# Control and synchronization of a Hyperchaotic Finance System via Single controller Scheme


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Project Report: Mathematical Modelling and Simulation , Under Dr. Subir Das

# Control and synchronization of a Hyperchaotic Finance System via Single controller Scheme

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**Abstract**

The purpose of this paper is to study the control and synchronization of the hyperchaotic finance system.
**Design methodology approach** – A single controller scheme is introduced. The Routh-Hurwitz criteria and the structure of solution of first-order linear differential equations are adopted in analysis of control and synchronization.
**Findings** – A single controllers is designed and added to the new hyperchaotic finance system. An effective controller is designed for the global asymptotic synchronization on the hyperchaotic finance system based on the structure of solution of first-order linear differential equations. Numerical simulations are demonstrated to verify the effectiveness of the proposed single controller scheme.
**Originality/value** – The introduced approach is interesting for control and synchronization the hyperchaotic finance system.
**Keywords** Routh-Hurwitz criteria, Hyperchaotic finance system, Synchronization ,Single controller scheme
**Paper type** Project Report

## 1 Introduction

Chaos in a finance system was shown firstly in 2001 (Ma and Chen, 2001a, b). Then, a new modified chaotic finance attractor was proposed in 2007 (Cai and Huang, 2007). Control of chaos in chaotic finance systems was implemented with several methods. Linear feedback, speed feedback, selection of gain matrix, revision of gain matrix controllers (Yang and Cai, 2011), time-delayed feedback controllers (Chen, 2008) and a passive controller (Selçuk, 2012) were used for the control of chaotic finance system. The control of modified chaotic finance system was applied by means of linear feedback, speed feedback and adaptive control methods (Cai et al., 2011). Afterwards, a hyperchaotic finance system from the modified finance system was presented (Ding et al., 2009), and a new hyperchaotic finance system was introduced from the first chaotic finance system (Cai et al., 2011).

In this report, further investigation on the control and synchronization of the new hyperchaotic finance system is explored. The rest of this report is organized as follows. In Section 2, the related hyperchaotic finance systems is described. In Section 3, the single controller have been employed for achieving the synchronization of the new hyperchaotic finance system. In Section 4, numerical simulations have been presented in figures to confirm the effectiveness of the single controller scheme. Finally, conclusions are given in Section 5.

## 2 Hyperchaotic Finance System

We mainly investigate a hyperchaotic finance system which can be described as (Yu et al., 2012):

$$\begin{cases} \dot{x} = z + (y - a)x \\ \dot{y} = 1 - by - x^2 \\ \dot{z} = -x - cz \\ \dot{w} = -dxy - fw, \end{cases} \tag{1}$$

where $x$ denotes the interest rate, $y$ denotes the investment demand, $z$ denotes the price index and $w$ denotes the average profit margin, respectively. The parameter $a$ denotes the savings, $b$ denotes the investment cost and $c$ denotes the commodities demand elasticity, $d$ and $f$ denote additional finance parameters, respectively.

When the parameter values are taken as $a = 0.9$ ,$b = 0.2$ , $c = 1.5$ , $d = 0.2$ and $f = 0.17$, this non linear system exhibits chaotic behaviour . The 3D phase plane of the hyperchaotic finance system (1) under the initial conditions $x(0) = 1$ ,$y(0) = 2$ ,$z(0) = 0.5$ ,$w(0) = 0.5$ are demonstrated in Figures 1-4, the time series response is presented in Figure 5

