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**UNITED'S**  
**SPEEDY**  
**MATHS**

**Revised and Enlarged Edition** **10**

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**United Nepal Publication (P) Ltd.**  
Kathmandu, Nepal

# Speedy Maths

## Book 10

Revised and Enlarged Edition

### Author

Hukum Pd. Dahal

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**First Edition: B.S. 2069 (2012 A.D.)**

**Revised and Enlarged Edition: B.S. 2074 (2017 A.D.)**

**Re-print: B.S. 2076 (2019 A.D.)**

*Layout, Design and Illustration*

**Pradeep Kandel**

**Printed in NEPAL**

### Published by:

United Nepal Publication (P) Ltd.

Kathmandu, Nepal

Phone No.: 01-4385020/4385021

E-mail: [united.npublication@gmail.com](mailto:united.npublication@gmail.com)

[www.unitednepalpublication.com.np](http://www.unitednepalpublication.com.np)

# Foreword

**Speedy Maths** is an innovative and activity-oriented school level mathematics series. The series has incorporated applied constructivism – the latest trend of learner-centred teaching-learning pedagogy. Every chapter of the whole series is designed in such a manner that makes the classes constructive automatically. As a result, the learners actively participate in the learning processes. Furthermore, they are enabled to construct knowledge themselves rather than just receiving readymade information from their instructor. The teachers no more remain the traditionally imposing instructors. They would rather play the role of accessible facilitators and guides.

The series of **Speedy Maths** is a unique set of mathematics books as it is the fusion of mental and manual mathematical work and activities. In the series, following the appropriate approach ‘Mental operation and judgement’ based on the latest pedagogy of teaching/learning mathematics, mental mathematical problems are so designed as to achieve knowledge, understanding and skill based learning objectives. Additionally, the mental maths section in every exercise of each book of the series enables the learners to enhance their instant computational skill. Likewise, manual mathematics fulfils the achievement of overall learning objectives such as knowledge, understanding, skill and higher ability.

In each of the **Speedy Maths**’ exercises in grades 4 and 5 in primary level, and in lower secondary level, classwork section is provided for the immediate diagnostic evaluation of the learners. The approach helps teachers to judge whether the learners are able to construct the fundamental concept in the related mathematical content or not. The problems in this section are designed to evaluate the achievement of knowledge and understanding based objectives. Furthermore, the problems in this section develop the computational skill of the learners.

**Speedy Maths**, in fact, has drawn remarkable impression and inspiration from real teaching-learning situations, teachers’ training programmes, workshops, seminars and symposia. As a result, the series, I believe, has become lucid. So, learning mathematics is going to be joyful for students.

The series is completely revised according to the latest curriculum of mathematics, designed and developed by the Curriculum Development Centre (CDC), the Government of Nepal.

I hope both the students and teachers will be highly benefitted from Speedy Maths.

For further improvement and enhancement of the series, constructive comments and suggestions are always welcome.

The Author

# Acknowledgments

**Speedy Maths** is an absolutely modified and revised edition of my previous two series 'Elementary Mathematics' and 'Maths in Action'. In making effective modification and revision, I'm highly grateful to the principals, mathematics teachers and experts, PABSON, NPABSAN, PETSAN, ISAN and independent bodies of some Private Boarding Schools, that provided me with opportunities to participate in many workshops, Seminars, Teachers' training, Interaction programmes and symposia as the resource person. Such programmes helped me a lot to investigate the teaching-learning problems and to research the possible remedies.

I'm very proud of my wife Rita Rai Dahal who always encourages me to write the texts in a more effective way so that the texts stand as useful and unique in all respects. I'm equally grateful to my son Bishwant Dahal and my daughter Sunayana Dahal for their important roles as my 'Model Learners'.

I'm extremely grateful to Dr. Ruth Green, a retired professor from Leeds University, England who provided me very valuable suggestions about the effective methods of teaching-learning mathematics and many reference materials.

Grateful thanks are due to Mr. Bijay Kumar Basnet and Mr. Shaligram Ghimire, the Former English News Reader, the Nepal Television, for painstakingly editing the language of the series. I am thankful to Mr. Rameshwor Prasad Yadav for helping me in preparing answers to the problems.

Moreover, I gratefully acknowledge all Mathematics Teachers throughout the country who encouraged me and provided me the necessary feedback during the workshops/interactions and teachers' training programmes in order to prepare the series in this shape.

I'm profoundly grateful to the United Nepal Publication (P) Ltd. to get this series published. I would like to thank Chairperson Mr. Sarba Kalyan Kandel, Managing Director Mr. Jiwan Shrestha, Former M. D. Mr. Dhana Raj Acharya, Production Director Mr. Suresh Kumar Regmi, Marketing Director Mr. Manoj Kumar Regmi for their invaluable suggestions during the preparation of the series.

Last but not the least, I'm heartily thankful to Mr. Pradeep Kandel, the Senior Designer of the publication house for his skill in designing the series in such an attractive form.

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# 1

## Chapter

## Set

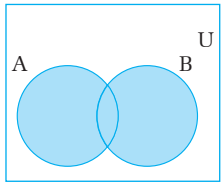
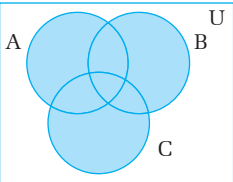
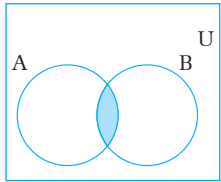
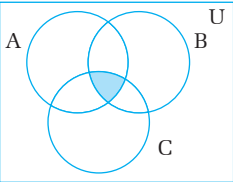
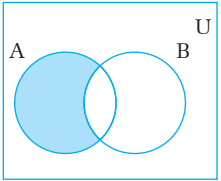
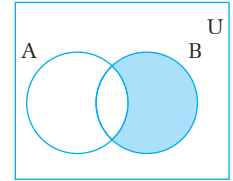
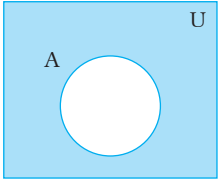
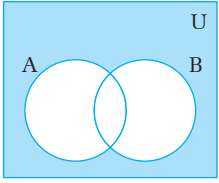
### 1.1 Set operations and use of Venn-diagrams

There are four main set operations. They are,

- (i) Union of sets                      (ii) Intersection of sets
- (iii) Difference of sets              (iv) Complement of sets

Sets and set operations can also be represented by diagrams. Such diagrams are called Venn-diagrams. The idea of representation of sets and set operations in diagrams was first introduced by **Euler John Venn** in 20<sup>th</sup> century.

Let's learn the operations of sets by using Venn-diagrams.

Set operations	Use of Venn-diagrams
<b>(i) Union of sets</b>  The union of sets A and B is the set of all members that belong either to A or to B or to both A and B.  $A \cup B = \{x : x \in A \text{ or } x \in B\}$	 $A \cup B$  $A \cup B \cup C$
<b>(ii) Intersection of sets</b>  The intersection of sets A and B is the set that contains the elements common to A and B.  $A \cap B = \{x : x \in A \text{ and } x \in B\}$	 $A \cap B$  $A \cap B \cap C$
<b>(iii) Difference of two sets</b>  The difference of sets A and B is the set of the elements of A which do not belong to B. $A - B = \{x : x \in A, \text{ but } x \notin B\}$ $B - A = \{x : x \in B, \text{ but } x \notin A\}$	 $A - B$  $B - A$
<b>(iv) Complement of a set</b>  If A be a subset of a universal set U, then the complement of A is the set of the elements of U which do not belong to A.  $\bar{A} = \{x : x \in U, \text{ but } x \notin A\}$	 $\bar{A}$  $\overline{A \cup B}$

## 1.2 Cardinality relations of sets

The cardinal number of a set is called its cardinality. Certain relations can be generalised by taking the cardinalities of different sets. Such relations are very much important as the application of set theory.

### 1. Cardinality relations of union of two sets

If A and B are two overlapping sets in a universal set U, then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

**Proof**

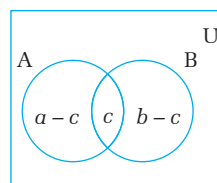
Let,  $n(A) = a$ ,  $n(B) = b$  and  $n(A \cap B) = c$

Then, from the Venn-diagram,

$$n(A \cup B) = (a - c) + c + (b - c)$$

$$= a + b - c$$

$$= n(A) + n(B) - n(A \cap B) \quad \text{Proved}$$



Let's learn a few more relations which are derived from the cardinality relation of union of two sets.

(i) If A and B are disjoint sets, then

$$n(A \cup B) = n(A) + n(B)$$

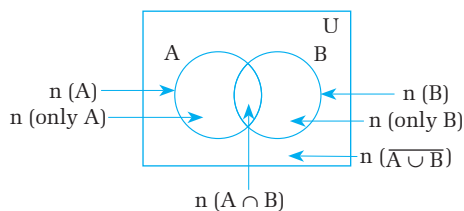
(ii)  $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

(iii)  $n(A \cup B) = n_o(A) + n_o(B) + n(A \cap B)$

(iv)  $n(\text{only } A) = n_o(A) = n(A) - n(A \cap B)$

(v)  $n(\text{only } B) = n_o(B) = n(B) - n(A \cap B)$

(vi)  $n(\overline{A \cup B}) = n(U) - n(A \cup B)$



### Worked-out examples

**Example 1:** If  $n(U) = 120$ ,  $n(A) = 55$ ,  $n(B) = 75$  and  $n(A \cup B) = 100$ , find

(i)  $n(\overline{A \cup B})$       (ii)  $n(A \cap B)$       (iii)  $n_o(A)$       (iv)  $n_o(B)$ .

Draw a Venn-diagram to illustrate the above information.

**Solution:**

Here,  $n(U) = 120$

$n(A) = 55$ ,  $n(B) = 75$

$n(A \cup B) = 100$

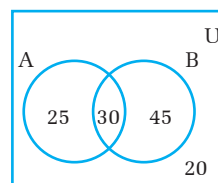
(i) Now,  $n(\overline{A \cup B}) = n(U) - n(A \cup B)$   
 $= 120 - 100 = 20$

(ii) Also,  $n(A \cap B) = n(A) + n(B) - n(A \cup B)$   
 $= 55 + 75 - 100 = 30$

(iii) Again,  $n_o(A) = n(A) - n(A \cap B) = 55 - 30 = 25$

(iv) And,  $n_o(B) = n(B) - n(A \cap B) = 75 - 30 = 45$

**Venn-diagram**





**Example 2:** In an examination, 85 % students got grade A in English, 75 % got grade A in Science and every student got grade A at least one subject. Find the percent of students who got grade A

- (i) in both subjects.
- (ii) What percent of students got grade A in Science only?
- (iii) Show the information in a Venn-diagram.

**Solution:**

Let, E and S denote the sets of students who got grade A in English and Science respectively.

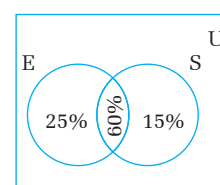
Here,  $n(E) = 85\%$ ,  $n(S) = 75\%$   $n(U) = n(E \cup S) = 100\%$

$$\begin{aligned} \text{(i) Now, } n(E \cap S) &= n(E) + n(S) - n(E \cup S) \\ &= 85\% + 75\% - 100\% = 60\% \end{aligned}$$

$$\begin{aligned} \text{(ii) Again, } n_o(S) &= n(S) - n(E \cap S) \\ &= 75\% - 60\% = 15\% \end{aligned}$$

So, 60 % of the students got grade A in both subjects and 15 % got grade A in Science only.

**(iii) Venn-diagram**



**Example 3:** In a group of 100 students, 68 like football, 60 like volleyball and 2 of them do not like any of the games. Find the number of students who like both the games and only football by using a Venn-diagram.

**Solution:**

Let, F and V denote the sets of students who like football and volleyball respectively.

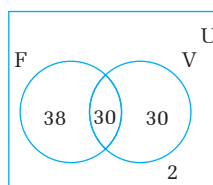
Here,  $n(U) = 100$ ,  $n(F) = 68$ ,  $n(V) = 60$ ,  $(\overline{F \cup V}) = 2$ .

$$\begin{aligned} \text{Now, } n(F \cup V) &= n(U) - n(\overline{F \cup V}) \\ &= 100 - 2 = 98 \end{aligned}$$

$$\begin{aligned} \text{Again, } n(F \cap V) &= n(F) + n(V) - n(F \cup V) \\ &= 68 + 60 - 98 = 30 \end{aligned}$$

$$\text{Also, } n_o(F) = n(F) - n(F \cap V) = 68 - 30 = 38$$

So, 30 students like both the games and 38 like only football.



**Example 4:** In a group of 115 people, 55 like tea only, 48 like coffee only and each person likes at least one of the two drinks.

- (i) Draw a Venn-diagram to illustrate the above information.
- (ii) How many people like both the drinks?
- (iii) How many people like tea?
- (iv) How many people like coffee?

**Solution:**

Let, T and C denote the set of students who like tea and coffee respectively.

Here,  $n(U) = n(T \cup C) = 115$ ,  $n_o(T) = 55$  and  $n_o(C) = 48$

(i) Now,  $n(T \cup C) = n_o(T) + n_o(C) + n(T \cap C)$

$$\text{or, } 115 = 55 + 48 + n(T \cap C)$$

$$\text{or, } n(T \cap C) = 12$$

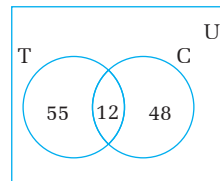
(ii) From the Venn-diagram, 12 people like both the drinks.

$$\begin{aligned} \text{(iii) } n(T) &= n_o(T) + n(T \cap C) \\ &= 55 + 12 = 67 \end{aligned}$$

So, 67 people like tea.

$$\begin{aligned} \text{(iv) } n(C) &= n_o(C) + n(T \cap C) \\ &= 48 + 12 = 60 \end{aligned}$$

So, 60 people like coffee.



**Example 5:** 44 students in a class like Mathematics or Science or both. Out of them 12 like both subjects. The ratio of the number of students who like Mathematics to those who like Science is 4 : 3.

(i) Find the number of students who like Mathematics.

(ii) Find the number of students who like Science only.

(iii) Represent the above information in a Venn-diagram.

**Solution:**

Let M and S denote the sets of students who like Mathematics and Science respectively.

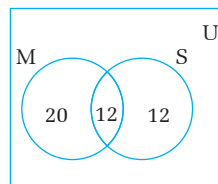
Here,  $n(U) = n(M \cup S) = 44$  and  $n(M \cap S) = 12$

Let,  $n(M) = 4x$  and  $n(S) = 3x$

Now,  $n(M \cup S) = n(M) + n(S) - n(M \cap S)$

$$\text{or, } 44 = 4x + 3x - 12$$

$$\text{or, } x = 8$$



(i) The number of students who like Mathematics  $= n(M) = 4 \times 8 = 32$

(ii) Also, the number of students who like Science  $= n(S) = 3 \times 8 = 24$

$$\begin{aligned} \therefore \text{The number of students who like Science only} &= n_o(S) = n(S) - n(M \cap S) \\ &= 24 - 12 = 12 \end{aligned}$$

**Example 6:** In a survey, it was found that the ratio of the people who liked modern songs and folk songs is 8 : 9. Out of which, 50 people liked both the songs, 40 liked folk songs only and 80 liked none of the songs.

(i) Represent the above data in a Venn-diagram.

(ii) Find the number of people who participated in the survey.

**Solution:**

Let M and F denote the sets of people who liked modern and folk songs respectively.

Here,  $n(M \cap F) = 50$ ,  $n_o(F) = 40$  and  $n(\overline{M \cup F}) = 80$

$$\begin{aligned}\text{Now, } n(F) &= n_o(F) + n(M \cap F) \\ &= 40 + 50 = 90\end{aligned}$$

$$\text{Let } n(M) = 8x \text{ and } n(F) = 9x$$

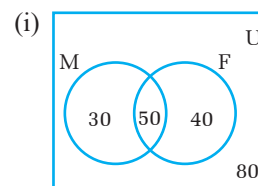
$$\text{Then, } 9x = 90$$

$$\text{or, } x = 10$$

$$\therefore n(M) = 8x = 8 \times 10 = 80$$

(ii) From the Venn-diagram,  $n(U) = 30 + 50 + 40 + 80 = 200$

So, 200 people participated in the survey.



**Example 7:** In a survey of some students, it was found that 45 % of them like football, 60 % like cricket and 15 % of them like none of the games.

(i) Represent the above information in a Venn-diagram.

(ii) If there are 60 students who like both the games, find the number of students who participated in the survey.

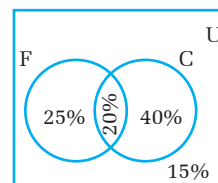
**Solution:**

Let F and C be the sets of the students who like football and cricket respectively.

Here,  $n(U) = 100\%$ ,  $n(F) = 45\%$ ,  $n(C) = 60\%$  and  $n(\overline{F \cup C}) = 15\%$

$$\begin{aligned}\text{(i) Now, } n(F \cup C) &= n(U) - n(\overline{F \cup C}) \\ &= 100\% - 15\% \\ &= 85\%\end{aligned}$$

$$\begin{aligned}\text{Again, } n(F \cap C) &= n(F) + n(C) - n(F \cup C) \\ &= 45\% + 60\% - 85\% \\ &= 20\%\end{aligned}$$



(ii) Let the number of students who participated in the survey be x.

Then, 20% of x = 60

$$\text{or, } x = 300$$

So, 300 students participated in the survey.

**Example 8:** In an examination, 60 % examinees failed in Mathematics, 55 % failed in English and 24 failed in both the subjects. If none of the examinees passed in both the subjects, find

(i) the number of examinees who passed in Mathematics only

(ii) Represent the above information in a Venn-diagram.

**Solution:**

Let the sets of examinees who failed in Mathematics and English are M and E respectively.

