡

---

# Outline

- A. Additional modeling techniques
  - 1. Equational Theory vs Rewrite rules
  - 2. Equivalence properties
  - 3. Memory cell and locks for stateful protocols
- B. Dealing with « cannot be proved »
  - 1. Adding precision
  - 2. Trace with assumption
  - 3. Proof by induction
  - 4. Lemma
- C. Dealing with non-termination



# How to determine if ProVerif does not terminate ?

## The first clues

```
Translating the process into Horn clauses...
Completing...
200 rules inserted. Base: 200 rules (97 with conclusion selected). Queue: 679 rules.
400 rules inserted. Base: 400 rules (133 with conclusion selected). Queue: 481 rules.
600 rules inserted. Base: 600 rules (133 with conclusion selected). Queue: 291 rules.
800 rules inserted. Base: 800 rules (133 with conclusion selected). Queue: 135 rules.
1000 rules inserted. Base: 997 rules (157 with conclusion selected). Queue: 184 rules.
1200 rules inserted. Base: 1093 rules (204 with conclusion selected). Queue: 134 rules.
1400 rules inserted. Base: 1253 rules (293 with conclusion selected). Queue: 208 rules.
1600 rules inserted. Base: 1420 rules (352 with conclusion selected). Queue: 281 rules.
1800 rules inserted. Base: 1596 rules (382 with conclusion selected). Queue: 315 rules.
2000 rules inserted. Base: 1790 rules (394 with conclusion selected). Queue: 369 rules.
2200 rules inserted. Base: 1970 rules (400 with conclusion selected). Queue: 387 rules.
2400 rules inserted. Base: 2166 rules (400 with conclusion selected). Queue: 393 rules.
2600 rules inserted. Base: 2323 rules (402 with conclusion selected). Queue: 423 rules.
2800 rules inserted. Base: 2507 rules (402 with conclusion selected). Queue: 447 rules.
3000 rules inserted. Base: 2644 rules (416 with conclusion selected). Queue: 484 rules.
3200 rules inserted. Base: 2790 rules (416 with conclusion selected). Queue: 500 rules.
3400 rules inserted. Base: 2933 rules (443 with conclusion selected). Queue: 547 rules.
3600 rules inserted. Base: 3068 rules (443 with conclusion selected). Queue: 571 rules.
3800 rules inserted. Base: 3209 rules (464 with conclusion selected). Queue: 617 rules.
4000 rules inserted. Base: 3320 rules (484 with conclusion selected). Queue: 715 rules.
4200 rules inserted. Base: 3408 rules (484 with conclusion selected). Queue: 747 rules.
4400 rules inserted. Base: 3529 rules (498 with conclusion selected). Queue: 756 rules.
4600 rules inserted. Base: 3637 rules (530 with conclusion selected). Queue: 804 rules.
4800 rules inserted. Base: 3705 rules (530 with conclusion selected). Queue: 882 rules.
...
...
```

Number of rules  
generated

Current size of the set  
of rules

Number of rules left to  
handle

- Size of the queue seems to always increase
- Size of the queue seems to be cyclic
- No rule inserted for ages (can happen with lemmas)
- Termination warnings



Be patient 😊

On TLS 1.3, terminates  
with 200k rules inserted.