# Figures

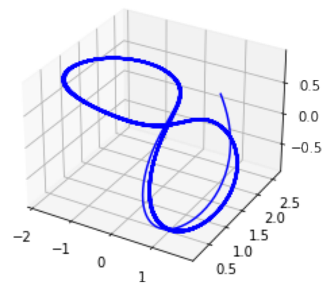


Figure 1: 3D phase plane of $x$, $y$ and $z$ in the hyperchaotic finance system

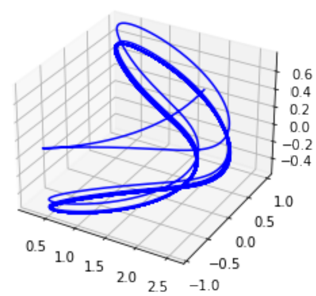


Figure 2: 3D phase plane of $y$, $z$ and $w$ in the hyperchaotic finance system

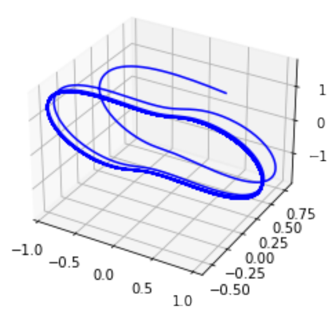


Figure 3: 3D phase plane of $z$, $w$ and $x$ in the hyperchaotic finance system

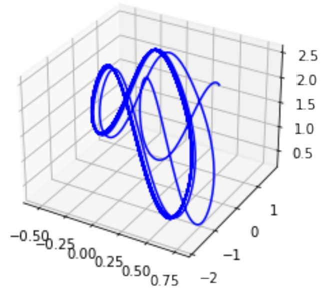


Figure 4: 3D phase plane of $w$, $x$ and $y$ in the hyperchaotic finance system

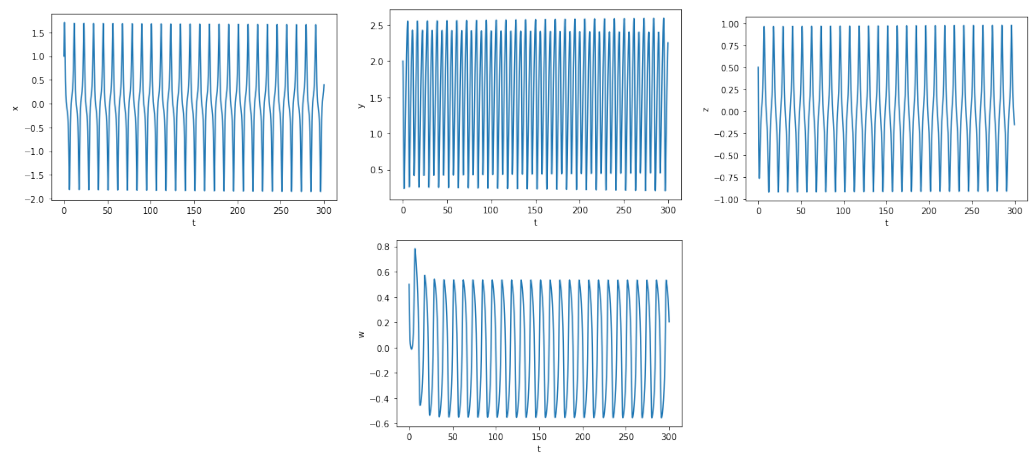


Figure 5: State response of hyperchaotic finance system

## 3 Synchronization of a hyperchaotic finance system

In this section, a single controller is applied to synchronize two hyperchaotic finance systems.

First, suppose the drive system takes the following form:

$$\begin{cases} \dot{x_1} = x_3 + (x_2 - a)x_1 \\ \dot{x_2} = 1 - bx_2 - x_1^2 \\ \dot{x_3} = -x_1 - cx_3 \\ \dot{x_4} = -dx_1x_2 - fx_4, \end{cases} \tag{2}$$

So we can also give the response system as follows:

$$\begin{cases} \dot{y_1} = y_3 + (y_2 - a)y_1 + u_1 \\ \dot{y_2} = 1 - by_2 - y_1^2 \\ \dot{y_3} = -y_1 - cy_3 \\ \dot{y_4} = -dy_1y_2 - fy_4, \end{cases} \tag{3}$$

where $u_1$ is is a controller to be determined for achieving synchronization between the system (2) and (3).