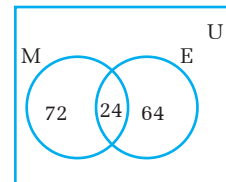
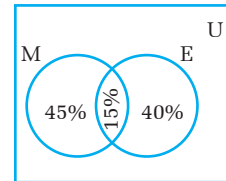
Here,  $n(U) = n(M \cup E) = 100\%$ ,  $n(M) = 60\%$  and  $n(E) = 55\%$

$$\begin{aligned}\text{Now, } n(M \cap E) &= n(M) + n(E) - n(M \cup E) \\ &= 60\% + 55\% - 100\% \\ &= 15\%\end{aligned}$$

Let the total number of examinees be x.

Then, 15 % of x = 24

or,  $x = 160$



Again, percent of examinees who passed in Mathematics only =  $(55 - 15)\%$  or  $(100 - 60)\%$   
 $= 40\%$

The number of examinees who passed in Mathematics only =  $40\%$  of 160  
 $= 64$

**Example 9:** In an examination, 48 % students got grade B in History only and 35 % got grade B in Geography only. If 12 % students got grade D in both the subjects, find

- (i) the percent of students who got grade B in both the subjects.
- (ii) What percent of students got grade B in History?
- (iii) Represent all the results in a Venn-diagram.

**Solution:**

Let the sets of examinees who got grade B in History and Geography are H and G respectively.

Here,  $n(U) = 100\%$ ,  $n_o(H) = 48\%$ ,  $n_o(G) = 35\%$  and  $n(\overline{H \cup G}) = 12\%$

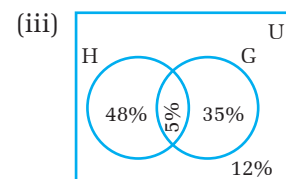
$$\begin{aligned}\text{(i) Now, } n(H \cup G) &= n(U) - n(\overline{H \cup G}) \\ &= 100\% - 12\% \\ &= 88\%\end{aligned}$$

$\therefore$  Percentage of students who got grade B in both subjects

$$\begin{aligned}n(H \cap G) &= n(H \cup G) - n_o(H) - n_o(G) \\ &= 88\% - 48\% - 35\% \\ &= 5\%\end{aligned}$$

(ii) Again, percentage of students who got grade B in History

$$\begin{aligned}&= n_o(H) + n(H \cap G) \\ &= 48\% + 5\% \\ &= 53\%\end{aligned}$$



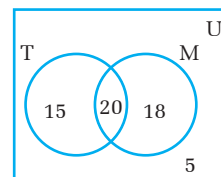
## EXERCISE 1.1

### General section

1. a) In the adjoining Venn-diagram, T and M are sets of students who like tea and milk respectively. Find

(i)  $n(T \cup M)$     (ii)  $n(T \cap M)$     (iii)  $n(\overline{T \cup M})$

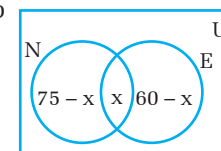
(iv)  $n_o(T)$     (v)  $n_o(M)$     (vi)  $n(U)$



- b) In the given Venn-diagram, N and E are the sets of people who speak Nepali and English languages respectively.

If  $n(U) = 150$  and  $n(N \cup M) = 110$ , find

(i)  $n(N \cap M)$     (ii)  $n(\overline{N \cup E})$     (iii)  $n_o(N)$     (iv)  $n_o(E)$



2. a) If  $n(U) = 50$ ,  $n(A) = 25$ ,  $n(B) = 27$  and  $n(A \cap B) = 10$ , find  
 (i)  $n(A \cup B)$     (ii)  $n(\overline{A \cup B})$     (iii)  $n(\text{only } A)$     (iv)  $n_o(B)$
- b) If  $n(U) = 60$ ,  $n(A) = 30$ ,  $n(B) = 40$  and  $n(A \cup B) = 55$ , find  
 (i)  $n(A \cap B)$     (ii)  $n(\overline{A \cup B})$     (iii)  $n_o(A)$     (iv)  $n(\text{only } B)$
- c) P and Q are the subsets of a universal set U. If  $n(U) = 90$ ,  $n(P) = 45$ ,  $n(Q) = 35$  and  $n(\overline{P \cup Q}) = 15$ , illustrate this information in a Venn-diagram and find  
 (i)  $n(P \cup Q)$     (ii)  $n(P \cap Q)$     (iii)  $n_o(P)$     (iv)  $n_o(Q)$
- d) If  $n(A) = 65$ ,  $n(B) = 80$  and  $A \subset B$ , then find  
 (i)  $n(A \cup B)$     (ii)  $n(A \cap B)$     (iii)  $n(A - B)$     (iv)  $n(B - A)$
- e) If  $n_o(A) = 12$ ,  $n_o(B) = 15$ ,  $n(\overline{A \cup B}) = 11$  and  $n(U) = 45$ , find  
 (i)  $n(A \cup B)$     (ii)  $n(A \cap B)$     (iii)  $n(A)$     (iv)  $n(B)$

### Creative section

3. a) In a survey of 350 students of a school, 200 liked Pokhara, 220 liked Chitwan and 120 liked both the places.  
 (i) Show the above information in a Venn-diagram.  
 (ii) Find the number of students who liked neither of two places.  
 (iii) Find the number of students who liked only Pokhara.
- b) Among 1,200 Chinese tourists who are visiting Nepal, 45% of them have already visited India, 40 % have visited Pakistan and 15 % have visited both the countries.  
 (i) Illustrate this information in a Venn-diagram.  
 (ii) How many tourists have not visited any of these two countries?

- c) In a group of 200 game lover students, 120 like cricket and 105 like football. By drawing a Venn-diagram, find
- how many students like both the games?
  - How many students like only cricket?
- d) In a school, every student has to participate at least one of the activities, athletics or music. In a class of 50 students 30 participated in athletics and 25 participated in music.
- Draw a Venn-diagram to illustrate the above information.
  - How many students participated in only one activity?
- e) In an election of a municipality, A and B were two candidates for the post of the Mayor and 25000 voters were in the voter list. Voters were supposed to cast the vote for a single candidate. 12000 people cast vote for A, 10000 people cast for B and 1000 people cast vote even for both the candidates.
- Show these information in a Venn-diagram
  - How many people didn't cast the vote?
  - How many votes were valid?
4. a) In a survey of 900 students in a school, it was found that 600 students liked tea, 500 liked coffee and 125 did not like both the drinks.
- Draw a Venn-diagram to illustrate the above information.
  - Find the number of students who liked both the drinks.
  - Find the number of students who didn't like tea only.
- b) In a group of 120 people, 90 play volleyball, 72 play football and 10 play neither of games.
- Draw a Venn-diagram to represent the given information.
  - How many people play only one game?
- c) In a class of 60 students, 15 students liked maths only, 20 liked English only and 5 didn't like any subject.
- Represent the above information in a Venn-diagram.
  - Find the number of students who liked both the subjects.
  - How many students liked maths?
- d) In a survey of a group of people, 50 % liked to listen radio and 60% liked to watch television. If 30 % of them neither listen radio nor watch television, then
- Represent the above information in a Venn-diagram.
  - Find the percent of people who like to listen radio as well as watch television.
- e) In a survey of a community, 60 % people read Daily-newspaper, 50 % read Weekly-newspaper and 20 % people do not read any type of newspaper.
- Illustrate the above information in a Venn-diagram.
  - If 1500 people read both types of paper, how many people were surveyed?
- f) Out of some students appeared in an examination, 80% passed in Mathematics, 75% passed in English and 5% failed in both subjects. If 300 of them were passed in both subjects, how many students were appeared in the examination, find by using a Venn-diagram.

- g) In a survey of 15,000 students of different schools, 40 % of them were found to have tuition before the SEE examination. Among them 50 % studied only Mathematics, 30 % only Science and 10 % studied other subjects.
- (i) Represent the above information in a Venn-diagram,  
(ii) How many students studied Mathematics as well as Science?
- h) In an examination 45 % students passed in Science only, 25 % passed in English only and 5 % students failed in both subjects. If 200 students passed in English, find the total number of students by using Venn-diagram.
5. a) In a survey of some students, it was found that 60 % of them studied Commerce and 40 % studied Science. If 40 students studied both the subjects and 10 % didn't study any of the subjects, by drawing a Venn-diagram,
- (i) find the total number of students.  
(ii) Find the number of students who studied Science only.
- b) In an examination, 50 % examinees got grade A in Mathematics, 75 % got grade A in Science and 60 got grade A in both the subjects. If none of the examinees got grade B in both the subjects,
- (i) represent the given information in a Venn-diagram.  
(ii) Find the number of examinees who got grade A in Science only.
- c) In an examination, 80 % examinees passed in English, 70 % in Mathematics and 60 % in both the subjects. If 45 examinees failed in both subjects.
- (i) draw a Venn-diagram to represent the above information.  
(ii) Find the number of examinees who passed only one subject.  
(iii) Find the number of students who failed Mathematics.
- d) In an examination, 56 % examinees failed in Science, 54 % failed in Nepali and 25 students failed in both the subjects. If none of the examinees passed in both the subjects,
- (i) find the number of examinees who passed in Science only.  
(ii) Find the number of examinees who passed in Nepali only.  
(iii) Represent the above information in a Venn-diagram.
6. a) 75 students in a class like tea or coffee or both. Out of them 10 like both the drinks. The ratio of the number of students who like tea to those who like coffee is 2 : 3.
- (i) Find the number of students who like tea.  
(ii) Find the number of students who like coffee only.  
(iii) Represent the above information in a Venn-diagram.
- b) In a survey of 825 farmers, 125 of them were not using any type of fertilizer in their vegetable farming. 150 of them were using chemical as well as organic fertilizers. The ratio of the number of only chemical fertilizer users to that of only organic fertilizer users is 5 : 6.

- (i) Find the number of chemical fertilizer users.
  - (ii) How many of them were using only one type of fertilizer?
  - (iii) How many of them were not using organic fertilizer?
  - (iv) Illustrate the above information in a Venn-diagram.
- c) In a group of students, the ratio of the number of students who liked music and sports is 9 : 7. Out of which 25 liked both the activities, 20 liked music only and 15 liked none of the activities.
- (i) Represent the above information in a Venn-diagram.
  - (ii) Find the total number of students in the group.
- d) In a survey of 80 people, it was found that 60 liked oranges only and 10 liked both oranges and apples. The number of people who liked oranges is five times the number of people who liked apples. By using a Venn-diagram, find the number of people who liked apples only and who didn't like any of these fruits.
- e) Out of 120 students appeared in an examination, the number of students who passed in Mathematics only is twice the number of students who passed in Science only. If 50 students passed in both the subjects and 40 students failed in both the subjects,
- (i) find the number of students who passed in Mathematics.
  - (ii) Find the number of students who passed in Science.
  - (iii) Show the result in a Venn-diagram.

## 2. Cardinality relation of union of three sets

If A, B and C are any three overlapping sets in a universal set U, then,

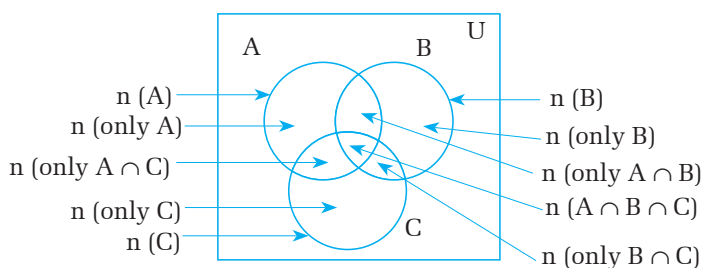
$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

**Proof**

$$\begin{aligned}
 n(A \cup B \cup C) &= n[A \cup (B \cup C)] \\
 &= n(A) + n(B \cup C) - n[A \cap (B \cup C)] \\
 &= n(A) + n(B) + n(C) - n(B \cap C) - n[(A \cap B) \cup (A \cap C)] \\
 &= n(A) + n(B) + n(C) - n(B \cap C) - n[(A \cap B) + n(A \cap C) - n(A \cap B \cap C)] \\
 &= n(A) + n(B) + n(C) - n(B \cap C) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C) \\
 &= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)
 \end{aligned}$$

*Proved*





Let's learn a few more relations from the cardinality relation of union of three sets.

$$(i) \quad n(A \cap B \cap C) = n(A \cup B \cup C) - n(A) - n(B) - n(C) + n(A \cap B) + n(B \cap C) + n(A \cap C)$$

If the cardinal value of the complement of  $(A \cup B \cup C)$  is given, then

$$(ii) \quad n(U) = n(A \cup B \cup C) + n(\overline{A \cup B \cup C})$$

$$(iii) \quad n_o(A) = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$$

$$(iv) \quad n_o(B) = n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$$

$$(v) \quad n_o(C) = n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

### Worked-out examples

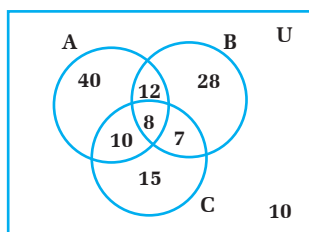
**Example 1:** A, B and C are subsets of a universal set U. If  $n(U) = 130$ ,  $n(A) = 70$ ,  $n(B) = 55$ ,  $n(C) = 40$ ,  $n(A \cap B) = 20$ ,  $n(B \cap C) = 15$ ,  $n(A \cap C) = 18$  and  $n(A \cap B \cap C) = 8$ , illustrate this information in a Venn-diagram and find

- (i)  $n(\text{only } A)$       (ii)  $n(\text{only } B)$       (iii)  $n(\text{only } C)$       (iv)  $n(\text{only } A \cap B)$   
 (v)  $n(\text{only } B \cap C)$       (vi)  $n(\text{only } A \cap C)$       (vii)  $n(\overline{A \cup B \cup C})$ .

**Solution:**

Here,  $n(U) = 130$ ,  $n(A) = 70$ ,  $n(B) = 55$ ,  $n(C) = 40$ ,  $n(A \cap B) = 20$ ,  
 $n(B \cap C) = 15$ ,  $n(A \cap C) = 18$  and  $n(A \cap B \cap C) = 8$ .

**Venn-diagram**



### Hints:

At first, insert

$$\begin{aligned} n(A \cap B \cap C) &= 8, \\ \text{then } n(\text{only } A \cap B) &= 20 - 8 &= 12, \\ \text{then } n(\text{only } B \cap C) &= 15 - 8 &= 7, \\ \text{then } n(\text{only } A \cap C) &= 18 - 8 &= 10, \\ \text{then } n(\text{only } A) &= 70 - 12 - 8 - 10 = 40, \\ \text{then } n(\text{only } B) &= 55 - 12 - 8 - 7 = 28, \\ \text{then } n(\text{only } C) &= 40 - 10 - 8 - 7 = 15 \end{aligned}$$

From the Venn-diagram,

$$n(\text{only } A) = 40 \quad [n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)]$$

$$n(\text{only } B) = 28 \quad [n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)]$$

$$n(\text{only } C) = 15 \quad [n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)]$$

$$n(\text{only } A \cap B) = 12 \quad [n(A \cap B) - n(A \cap B \cap C)]$$

$$n(\text{only } B \cap C) = 7 \quad [n(B \cap C) - n(A \cap B \cap C)]$$

$$n(\text{only } A \cap C) = 10 \quad [n(A \cap C) - n(A \cap B \cap C)]$$

$$\begin{aligned} n(\overline{A \cup B \cup C}) &= n(U) - n(A \cup B \cup C) \\ &= 130 - (40 + 28 + 15 + 8 + 12 + 10 + 7) = 130 - 120 = 10 \end{aligned}$$

### Alternative process

$$\begin{aligned} n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) \\ &= 70 + 55 + 40 - 20 - 15 - 18 + 8 \\ &= 173 - 53 = 120 \end{aligned}$$

$$\therefore n(\overline{A \cup B \cup C}) = n(U) - n(A \cup B \cup C) = 130 - 120 = 10$$

**Example 2:** In a survey, 500 people in a locality were asked what languages they can speak. 200 of them replied they can speak Nepali, 150 replied Newari, 120 replied Bhojpuri, 90 replied Nepali as well as Newari, 85 replied Nepali as well as Bhojpuri, 60 replied Newari as well as Bhojpuri, and 35 replied they can speak all the three languages.

- Draw a Venn-diagram to illustrate the above information
- Find how many people do not speak any of the three languages.

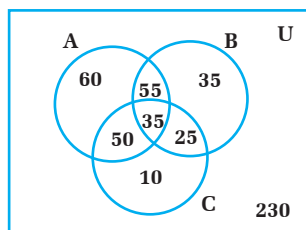
### Solution:

Let A, B and C be the sets of people who can speak Nepali, Newari and Bhojpuri languages respectively.

$$\text{Here, } n(U) = 500, \quad n(A) = 200, \quad n(B) = 150, \quad n(C) = 120$$

$$n(A \cap B) = 90, \quad n(A \cap C) = 85, \quad n(B \cap C) = 60, \quad n(A \cap B \cap C) = 35$$

(i) Illustration in a Venn diagram



$$\begin{aligned} \text{(ii) } n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C) \\ &= 200 + 150 + 120 - 90 - 85 - 60 + 35 = 270 \end{aligned}$$

$$\therefore n(\overline{A \cup B \cup C}) = n(U) - n(A \cup B \cup C) = 500 - 270 = 230$$

So, 230 people do not speak any of the three languages.

**Example 3:** In a group of students, 25 study Computer, 28 study Health, 20 study Mathematics, 9 study Computer only, 12 study Health only, 8 study Computer and Health only, and 5 students study Health and Mathematics only.

- (i) Draw a Venn-diagram to illustrate the above information
- (ii) Find how many students study all the subjects.
- (iii) How many students are there altogether?

**Solution:**

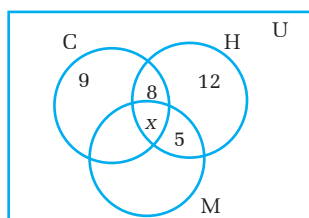
Let C, H and M be the sets of students who study Computer, Health and Mathematics respectively.

Here,  $n(C) = 25$ ,  $n(H) = 28$ ,  $n(M) = 20$ ,  $n_o(C) = 9$ ,  $n_o(H) = 12$ ,

$n_o(C \cap H) = 8$ ,  $n_o(H \cap M) = 5$

Let, the number of students who study all subjects be  $x$ .

