Economics 8185
Advanced Topics in Macroeconomics–Computation
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Homework 1.

1. Compute equilibria of the following growth model:

$$\max_{\{c_t, x_t, \ell_t\}} E \sum_{t=0}^{\infty} \beta^t \{\log(c_t) + \psi \log(\ell_t)\} N_t$$

$$\begin{aligned} \text{subj. to} \quad & c_t + x_t = k_t^\theta \left((1+\gamma_z)^t z_t h_t\right)^{1-\theta} \\ & N_{t+1} k_{t+1} = [(1-\delta) k_t + x_t] N_t \\ & \log z_t = \rho \log z_{t-1} + \epsilon_t, \quad \epsilon \sim N\left(0, \sigma_\epsilon^2\right) \\ & h_t + \ell_t = 1 \\ & c_t, x_t \geq 0 \quad \text{in all states} \end{aligned}$$

where $N_t = (1+\gamma_n)^t$ using the following methods:
   a. Iterate on Bellman's equation;
   b. Map it to a linear quadratic problem and try three different versions as discussed in class, that is, with the return function depending on either
      — consumption and leisure;
      — hours, capital today, and capital tomorrow; or
      — capital today and capital tomorrow;
   c. Apply Vaughan's method.

   If you have trouble with this problem, start with the simpler problem with inelastic labor ($\psi = 0$), without growth ($\gamma_z = \gamma_n = 0$), and with full depreciation ($\delta = 1$). This problem has a known solution and serves as a good test case for your codes.

2. Discuss the properties of the solution (e.g., value and decision functions) for the parameter sets below and evaluate the computational procedures in light of these properties:
   a. $\psi = 0,\ \delta = 1,\ \gamma_n = 0,\ \gamma_z = 0$
   b. $\psi = 0,\ \gamma_n = 0,\ \gamma_z = 0$
   c. $\psi = 0,\ \gamma_n = 0$
   d. $\psi = 0,\ \gamma_z = 0$

3. Modify the preferences so that

$$U(c_t, \ell_t) = \left(c_t \ell_t^\psi\right)^{1-\sigma} / (1-\sigma)$$

   and add two more variations on the parameter set:
   e. $\sigma = 0$
   f. σ large.