Define the synchronization errors as follows:

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \\ e_4 = y_4 - x_4, \end{cases} \tag{4}$$

Substituting systems (2)-(3) into system (4), we can know:

$$\begin{cases} \dot{e_1} = e_3 - ae_1 + x_1e_2 + x_2e_1 + e_1e_2 + u_1 \\ \dot{e_2} = -be_2 - 2x_1e_1 - e_1^2 \\ \dot{e_3} = -e_1 - ce_3 \\ \dot{e_4} = -fe_4 - d(x_1e_2 + x_2e_1 + e_1e_2), \end{cases} \tag{5}$$

We construct the controller $u_1$ as follows:

$$u_1 = -e_3 - x_1e_2 - x_2e_1 - e_1e_2 \tag{6}$$

Substituting the controller (6) into the error dynamics (5), yield:

$$\begin{cases} \dot{e_1} = -ae_1 \\ \dot{e_2} = -be_2 - 2x_1e_1 - e_1^2 \\ \dot{e_3} = -e_1 - ce_3 \\ \dot{e_4} = -fe_4 - d(x_1e_2 + x_2e_1 + e_1e_2), \end{cases} \tag{7}$$

It is easy to know that:

$$e_1(t) = e_1(0)e^{-at} \tag{8}$$

Because of $a > 0$, that is, yield

$$\lim_{t\to\infty} e_1(t) = 0 \tag{9}$$

Thus, the system (7) can be reduced as follows:

$$\begin{cases} \dot{e_2} = -be_2 \\ \dot{e_3} = -ce_3 \\ \dot{e_4} = -fe_4 - dx_1e_2 \end{cases} \tag{10}$$

It is easy to know that:

$$\begin{aligned} e_2(t) = e_2(0)e^{-bt} \\ e_3(t) = e_3(0)e^{-ct} \end{aligned} \tag{11}$$

Because of $b > 0$, $c > 0$, that is, yield:

$$\begin{aligned} \lim_{t\to\infty} e_2(t) = 0 \\ \lim_{t\to\infty} e_3(t) = 0 \end{aligned} \tag{12}$$

Thus, the system (10) can be reduced as follows:

$$\dot{e_4} = -fe_4 \tag{13}$$

It is easy to know that:

$$e_4(t) = e_4(0)e^{-ft} \tag{14}$$

Because of $f > 0$, that is, yield

$$\lim_{t\to\infty} e_4(t) = 0 \tag{15}$$

Therefore, we have a theorem as follows:

*Theorem:* The controller (6) can synchronize the drive system (2) and the response system (3).

# 4 Simulation

Setting the initial condition as: $\begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \\ y_4(0) \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ -2 \\ -1 \end{bmatrix}$

The controller $u_1$ is also activated at the 100th second, the curves of the output of the controller and synchronization errors are shown in Figures 6-10. From Figures 6-10, we can know that Theorem is effective:

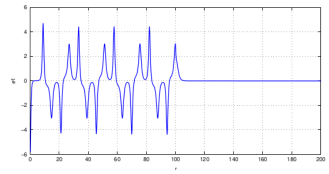

Figure 6: The curve of synchronization error $e_1$

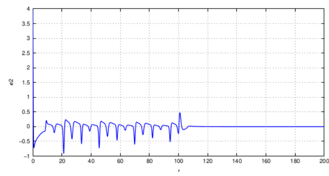

Figure 7: The curve of synchronization error $e_2$

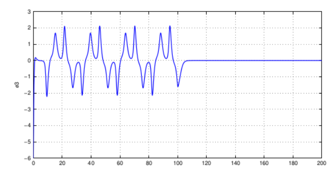

Figure 8: The curve of synchronization error $e_3$

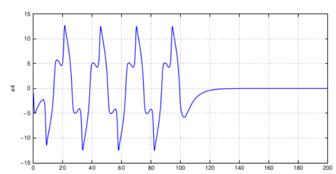

Figure 9: The curve of synchronization error $e_4$

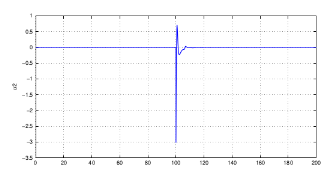

Figure 10: Output $u_1$ of the controller of hyperchaotic finance system

# 5 Conclusion

This project proposes a single controllers for control and synchronization of a hyperchaotic finance system has been considered. Some control and synchronization conditions have been derived via the single controller scheme.Based on the structure of solution of first-order linear differential equations, the corresponding controller has been designed to achieve synchronization between two identical hyperchaotic finance systems. In addition, the extending of active control can be applied in other chaotic systems or hyperchaotic systems. The numerical simulation in Python Notebook is then provided to show the effectiveness and feasibility of the proposed method. In fact, the proposed method in this project at least has two advantages: first, the controller obtained in this paper is simpler than the controller obtained by previous methods, the utility of the proposed method is easy to realize; second, the designed controllers in this paper have faster convergence speed .