(i)



(ii) From the Venn-diagram,  $n(H) = 8 + x + 5 + 12$

$$\text{or, } 28 = x + 25$$

$$\text{or, } x = 3$$

$\therefore$  The number of students who study all the subjects,  $n(C \cap H \cap M) = x = 3$

(iii) Again, the number of students who study Computer and Mathematics only  $= n_o(C \cap M)$

$$= n(C) - (9 + 8 + x)$$

$$= 25 - 17 - 3 = 5$$

Also, the number of students who study Mathematics only  $= n_o(M) = n(M) - (5 + x + 5)$

$$= 20 - 10 - 3 = 7$$

$\therefore$  The total number of students,  $n(U) = n(C \cup H \cup M) = 9 + 8 + 3 + 5 + 12 + 5 + 7 = 49$

## EXERCISE 1.2

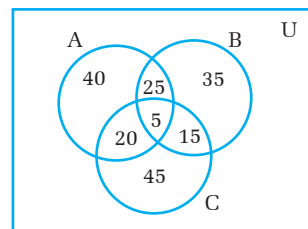
### General section

1. a) A, B and C are the subsets of a universal set U. Draw a Venn-diagram and insert the cardinality of the following information.

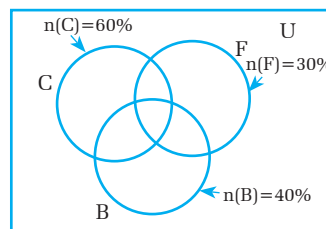
$$n(U) = 45, n(A) = 15, n(B) = 20, n(C) = 16, n(A \cap B) = 4, n(B \cap C) = 3, \\ n(A \cap C) = 5 \text{ and } n(A \cap B \cap C) = 2.$$

- b) In the given Venn-diagram, A, B and C represent the sets of people who like cricket, football and basketball respectively. If  $n(U) = 200$ , find

- (i)  $n(A)$ , (ii)  $n(B)$ , (iii)  $n(C)$   
 (iv)  $n(A \cup B \cup C)$  (v)  $n(\overline{A \cup B \cup C})$  (vi)  $n(A \cap B \cap C)$   
 (vii)  $n(\text{only } A)$  (viii)  $n(\text{only } B)$  (ix)  $n(\text{only } C)$   
 (x)  $n(A \cap B)$  (xi)  $n(B \cap C)$  (xii)  $n(A \cap C)$   
 (xiii)  $n_o(A \cap B)$  (xiv)  $n_o(B \cap C)$  (xv)  $n_o(A \cap C)$



- c) The given Venn-diagram shows the set of students C, F and B who plays cricket, football and basketball respectively. If  $C \cup F \cup B = 100\%$ ,  $n(C \cap F) = 10\%$ ,  $n(F \cap B) = 10\%$  and  $n(C \cap B) = 15\%$ , find the percentage of the students who plays all three types of game.



### Creative section

2. a) If  $n(A) = 48$ ,  $n(B) = 51$ ,  $n(C) = 40$ ,  $n(A \cap B) = 11$ ,  $n(B \cap C) = 10$ ,  $n(C \cap A) = 9$ ,  $n(A \cap B \cap C) = 4$  and  $n(U) = 120$ , find the value of  $n(A \cup B \cup C)$  and  $n(\overline{A \cup B \cup C})$ . Present the above information in a Venn-diagram.
- b) Given that,  $n(U) = 100$ ,  $n(P) = 45$ ,  $n(Q) = 48$ ,  $n(R) = 40$ ,  $n_o(P) = 15$ ,  $n_o(Q) = 18$ ,  $n_o(P \cap Q) = 10$ ,  $n_o(Q \cap R) = 8$ . Find
- (i)  $n(P \cap Q \cap R)$  (ii)  $n_o(P \cap R)$  (iii)  $n_o(R)$  (iv)  $n(\overline{P \cup Q \cup R})$
- Illustrate the above information in a Venn-diagram.
3. a) The survey of a group of people showed that 60 liked tea, 45 liked coffee, 30 liked milk, 25 liked coffee as well as tea, 20 liked tea as well as milk, 15 liked coffee as well as milk and 10 liked all three. If every person liked at least one drink, how many people were asked this question? Solve it by drawing the Venn-diagram.
- b) In an examination, out of 200 students, 70 succeeded in English, 80 in Mathematics, 60 in Nepali, 35 in English as well as in Mathematics, 25 in English as well as in Nepali, 35 in Nepali as well as in Mathematics and 10 succeeded in all three subjects.
- (i) Draw a Venn-diagram showing the above information.  
 (ii) Find also the number of students who didn't succeed in all three subjects.
- c) Of the total candidates in an examination, 40% students passed in Maths, 45% in Science and 55% in Health. 10% students passed in both Maths and Science, 20% in Science and Health and 15% in Health and Maths. If every student passed at least one subject,
- (i) draw a Venn-diagram to show the above information.  
 (ii) Calculate the percent who passed in all the three subjects.

4. a) In a survey of 750 tourists who arrived in Nepal, 450 preferred to go trekking, 300 preferred rafting, 250 preferred forest safari and 30 preferred none of these activities. If 150 preferred trekking and rafting, 90 preferred trekking and safari, 75 preferred rafting and safari,
- (i) how many of them preferred all of these activities?
- (ii) Show the above information in a Venn-diagram.
- b) In a survey of people who like different languages in films, 50 liked Nepali, 40 liked English, 30 liked Hindi, 24 liked Nepali and English, 19 liked Nepali and Hindi, 13 liked Hindi and English, 6 liked all three and 21 people were found not interested in any language. Find
- (i) How many people did not like Hindi films?
- (ii) How many people did not like both Nepali and Hindi films?
- c) In a group of students, 20 study Economics, 18 study History, 21 study Science, 7 study Economics only, 10 study Science only, 6 study Economics and Science only and 3 study Science and History only.
- (i) Represent the above information in Venn-diagram
- (ii) How many students study all the subjects?
- (iii) How many students are there altogether?
- d) In a group of students, 30 study English, 35 study Geography, 25 study Mathematics, 12 study English only, 15 study Geography only, 10 study English and Geography only and 6 students study Geography and Mathematics only.
- (i) Draw a Venn-diagram to illustrate the above information
- (ii) Find how many students study all the subjects.
- (iii) How many students are there altogether?

## Project Work

**Make the group of your friends and conduct a survey inside your classroom: how many of your friends like Tea, Coffee and Tea as well as Coffee. Then, tabulate the collected data as shown in the table given below:**

Total No. of Student	No. who like Tea	No. who like Coffee	No. who like Tea as well as coffee	No. who like Tea only	No. who like Coffee only	No. who like Tea and Coffee	No. who don't like both drinks

- (i) Show the tabulated data in Venn-diagram
- (ii) Calculate the number of students of columns 5, 6, 7 and 8 by using the cardinality relations of sets.
- (iii) Compare the results with the surveyed data.

**Note: Dear students, please go through the appendix given at the end of the book for more practice.**

# 2 Chapter

## Tax and Money Exchange

### 2.1 Taxation - Review

Let's answer the following questions about different types of taxes.

1. Suppose the annual income of an individual is Rs 3,80,000. How much social security tax does he/she pay to the state at the rate of 1%.
2. If 15% income tax is levied on the annual income of Rs 3,50,000 to Rs 4,50,000, how much income tax does the individual pay to the state?
3. The annual income of a Private School is Rs 2,20,00,000. How much tax should be paid by the school in a year at the rate of 1% Education Service Charge?
4. When you purchased a pair of shoes you paid Rs 2825 including Value Added Tax (VAT) at the rate of 13%. Calculate, how much sales tax did you pay as VAT.

These are some of the various cases of taxation. A tax is a financial charge or other levy imposed upon a taxpayer by a state. The government does not have its own money. Its receipts come from individual income taxes, corporate income taxes, estate and gift taxes, education service taxes, excise tax, etc. The taxes we pay to the state are used by government for transport, education, health, law and order, culture, civil service, trade and industry, etc.

The Inland Revenue Department (IRD) under the Ministry of Finance of Government of Nepal is responsible for the administration of Value Added Tax, income tax and excise duty. We have already discussed about the various types of taxes in grade 9. Here, we are further discussing about Value Added Tax (VAT).

### 2.2 Value Added Tax (VAT) - Review

Let's study the adjoining bill given by a shopkeeper to a customer.

- (i) How much is the selling price of the articles without VAT?
- (ii) What is the rate of VAT?
- (iii) How much price should the customer pay with VAT?

TAX INVOICE		ABC Enterprises		
PAN NO. 301334983		Anamnagar-32, Kathmandu, Nepal		
		Ph. No.: 01-4385020, 4385021		
Bill Issued Date: .....		Transaction Date: .....		
Customer's Name: .....				
Address: .....		Phone No.: .....		
Customer's PAN [ ] [ ] [ ] [ ] [ ] [ ]		No. ....		
Mode of Payment: Cash/Cheque/Credit/Other				
S.No.	Descriptions	Quantity	Rate	Amount
1.	Cartiage	1	2380	2380
Rs. In words: .....				Total 2380
Two thousand six hundred eighty-nine				.....%
rupees four paisa only.				Taxable Amount
1. 2% per month will be charged on our due amount.				Ownership Tax
2. Goods once sold cannot be returned or exchanged.				Sub Total 2380
3. E. & O.E.				13 % VAT 309.4
				Grand Total 2689.4
Customer's Signature				For: ABC Enterprises

In Nepal, the sales tax rate is a tax charged to consumers based on the purchase price of certain goods and services. Revenues from the Sales Tax Rate are an important source of income for the government of Nepal. The sales tax is also known as the Value Added Tax (VAT). The VAT rate is given in percent and it is decided by the concern authority of the government. The VAT rate may vary from country to country. Even in a particular country it may be changed from time to time. For example, when VAT was introduced in Nepal for the first time on 16 November, 1997, the VAT rate was 10% and it is now 13%.

There are certain goods and services for which VAT is exempted. For example, educational items, social welfare services, etc. are VAT-free goods and services. VAT is levied on the actual selling price.

$$\text{VAT amount} = \text{Rate of VAT (in \%)} \times \text{Selling price.}$$

$$\text{S.P. with VAT} = \text{S.P.} + \text{VAT \% of S.P.}$$

### ***Worked-out examples***

**Example 1: Calculate the amount of VAT on the selling price of Rs 1,500 at the rate of 13%.**

**Solution:**

$$\begin{aligned}\text{Here, the amount of VAT} &= \text{VAT\% of S.P.} \\ &= 13\% \text{ VAT of Rs 1,500} \\ &= \text{Rs 195}\end{aligned}$$

**Example 2: The cost of an electric item is Rs 4,250. How much should a customer pay for it with 13% VAT?**

**Solution:**

Here, the cost of the electric item (i.e. selling price) = Rs 4,250

VAT rate = 13%

$$\begin{aligned}\text{Now, the cost of the item with VAT} &= \text{S.P.} + \text{VAT\% of S.P.} \\ &= \text{Rs 4,250} + 13\% \text{ of Rs 4,250} \\ &= \text{Rs 4,250} + \frac{13}{100} \times \text{Rs 4,250} \\ &= \text{Rs 4,802.50}\end{aligned}$$

So, the customer should pay Rs 4,802.50

**Example 3: Mrs. Tamang had some snacks in a restaurant. If the cost of the snacks is Rs 500, how much should she pay with 10% service charge and 13% VAT?**

**Solution:**

Here, cost of the snacks = Rs 500

Service charge = 10%

VAT rate = 13%

Now, the cost of the snacks with service charge = S.P. + VAT% of S.P.  
= Rs 550

Again, the cost of the snacks with service charge and VAT = Rs 550 + 13% of Rs 500  
= Rs 621.50

So, she should pay Rs 621.50

**Example 4: A customer purchased a furniture for Rs 3,390 with 13% VAT. Find the cost of the furniture without VAT?**

**Solution:**

Here, the cost of the furniture with 13% VAT = Rs 3,390

Let, the cost of the furniture without VAT = Rs x

Now,  $x + 13\% \text{ of } x = \text{Rs } 3,390$

or,  $\frac{113x}{100} = \text{Rs } 3,390$

or,  $x = \text{Rs } 3,000$

So, the cost of the furniture without VAT is Rs 3,000.

**Example 5: If the cost of an article with VAT is Rs 7,475 and without VAT is Rs 6,500, find the VAT rate.**

**Solution:**

Here, the cost of the article with VAT = Rs 7,475

And, the cost of the article without VAT = Rs 6,500

Now, the amount of VAT = Rs 7,475 – Rs 6,500

= Rs 975

$$\begin{aligned}\therefore \text{Rate of VAT} &= \frac{\text{VAT amount}}{\text{cost without VAT}} \times 100\% \\ &= \frac{\text{Rs } 975}{\text{Rs } 6500} \times 100\% \\ &= \text{Rs } 15\%\end{aligned}$$

So, the required VAT rate is 15%.



**Example 6: A shopkeeper bought a radio for Rs 10,000 and sold at a profit of 10% to a customer with 13% VAT. How much did the customer pay for the radio?**

**Solution:**

$$\begin{aligned}\text{Here, C.P. of the radio} &= \text{Rs } 10,000. \\ \text{Profit percent} &= 10\% \\ &= \text{Rs } 10,000 + \frac{10}{100} \times \text{Rs } 10,000 \\ &= \text{Rs } 11,000\end{aligned}$$

$$\begin{aligned}\text{Now, S.P. of the radio with 13\% VAT} &= \text{S.P.} + 13\% \text{ of S.P.} \\ &= \text{Rs } 11,000 + \frac{13}{100} \times \text{Rs } 11,000 \\ &= \text{Rs } 12,430\end{aligned}$$

So, the customer paid Rs 12,430 for the radio.

**Example 7: A retailer bought a mobile set for Rs 15,000 and fixed its price to make 20% profit. If the set was sold for Rs 20,340 with VAT, calculate the rate of VAT.**

**Solution:**

$$\begin{aligned}\text{Here, S.P. for the retailer} &= \text{Rs } 15,000 + 20\% \text{ of Rs } 15,000 \\ &= \text{Rs } 15,000 + \text{Rs } 3,000 \\ &= \text{Rs } 18,000\end{aligned}$$

$$\text{Also, S.P. with VAT} = \text{Rs } 20,340$$

$$\begin{aligned}\therefore \text{VAT amount} &= \text{Rs } 20,340 - \text{Rs } 18,000 \\ &= \text{Rs } 2,340\end{aligned}$$

$$\begin{aligned}\text{Again, rate of VAT} &= \frac{\text{VAT amount}}{\text{S.P.}} \times 100\% \\ &= \frac{\text{Rs } 2,340}{\text{Rs } 18,000} \times 100\% \\ &= 13\%\end{aligned}$$

So, the required rate of VAT is 13%.

**Example 8: A trader bought an electric fan for Rs 4,000 and sold it to a customer for Rs 5,424 with 13% VAT. Find his/her profit or loss percent.**

**Solution:**

$$\begin{aligned}\text{Here, C.P. for the fan} &= \text{Rs } 4,000 \\ \text{S.P. of the fan with VAT} &= \text{Rs } 5,424 \\ \text{Rate of VAT} &= 13\%\end{aligned}$$

Let, the S.P. of the fan without VAT be Rs x.

$$\therefore x + 13\% \text{ of } x = \text{Rs } 5,424$$

$$\text{or, } x + \frac{13x}{100} = \text{Rs } 5,424$$

$$\text{or, } \frac{113x}{100} = \text{Rs } 5,424$$

$$\text{or, } x = \text{Rs } 4,800$$

Thus, S.P. without VAT = Rs 4,800

$$\begin{aligned}\text{Now, profit} &= \text{S.P.} - \text{C.P.} \\ &= \text{Rs } 4,800 - 4,000 \\ &= \text{Rs } 800\end{aligned}$$

$$\begin{aligned}\text{Also, profit percent} &= \frac{\text{Actual profit}}{\text{C.P.}} \times 100\% \\ &= \frac{\text{Rs } 800}{\text{Rs } 4,000} \times 100\% \\ &= 20\%\end{aligned}$$

So, the required profit percent is 20%.

**Example 9: A shopkeeper bought a washing machine and sold it at a profit of 25% to a customer. If the customer paid Rs 28,250 for it with 13% VAT, at what price did the shopkeeper buy the machine?**

**Solution:**

Here, S.P. of the machine with VAT = Rs 28,250

$$\text{Rate of VAT} = 13\%$$

Let, S.P. of the machine without VAT be Rs x.

$$\therefore x + 13\% \text{ of } x = \text{Rs } 28,250$$

$$\text{or, } \frac{113x}{100} = \text{Rs } 28,250$$

$$\text{or, } x = \text{Rs } 25,000$$

Thus, S.P. of the machine without VAT = Rs 25,000.

Also, the profit of the shopkeeper = 25%

And, C.P. of the machine for the shopkeeper = ?

$$\text{Now, C.P.} + 25\% \text{ of C.P.} = \text{Rs } 25,000$$

$$\text{or, } \frac{125 \text{ C.P.}}{100} = \text{Rs } 25,000$$

$$\text{or, C.P.} = \text{Rs } 20,000$$

So, the shopkeeper bought the machine for Rs 20,000.

**Example 10: A dealer sold a machine levying 13% VAT on Rs 2,50,000 to a retailer. The retailer spent Rs 5,000 for transportation and Rs 2,500 for the local tax. If the retailer sold it at a profit of Rs 10,000 to a customer, how much did the customer pay for it with 13% VAT?**

**Solution:**

$$\begin{aligned}\text{Here, S.P. for the dealer with 13\% VAT} &= \text{Rs } 2,50,000 + 13\% \text{ of Rs } 2,50,000 \\ &= \text{Rs } 2,82,500\end{aligned}$$

$$\therefore \text{C.P. for the retailer} = \text{Rs } 2,82,500$$

Also, C.P. for the retailer including transportation cost and the local tax

$$= \text{Rs } 2,82,500 + \text{Rs } 5,000 + \text{Rs } 2,500$$

$$= \text{Rs } 2,90,000$$

Again,

$$\begin{aligned} \text{S.P. for the retailer} &= \text{C.P.} + \text{Profit} \\ &= \text{Rs } 2,90,000 + \text{Rs } 10,000 \\ &= \text{Rs } 3,00,000 \end{aligned}$$

$$\begin{aligned} \text{Now, cost of the machine for the customer} &= \text{Rs } 3,00,000 + 13\% \text{ of } 3,00,000 \\ &= \text{Rs } 3,39,000 \end{aligned}$$

So, the customer paid Rs 3,39,000 for the machine.