# How to determine if ProVerif does not terminate ?

The real way to do it...

```
set verboseRules = true.
```

Display all the rules generated

```
Rule with hypothesis fact 0 selected: mess(cellQ[],i_2)
mess(cellQ[],i_2) -> mess(cellQ[],i_2)
The hypothesis occurs before the conclusion.
1 rules inserted. Base: 1 rules (0 with conclusion selected). Queue: 3 rules.

Rule with hypothesis fact 0 selected: mess(cellQ[],i_2)
is_nat(i_2) && mess(cellQ[],i_2) -> mess(cellQ[],i_2 + 1)
The hypothesis occurs strictly before the conclusion.
2 rules inserted. Base: 2 rules (0 with conclusion selected). Queue: 5 rules.

Rule with conclusion selected:
mess(cellQ[],0)
3 rules inserted. Base: 3 rules (1 with conclusion selected). Queue: 4 rules.

Rule with hypothesis fact 0 selected: attacker(cellQ[])
attacker(cellQ[]) && attacker(i_2) -> mess(cellQ[],i_2)
The 1st, 2nd hypotheses occur before the conclusion.
4 rules inserted. Base: 4 rules (1 with conclusion selected). Queue: 3 rules.

Rule with hypothesis fact 0 selected: mess(cellQ[],i_2)
is_nat(i_2) && mess(cellQ[],i_2) -> mess(cellQ[],i_2 + 2)
The hypothesis occurs strictly before the conclusion.
5 rules inserted. Base: 5 rules (1 with conclusion selected). Queue: 5 rules.

Rule with conclusion selected:
mess(cellQ[],1)
6 rules inserted. Base: 6 rules (2 with conclusion selected). Queue: 4 rules.

Rule with hypothesis fact 0 selected: attacker(cellQ[])
is_nat(i_2) && attacker(cellQ[]) && attacker(i_2) -> mess(cellQ[],i_2 + 1)
The 1st, 2nd hypotheses occur strictly before the conclusion.
7 rules inserted. Base: 7 rules (2 with conclusion selected). Queue: 3 rules.
```

## Signs of a cycle

- Size of the term in the conclusion increases
- Number of hypotheses increases



Very long and painful to read



Best way to find the problem



Best way to understand how to solve it

# Not attacker declaration and lemmas

Look if some facts should not be true

Rule with hypothesis fact 0 selected: attacker(cellQ[])  
attacker(cellQ[]) && attacker(i\_2) -> mess(cellQ[],i\_2)  
The 1st, 2nd hypotheses occur before the conclusion.  
4 rules inserted. Base: 4 rules (1 with conclusion selected). Queue: 3 rules.

The cell should be private

Rule with hypothesis fact 1 selected: attacker(h(i))  
is\_not\_nat(i\_2) && event(Accept(i\_2)) && attacker(h(i)) -> mess(cellQ[],i\_2)  
The 1st, 2nd hypotheses occur before the conclusion.  
14 rules inserted. Base: 3 rules (2 with conclusion selected). Queue: 3 rules.

The content of the cell should be natural numbers

`not attacker(cellQ).`

Faster but semantically equivalent to

`Lemma attacker(cellQ).`

`lemma i:nat; mess(cellQ,i) ==> is_nat(i).`

# Playing with the selection function

The fact that will be selected for resolution

Rule with hypothesis fact 1 selected: `attacker(h(i))`  
`is_not_nat(i_2) && event(Accept(i_2)) && attacker(h(i)) -> mess(cellQ[],i_2)`  
The 1st, 2nd hypotheses occur before the conclusion.  
14 rules inserted. Base: 3 rules (2 with conclusion selected). Queue: 3 rules.

The automatic detection of selections to avoid is not perfect

If you think this fact will lead to non-termination, you can tell it to ProVerif

```
noselect i:nat, attacker(h(i)).  
noselect i:nat, attacker(h(*i)).
```

`*i` means « any term »

```
select mess(cellQ[],i_2)/-5000
```

Clauses generated from the process  $Q$

$$\text{mess}(\text{cell}Q, i) \rightarrow \text{mess}(\text{cell}Q, i + 1)$$
$$\text{mess}(\text{cell}Q, i_1) \rightarrow \text{mess}(\text{cell}Q, i_1 + 2)$$
$$\text{mess}(\text{cell}Q, i_2) \rightarrow \text{mess}(\text{cell}Q, i_2 + 3)$$

⋮



---

## What's next ?

### **Development of GSVerif as a frontend and backend**

Current: Frontend for stateful protocol

Futur: Backend of ProVerif to suggest user-features to apply

### **Going beyond diff-equivalence**

#### **Modulo XOR / Abelian group**

Current: Not accepted by ProVerif

Main concerns: Efficiency and non-termination

#### **Certified ProVerif (very long term)**

ProVerif would generate a certificate of a successful verification

Certificate verifier written in  $F^*$

### **Improve memory consumption and efficiency**