Economics 8185
Advanced Topics in Macroeconomics–Computation
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Homework 2.

1. Compute equilibria of the following growth model:

$$\max_{\{c_t,x_t,\ell_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t,\ell_t\right) N_t$$

$$\begin{aligned}
\text{subj. to}\quad c_t + x_t &= r_t k_t + w_t h_t + \kappa_t \\
&\quad - \tau_{ct} c_t - \tau_{ht} w_t h_t - \tau_{pt}\left(r_t k_t - \delta k_t\right) \\
&\quad - \tau_{dt}\left(r_t k_t - x_t - \tau_{pt}\left(r_t k_t - \delta k_t\right)\right) \\
N_{t+1} k_{t+1} &= \left[\left(1-\delta\right) k_t + x_t\right] N_t \\
h_t + \ell_t &= 1 \\
S_t &= P S_{t-1} + Q\epsilon_t, \quad S_t = [\log z_t, \tau_{ct}, \tau_{ht}, \tau_{dt}, \tau_{pt}, \log g_t] \\
c_t, x_t &\geq 0 \quad \text{in all states,}
\end{aligned}$$

where $N_t = (1+\gamma_n)^t$ and firm technology is $Y_t = K_t^{\theta}(Z_t L_t)^{1-\theta}$. Factors are paid their marginal products r and w, and revenues in excess of government purchases of goods and services, $N_t g_t$, are lump-sum transferred to households in amount κ_t. The stochastic shocks hitting this economy affect technology, tax rates, and government spending and the stochastic processes are modeled as a VAR(1) process. The resource constraint in this economy is $Y_t = N_t(c_t + x_t + g_t)$ and the government budget constraint is:

$$\begin{aligned}
G_t + N_t \kappa_t = \tau_{ct} C_t + \tau_{ht} w_t H_t + \tau_{pt}\left(r_t K_t - \delta K_t\right) \\
+ \tau_{dt}\left(r_t K_t - X_t - \tau_{pt}\left(r_t K_t - \delta K_t\right)\right).
\end{aligned}$$

Notice that this is the same as Homework 1 except that now the economy is distorted and cannot (except in a few special cases) be mapped to a concave programming problem. Use the following methods to compute the equilibrium for general parameters:

   a. Map it to a linear quadratic problem;
   b. Apply Vaughan's method.

Additionally, set up the problem as a dynamic program and discuss why it would be hard to solve it in a "brute force" way.

2. Simulate time series for all variables listed above assuming $\epsilon \sim N(0, \Sigma)$. In addition, construct time series for dividends, accounting profits, and stock valuations. Construct some interesting examples, explaining in detail why you think they are interesting.