Benchmarks

Chao Huang[1], Jiameng Fan[2], Xin Chen[3], Wenchao Li[2], and Qi Zhu[4]

[1] University of Liverpool, Liverpool, UK, chao.huang2@liverpool.ac.uk
[2] Boston University, Boston, USA, {jmfan,wenchao}@bu.edu
[3] University of Dayton, Dayton, USA, xchen4@udayton.edu
[4] Northwestern University, Evanston, USA, qzhu@northwestern.edu

1 Benchmark: Attitude Control

We consider the attitude control of a rigid body with six states and three inputs[2]. The system dynamics is

$$\begin{cases} \dot{\omega}_1 = 0.25(u_0 + \omega_2\omega_3), \qquad \dot{\omega}_2 = 0.5(u_1 - 3\omega_1\omega_3), \qquad \dot{\omega}_3 = u_2 + 2\omega_1\omega_2, \\ \dot{\psi}_1 = 0.5\left(\omega_2(\psi_1^2+\psi_2^2+\psi_3^2-\psi_3)+\omega_3(\psi_1^2+\psi_2^2+\psi_2+\psi_3^2)+\omega_1(\psi_1^2+\psi_2^2+\psi_3^2+1)\right), \\ \dot{\psi}_2 = 0.5\left(\omega_1(\psi_1^2+\psi_2^2+\psi_3^2+\psi_3)+\omega_3(\psi_1^2-\psi_1+\psi_2^2+\psi_3^2)+\omega_2(\psi_1^2+\psi_2^2+\psi_3^2+1)\right), \\ \dot{\psi}_3 = 0.5\left(\omega_1(\psi_1^2+\psi_2^2-\psi_2+\psi_3^2)+\omega_2(\psi_1^2+\psi_1+\psi_2^2+\psi_3^2)+\omega_3(\psi_1^2+\psi_2^2+\psi_3^2+1)\right). \end{cases}$$

wherein the state $\vec{x}=(\omega,\psi)$ consists of the angular velocity vector in a body-fixed frame $\omega\in\mathbb{R}^3$, and the Rodrigues parameter vector $\psi\in\mathbb{R}^3$.

The control torque $u\in\mathbb{R}^3$ is updated every 0.1 second by a neural network with 3 hidden layers, each of which has 64 neurons. The activations of the hidden layers are sigmoid and identity, respectively. We train the neural-network controller using supervised learning methods to learn from a known nonlinear controller [2]. The initial state set is:

$$\omega_1 \in [-0.45,-0.44], \omega_2 \in [-0.55,-0.54], \omega_3 \in [0.65,0.66],$$
$$\psi_1 \in [-0.75,-0.74], \psi_2 \in [0.85,0.86], \psi_3 \in [-0.65,-0.64].$$

We would like to verify whether the system will reach the following unsafe set X_u in 3 seconds (30 time steps):

$$\omega_1 \in [-0.2,0], \omega_2 \in [-0.5,-0.4], \omega_3 \in [0,0.2],$$
$$\psi_1 \in [-0.7,-0.6], \psi_2 \in [0.7,0.8], \psi_3 \in [-0.4,-0.2].$$

2 Benchmark: QUAD

We study a neural-network controlled quadrotor (QUAD) with 12 states [1]. For the states, we have the inertial (north) position x_1, the inertial (east) position x_2, the altitude x_3, the longitudinal velocity x_4, the lateral velocity x_5, the vertical velocity x_6, the roll angle x_7, the pitch angle x_8, the yaw angle x_9, the roll rate x_{10}, the pitch rate x_{11}, and the yaw rate x_{12}. The control torque $u \in \mathbb{R}^3$

is updated every 0.1 second by a neural network with 3 hidden layers, each of which has 64 neurons. The activations of the hidden layers and the output layer are sigmoid and identity, respectively.

$$\begin{cases} \dot{x}_1 = \cos(x_8)\cos(x_9)x_4 + (\sin(x_7)\sin(x_8)\cos(x_9) - \cos(x_7)\sin(x_9))\, x_5 \\ \qquad + (\cos(x_7)\sin(x_8)\cos(x_9) + \sin(x_7)\sin(x_9))\, x_6 \\ \dot{x}_2 = \cos(x_8)\sin(x_9)x_4 + (\sin(x_7)\sin(x_8)\sin(x_9) + \cos(x_7)\cos(x_9))\, x_5 \\ \qquad + (\cos(x_7)\sin(x_8)\sin(x_9) - \sin(x_7)\cos(x_9))\, x_6 \\ \dot{x}_3 = \sin(x_8)x_4 - \sin(x_7)\cos(x_8)x_5 - \cos(x_7)\cos(x_8)x_6 \\ \dot{x}_4 = x_{12}x_5 - x_{11}x_6 - g\sin(x_8) \\ \dot{x}_5 = x_{10}x_6 - x_{12}x_4 + g\cos(x_8)\sin(x_7) \\ \dot{x}_6 = x_{11}x_4 - x_{10}x_5 + g\cos(x_8)\cos(x_7) - g - u_1/m \\ \dot{x}_7 = x_{10} + \sin(x_7)\tan(x_8)x_{11} + \cos(x_7)\tan(x_8)x_{12} \\ \dot{x}_8 = \cos(x_7)x_{11} - \sin(x_7)x_{12} \\ \dot{x}_9 = \dfrac{\sin(x_7)}{\cos(x_8)}x_{11} - \dfrac{\cos(x_7)}{\cos(x_8)}x_{12} \\ \dot{x}_{10} = \dfrac{J_y - J_z}{J_x}x_{11}x_{12} + \dfrac{1}{J_x}u_2 \\ \dot{x}_{11} = \dfrac{J_z - J_x}{J_y}x_{10}x_{12} + \dfrac{1}{J_y}u_3 \\ \dot{x}_{12} = \dfrac{J_x - J_y}{J_z}x_{10}x_{11} + \dfrac{1}{J_z}\tau_\psi \end{cases}$$

where

$$g = 9.81, \quad m = 1.4, \quad J_x = 0.054,$$
$$J_y = 0.054, \quad J_z = 0.104, \quad \tau_\psi = 0.$$

The initial set is:

$$x_1 \in [-0.4, 0.4], x_2 \in [-0.4, 0.4], x_3 \in [-0.4, 0.4], x_4 \in [-0.4, 0.4],$$
$$x_5 \in [-0.4, 0.4], x_6 \in [-0.4, 0.4], x_7 = 0, x_8 = 0, x_9 = 0, x_{10} = 0, x_{11} = 0, x_{12} = 0$$

The control goal is to stabilize the attitude x_3 to a goal region $[0.94, 1.06]$ in 5 seconds (50 time steps).

References

1. Beard, R.: Quadrotor dynamics and control rev 0.1 (2008)
2. Prajna, S., Parrilo, P.A., Rantzer, A.: Nonlinear control synthesis by convex optimization. IEEE Transactions on Automatic Control **49**(2), 310–314 (2004)