Computational Macro Prelim

Bhandari and McGrattan

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Consider the following economy. There is a continuum of ex-ante identical households who have preferences over consumption and leisure:

$$E_0 \sum \beta^t \left\{ \frac{c_{i,t}^{1-\mu} - 1}{1-\mu} - \psi \frac{(1-l_{i,t})^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right\}$$

Households supply labor and save in risk-free bonds subject to a debt limit. They also pay income taxes at rate τ_y and receive transfers T. Let w and r be the pre-tax wage rate and return on savings. The budget constraint for a typical household is give by

$$c_{i,t} + a_{i,t+1} \leq (1-\tau_y)\, e_{i,t} w_t \,(1-l_{i,t}) + T_t + (1 + (1-\tau_y)\, r_t)\, a_{i,t},$$

where $a_{i,t+1} \geq \underline{a}$, $l_{i,t} \leq 1$ and $c_{i,t} \geq 0$. The skill process $e_{i,t}$ is:

$$e_{i,t} = \rho e_{i,t-1} + \sigma \epsilon_{i,t}.$$

The aggregate asset supply is $A_t = \int a_{i,t} di$ and the labor supply

$$N_t = \int e_{i,t}\,(1-l_{i,t})\,.$$

The government budget constraint is

$$G_t + T_t + r_t B_t = B_{t+1} - B_t + \tau_y \left(w_t N_t + r A_t \right),$$

where G_t is government spending and B_t is debt. With respect to technology, there is a representative firm that uses capital, K_t and labor L_t to operate a CRS technology that produces output

$$Y_t = F\left(K_t, N_t\right) = A K_t^{\theta} N_t^{1-\theta}.$$

The choice of production pins down the rental rate $r_t = F_K - \delta$ and wage rate $w_t = F_N$. In terms of asset markets, agents can trade claims to one-period risk-free bonds, capital and government bonds:

$$A_t = K_t + B_t.$$

For the baseline calculations set $\beta = 0.98$, $\mu = 2$, $\gamma = .5$ or $\gamma = 2$, $\tau_y = 0.4$, $\rho = 0.6$, $\sigma = 0.3$, $\delta = 0.075$, $\theta = 0.3$. It would be useful to set A such that steady state $Y = 1$ and ψ such that the steady state fraction of time at work is 0.28. Set the ratios of B_t/Y_t and G_t/Y_t equal to 100% and 20%, respectively, with transfers T_t set to satisfy the budget constraint of the government. Discretize the AR(1) process for skills using 5 states.

In answering questions below, use the following welfare criterion:

$$\Omega = \int \int V(a, e)\, dH(a, e),$$

where $V(a, e)$ is the optimal value function and H is the steady-state joint distribution of assets and productivity. Measure welfare gains/losses in units of consumption by computing the percentage increase in baseline consumption at every date and state (with leisure at every date and state held at baseline values) that leads to the same value of the welfare standard as its value in the new economy.

1. Compare the stationary equililibrium of the baseline economy for the two different labor elasticities: $\gamma = 0.5$ and $\gamma = 2$ (with parameters recalibrated to get the relevant aggregates correct).

   a. What are the main differences in these two economies?

   b. What data would you want to have to estimate γ?

2. Compute the consumption-equivalent welfare when comparing the baseline economy with $\tau_y = 0.4$ with an alternative that has $\tau_y = 0.6$ (holding debt to output at 100% and setting transfers residually).

   a. What is your estimate of the gain or loss?

   b. What drives the estimate? (Hint: think of decomposing gains/losses into changes in efficiency, redistribution, or insurance.)