

Assignment 11
IC252 - IIT Mandi
Submission Deadline: 5 June, 2021

1. A sociologist is concerned about the effectiveness of a training course designed to get more drivers to use seat belts in automobiles.
 - (a) What hypothesis is she testing if she commits a type I error by erroneously concluding that the training course is ineffective? [1]
 - (b) What hypothesis is she testing if she commits a type II error by erroneously concluding that the training course is effective? [1]
2. A fabric manufacturer believes that the proportion of orders for raw material arriving late is $p = 0.6$. If a random sample of 10 orders shows that 3 or fewer arrived late, the hypothesis that $p = 0.6$ should be rejected in favor of the alternative $p < 0.6$. Use the binomial distribution.
 - (a) Find the probability of committing a type I error if the true proportion is $p = 0.6$. [2]
 - (b) Find the probability of committing a type II error for the alternatives $p = 0.3, p = 0.4$, and $p = 0.5$. [3]
3. An electrical firm manufactures light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours. Test the hypothesis that $\mu = 800$ hours against the alternative, $\mu \neq 800$ hours, if a random sample of 30 bulbs has an average life of 788 hours. Use a p -value for hypothesis testing. [2]
4. A dry cleaning establishment claims that a new spot remover will remove more than 70% of the spots to which it is applied. To check this claim, the spot remover will be used on 12 spots chosen at random. If fewer than 11 of the spots are removed, we shall not reject the null hypothesis that $p = 0.7$; otherwise, we conclude that $p > 0.7$.
 - (a) Evaluate α , assuming that $p = 0.7$. [2]
 - (b) Evaluate β for the alternative $p = 0.9$. [2]
5. Let X be exponentially distributed with parameter λ . Suppose that we wish to test the hypotheses

$$H_0 : \lambda \geq 1,$$

$$H_1 : \lambda < 1.$$

Consider the test procedure that rejects H_0 if $X \geq 1$. Determine the power of the test. [2]

6. Suppose that the proportion p of defective items in a large population of items is unknown, and that it is desired to test the following hypothesis:

$$H_0 : p = 0.2,$$

$$H_1 : p \neq 0.2.$$

Consider a random sample of 20 items. Let Y be the number of defective items in the sample, and consider a test procedure such that the critical region contains all the outcomes for which either $Y \geq 7$ or $Y \leq 1$.

Determine the value of the power of the test at the points $p = 0, 0.4, 0.8$ and 1 . [4]

7. A biologist claims that mice with an average life span of 32 months will live to be about 40 months old when 40% of the calories in their diet are replaced by vitamins and protein. Is there any reason to believe that $\mu < 40$ if 64 mice that are placed on this diet have an average life of 38 months with a standard deviation of 5.8 months? Use a p -value for making the conclusion. [2]