**Example 11:** A wholesaler purchased some building materials for Rs 1,80,000. He/she sold them at 5% profit and levying 15% VAT to a supplier. The supplier spent Rs 7,500 for transportation and sold at a profit of 10% to a customer. How much did the customer pay for the materials with 15% VAT?

**Solution:**

$$\begin{aligned} \text{Here, S.P. for the wholesaler at 5\% profit} &= \text{Rs } 1,80,000 + 5\% \text{ of Rs } 1,80,000 \\ &= \text{Rs } 1,89,000 \\ \text{S.P. for the wholesaler with VAT} &= \text{Rs } 1,89,000 + 15\% \text{ of Rs } 1,89,000 \\ &= \text{Rs } 2,17,350 \\ \text{C.P. for the supplier} &= \text{Rs } 2,17,350 \\ \text{C.P. for the supplier with transportation cost} &= \text{Rs } 2,17,350 + \text{Rs } 7,500 \\ &= \text{Rs } 2,24,850 \\ \text{S.P. for the supplier at 10\% profit} &= \text{Rs } 2,24,850 + 10\% \text{ of Rs } 2,24,850 \\ &= \text{Rs } 2,47,335 \\ \text{C.P. for the customer with 15\% VAT} &= \text{Rs } 2,47,335 + 15\% \text{ of Rs } 2,47,335 \\ &= \text{Rs } 2,84,435.25 \end{aligned}$$

So, the customer paid Rs 2,84,435.25 for the materials.

## EXERCISE 2.1

### General Section

1. a) Write the formula to find amount of VAT.
- b) Write the formula to find VAT percent.
- c) Write the relation to find a selling price with the given rate of VAT percent.

2. a) Find the selling price with VAT from the table given below.

S.No.	S.P. without VAT	VAT%	S.P. with VAT
(i)	Rs 1,500	13%	.....
(ii)	Rs 7,200	13%	.....
(iii)	Rs 25,000	15%	.....

- b) Find the selling price without VAT from the table given below.

S.No.	S.P. with VAT	VAT%	S.P. without VAT
(i)	Rs 1,958	10%	.....
(ii)	Rs 5,085	13%	.....
(iii)	Rs 36,160	13%	.....

3. a) The cost of a mobile set is Rs 18,000. If Mrs. Sharma purchased it with 13% VAT, how much did she pay for it.
- b) A family had dinner in a restaurant. If the cost of the dinner was Rs 2,500, how much did the family pay with 10% service charge and 13% VAT?
4. a) The cost of an article with 13% VAT is Rs 5,424. Find its cost without VAT.
- b) Mr. Rai purchased a watch for Rs 8,625 with 15% VAT. Find the cost of the watch without VAT and also calculate the VAT amount.
- c) A woman bought a television for Rs 22,600 with 13% VAT. How much did she pay for the VAT?
5. a) If the cost of an item with VAT is Rs 1,356 and without VAT is Rs 1,200, find the VAT rate.
- b) A customer purchased a bicycle for Rs 10,716 with VAT. If its cost without VAT is Rs 9,400, calculate the rate of VAT.

### Creative Section

6. a) A shopkeeper purchased a laptop for Rs 40,000 and sold at a profit of 20% to a customer. How much did the customer pay for it with 13% VAT?
- b) A retailer bought an electric stove for Rs 12,500 and fixed its price to make 10% profit. How much should a customer pay for it with 13% VAT?
7. a) A trader purchased a camera for Rs 20,000 and marked its price to make 25% profit. If he/she sold it for Rs 27,500 with VAT, calculate the rate of VAT.
- b) A wholesaler purchased an electronic item for Rs 50,000 and sold it to a retailer at 5% profit. If the retailer purchased it for Rs 59,325 with VAT, find the VAT percent.
8. a) The cost of an article is Rs 10,000 and it is sold at same profit. If a customer bought it for Rs 12,430 with 13% VAT, calculate the profit percent on the article.

- b) A trades person bought an electric item for Rs 5,000 and sold it to a customer for Rs 5,085 with 13% VAT. Find his/her profit or loss percent.
9. a) A shopkeeper bought a sunglasses and sold it at a profit of 20% to a customer. If the customer paid Rs 21,696 for it with 13% VAT, at what price did the shopkeeper buy it?
- b) A retailer sold an article to a customer at a loss of 10%. The customer purchased it for Rs 25,425 including 13% VAT. Calculate the cost price of the article to the retailer.
10. a) A retailer purchased a photocopy machine for Rs 1,60,000 from a dealer. He/she spent Rs 2,500 for transportation and Rs 1,500 for the local tax. If he/she sold it at a profit of Rs 6,000 to a customer, how much did the customer pay for it with 13% VAT?
- b) A supplier sold a machine levying 13% VAT on Rs 3,20,000 to a retailer. The retailer spent Rs 10,000 for transportation and Rs 5,000 for the local tax. If the retailer sold it at a profit of Rs 15,000 to a customer, how much did the customer pay for the machine with 13% VAT?
- c) A supplier purchased a machine for Rs 2,00,000 and spent Rs 5,000 for transportation and Rs 2,000 for local tax. He/she sold it to a customer at 10% profit. At what price did the customer purchase the machine with 13% VAT?
- d) A wholesaler purchased construction materials for Rs 3,50,000 and sold them to a supplier at 5% profit and levying 13% VAT. The supplier spent Rs 2,725 for transportation and Rs 2,000 for the local tax. The supplier sold the materials to a customer at 10% profit. How much did the customer pay for the materials with 13% VAT?

### 2.3 Marked Price, Discount and VAT

The price which is listed or fixed on an article is called its **marked price (M.P.)**. When a shopkeeper reduces the marked price of any article and sells it to customers, the reduced amount is called **discount**. Discount is usually given as a certain percent of marked price.

Thus,

$$\text{Discount amount} = \text{discount percent of M.P.}$$

$$\text{Selling price (S.P.)} = \text{M.P.} - \text{Discount amount or M.P.} - \text{D\% of M.P.}$$

$$\text{Discount percent} = \frac{\text{Discount amount}}{\text{M.P.}} \times 100 \%$$

In the case of finding selling price with VAT, at first, discount is to be deducted from the given marked price (M.P.) to find selling price (S.P.). Then, VAT is levied on the selling price.

Thus,

$$\begin{aligned} \text{S.P. with VAT} &= (\text{M.P.} - \text{D\% of M.P.}) + \text{VAT\% of S.P.} \\ &= \text{S.P.} + \text{VAT\% of S.P.} \end{aligned}$$

## Worked-out examples

**Example 1:** The marked price of a motorcycle was Rs 1,50,000. What was the price of the motorcycle after 10 % discount and 13 % VAT included in its price?

**Solution:**

Here, M.P. of the motorcycle = Rs 1,50,000

Discount percent (D%) = 10 %

VAT = 13 %

Now,

S.P. of the motorcycle = M.P. – D% of M.P.

= Rs 1,50,000 – 10% of Rs 1,50,000

= Rs 1,35,000

Again, S.P. with VAT = S.P. + VAT % of S.P.

= Rs 1,35,000 + 13 % of Rs 1,35,000

= Rs 1,52,550

So, the required price of the motorcycle is Rs 1,52,550.

**Example 2:** A retailer allows a discount of 10 % and still makes a profit of 20 % by selling a radio. If the marked price of the radio is Rs 1,200, find his/her profit.

**Solution:**

Here, M.P. of the radio = Rs 1,200

Discount percent (D%) = 10 %

Profit percent (P%) = 20 %

Now, S.P. of the radio = M.P. – D% of M.P.

= Rs 1,200 – 10% of Rs 1,200

= Rs 1,080

Again, in 20% of profit,

120% of C.P. = S.P.

or,  $\frac{120}{100} \times \text{C.P.} = \text{Rs } 1,080$

or, C.P. = Rs 900

So, his/her profit is Rs 180.

Again, profit = 20% of C.P.

=  $\frac{20}{100} \times \text{Rs } 900$

= Rs 180

**Example 3:** A shopkeeper fixed the price of his articles 30% above the cost price. If he/she sold an article allowing 10% discount, find his/her profit percent.

**Solution:**

Let the C.P. of the article be Rs x.

Then, the M.P. of the article = Rs x + 30% of Rs x

= Rs x +  $\frac{30}{100} \times \text{Rs } x = \text{Rs } \frac{13x}{10}$

**Alternative calculation**

M.P. = 130 % of C.P.

=  $\frac{130}{100} \times \text{Rs } x$

= Rs  $\frac{13x}{10}$

Now, S.P. of the article = M.P. – discount % of M.P.

$$= \text{Rs } \frac{13x}{10} - \frac{10}{100} \times \text{Rs } \frac{13x}{10} = \text{Rs } \frac{117x}{100}$$

Again, profit = S.P. – C.P.

$$= \text{Rs } \frac{117x}{100} - x = \frac{17x}{100}$$

$$\therefore \text{Profit percent} = \frac{\text{Actual profit}}{\text{C.P.}} \times 100\% = \frac{17x}{100 \times x} \times 100\% = 17\%$$

So, his/her profit percent is 17%.

#### Alternative calculation

$$\text{S.P.} = 90\% \text{ of M.P.}$$

$$= \frac{90}{100} \times \text{Rs } \frac{13x}{10}$$

$$= \text{Rs } \frac{117x}{100}$$

#### Alternative process

Let the C.P. of the article be Rs 100

Then, the M.P. of the article = Rs 100 + Rs 30 = Rs 130

Now, the S.P. of the article = M.P. – discount % of M.P. = Rs 130 –  $\frac{10}{100} \times \text{Rs } 130 = \text{Rs } 117$

Again, profit = S.P. – C.P. = Rs 117 – Rs 100 = Rs 17

$$\therefore \text{Profit percent} = \frac{\text{Actual profit}}{\text{C.P.}} \times 100\% = \frac{17}{100} \times 100\% = 17\%$$

**Example 4:** The marked price of a radio is 40% above the cost price. When it is sold allowing 30% discount on it, there was a loss of Rs 100. What was the marked price of the radio?

**Solution:**

Let the C.P. of the radio be Rs  $x$ .

Then, the M.P. of the radio = Rs  $x$  + 40% of Rs  $x$  = Rs  $x$  +  $\frac{40}{100} \times \text{Rs } x = \text{Rs } \frac{7x}{5}$

Now, S.P. of the radio = M.P. – discount % of M.P.

$$= \text{Rs } \frac{7x}{5} - \frac{30}{100} \times \text{Rs } \frac{7x}{5} = \text{Rs } \frac{49x}{50}$$

Again, loss = C.P. – S.P.

$$\text{or, Rs } 100 = x - \frac{49x}{50}$$

$$\text{or, Rs } 100 = \frac{x}{50}$$

$$\text{or, } x = \text{Rs } 5,000$$

$$\text{Now, M.P.} = \text{Rs } \frac{7x}{5} = \text{Rs } \frac{7 \times 5,000}{5} = \text{Rs } 7,000$$

So, the marked price of the radio was Rs 7,000.

**Example 5:** After allowing 5% discount on the marked price of a watch, 10% VAT is charged on it, then its price becomes Rs 2,090. Calculate the amount of discount.

**Solution:**

Let the marked price (M.P.) of the watch be Rs  $x$ .

Here, discount percent (D%) = 5 %

VAT = 10%

Now, S.P. = M.P. – D% of M.P.

$$= x - 5\% \text{ of } x$$

$$= \frac{19x}{20}$$

Also, S.P. with VAT = S.P. + VAT% of S.P.

$$= \frac{19x}{20} + 10\% \text{ of } \frac{19x}{20}$$

$$= \frac{19x}{20} \times \frac{11}{10}$$

According to the question,

$$\frac{19x}{20} \times \frac{11}{10} = 2,090$$

$$\text{or, } x = \text{Rs } 2,000$$

$$\therefore \text{M.P.} = \text{Rs } 2,000$$

So, the required amount of discount is Rs 100.

### Alternative process

Here, S.P. with 10% VAT = Rs 2,090

Let S.P. without VAT = Rs x

Then,  $x + 10\% \text{ of } x = \text{Rs } 2,090$

or,  $x = \text{Rs } 1,900$

i.e. S.P. = Rs 1,900

Again, discount = 5% of M.P. = 5% of Rs 2000 = Rs 100

### Alternative process

S.P. = 95 % of M.P.

$$= \frac{95}{100} \times x = \text{Rs } \frac{19x}{20}$$

### Alternative process

S.P. with VAT

$$= 110\% \text{ of } \frac{19x}{20} = \frac{11}{10} \times \frac{19x}{20}$$

Now, discount = 5% of M.P.

$$= \frac{5}{100} \times \text{Rs } 2,000$$

$$= \text{Rs } 100$$

Let M.P. of the watches be Rs y.

Then, S.P. = M.P. – D% of M.P.

or,  $1,900 = y - 5\% \text{ of } y$

or,  $y = \text{Rs } 2,000$

i.e. M.P. = Rs 2,000

**Example 6:** A watch was sold on its marked price at a gain of 20%. But allowing 5% discount, there would have been a gain of Rs 140. Find the cost price of the watch.

### Solution:

Let the M.P. of the watch be Rs x.

Then, the (S.P.)<sub>1</sub> of the watch is also Rs x

Also, in 20% profit, 120% of C.P. = (S.P.)<sub>1</sub>

$$\text{or, } \frac{120}{100} \times \text{C.P.} = \text{Rs } x$$

$$\text{or, } \text{C.P.} = \text{Rs } \frac{5x}{6} \quad \dots\dots\dots (i)$$

Again, (S.P.)<sub>2</sub> = M.P. – discount % of M.P.

$$= \text{Rs } x - \frac{5}{100} \times \text{Rs } x = \text{Rs } \frac{19x}{20}$$

When profit is Rs 140, C.P. = (S.P.)<sub>2</sub> – Rs 140

$$\text{or, } \text{C.P.} = \text{Rs } \frac{19x}{20} - \text{Rs } 140 \quad \dots\dots\dots (ii)$$

From equations (i) and (ii),

$$\text{Rs } \frac{19x}{20} - \text{Rs } 140 = \text{Rs } \frac{5x}{6}$$



$$\begin{aligned}\text{or, } \text{Rs } \frac{19x}{20} - \text{Rs } \frac{5x}{6} &= \text{Rs } 140 \\ \text{or, } x &= \text{Rs } 1,200\end{aligned}$$

$$\therefore \text{C.P.} = \text{Rs } \frac{5x}{6} = \frac{5 \times 1,200}{6} = \text{Rs } 1,000$$

So, the cost price of the watch is Rs 1,000.

**Example 7:** An electric fan, after allowing a discount of 20% on its marked price, was sold at a gain of 20%. Had it been sold after allowing 25% discount there would have been a gain of Rs 125. Find the marked price of the fan.

**Solution:**

Let, the M.P. of the fan be Rs  $x$  and its C.P. be Rs  $y$ .

$$\text{Now, S.P. in 20\% discount} = \text{Rs } x - 20\% \text{ of Rs } x = \frac{4x}{5}$$

$$\therefore \text{C.P.} = \text{S.P.} - \text{profit \% of C.P.}$$

$$y = \frac{4x}{5} - 20\% \text{ of } y$$

$$\text{or, } y = \frac{4x}{5} - \frac{y}{5}$$

$$\text{or, } \frac{6y}{5} = \frac{4x}{5}$$

$$\therefore y = \text{Rs } \frac{4x}{6} = \text{Rs } \frac{2x}{3} \dots\dots\dots (i)$$

$$\text{Again, S.P. in 25\% discount} = \text{M.P.} - \text{discount \% of M.P.} = \text{Rs } x - 25\% \text{ of Rs } x = \text{Rs } \frac{3x}{4}$$

$$\therefore \text{C.P.} = \text{S.P.} - \text{profit}$$

$$y = \text{Rs } \frac{3x}{4} - \text{Rs } 125 \dots\dots\dots (ii)$$

From equations (i) and (ii)

$$\frac{3x}{4} - \text{Rs } 125 = \frac{2x}{3}$$

$$\text{or, } \frac{3x}{4} - \frac{2x}{3} = \text{Rs } 125$$

$$\text{or, } x = \text{Rs } 1,500$$

So, the marked price of the fan is Rs 1,500.

**Example 8:** After allowing 20% discount on the marked price and then levying 10% VAT, a radio was sold. If buyer had paid Rs 320 for VAT, how much would he have got the discount?

**Solution:**

Let the marked price of the radio be Rs  $x$ .

$$\text{S.P. of radio} = \text{M.P.} - \text{discount \% of M.P.} = \text{Rs } x - 20\% \text{ of Rs } x = \text{Rs } \frac{4x}{5}$$

Now, the given amount of VAT = Rs 320

$$\text{or, } 10\% \text{ of S.P.} = \text{Rs } 320$$

$$\begin{aligned} \text{or, } \quad \frac{10}{100} \times \text{Rs } \frac{4x}{5} &= \text{Rs } 320 \\ \text{or, } \quad x &= \text{Rs } 4,000 \end{aligned}$$

∴ M.P. of the radio is Rs 4,000.

$$\text{Again, discount} = 20\% \text{ of M.P.} = \frac{20}{100} \times \text{Rs } 4,000 = \text{Rs } 800$$

So, he had got the discount of Rs 800.

