Problem set 2

November 8, 2021

Consider an economy like we studied in the class: Output is linear in labor and subject to an exogenous TFP process $\theta_t(s^t) = \theta(s_t)$. The government consumption $g_t(s^t) = g(s_t)$ is also exogenous and the resource constraint is given by

$$y_t(s^t) = c_t(s^t) + g(s_t)$$

Rep agent has expected utility GHH preferences of the form

$$\mathbb{E}_0 \sum_{t=0}^{\infty} u\left(c_t(s^t) - v\left(\frac{y_t(s^t)}{\theta(s_t)}\right)\right)$$

Assume the following quarterly calibration:

1. Functional forms: $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ and $v(x) = \frac{x^{1+\gamma^{-1}}}{1+\gamma^{-1}}$. Set $\sigma = 2$ and $\gamma = 1$

2. Discount factor $\beta = 0.99$

3. Parameterize the shocks as follows

   (a) Suppose that g and θ are AR(1) in logs with autocorrelation $\rho_g = \rho_\theta = 0.95$, mean of $\mu_g = \ln\left(\frac{15}{100}\right)$, $\mu_\theta = \ln(1)$ and one period ahead conditional std. deviations $\sigma_g = \frac{1.2}{15}$ and $\sigma_\theta = \frac{2}{400}$.

   (b) Discretize $s = (\theta, g)$ with a grid of 5 points on g and 5 points on θ

4. Set b_0 to about 4 times y_0 so that we have an annual debt to gdp ratio of about 100%

For this economy:

1. Solve for the Ramsey allocation assuming the government sets a linear tax on labor at all histories $\{\tau_t(s^t)\}$ and trades in

   (a) A set of state-contingent assets: $\{b_t(s_{t+1}|s^t)\}_{s_{t+1}}$

   (b) A risk-free bond only: $b_t(s^t)$

2. Compute the portfolio of risk-free bonds of maturities $1 \ldots N$ that implements the allocation in $1(a)$

3. Simulate the allocation by drawing shocks s and using policy functions computed in $1(a)$ and $1(b)$. What do you notice as the main differences in allocations and the path for implied taxes and debt across $1(a)$ and $1(b)$?