## Link

Link to github

## Refrences


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# project_outputs

December 1, 2022

[132]:
```python
from pylab import *
from mpl_toolkits.mplot3d import Axes3D
```

[133]:
```python
a = 0.9
b = 0.2
c = 1.5
d = 0.2
f = 0.17
Dt = 0.05
```

[134]:
```python
def intialize():
    global x , xresult , y , yresult ,z , zresult , w , wresult , t , timesteps
    x = 1
    y = 2
    z = 0.5
    w =0.5

    xresult = [x]
    yresult = [y]
    zresult = [z]
    wresult = [w]
    t = 0
    timesteps = [t]
```

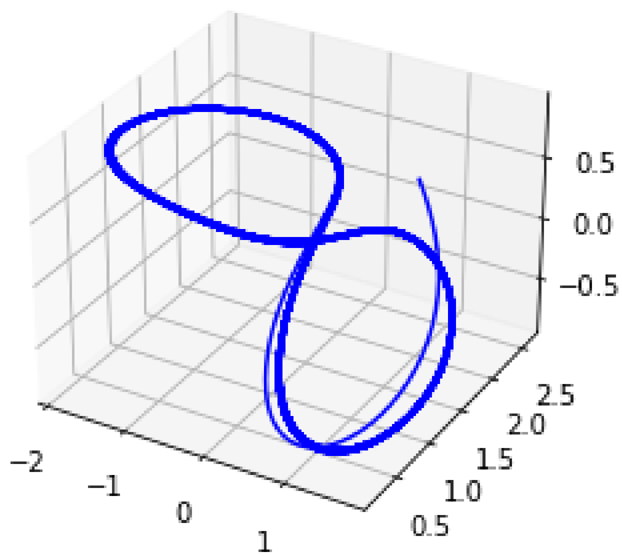
[135]:
```python
def observe():
    global x , xresult , y , yresult ,z , zresult , w , wresult , t , timesteps
    xresult.append(x)
    yresult.append(y)
    zresult.append(z)
    wresult.append(w)
    timesteps.append(t)
```

[136]:
```python
def update():
    global x , xresult , y , yresult ,z , zresult , w , wresult , t , timesteps
    nextx = x + (z + (y-a)*x)*Dt
    nexty = y + (1-b*y-x*x)*Dt
    nextz = z + (-x -c*z)*Dt
    nextw = w + (-d*x*y - f*w)*Dt
```

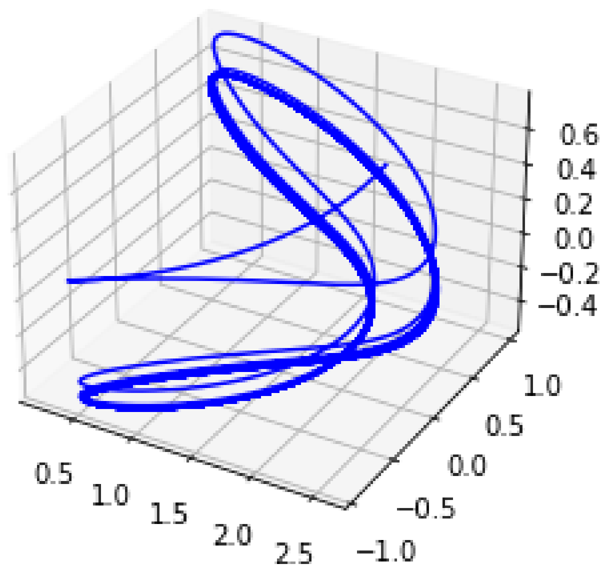
```
    x,y,z,w = nextx,nexty,nextz,nextw
    t = t + Dt
```