## EXERCISE 2.2

1.
  - a) If marked price is M.P. and amount of discount is D, write the formula to find S.P.
  - b) If marked price is M.P., selling price after discount is S.P., write the formula to find the amount of discount.
  - c) When the amount of discount and marked price are given, write the formula to find the rate of discount.
  - d) When the marked price and the selling price after discount are given, write the formula to find discount percent.
2.
  - a) If M.P. = Rs 750, discount percent = 10%, find S.P.
  - b) If M.P. = Rs 1,200, S.P. = Rs 1,020, find discount percent.
  - c) If S.P. = Rs 640, discount percent = 20 %, find M.P.
  - d) If S.P. = Rs 900, VAT percent = 13 %, find S.P. with VAT.
  - e) If S.P. = Rs 1,600, S.P. with VAT = Rs 1,840, find the rate of VAT.
  - f) If S.P. with VAT = Rs 1,695, VAT percent = 13 %, find S.P. without VAT.
3.
  - a) The cost of an article is Rs 550. If a shopkeeper sells the article by allowing 10 % discount, find the selling price of the article.
  - b) The selling price of an article is Rs 270. If the article was sold at 10 % discount from the marked price, find the marked price.
  - c) A shopkeeper sold a watch for Rs 992 allowing 20 % discount. Find its marked price.
  - d) What percent of discount should be given in a doll costing Rs 180 so that a customer has to buy it for Rs 160?
  - e) A man purchased an article at 5 % discount. If he got the discount amount of Rs 74, at what price did he purchase the article?
  - f) Find the net selling price of a shirt marked at Rs 1,200 when VAT of 13 % is levied.
  - g) When the VAT rate is 10 % a customer pays Rs 2,816 to buy a mobile set. Find the cost of mobile without VAT.
  - h) While paying a bill, a man paid Rs 189.80 as the amount of VAT at 13 %. How much was the actual bill amount?
4.
  - a) The marked price of an article is Rs 400 and 10 % discount is allowed.
    - (i) Find the discount amount.      (ii) Find the selling price of the article.
    - (iii) How much is its price with 13 % VAT?

- b) The marked price of a watch is Rs 1,000 and 20 % discount is allowed.  
 (i) Find its selling price.  
 (ii) How much should a customer pay for it with 13 % VAT?

### Creative section

5. a) The marked price of a bicycle is Rs 3,500. If a customer is given 12 % discount, how much does he/she pay with 10 % VAT?  
 b) The marked price of a radio is Rs 5,000. What will be the price of the radio if 13 % VAT is levied after allowing 10 % discount on it?
6. a) The marked price of an electric heater is Rs 1,500. If the shopkeeper allows 15 % discount and makes a profit of Rs 175, at what price did he purchase the heater?  
 b) The marked price of a calculator is Rs 1,800. The shopkeeper allows 10 % discount and still makes 8 % profit. At what price did the shopkeeper purchase the calculator?  
 c) A man bought a radio for Rs 2,000 and fixed its price so that after giving 20 % discount he made 10 % profit. Find the fixed price of the radio.  
 d) Mr. Limbu bought a bicycle for Rs 2,500 and he labelled its price 20 % above the cost price. If he allows 10 % discount to a customer, find his profit percent.  
 e) The marked price of an article is Rs 2,800 which is 40 % above the cost price. If it is sold by allowing 20 % discount, what will be the profit percent?  
 f) A shopkeeper purchased a television costing Rs 5,600 from a dealer at 5 % discount and sold at a profit of 10 %. If he/she had sold it at 5 % discount, find its marked price.
7. a) A shopkeeper marks the price of his/her goods 40 % above the cost price and allows 20 % discount. If his/her purchase price of an item is Rs 6,000, how much should a customer pay for it levying 13 % VAT?  
 b) The cost price of an electronic fan is Rs 2,800. If the shopkeeper marks its price 30% above the cost price and sells it at 10 % discount, how much should a customer pay for it with 15 % VAT?  
 c) The marked price of an article is Rs 4,500. After allowing some percent of discount and levying 10 % VAT it is sold at Rs 4,400, find the discount percent.  
 d) The marked price of an electric item is Rs 2,400 and the shopkeeper allows 20% discount. After levying VAT, if a customer pays Rs 2,208 for it, find the VAT percent.
8. a) A shopkeeper fixed the price of his articles 25 % above the cost price. If he sold an article allowing 5 % discount, find his profit percent.  
 b) A shopkeeper fixed the marked price of his radio to make a profit of 30 %. Allowing 15 % discount on the marked price, the radio was sold. What percent profit did he make?  
 c) The marked price of a watch is 30 % above the cost price. When it is sold allowing 20 % discount on it, there is a gain of Rs 150. Find the marked price and the cost price of the watch.  
 d) Mrs. Sharma fixed the price of cosmetic items 30 % above the cost price. When she sold an item at 25 % discount, there was a loss of Rs 15. Find the cost price and marked price of the item.

- e) The selling price of an article is 20 % less than its marked price and the marked price is 30 % above the cost price. Find the profit percent.
  - f) The marked price of a radio is 25 % above the selling price and the cost price is 30 % less than its marked price. Find the discount percent and gain percent.
  - g) A shopkeeper fixed the price of a pen to make a profit of 10 %. But he sold it allowing a discount of Rs 7.50 and lost 5 %. At what price did he purchase the pen?
9. a) When an article is sold at a discount of 10 %, a profit of Rs 8 is earned. If the same article is sold without allowing a discount, there will be a profit of Rs 20. Find the cost price of the article.
- b) A watch was sold on its marked price at a gain of 20 %. But allowing 5 % discount, there would have been a gain of Rs 140. Find the cost price of the watch.
- c) A shopkeeper sold an article at 20 % discount and made a loss of Rs 90. If he had sold it at 5 % discount, he would have gained Rs 90. Find the cost price and the marked price of the article.
- d) An article, after allowing a discount of 20 % on its marked price, was sold at a gain of 20 %. Had it been sold after allowing 25 % discount, there would have been a gain of Rs 125. Find the marked price of the article.
10. a) A bicycle is sold at Rs 9,040 after allowing 20 % discount and imposing 13 % VAT. Find the marked price of the bicycle.
- b) After allowing 15 % discount on the marked price of an article 13 % VAT was levied on the remaining amount, then the price of the article becomes Rs 13,447. Find the marked price of the article.
- c) The marked price of an article is Rs 4,000. If the price of the article including 13 % VAT is Rs 3,616, find the discount percent given in it.
- d) A colour TV is sold at Rs 20,700 after 10 % discount with 15 % VAT. Find the VAT amount.
- e) After allowing 10 % discount on the marked price of an article and then 15 % VAT is charged, its price becomes Rs 16,720. How much amount was given as discount?
11. a) A shopkeeper purchased a bicycle for Rs 5,000 and marked its price a certain percent above the cost price. Then, he sold it at 10% discount. If a customer paid Rs 6,356.25 with 13% VAT to buy it, how many percent is the marked price above the cost price?
- b) The price of an article is fixed a certain percent above the cost price and sold it at 5% discount. If the cost price of the article is Rs 16,000 and sold it for Rs 20,064 with 10% VAT by how many percent is the marked price above the cost price?
12. a) When an article was sold at a discount of 10%, a customer paid Rs 9,153 with 13% VAT. If 8% profit was made in this transaction by how many percent was the marked price above the cost price?
- b) A retailer marked the price of a watch a certain percent above the cost price. Then, he allowed 20% discount to make 12% profit. If the watch was sold for Rs 5,062.40 with 13% VAT, by what percent is the marked price above the cost price?
13. a) The marked price of an item is Rs 4,000 and 10% discount is given to make 20% profit. By what percent is the discount to be increased to get only 12% profit?
- b) A shopkeeper marked the price of a jacket as Rs 10,000. Then, he allowed 40% discount to make 20% profit. By what percent is the discount to be reduced to increased the profit by 10%?

14. a) A shopkeeper allowed 10% discount on his goods to make 20% profit. If he sold a watch for Rs 10,170 with 13% VAT, by what percent is the discount to be increased to make only 12% profit?
- b) A retailer allowed 20% discount and sold a mobile set for Rs 4,520 with 13% VAT and made a profit of 25%. By what percent is the discount to be reduced to increase the profit by 5%?
15. a) After allowing 25 % discount on the marked price and then levying 10 % VAT, a cycle was sold. If the discount amount was Rs 750, how much VAT was levied on the price of the cycle?
- b) A tourist buys a Nepalese flag at a discount of 15 % but pays 10 % VAT. If he/she pays Rs 170 for VAT, calculate the discount amount.
- c) A person buys an article at a discount of 13 % and pays 16 % VAT. If he/she pays Rs 261 for VAT, find the marked price of the article and also the amount paid by him/her to buy the article.
- d) After allowing 20% discount on the marked price of a mobile 15% VAT was levied and sold it. If the difference between the selling price with VAT and selling price after discount is Rs 1800, find the marked price of the mobile.

## Project Work

**Make groups of your friends. Collect different types of bills such as goods purchased bills, electricity bills, water bills, telephone bills, etc. Study about the marked price rate of discount, rate of rebate, rate of VAT or other rates of taxes mentioned in the bills. Prepare the reports and present in your class.**



## 2.4 Money exchange- Introduction

Different countries have their own currencies. The value of one nation's currency may be higher or lower than the currency of another nation.

A Currency gets stronger when money is invested in the country. When we buy products that are made in Nepal, we are investing in Nepali business and keeping the money in the country. When we buy products imported from other countries, we are investing money in those countries and as a result the Nepali rupees will weaken. The more Nepali products we buy, the greater the demand for them will be and more jobs will become available for the people.

International trade requires an organised system for exchanging money. The exchanging rate between two currencies is used for this purpose. The exchanging rate is either the bank selling or the bank buying rate.

Exchange Rates Fixed by Nepal Rastra Bank			
Currency	Unit	Buying/Rs.	Selling/Rs.
Indian Rupee	100	160.00	160.15
Open Market Exchange Rates			
(For the purpose of Nepal Rastra Bank)			
Currency	Unit	Buying/Rs.	Selling/Rs.
U.S. Dollar	1	106.52	107.12
European Euro	1	118.02	118.68
UK Pound Sterling	1	130.61	131.34
Swiss Franc	1	109.55	110.16
Australian Dollar	1	81.34	81.79
Canadian Dollar	1	79.57	80.02
Singapore Dollar	1	76.77	77.20
Japanese Yen	10	10.28	10.34
Chinese Yuan	1	15.75	15.84
Saudi Arabian Riyal	1	28.40	28.56
Qatari Riyal	1	29.26	29.42
Thai Baht	1	3.04	3.06
UAE Dirham	1	29.00	29.16
Malaysian Ringgit	1	25.42	25.56
South Korean Won	100	9.30	9.35
Swedish Kroner	1	11.91	11.98
Danish Kroner	1	15.87	15.96
Hong Kong Dollar	1	13.74	13.81
Kuwaiti Dinar	1	352.03	354.01
Bahrain Dinar	1	282.52	284.11
Note: Under the present system the open market exchange rates quoted by different banks may differ. (The above mentioned exchange rates are on 2073-08-17)			

An exchanging rate is the price of one nation's currency in terms of another nation's currency. The table given alongside is a display of exchange rates in a commercial bank on a day.

Most of the world's currencies, including the euro (Eur), the US dollar (USD), the Canadian dollar (CAD), the Australian dollar (AUD), and the British pound (GBP), are floating, or variable. This means their values and their exchanging rates depend on the international money market.

### Worked-out examples

**Example 1:** By using the exchange rate table given above, find the difference of selling and buying rates of US \$ 500 in Nepali rupee.

**Solution:**

From the above exchange rate table:

The buying rate of US\$ 1 = Rs 106.52

The buying rate of US\$ 500 = Rs  $106.52 \times 500$  = Rs 53,260

Alos, the selling rate of US\$ 1 = Rs 107.12

the selling rate of US\$ 500 = Rs  $107.12 \times 500$  = Rs 53,560

∴ The difference due to the selling and buying rates = Rs 53,560 – Rs 53,260 = Rs 300

**Example 2:** The exchange rate of USD (\$) and Nepali rupees (NRs) on a day is \$ 1 = NRs 105.

a) How many dollars can be exchanged for NRs 76,125?

b) How many rupees can be exchanged for \$ 2,500?

**Solution:**

a) Here, NRs 105 = \$ 1

$$\text{NRs } 1 = \$ \frac{1}{105}$$

$$\text{NRs } 76,125 = \$ \frac{1}{105} \times 76,125 = \$ 725$$

b) \$ 1 = NRs 105

$$\$ 2,500 = \text{NRs } 105 \times 2,500 = \text{NRs } 2,62,500$$

**Example 3:** Study the following exchange rate table:

Country	Currency	Exchange rate
United Kingdom	Pounds (£)	1 £ = NRs 131
United States	Dollars (\$)	1 \$ = NRs 107

In Nepal the cost of a television is Rs 1,40,000. In England the same television costs £ 900 and in the USA \$1,240. In which country is the television the cheaper?

**Solution:**

The cost of the television in Nepal = NRs 1,40,000

The cost of the television in England in NRs = NRs  $131 \times 900$  = NRs 1,17,900

The cost of the television in USA is NRs = NRs  $107 \times 1,240$  = NRs 1,32,680

So, the television is cheaper in England.

**Example 4:** Use the exchange rate given in the table and solve the following problems.

	USD (\$)	GBP (£)	CAD (\$)	EUR (€)	AUD (\$)
USD (\$)	1	0.80	1.34	0.90	1.30
GBP (£)	1.25	1	1.67	1.12	1.62
CAD (\$)	0.75	0.60	1	0.67	0.97
EUR (€)	1.10	0.90	1.48	1	1.44
AUD (\$)	0.76	0.62	1.03	0.69	1

a) Convert USD 2,000 into GBP and EUR

b) Convert EUR 4,000 into AUD and CAD.

**Solution:**

a) From the table,

$$\text{US\$ } 1 = \text{GBP£ } 0.80$$

$$\therefore \text{US\$ } 2,000 = 2,000 \times \text{£ } 0.80 \\ = \text{£ } 1,600$$

$$\text{US\$ } 1 = (\text{€}) 0.90$$

$$\therefore \text{US\$ } 2,000 = 2,000 \times \text{€ } 0.90 \\ = \text{€ } 1,800$$

b) From the table,

$$\text{€ } 1 = \text{AUD\$ } 1.44$$

$$\therefore \text{€ } 4,000 = 4,000 \times \text{AUD\$ } 1.44 \\ = \text{AUD\$ } 5,760$$

$$\text{€ } 1 = \text{CAD\$ } 1.48$$

$$\therefore \text{€ } 4,000 = 4,000 \times \text{CAD\$ } 1.48 \\ = \text{CAD\$ } 5,920$$

**Example 5:** A trader purchased 200 pieces of Nepali carpet at Rs 5,000 per piece. He exported them to United Kingdom with 5% export tax. If he sold them at £ 80 per piece in UK, calculate his profit or loss percent. (£ 1 = NPR 130.00)

**Solution:**

$$\text{Here, C.P. of 200 piece of carpets} = 200 \times \text{Rs } 5,000 \\ = \text{Rs. } 10,00,000$$

$$\text{Also, C.P. with 5\% export tax} = \text{Rs } 10,00,000 + 5\% \text{ of Rs } 10,00,000 \\ = \text{Rs } 10,50,000$$

$$\text{Again, S.P. of 200 pieces of carpet in UK} = 200 \times \text{£ } 80 = \text{£ } 16,000$$

$$\text{Now, £ } 1 = \text{Rs } 130.00$$

$$\text{£ } 16,000 = 16,000 \times \text{Rs } 130 \\ = \text{Rs } 20,80,000$$

$$\therefore \text{Profit} = \text{Rs } 20,80,000 - \text{Rs } 10,50,000 \\ = \text{Rs } 10,30,000$$

$$\text{And, profit percent} = \frac{\text{Actual profit}}{\text{C.P.}} \times 100\% \\ = \frac{\text{Rs } 10,30,000}{\text{Rs } 10,50,000} \times 100\% = 98.09\%$$

So, his profit percent is 98.09%.

## 2.2 Money exchange by using chain rule

Chain rule is an alternative method of unitary method and ratio and proportion method to find the value of unknown variable in a compound proportion. Study the following examples and learn about this method.

If  $A = B$  and

$B = C$ , then

$C = A$

$A \times B \times C = B \times C \times A$

**Example 6:** If US \$ 1 = NPR 104.00 and GBP £ 1 = NPR 130.00, convert \$ 60 into pound.

**Solution:**

Here, \$ 1 = Rs 104

Rs 130 = £ 1

£ x = \$ 60

Now, using chain rule, we get,

$$1 \times 130 \times x = 104 \times 1 \times 60$$

$$\text{or, } x = \frac{104 \times 1 \times 60}{1 \times 130} = \text{£ } 48$$

So, \$ 60 is equal to £ 48 according to the given exchange rate.

## EXERCISE 2.3

### General Section

The exchange rates of foreign currencies on a day is given below in a table. Use the exchange rates to help you solve the problems.

Country	Currency	Exchange rate
India	Rupee (₹)	₹ 100 = NRs 160.00
United States	Dollar (\$)	\$1 = NRs 106.00
European Union	EURO (€)	1 = NRs 118.00
United Kingdom	Pound Sterling (GBP)	£ 1 = NRs 130.00
Australia	AUD (\$) Australian Dollar	AUD 1 = NRs 82.00
Canada	CAD (\$) Canadian Dollar	CAD 1 = NRs 80.00
Singapore	SGD (\$)	1 = NRs 77.00
Japan	Yen (Y)	10Y = NRs 10.50
Saudi Arab	Riyal (SAR)	SAR 1 = NRs 28.00
Qatar	Qatar Riyal (QAR)	QAR 1 = NRs 29.00
Malaysia	Ringgit (RM)	RM 1 = NRs 25.00
South Korea	Won (KRW)	KRW 100 = NRs 9.00
Hongkong	Hongkong Dollar (HKD\$)	1 = NRs 14.00
China	Yuan (¥)	1 = 15.75.00



- Convert the following currencies into Nepali Currency (NRs)
  - IN ₹ 12,500
  - US\$ 640
  - GBP£ 980
  - SAR 2,160
  - QAR 1,890
  - KRW 50,000
  - RM 2,460
  - AU\$ 1,230
- Convert NRs 84,800 into
  - INR
  - USD
  - GBP
  - RM
- Use the exchange rates given in the table and solve the following problems.

