# Problem set 1

November 1, 2021

1. Download the data for annual US real GDP for the years 1947-2021 ( u can download it from the BEA website). Plot the growth rate of the series

2. Suppose the statistical model for the change in log output is

$$y_t = \mu_t + \sigma_\epsilon \epsilon_t$$

$$\mu_{t+1} = \mu_t + \sigma_\nu \nu_{t+1}$$

   where $\epsilon_t, \nu_{t+1}$ are i.i.d standard Gaussian. Use a prior that $\mu_0$ is Gaussian with mean and variance of about 2%.

   (a) For a given $\sigma_\epsilon, \sigma_\nu$ and apply the Kalman recursions to obtain the best estimate of $\mu_t$ given past data $y^{t-1}$

   (b) Check the if code is correct:

      i. Instead of using actual data on output that you downloaded generate a "test" sample by simulating the system $y_t, \mu_t$ using the state space representation

      ii. Using only your simulated $y^t$ apply the Kalman filter code to obtain $\mathbb{E}\mu_{t+1}|y^t$ and $\sigma(\mu_{t+1}|y^t)$

      iii. Plot the $\mathbb{E}\mu_{t+1}|y^t, \mathbb{E}\mu_{t+1}|y^t \pm 2\sigma(\mu_{t+1}|y^t)$ against the simulated $\mu_t$ series from step (i)

3. Now write code that takes as input $\sigma_\epsilon, \sigma_v$ and yields the log likelihood of the actual data that you downloaded.

   (a) Estimate $\sigma_\epsilon, \sigma_v$ by maximizing the likelihood function.

   (b) Plot the $\mathbb{E}\mu_{t+1}|y^t$ at the estimated $\sigma_\epsilon, \sigma_\nu$

4. Instead of directly maximizing the likelihood estimate $\{\sigma_\epsilon, \sigma_v\}$ using the expected maximization method that we discussed in the class.