[137]:
```
intialize()
while t<300.:
    update()
    observe()
```

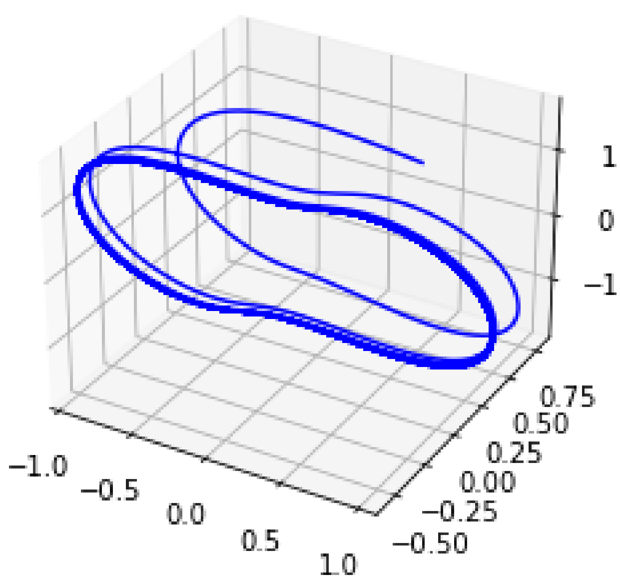
[138]:
```
figure()
ax = gca(projection='3d')
ax.plot(xresult,yresult,zresult,'b')
show();
```



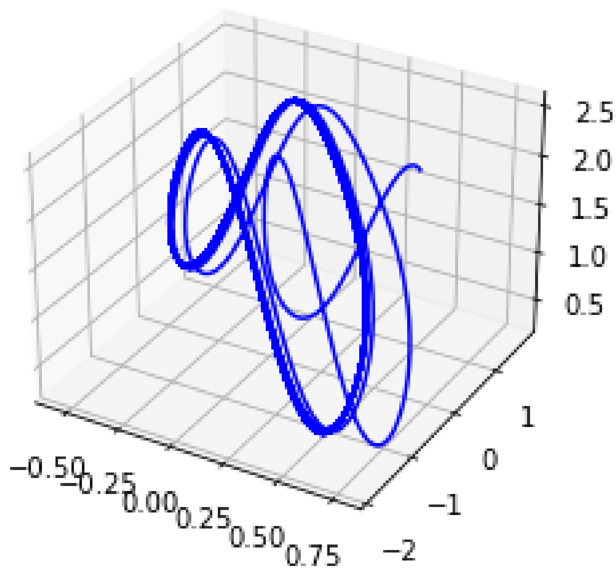

[139]:
```
figure()
ax = gca(projection='3d')
ax.plot(yresult,zresult,wresult,'b')
show();
```