	USD (\$)	GBP (£)	CAD (\$)	EUR (€)	AUD (\$)
USD (\$)	1	0.80	1.34	0.90	1.30
GBP (£)	1.25	1	1.67	1.12	1.62
CAD (\$)	0.75	0.60	1	0.67	0.97
EUR (€)	1.10	0.90	1.48	1	1.44
AUD (\$)	0.76	0.62	1.03	0.69	1

- Convert USD 5000 into
    - GBP
    - CAD
    - EUR
    - AUD
  - Covert GBP 3,000 into
    - USD
    - CAD
    - EUR
    - AUD
  - Convert EUR 2,000 into
    - USD
    - GBP
    - CAD
    - AUD
  - Convert CAD 4,000 into
    - USD
    - GBP
    - EUR
    - AUD
  - Convert AUD 6,000 into
    - USD
    - GBP
    - EUR
    - CAD
- A Nepali worker earns Quatrai Riyal (QAR) 1,800 per month in Quatar. If the exchange rate is QAR 1 = NPR 29.25, how much is his monthly income in Nepali rupee?
    - Bishu Rai is a primary teacher in Australia. She earns AUD\$ 30 per hour. She teaches 5 hours per day except Sunday. The exchange rate in AUD\$ 1 = NPR 80.50.
      - Find her earning per week in Nepali rupees.
      - How much Nepali rupee does she earn in 4 weeks?
    - Sunayana wants to buy an ipad that costs US\$ 750, with the exchange rate currently at US\$ 1 = NPR 105. She estimates that the exchange rate will drop to NPR 102 in a month.
      - How much will the ipad cost in Nepali rupees, if she buy it now?
      - How much will she save if the exchange rate drops to NPR 102?
      - How much will she lose if the exchange rate moves to NPR 107?
    - Mr. Pariyar is going to visit Thiland for his family trip. He estimated to exchange US \$ 5,000 in a bank. If the bank charges 1.5% commission for exchanging the money, how much Nepali rupees is required for him? (US\$ 1 = NPR 106.50)
    - Rajani Chaudhary bought some EURO (€) for NPR 3,00,000 at the exchange rate of € 1 = NPR 118 to visit a few European countries. Unfortunately, because of her Visa problem, she cancelled her trip. Within a week Nepali rupee is devaluated by 2%. She again exchanged her EURO to Nepali rupee after a week. How much did she gain?

5. a) In Nepal the cost of a mobile set is Rs 1,20,000. In England the same set costs £ 950 and in America US\$ 1,200. In which country is the mobile set the cheaper?  
(£ 1 = NPR 130.00, \$ 1 = NPR 106.00)
- b) Mrs. Magar wants to buy a book online. She finds a publisher in London selling the book for £ 15. This publisher is offering free transportation on the product. She then finds the same book from a publisher in New York for \$17 with a transportation fee of \$2. Which publisher should she buy the book from?  
(Exchange rate: £1 = NPR 132 and \$ 1 = NPR 106.00)
6. a) Mr. Gurung bought 100 g of gold in Hong Kong for KHD\$ 30,000 and bought to Nepal paying 20% custom duty. If he sold the gold with 13% VAT in Nepal, how much Nepali rupee did he get? (HKD\$ 1 = NPR 14.00)
- b) A merchant purchased 500 pieces of 'Nepali Pasmina' at Rs 2,500 per piece. He exported them to USA with 5% export tax. If he sold them at \$ 40 per piece in USA, calculate his profit or loss percent. (\$ 1 = NPR 105.00)
- c) Laxmi Yadav bought a television for RM 2,000 and sold to her friend at 10% profit in Malaysia. Her friend brought it to Nepal and sold at a profit of 25%. Find the selling price of the television in Nepal. (RM 1 = NPR 25.00)
7. a) If 176 dollars can be exchanged for 140 pounds and 1 pound is exchanged for Rs 130, how many dollars can be exchanged for Rs 8925?
- b) If \$ 5 = £ 4 and Rs 315 = \$ 3, how many pounds (£) will be equal to Rs 5,625?
- c) The exchange rate of 10 Japanese Yen on a certain day was Rs 10.25. On the same day, if \$ 3 was exchanged for Rs 318, how many Yens could be exchanged for \$ 9?
- d) If the exchange rate of £ 1 is Rs 131.50 and the exchange rate of US \$ 1 is Rs 106.50, how many dollars can be exchanged for 10 sterling pounds?
- e) If Rs 1,785 = \$ 17 and Rs 1,950 = £ 15, how many dollars can be exchanged for £ 195?
- f) If QAR 16 = \$ 4.50 and \$ 5 = Rs 525, how many QAR can be exchanged for Rs 31,200?
- g) If the exchange rate of RM 1 is Rs 25 and the exchange rate of \$ 1 is Rs 105, find the exchange rate of dollars for RM 100.

### **Project Work**

**Collect the information about today's exchange rates of different currencies in terms of Nepali currency. Also, make a note about the cost of oil per barrel and cost of gold per 10 gram in the international market on the same day. Calculate the selling price of these materials in Nepali market according to the present rates of custom duty, VAT and other rates of taxes.**

**Note: Dear students, please go through the appendix given at the end of the book for more practice.**

### 3.1 Compound interest - Introduction

When we borrow a loan from a bank, we should pay some charge for the use of borrowed money under the certain condition. Such charge paid for using other's money is called Interest. Similarly, if we deposit money in a bank, the bank will pay us interest. Interest can be paid in two ways - simple interest or compound interest.

#### (i) Simple interest

The interest which is calculated always from the original principal for any interval of time and rate is known as **simple interest**. We calculate simple interest by using the following formula,

$$\text{Simple interest (I)} = \frac{P \times T \times R}{100}$$

Where, P is the principal, T is the time and R is the rate of interest.

#### (ii) Compound interest

When the interest of a principal for one year is added to the principal and the interest for the next year is calculated from the new principal, it is called **compound interest**.

In this way, in the case of simple interest, the principal always remains the same; however, the principal is increased every year in case of compound interest.

Now, let's derive the formula to calculate compound interest.

Let the original principal = Rs P  
 the rate of interest = R% p.a.  
 the time interval = T years

Now, the interest for the 1<sup>st</sup> year =  $P \times T \times R\% = P \times 1 \times R\% = PR\%$

∴ The amount in the 1<sup>st</sup> year =  $P + PR\% = P(1 + R\%)$

Again, the principal for 2<sup>nd</sup> year =  $P(1 + R\%)$

The interest for 2<sup>nd</sup> year =  $P(1 + R\%) \times 1 \times R\% = P(1 + R\%)R\%$

Then, the amount in 2<sup>nd</sup> year =  $P(1 + R\%) + P(1 + R\%)R\%$   
 $= P(1 + R\%)(1 + R\%) = P(1 + R\%)^2$

Similarly, the amount in 3<sup>rd</sup> year =  $P(1 + R\%)^3$

the amount in 4<sup>th</sup> year =  $P(1 + R\%)^4$

$$\therefore \text{The amount in T year} = P \left(1 + \frac{R}{100}\right)^T = P \left(1 + \frac{R}{100}\right)^T$$

$$\text{Also, the interest in T year} = P \left(1 + \frac{R}{100}\right)^T - P = P \left[\left(1 + \frac{R}{100}\right)^T - 1\right]$$

Thus,

$$\text{Compound amount} = P \left(1 + \frac{R}{100}\right)^T \quad \text{and} \quad \text{Compound interest} = P \left[\left(1 + \frac{R}{100}\right)^T - 1\right]$$

1. When the interest is compounded annually but the rate being different in different years, for example,  $R_1\%$  in the first year,  $R_2\%$  in the second year,  $R_3\%$  in the third years, then the amount in 3 years is calculated as,

$$\text{Compound Amount} = P \left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \left(1 + \frac{R_3}{100}\right)$$

$$\text{Also compound Interest} = P \left[\left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \left(1 + \frac{R_3}{100}\right) - 1\right]$$

2. When the interest is compounded annually but the time is given in 'T' years and 'M' months, then the amount is calculated as

$$\text{Compound Amount} = P \left(1 + \frac{R}{100}\right)^T \left(1 + \frac{MR}{1200}\right)$$

$$\text{Also, compound Interest} = P \left[\left(1 + \frac{R}{100}\right)^T \left(1 + \frac{MR}{1200}\right) - 1\right]$$

### 3.2 Interest compounded half-yearly and quarter-yearly

If the compound interest is payable half yearly, then **rate** =  $\frac{R}{2}$  % per half yearly and **time** =  $2T$  half years.

$$\text{Now, the compound amount half yearly} = P \left(1 + \frac{R}{2 \times 100}\right)^{2T}$$

$$\text{Also, the compound interest half yearly} = P \left[\left(1 + \frac{R}{2 \times 100}\right)^{2T} - 1\right]$$

Similarly, if the compound interest is payable quarter-yearly (every 3 months),

then rate =  $\frac{R}{4}$  % per quarter-yearly and time =  $4T$  quarter years.

$$\text{So, the compound amount quarter-yearly} = P \left(1 + \frac{R}{4 \times 100}\right)^{4T}$$

$$\text{And the compound interest quarter-yearly} = P \left[\left(1 + \frac{R}{4 \times 100}\right)^{4T} - 1\right]$$

### Worked-out examples

**Example 1:** Calculate the compound interest of Rs 5,000 at 10% per annum for 3 years without using the formula.

**Solution:**

Here, principal (P) = Rs 5,000

Rate of interest (R) = 10% p.a.

Time (T) = 3 years

$$\begin{aligned} \text{Now, the interest in the first year (I}_1\text{)} &= \frac{PTR}{100} = \frac{5,000 \times 1 \times 10}{100} \\ &= \text{Rs } 500 \end{aligned}$$

$$\begin{aligned}\therefore \quad \text{The principal for the second year} &= \text{Rs } 5,000 + \text{Rs } 500 \\ &= \text{Rs } 5,500\end{aligned}$$

$$\begin{aligned}\text{Now, the interest in the second year (I}_2\text{)} &= \frac{5,500 \times 1 \times 10}{100} \\ &= \text{Rs } 550\end{aligned}$$

$$\begin{aligned}\therefore \quad \text{The principal for the third year} &= \text{Rs } 5,500 + \text{Rs } 550 \\ &= \text{Rs } 6,050\end{aligned}$$

$$\begin{aligned}\text{Again, the interest in the third year (I}_3\text{)} &= \frac{6,050 \times 1 \times 10}{100} \\ &= \text{Rs } 605\end{aligned}$$

$$\begin{aligned}\therefore \quad \text{The compound interest (C.I.)} &= I^1 + I^2 + I^3 \\ &= \text{Rs } 500 + \text{Rs } 550 + \text{Rs } 605 \\ &= \text{Rs } 1,655\end{aligned}$$

**Example 2: Find the difference between the simple interest and compound interest of Rs 32000 at the rate of 5 % p.a. for 3 years.**

**Solution:**

Here, principal (P) = Rs 32000, time (T) = 3 years and rate (R) = 5 % p.a.

$$\text{Now, simple interest} = \frac{PTR}{100} = \text{Rs } \frac{32,000 \times 3 \times 5}{100} = \text{Rs } 4,800$$

$$\begin{aligned}\text{Also, compound interest} &= P \left[ \left( 1 + \frac{R}{100} \right)^T - 1 \right] \\ &= \text{Rs } 32,000 \left[ \left( 1 + \frac{5}{100} \right)^3 - 1 \right] \\ &= \text{Rs } 32,000 \left[ \left( 1 + \frac{1}{20} \right)^3 - 1 \right] \\ &= \text{Rs } 32,000 \left[ \left( \frac{21}{20} \right)^3 - 1 \right] \\ &= \text{Rs } 32,000 \left[ \frac{21 \times 21 \times 21 - 8,000}{8,000} \right] = \text{Rs } 4 \times 1261 = \text{Rs } 5,044\end{aligned}$$

$$\therefore \text{ Difference of compound and simple interest} = \text{Rs } 5,044 - \text{Rs } 4,800 = \text{Rs } 244$$

**Example 3: Find the difference between compound interest compounded semi-annually and the interest compounded annually on Rs 8,000 at 10 % per annum in  $1\frac{1}{2}$  years.**

**Solution:**

Here, principal (P) = Rs 8,000, time (T) =  $1\frac{1}{2}$  years and rate (R) = 10 % p.a.

$$\begin{aligned}\text{Now, interest compounded semi annually (I}_1\text{)} &= P \left[ \left( 1 + \frac{R}{2 \times 100} \right)^{2T} - 1 \right] \\ &= \text{Rs } 8,000 \left[ \left( 1 + \frac{10}{200} \right)^3 - 1 \right] \\ &= \text{Rs } 8,000 \left[ \frac{21 \times 21 \times 21}{20 \times 20 \times 20} - 1 \right] \\ &= \text{Rs } 1,261\end{aligned}$$

## Compound Interest

Also, interest compounded annually ( $I_2$ )

$$\begin{aligned} &= P \left[ \left( 1 + \frac{R}{100} \right)^T \left( 1 + \frac{\frac{1}{2}R}{100} \right) - 1 \right] \\ &= \text{Rs } 8,000 \left[ \left( 1 + \frac{10}{100} \right)^1 \left( 1 + \frac{10}{200} \right) - 1 \right] \\ &= \text{Rs } 8,000 \left[ \frac{11}{10} \times \frac{21}{20} - 1 \right] \\ &= \text{Rs } 8,000 \left[ \frac{231 - 200}{200} \right] \\ &= \text{Rs } 1,240 \end{aligned}$$

∴ Difference between  $I_1$  and  $I_2 = I_1 - I_2$

$$= \text{Rs } 1,261 - \text{Rs } 1,240$$

$$= \text{Rs } 21$$

So, the required difference between the interest compounded semi annually and the interest compounded annually is Rs 21.

### Alternative process

$$\begin{aligned} I_2 &= P \left[ \left( 1 + \frac{R}{100} \right)^T \left( 1 + \frac{MR}{1200} \right) - 1 \right] \\ &= 8,000 \left[ \left( 1 + \frac{10}{100} \right)^1 \left( 1 + \frac{6 \times 10}{1,200} \right) - 1 \right] \\ &= 8,000 \left[ \frac{11}{10} \times \frac{21}{20} - 1 \right] \\ &= \text{Rs } 1,240 \end{aligned}$$

**Example 4: The compound interest on the sum of money in 2 years at the rate of 10 % per annum will be Rs 420 more than simple interest. Find the sum.**

### Solution:

Here, time ( $T$ ) = 2 years, rate ( $R$ ) = 10 %, let Principal be Rs  $P$ .

$$\begin{aligned} \text{Now, C.I.} &= P \left[ \left( 1 + \frac{R}{100} \right)^T - 1 \right] \\ &= P \left[ \left( 1 + \frac{10}{100} \right)^2 - 1 \right] = \frac{21P}{100} \end{aligned}$$

$$\text{Also, S.I.} = \frac{PTR}{100} = \frac{P \times 2 \times 10}{100} = \frac{P}{5}$$

From the question, C.I. – S.I. = Rs 420

$$\text{or, } \frac{21P}{100} - \frac{P}{5} = \text{Rs } 420$$

$$\text{or, } \frac{P}{100} = \text{Rs } 420$$

$$\text{or, } P = \text{Rs } 42,000$$

So, the required sum is Rs 42,000.

**Example 5: At what rate percent per annum compound interest will Rs 576 amounts to Rs 625 in 2 years?**

### Solution:

Here, principal ( $P$ ) = Rs 576, time ( $T$ ) = 2 years, C.A. = Rs 625

$$\begin{aligned} \text{Now, C.A.} &= P \left( 1 + \frac{R}{100} \right)^T \\ 625 &= 576 \left( 1 + \frac{R}{100} \right)^2 \end{aligned}$$

$$\text{or, } \frac{625}{576} = \left( \frac{100 + R}{100} \right)^2$$

$$\text{or, } \frac{100 + R}{100} = \frac{25}{24}$$

$$\text{or, } R = \frac{2,500}{24} - 100 = \frac{100}{24} = 4\frac{1}{6} \%$$

So, the required rate of interest is  $4\frac{1}{6} \%$  p.a.

**Example 6:** A person took a loan of Rs 46,875. If the rate of compound interest is 4 paise per rupee per year, in how many years will the compound interest be Rs 5,853?

**Solution:**

Here, the rate of interest (R) = 4 paise per Re 1 per 1 year  
 = 400 paise, i.e. Rs 4 per Rs 100 per 1 year  
 = 4 % per annum

Also, principal (P) = Rs 46,875 and C.I. = Rs 5,853

$$\text{Now, C.I.} = P \left[ \left( 1 + \frac{R}{100} \right)^T - 1 \right]$$

$$\text{or, } 5,853 = 46,875 \left[ \left( 1 + \frac{R}{100} \right)^T - 1 \right]$$

$$\text{or, } \frac{58,53}{46,875} = \left( \frac{26}{25} \right)^T - 1$$

$$\text{or, } \left( \frac{26}{25} \right)^T = \frac{1,951}{15,625} + 1$$

$$\text{or, } \left( \frac{26}{25} \right)^T = \frac{17,576}{15,625} = \left( \frac{26}{25} \right)^3$$

$$\text{or, } T = 3 \text{ years}$$

So, the required time is 3 years.

**Example 7:** Find the compound interest of Rs 50,000 for 3 years if the rates of interest for three years are 4 %, 5 % and 6 % p.a. respectively.

**Solution:**

Here, principal (P) = Rs 50,000

Rate are,  $R_1 = 4 \%$ ,  $R_2 = 5 \%$  and  $R_3 = 6 \%$  p.a.

$$\begin{aligned} \text{Now, compound amount (C.A.)} &= P \left( 1 + \frac{R_1}{100} \right) \left( 1 + \frac{R_2}{100} \right) \left( 1 + \frac{R_3}{100} \right) \\ &= 50,000 \left( 1 + \frac{4}{100} \right) \left( 1 + \frac{5}{100} \right) \left( 1 + \frac{6}{100} \right) \\ &= 50,000 \times \frac{104}{100} \times \frac{105}{100} \times \frac{106}{100} = \text{Rs } 57,876 \\ \therefore \text{Compound interest (C.I.)} &= \text{C.A.} - P \\ &= \text{Rs } 57,876 - \text{Rs } 50,000 \\ &= \text{Rs } 7,876 \end{aligned}$$

**Example 8:** The compound amount of a sum of money in 3 years is Rs 13,310 and in 4 years is Rs 14,641. Find the compound rate of interest per annum and the sum.