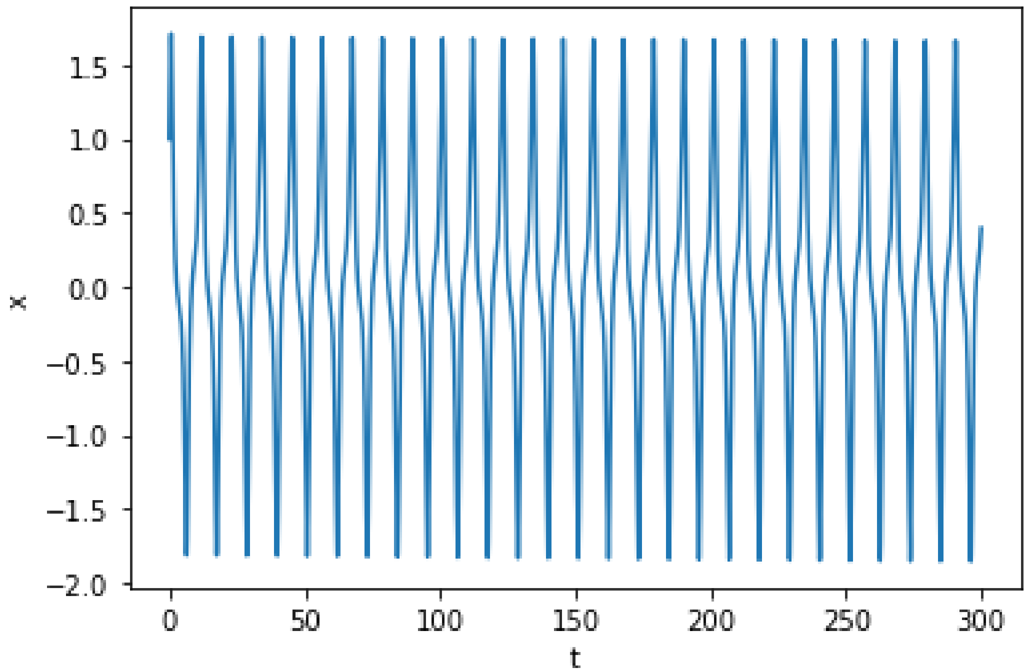

[140]:
```
figure()
ax = gca(projection='3d')
ax.plot(zresult,wresult,xresult,'b')
show();
```



[141]:
```
figure()
ax = gca(projection='3d')
ax.plot(wresult,xresult,yresult,'b')
show();
```

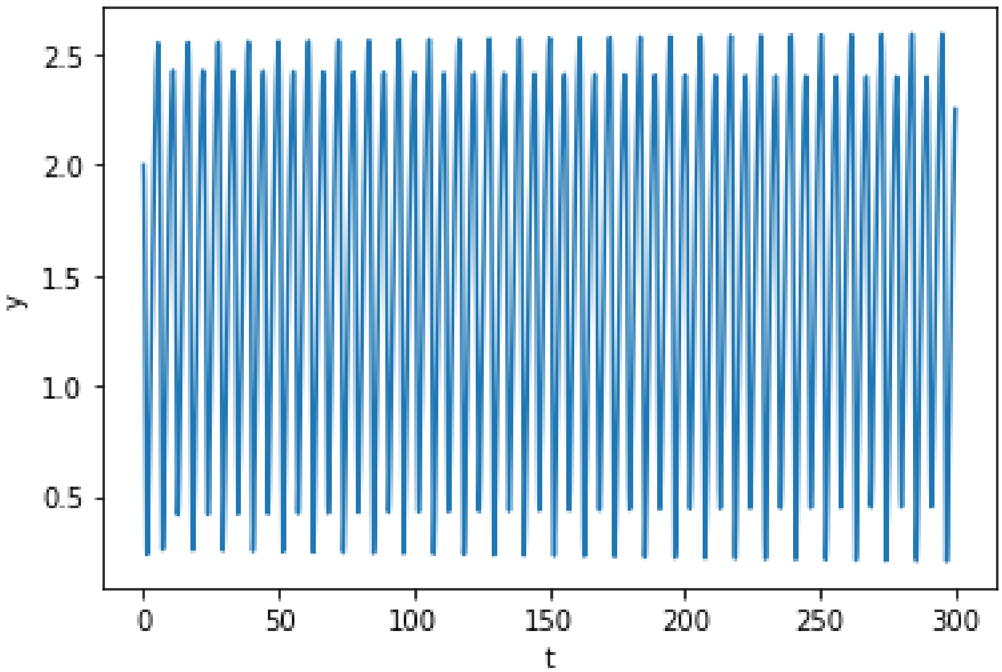



[142]:
```
subplot(1,1,1)
plot(timesteps,xresult)
xlabel('t')
ylabel('x')
show()
```

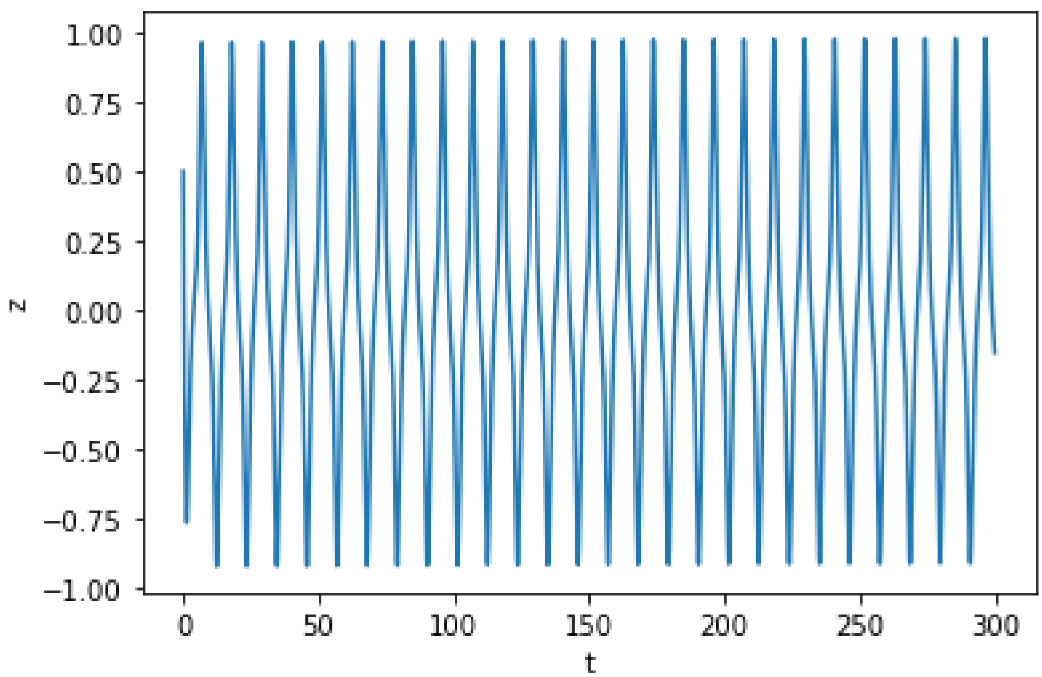



[143]:

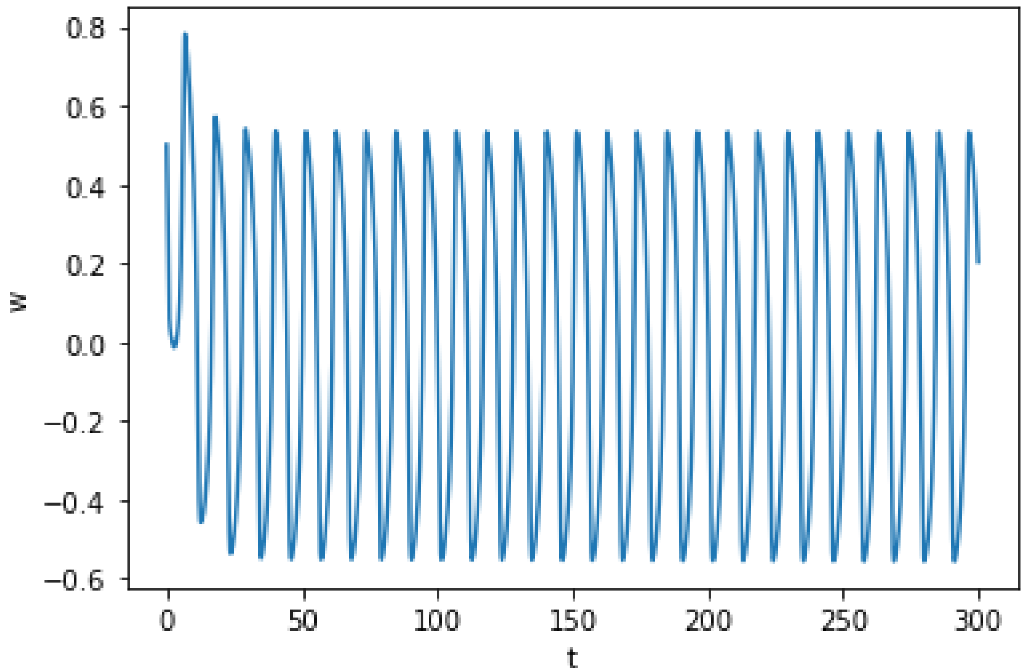
```
subplot(1,1,1)
plot(timesteps,yresult)
xlabel('t')
ylabel('y')
show()
```




[144]:
```
subplot(1,1,1)
plot(timesteps,zresult)
xlabel('t')
ylabel('z')
show()
```




[145]:
```
subplot(1,1,1)
plot(timesteps,wresult)
xlabel('t')
ylabel('w')
show()
```


0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
w
0
50
100
150
200
250
300
t

```
[ ]:
```