**Solution:**

Here, the C.A. in 3 years = Rs 13310

$$\text{or, } P \left( 1 + \frac{R}{100} \right)^3 = \text{Rs } 13310$$

$$\text{or, } P \left( \frac{100 + R}{100} \right)^3 = \text{Rs } 13310 \dots\dots(i)$$

or, Also, the C.A. in 4 years = Rs 14641

$$\text{or, } P \left( 1 + \frac{R}{100} \right)^4 = \text{Rs } 14641$$

$$\text{or, } P \left( \frac{100 + R}{100} \right)^4 = \text{Rs } 14641 \dots\dots(ii)$$

Now, dividing equation (ii) by (i), we get,

$$\frac{P \left( \frac{100 + R}{100} \right)^4}{P \left( \frac{100 + R}{100} \right)^3} = \frac{14,641}{13,310}$$

$$\text{or, } \frac{100 + R}{100} = 1.1$$

$$\text{or, } R = 10 \%$$

Again, putting the value of R in equation (i), we get,

$$P \left( \frac{100 + 10}{100} \right)^3 = \text{Rs } 13,310$$

$$\text{or, } P \left( \frac{110}{100} \right)^3 = \text{Rs } 13,310$$

$$\text{or, } P = \text{Rs } 10,000$$

So, the required sum is Rs 10,000 and the rate of interest is 10 % p.a.

### Alternate process

Here, (C.A.)<sub>1</sub> = Rs 13,310

(C.A.)<sub>2</sub> = Rs 14,641

$$\begin{aligned} \text{Now, (C.A.)}_2 - (\text{C.A.})_1 &= \text{Rs } 14,641 - \text{Rs } 13,310 \\ &= \text{Rs } 1,331 \end{aligned}$$

Here, Rs 1,331 is the simple interest (S.I.) of Rs 13,310 for 1 year.

$$\begin{aligned} \therefore \text{Rate (R)} &= \frac{I \times 100}{P \times T} \\ &= \frac{1,331 \times 100}{13,310 \times 1} = 10 \% \end{aligned}$$

$$\begin{aligned} \text{Again, (C.A.)}_1 &= P \left( 1 + \frac{R}{100} \right)^3 \\ \text{Rs } 13,310 &= P \left( 1 + \frac{10}{100} \right)^3 \end{aligned}$$

$$\text{or, } P = \text{Rs } 10,000$$

**Example 9:** The yearly compound interest on a sum of money for two consecutive years are Rs 400 and Rs 440 respectively. Calculate the rate of interest and the sum.

**Solution:**

Here, taking the interest in the first year as principal,

$$I_1 = P = \text{Rs } 400$$

Also, taking the interest in the second year as compound amount,

$$I_2 = \text{C.A.} = \text{Rs } 440 \text{ and time (T) = 1 year}$$



$$\begin{aligned}
 \text{Now, } \text{C.A.} &= P \left(1 + \frac{R}{100}\right)^T \\
 \text{or, } 440 &= 400 \left(1 + \frac{R}{100}\right)^1 \\
 \text{or, } 100 + R &= 110 \\
 \text{or, } R &= 110 - 100 = 10 \% \text{ p.a.} \\
 \text{Again, } I_1 &= P \left[\left(1 + \frac{R}{100}\right)^T - 1\right] \\
 400 &= P \left[\left(1 + \frac{10}{100}\right)^T - 1\right] \\
 \text{or, } P &= \text{Rs } 4,000
 \end{aligned}$$

So, the required rate of interest is 10 % p.a. and the sum is Rs 4,000.

**Example 10:** Mohan lent altogether 6,600 to Ram and Shyam for 2 years. Ram agreed to pay simple interest at 15 % p.a. and Shyam agreed to pay compound interest at the same rate. If Ram paid Rs 112.50 more than Shyam as the interest, find how much did he lend to each of them?

**Solution:**

Suppose the money lent to Shyam =  $P_1 = \text{Rs } x$ .

$\therefore$  the money lent to Ram =  $P_2 = \text{Rs } (6,600 - x)$

Here, time (T) = 2 years and rate (R) = 15 % p.a.

$$\text{Now, the simple interest to Ram} = \frac{P_2 TR}{100} = \frac{\text{Rs } (6,600 - x) \times 2 \times 15}{100} = \text{Rs } \frac{19,800 - 3x}{10}$$

$$\begin{aligned}
 \text{Also, the compound interest to Shyam} &= P_1 \left[\left(1 + \frac{R}{100}\right)^T - 1\right] \\
 &= \text{Rs } x \left[\left(1 + \frac{15}{100}\right)^2 - 1\right] \\
 &= \text{Rs } x \left[\left(1 + \frac{3}{20}\right)^2 - 1\right] \\
 &= \text{Rs } x \left[\left(\frac{23}{20}\right)^2 - 1\right] \\
 &= \text{Rs } x \left[\frac{529}{400} - 1\right] \\
 &= \text{Rs } x \left[\frac{529 - 400}{400}\right] = \text{Rs } \frac{129x}{400}
 \end{aligned}$$

According to the question,

$$\text{or, } \frac{19,800 - 3x}{10} - \frac{129x}{400} = \text{Rs } 112.50$$

$$\therefore x = \text{Rs } 3,000$$

So, the money lent to Shyam =  $x = \text{Rs } 3,000$

The money lent to Ram =  $6,600 - x = 6,600 - 3,000 = \text{Rs } 3,600$

**EXERCISE 3.1****General section**

1. a) The compound interest and the compound amount on a sum  $P$  in  $T$  years at  $R$  % per annum are  $CI$  and  $CA$  respectively. Write down the relation among the following variables.
    - (i)  $P$ ,  $T$ ,  $R$ , and  $CA$  (Compounding annually)
    - (ii)  $P$ ,  $T$ ,  $R$  and  $CI$  (Compounding annually)
    - (iii)  $P$ ,  $T$ ,  $R$  and  $CA$  (Compounding semi-annually)
    - (iv)  $P$ ,  $T$ ,  $R$  and  $CI$  (Compounding semi-annually)
  - b) The compound amount on a sum  $P$  in  $T$  years  $M$  months at  $R$  % p.a. is  $CA$ . Write the relation among  $P$ ,  $T$ ,  $M$ ,  $R$  and  $CA$  compounding annually.
  - c) If  $CI$  is the compound interest of a sum  $P$  in  $T$  years  $M$  months at  $R$  % p.a., write the relation among  $P$ ,  $T$ ,  $M$ ,  $R$  and  $CI$  compounding annually.
  - d) If  $CA$  is the compound amount of a sum  $P$  at the different rates  $R_1$  %,  $R_2$  % and  $R_3$  % in the first, second and the third years respectively, write the relation among  $CA$ ,  $P$ ,  $R_1$ ,  $R_2$  and  $R_3$  (Compounding annually).
2. Calculate the compound interest without using the formula.
    - a)  $P = \text{Rs } 2,000$ ,  $T = 2$  years,  $R = 5\%$
    - b)  $P = \text{Rs } 10,000$ ,  $T = 3$  years,  $R = 10\%$
3. a) Find the compound interest compounding annually and the simple interest of the sum of  $\text{Rs } 5,000$  in 1 year at 10 % p.a. Are both types of interest same? Write your conclusion.
  - b) Find the amount compounded annually.
    - (i) Principal ( $P$ ) =  $\text{Rs } 2,000$ , time ( $T$ ) = 2 years, rate ( $R$ ) = 5 % p.a.
    - (ii) Principal ( $P$ ) =  $\text{Rs } 4,800$ , time ( $T$ ) =  $1\frac{1}{2}$  years, rate ( $R$ ) = 10 % p.a.
  - c) Find the interest compounded annually.
    - (i) Principal ( $P$ ) =  $\text{Rs } 8,000$ , time ( $T$ ) = 3 years, rate ( $R$ ) = 5 % p.a.
    - (ii) Principal ( $P$ ) =  $\text{Rs } 7,500$ , time ( $T$ ) =  $2\frac{1}{2}$  years, rate ( $R$ ) = 20 % p.a.
  - d) Find the amount compounded half yearly.
    - (i) Principal ( $P$ ) =  $\text{Rs } 6,250$ , time ( $T$ ) = 1 year, rate ( $R$ ) = 8 % p.a.
    - (ii) Principal ( $P$ ) =  $\text{Rs } 16,000$ , time ( $T$ ) =  $1\frac{1}{2}$  year, rate ( $R$ ) = 10 % p.a.
  - e) Find the interest compounded semi-annually.
    - (i) Principal ( $P$ ) =  $\text{Rs } 12,500$ , time ( $T$ ) = 1 year, rate ( $R$ ) = 12 % p.a.
    - (ii) Principal ( $P$ ) =  $\text{Rs } 64,000$  time ( $T$ )  $1\frac{1}{2}$  years, rate ( $R$ ) = 15 % p.a.

## Creative section

4. a) Ram borrowed Rs 4,800 from Sita at the rate of 10 % p.a. At the end of one year,  
 (i) how much simple interest will Ram have to pay?  
 (ii) How much compound interest semi-annually will Ram have to pay?
- b) Find the difference between simple interest and the annual compound interest on Rs 12,000 for 2 years at the rate of 20 % per annum.
- c) A person borrowed Rs 16,000 from a bank at 12.5 % per annum simple interest and lent the whole amount to a shopkeeper at the same rate of compound interest. How much will he gain after 3 years?
- d) A sum of Rs 1,50,000 amounts to Rs 2,62,500 at a certain rate of simple interest in 5 years. Find the sum of money that amounts to Rs 1,98,375 at the same rate of compound interest in 2 years.
- e) Mohan deposited Rs 5,000 at 8 % p.a. compound interest in a bank. Find the difference between the amounts compounded yearly and half-yearly in two years.
- f) Find the difference between compound interest compounded semi-annually and the interest compounded annually on Rs 16,000 at 10 % p.a. in  $1\frac{1}{2}$  years.
- g) Find the difference between compound interest compounded semi-annually and simple interest on Rs 8,000 at 10 % per annum in  $1\frac{1}{2}$  years.
- h) Sunayana borrowed a sum of Rs 12,500 at 12 % p.a. simple interest for 1 year 6 months and lent to Bishwant at the same rate of compound interest compounded half-yearly for the same interval of time. How much profit did she make?
- i) Ram borrowed Rs 1,50,000 from Sita at the rate of 21 % per year. At the end of nine months how much compound interest compounded half-yearly should he pay?

Hints: 9 months = 6 months + 3 months =  $\frac{1}{2}$  year + 3 months

$$\therefore T = \frac{1}{2}, M = 3 \text{ and } CI = P \left[ \left( 1 + \frac{R}{200} \right)^{2T} \left( 1 + \frac{MR}{1200} \right) - 1 \right]$$

5. a) Find the amount compounded annually on Rs 25,000 for 2 years if the rates of interest for two years are 10 % and 12 % respectively.
- b) Find the compound interest compounded annually on Rs 1,00,000 for 3 years if the rates of interest in the first, second and the third years are 4 %, 5 % and 7 % respectively.
6. a) The compound interest on the sum of money at 8 % per annum for 2 years is more than the simple interest on the same sum at the same rate for the same time by Rs 76.80. Find the sum.
- b) The simple interest on a sum of money in 2 years is Rs 36 less than the compound interest compounded annually. If the rate of interest is 12 % p.a., find the sum.

## Compound Interest

- c) If the compound interest on a sum of money compounded semi-annually in one year at 10 % per annum is Rs 40 more than the compound interest on the same sum compounded annually in the same time and at the same rate, find the sum.
  - d) The difference between the annual and semi-annual compound interest on a sum of money is Rs 482 at the rate of 20 % per annum for 2 years. Find the sum.
  - e) The sum of simple interest and compound interest after 2 years is Rs 202.50 and the rate of interest is 5% per annum. Find the principal.
7. a) In how many years will Rs 8,000 amount to Rs 13,824 at 20% per annum interest compounded annually?
- b) A man took a loan of Rs 40,000. If the rate of compound interest is 5 paise per rupee per year, in how many years will the compound interest be Rs 6,305?
8. a) At what percent rate of compound interest per annum will the compound interest on Rs 1,25,000 be Rs 91,000 in 3 years?
- b) At what rate percent per annum compound interest will Rs 2,500 amount to Rs 2,704 in 2 years?
9. a) If the compound amounts of a sum of money in 2 years and 3 years are Rs 13,000 and Rs 14,300 respectively, find the rate of interest.
- b) If a sum becomes Rs 6,480 in 3 years and Rs 7,776 in 4 years interest being compounded annually, find the sum and the rate of interest.
10. a) The compound interest of a sum of money in 1 year and 2 years are Rs 450 and Rs 945 respectively. Find the rate of interest compounded yearly and the sum.
- b) The compound interest of a sum of money in 1 year is Rs 350 and in 2 years is Rs 724.50. Find the rate of interest compounded yearly and the sum.
11. a) Ram lend altogether Rs 10,000 to Hari and Laxmi for 2 years. Hari agrees to pay simple interest at 12 % p.a. and Laxmi agrees to pay compound interest at the rate of 9 % p.a. If Laxmi paid Rs 596.70 more than Hari as the interest, find how much did he lend to each.
- b) The compound interest of a certain sum for 2 years is Rs 8,034 and the simple interest of the same sum for the same interval of time at the same rate of interest is Rs 7,800. Find the sum and the rate of interest.
- c) A bank has fixed the rate of interest 10% p.a. semi-annually compound interest in account P and 12% p.a., annually compound interest in account Q. If you are going to deposit Rs 50,000 for 2 years in the same bank, in which account will you deposit your money and why? Give your reason with calculation.

## Project Work

**Make the groups of your friends and visit different Banks and Finance Companies and collect their brochures. Study the of bank deals, bonuses, promotions offers mentioned in their brochures. Which plans and offers do you think the best and exciting to deposit your money?**

**Note: Dear students, please go through the appendix given at the end of the book for more practice.**

# 4

## Chapter

# Population Growth and Depreciation

## 4.1 Population growth - Introduction

The population of any country does not remain constant. The population of most of the countries has usually been found increasing. In some countries whether it is found to remain nearly constant or slightly decreasing.

The population structure and its growth rate is essentially required to the government of a country to make effective economical and social planning and policies.

The way of increasing population is exactly similar to the way of increasing compound amount. Because, every year population increases from the previously increased population.

Suppose, the population of a country or a place at a certain time =  $P$

the rate of growth of population =  $R$  % per annum,

the population of the country or place after  $T$  years =  $P_t$ , then,

$$P_t = P \left( 1 + \frac{R}{100} \right)^T$$

The growth of population is also affected by the number of deaths, in-migrants and out-migrants at the end of the given period of time.

In this case,

**The actual population in  $T$  years ( $P_t$ ) =  $P \left( 1 + \frac{R}{100} \right)^T - \text{no. of deaths} + M_{\text{in}} - M_{\text{out}}$**

Where,  $M_{\text{in}}$  and  $M_{\text{out}}$  are the number of in-migrants and out-migrants respectively at the end of the given period of time.

Similarly, the increased population in  $T$  years =  $P_t - P$

$$\begin{aligned} &= P \left( 1 + \frac{R}{100} \right)^T - P \\ &= P \left[ \left( 1 + \frac{R}{100} \right)^T - 1 \right] \end{aligned}$$

If the rate of growth of population every year is different, then the population after  $T$  years is calculated by using the following formula.

$$P_t = P \left( 1 + \frac{R_1}{100} \right) \left( 1 + \frac{R_2}{100} \right) \dots \left( 1 + \frac{R_T}{100} \right)$$

Where,  $R_1, R_2 \dots R_T$  are the rates of growth of population in the first year, second years, and ...  $T^{\text{th}}$  years.

**Worked-out examples**

**Example 1:** The population of a town before 2 years was 75,000 and the rate of annual growth of population of the town is 2%. If the numbers of in-migrants and out-migrants at the end of 2 years were 880 and 625 respectively and 550 people died within this time interval, find the present population of the town.

**Solution:**

Here, the population of the town before 2 years ( $P$ ) = 75,000

The rate of growth of population ( $R$ ) = 2%

The number of in-migrants ( $M_{in}$ ) = 880

The number of out-migrants ( $M_{out}$ ) = 625

The number of deaths ( $D$ ) = 550

Now, the present population of the town =  $P \left( 1 + \frac{R}{100} \right)^T - D + M_{in} - M_{out}$

$$= 75,000 \left( 1 + \frac{2}{100} \right)^2 - 550 + 880 - 625$$

$$= 75,000 \times \frac{51 \times 51}{50 \times 50} - 295 = 78,030 - 295 = 77,735$$

So, the present population of the town is 77,735.

**Example 2:** The present population of a town is 20,24,800. If the rate of growth of population is 5% per year, find the increased population in 2 years..

**Solution:**

Here, the present population of the town ( $P$ ) = 20,24,800

the rate of growth of population ( $R$ ) = 5% p.a.

Time ( $T$ ) = 2 years

Now, the increased population =  $P \left[ \left( 1 + \frac{R}{100} \right)^T - 1 \right]$

$$= 20,24,800 \left[ \left( 1 + \frac{5}{100} \right)^2 - 1 \right]$$

$$= 20,24,800 \left[ \left( \frac{21}{20} \right)^2 - 1 \right]$$

$$= 20,24,800 \times \frac{41}{400} = 2,07,542$$

So, the increased population of the town in 2 years is 2,07,542.

**Example 3:** Two years ago the population of a village was 40,000. If the birth rate is 4.5% p.a. and death rate is 2% p.a., find the present population of the village.

**Solution:**

Here, the population of the village before 2 years ( $P$ ) = 40,000

The annual growth rate of the population ( $R$ ) = 4.5% - 2% = 2.5%

$$\begin{aligned}\text{Now, the present population (P}_t\text{)} &= P \left(1 + \frac{R}{100}\right)^T \\ &= 40,000 \left(1 + \frac{2.5}{100}\right)^2 = 40,000 \times \frac{41}{40} \times \frac{41}{40} = 42,025.\end{aligned}$$

So, the present population of the village is 42,025.

**Example 4:** The population of a village increases every year by 5%. At the end of two years, if 1,025 people migrated to other places and the population of the village remained 10,000, what was the population of the village in the beginning?

**Solution:**

Here, the total population including the migrated number = 10,000 + 1,025 = 11,025

$$\text{We, know that, } P_t = P \left(1 + \frac{R}{100}\right)^T$$

$$\text{or, } 11,025 = P \left(1 + \frac{5}{100}\right)^2$$

$$\text{or, } 11,025 = P \left(\frac{21 \times 21}{20 \times 20}\right)$$

$$\text{or, } P = \frac{11025 \times 20 \times 20}{21 \times 21} = 10,000$$

So, the population of the village in the beginning was 10,000.

**Example 5:** The population of a town before 3 years was 1,50,000. If the annual growth rates of the population in the last 3 years were 2%, 4% and 5% respectively every year, find the population of the town at the end of 3 years.

**Solution:**

Here, the population of the town before 3 years (P) = 1,50,000

The annual growth rates in every year were  $R_1 = 2\%$ ,  $R_2 = 4\%$  and  $R_3 = 5\%$

Now, the population of the town at the end of 3 years is

$$\begin{aligned}P_t &= P \left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \left(1 + \frac{R_3}{100}\right) \\ &= 1,50,000 \left(1 + \frac{2}{100}\right) \left(1 + \frac{4}{100}\right) \left(1 + \frac{5}{100}\right) = 1,50,000 \times \frac{51}{50} \times \frac{26}{25} \times \frac{21}{20} = 1,67,076\end{aligned}$$

So, the population of the town at the end of 3 years is 1,67,076

**Example 6:** In the beginning of 2070 B.S., the population of a village was 50,000. The population growth rate of the village is 2% p.a. In the beginning of 2072 B.S. 420 people died due to earth quake. Find the population of the village in the beginning of 2073 B.S.

**Solution:**

Here, Population of the village

$$(P) = 50,000$$

Time

$$(T) = 2 \text{ years}$$

Population of the village at the end of 2 years ( $P_t$ ) = ?

$$\text{We have } P_t = P \left(1 + \frac{R}{100}\right)^T = 50,000 \left(1 + \frac{2}{100}\right)^2 = 50,000 \left(\frac{102 \times 102}{100 \times 100}\right) = 52,020$$

After 2 years, 420 people died due to earth quake.

So, the population of the village in the beginning of 2072 B.S. =  $52,020 - 420 = 51,600$

$$\begin{aligned}\text{Again, the population in the beginning of 2073 B.S.} &= P \left(1 + \frac{R}{100}\right)^T \\ &= 51,600 \left(1 + \frac{2}{100}\right)^1 = 51,600 \left(\frac{102}{100}\right) = 52,632\end{aligned}$$

So, the population of the village in the beginning of 2073 B.S. is 52,632.

**Example 7: 2 years ago the population of a town was 2,50,000. If the present population of the town is 2,70,400, find the rate of growth of population.**

**Solution:**

$$\begin{aligned}\text{Here, the population of the town before 2 years (P)} &= 2,50,000 \\ \text{The present population of the town (P}_t\text{)} &= 2,70,400 \\ \text{Time (T)} &= 2 \text{ years} \\ \text{The rate of growth of population (R)} &= ?\end{aligned}$$

$$\text{We have, } P_t = \left(1 + \frac{R}{100}\right)^T$$

$$\text{or, } 2,70,400 = 2,50,000 \left(1 + \frac{R}{100}\right)^2$$

$$\text{or, } \frac{2,704}{2,500} = \left(\frac{100 + R}{100}\right)^2$$

$$\text{or, } \left(\frac{52}{50}\right)^2 = \left(\frac{100 + R}{100}\right)^2$$

$$\text{or, } \frac{52}{50} = \frac{100 + R}{100}$$

$$\text{or, } 5000 + 50R = 5200$$

$$\text{or, } R = 4\%$$

So, the rate of growth of population of the town is 4% p.a.

## EXERCISE 4.1

### General Section

1. a) Write the formula to calculate the population of a region after T years if the initial population is P, rate of growth of population is R% p.a.  
b) The initial population of a town is P and the rate of growth of population is R% p.a., write the formula to find the increased population after T years.



- c) The rates of annual growth of population of a place in 3 years are  $R_1\%$ ,  $R_2\%$  and  $R_3\%$ . If the present population of the place is  $P$ , write the formula to find the population after 3 years.
2. a) The population of a village was 7,200. If 5% of the population out-migrated and 2% died due to different causes within a year, what would be the population of the village after a year?
- b) The population of a small village was 2,000. Within a year the population increased 3% by birthrate and 2% by immigration. Find the present population of the village.
- c) The population of a village increased from 10,000 to 11,000 in one year. Find the rate of growth of population.
- d) One year ago, the population of a village was 10,000. If the present population of the village is 10,210, find the population growth rate.

### Creative Section

3. a) The present population of a village is 30,000. If it is increased at the rate of 10 % per annum, what will be the population after 2 years?
- b) According to the students enrollment statistics published in 2072 B.S., the number of admitted students in 2071 B.S. in a region was 3,20,000. If the annual rate of growth of the number of students is 5%, how many students were admitted in 2074 B.S.?
- c) 3 years ago, the population of a village was 16,000. The rate of population growth of that village is 5%. What is the population at present?
- d) After two years the population of a town will be 33,620 at the population growth rate of 2.5 % p.a. Find the present population of the town.
- e) A village has population 1,75,760. If its population growth rate is 4 % p.a., find its population before 3 years.
4. a) In how many years will the population of a town be 2,09,475 from 1,90,000 at the growth rate of 5% per annum?
- b) The population of a town is 96,250. In how many years would it be 1,04,104 if the population increases at the rate of 4% every year?
- c) At present the population of a town is 80,000. At what growth rate of population would it be 88,200 in two years?
- d) The population of a municipality in the beginning of 2071 B.S. was 1,80,000 and at the end of 2073 B.S. was 2,39,580. Find the rate of growth of population per year.
5. a) The population of a town before 3 years was 3,75,000 and the annual growth rate is 2%. If the number of in-migrants and out-migrants at the end of 3 years were 1,480 and 875 respectively, and 2,750 people died within the times, find the present population of the town.

- b) The population of a village increases every year by 5%. At the end of two years, if 460 people were migrated to other village and the population of the village remained 26,000, what was the population of the village in the beginning?
  - c) The population of a town increases every year by 10%. At the end of two years, if 5,800 people were added by migration and the total population of the town became 30,000, what was the population of the town in the beginning?
  - d) The population of a village increases every year by 2%. If 950 people migrated to other places at the end of two years and the population of the village remained 9,454, what was the population of the village in the beginning?
  - e) In the beginning of 2071 B.S., the population of a town was 1,00,000 and the rate of growth of population is 2% every year. If 8,000 people migrated there from different places in the beginning of 2072 B.S., what will be the population of the town in the beginning of 2074 B.S.?
  - f) In the beginning of the year 2014, the population of a village was 25,000 and the rate of growth of population is 10% every year. In the beginning of 2015, if 500 people migrated to other places, what will be the population of the town in the beginning of 2018?
6. a) The population of a town before 3 years was 1,75,000. If the annual growth rates of the population in the last 3 years were 2%, 4% and 5% respectively every year, find the population of the town at the end of 3 years.
- b) The population of a village decreased by 5% in 2072 B.S. and by 10% in 2073 B.S. What would be the population of the town in the beginning of 2074 if its population in the beginning of 2072 B.S. was 32,000?
7. a) The population of a town in the beginning of 2016 was 1,68,000. If the annual rate of growth of population is 2.5%, find the increased population at the end of 2017.
- b) The present population of a state is 97,65,625. If the rate of growth of population is 4% p.a., find the increased population after 2 years.
8. a) The rate of growth of a plant is 5% every month. If the height of the plant in the beginning of Baisakh 2074 is 20 cm, find its height at the end of Asar 2074.
- b) The growth rate of a certain type of useful bacteria is 10% per day. During a research work in a laboratory, if the bacteria are grown upto the number of  $2.662 \times 10^{12}$  in 3 days, how many bacteria were there before 3 days?
- c) A house owner made an agreement to increase the house rent by 10% every year. If the rent of the house this year is Rs 16,000, find the house rent after 3 years.
- d) Due to the annual increment of price of land in a rapidly growing area by 25% the present value of a piece of land is Rs 12,50,000 per ropani. How much was the value of the land before 4 years?

### Project Work

**Make the different groups of your friends. Visit different parts of your Ward and collect the statistics of the present population of your Ward. Visit to the concerned Ward Administration Office and get the statistics of the population of the Ward in the last census report. Calculate the rate of growth of population of your Ward.**

## 4.2 Depreciation

When any asset such as a machinery item is being used for some time, its value is decreased. The reduction of value of an item due to its constant use is known as **depreciation**.

Depreciation may be simple or compound. In simple depreciation, the amount of reduction is constant every year from every depreciated value.

For example,

Suppose the original cost of machine is Rs 45,000 and every year it is depreciated by Rs 3,000.

Then, after 1 year its price = Rs 45,000 – Rs 3,000 = Rs 42,000

After, 2 years its price = Rs 42,000 – Rs 3,000 = Rs 39,000

After, 3 years its price = Rs 39,000 – Rs 3,000 = Rs 36,000 and so on.

On the other hand, in case of compound depreciation, the amount of reduction is increased every year from every depreciated value. For example, in the above case, if the value of the machine is depreciated by Rs 3,000, Rs 4,000, Rs 5,000, etc. in the first, second and third years respectively, it will be the compound depreciation.

The compound depreciation is calculated in the similar way of calculation of compound interest, but here the value is gradually decreasing.

$$\therefore P_t = P \left(1 - \frac{R}{100}\right)^T$$

Where, P is the original price, R is the rate of depreciation, T is the time period for depreciation and  $P_t$  is the depreciated value after T years.

Again, the depreciated amount =  $P - P_t$

$$\begin{aligned} &= P - P \left(1 - \frac{R}{100}\right)^T \\ &= P \left[1 - \left(1 - \frac{R}{100}\right)^T\right] \end{aligned}$$

Similarly, if the rates of depreciation are different in different years, then,

$$\text{the depreciated value } (P_t) = P \left(1 - \frac{R_1}{100}\right) \left(1 - \frac{R_2}{100}\right) \left(1 - \frac{R_3}{100}\right) \dots$$

### Worked-out examples

**Example 1:** A man purchased a bus for Rs 16,00,000. He earned a profit of Rs 2,10,000 in 3 years. If he sold it after 3 years at 5% p.a. compound depreciation, find his profit or loss.

**Solution:**

Here, the original cost of the bus (P) = Rs 16,00,000.

The time period for depreciation (T) = 3 years

The rate of compound depreciation (R) = 5% p.a.

$$\begin{aligned}
 \text{Now, the depreciated cost } (P_t) &= P \left(1 - \frac{R}{100}\right)^T \\
 &= \text{Rs } 16,00,000 \left(1 - \frac{5}{100}\right)^3 \\
 &= \text{Rs } 16,00,000 \times \frac{19}{20} \times \frac{19}{20} \times \frac{19}{20} = \text{Rs } 13,71,800
 \end{aligned}$$

Again, the profit in 3 years = Rs 2,10,000.

∴ The value of the bus = Rs 13,71,800 + Rs 2,10,000 = Rs 15,81,800

But the original cost of the bus = Rs 16,00,000

∴ Loss = Rs 16,00,000 – Rs 15,81,800 = Rs 18,200

So, his loss is Rs 18,200.

**Example 2:** The compound depreciation of shares of a business company for 2 years is at the rate of 2% p.a. If the present value of certain number of shares is Rs 24,010, how many shares at Rs 100 per share were sold before 2 years?

**Solution:**

Here the value of shares 2 years before = Rs P.

The rate of depreciation for 2 years = 2% p.a.

Now, the present value of the shares  $(P_t)$  = Rs 24,010

$$\text{or, } P \left(1 - \frac{R}{100}\right)^T = \text{Rs } 24,010$$

$$\text{or, } P \left(1 - \frac{2}{100}\right)^2 = \text{Rs } 24,010$$

$$\text{or, } P \times \frac{49}{50} \times \frac{49}{50} = \text{Rs } 24,010$$

$$\text{or, } P = \text{Rs } \frac{24,010 \times 50 \times 50}{49 \times 49} = \text{Rs } 25,000$$

$$\text{Again, the number of shares} = \frac{\text{Total value}}{\text{Rate of cost}} = \frac{25,000}{100} = 250$$

So, the required number of shares sold before 2 years were 250.

**Example 3:** Bishwant purchased a mountain bike for Rs 10,000. If he used it for 3 years and sold to Sunayana for Rs 7290, find the rate of compound depreciation.

**Solution:**

Here, the initial cost of the bike  $(P)$  = Rs 10,000

The cost of bike after 3 years  $(P_t)$  = Rs 7,290

Time  $(T)$  = 3 years

$$\text{We have, } P_t = P \left(1 - \frac{R}{100}\right)^T = 10,000 \left(\frac{100 - R}{100}\right)^3$$

$$\text{or, } \frac{729}{1000} = \left(\frac{100 - R}{100}\right)^3$$

$$\text{or,} \quad \left(\frac{9}{10}\right)^3 = \left(\frac{100 - R}{100}\right)^3$$

$$\text{or,} \quad \left(\frac{9}{10}\right) = \left(\frac{100 - R}{100}\right)$$

$$\text{or,} \quad 900 = 1,000 - 10R$$

$$\text{or,} \quad R = 10\%$$

So, the rate of compound depreciation is 10% p.a.

## EXERCISE 4.2

### General Section

1. a) Write the formula to calculate the depreciated value of a vehicle after T years if the original value is P and the rate of compound depreciation is R% p.a.
- b) The initial price of a machine is P and the rate of compound depreciation is R% p.a. Write the formula to calculate the amount of depreciation in T year.
- c) The rates of compound depreciation of a printing machine in 3 years are  $R_1\%$ ,  $R_2\%$  and  $R_3\%$ . If the initial value of the machine is P, write the formula to calculate its value after 3 years.
2. a) The value of a machine is Rs 2,60,000. If its value is depreciated by 5% every year, find its value after 1 year.
- b) After the depreciation at the rate of 10% p.a., the value of a vehicle became Rs 5,80,500 after one year. Find the original value of the vehicle.
- c) At what rate of depreciation is the price of an article reduced to Rs 42,500 from Rs 50,000 in 1 year?

### Creative Section

3. a) If the value of a computer which was bought for Rs 40,000 depreciates at 10% annually, find its value after 2 years.
- b) A cupboard costing Rs 16,800 is depreciated at the rate of 15% per year. What will be its cost after 2 years?
- c) The value of an article is depreciated every year by 10%. What will be the value of the article worth Rs 45,000 after 3 years?
- d) Mr. Tamang bought a tripper for Rs 32,00,000. He earned a profit of Rs 7,80,000 in 2 years and sold it at 10% p.a. compound depreciation. Find his profit or loss.
4. a) If the cost is depreciated at the rate of 12% per annum, the cost of a laptop computer becomes Rs 92,928 after 2 years. Find the original price of the computer.
- b) If the cost is depreciated at the rate of 10% per annum, the cost of a motorcycle after 3 years becomes Rs 92,583. Calculate the original price of the motorcycle.
- c) A woman sold a type-writer for Rs 13,718 in the system of compound depreciation at the rate of 5% p.a. If she had purchased it 3 years before, how much did she pay for it by that time?

- d) The compound depreciation of shares of business company for 2 years is at the rate of 2.5% p.a. If the present value of certain number of shares is Rs 45,630, how many shares at Rs 200 per share were sold before 2 years?
5. a) The original value of a computer is Rs 96,000. If it is depreciated by 15% every year with the compound depreciation, by how much is its value depreciated in 3 years?
- b) Mrs. Yadav bought a vehicle for Rs 12,50,000. She used it for 3 years and sold at the rate of compound depreciation of 12% p.a. By how much was the value of the vehicle depreciated?
6. a) The original price of a motorbike is 1,80,000. If the rate of depreciation is 10% per annum, after how many years will its price become Rs 1,31,220?
- b) A factory was bought for Rs 4,00,000 some years ago and now its value is Rs 1,96,000. If the value of the factory is depreciated at 30% p.a. compound depreciation when was the factory bought?
- c) A few years ago a man bought a computer for Rs 1,25,000. In the recent year, he sold it for Rs 64,000 at 20% annual rate of depreciation. How long did he use the computer?
- d) Mrs. Jha had purchased a scooter for Rs 1,20,000. In the recent year, she sold it for Rs 86,700 at 15% annual rate of compound depreciation. How long did she use the scooter?
7. a) A radio costing Rs 6,000 is depreciated per year and after 2 years its price becomes Rs 5,415. Find the rate of depreciation.
- b) If the value of an article depreciated from Rs 18,000 to Rs 14,580 in two years, find the rate of depreciation.
- c) A woman bought a motorcycle for Rs 1,60,000 and after using it for 3 years, she sold it for Rs 1,16,640. Find the rate of compound depreciation of the motorcycle.
- d) The value of a motorbike is depreciated from Rs 2,50,000 to Rs 1,70,368 in 3 years, find the rate of depreciation.
8. a) A man bought a computer 3 years before for Rs 75,000. If the value of the computer is depreciated by 2 %, 4 % and 5 % in the first, second and in the third year respectively, at what price did he sell the computer at the end of 3 years?
- b) The present value of a printing machine is Rs 2,40,000. If its value is depreciated by 5% in the first year, 10% in the second year and 15% in the third year, find its value after 3 years.

## Project Work

***Make the different groups of your friends. Visit different showrooms or shops where the second-hand vehicles are sold. Also visit different companies and organisations where the second-hand electronic items, electrical machines, vehicles, etc. are in sale. Ask the original price and the second-hand price of those items and calculate the rate of depreciation. Also calculate the profit or loss percent by selling those items.***

***Note: Dear students, please go through the appendix given at the end of the book for more practice.***