25/02/21

IC 252

MOHIT VERMA
B20215

Assignment 1

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Ans1 Possible Representative from labor = 3
" " " management = 2
" " " Public = 4

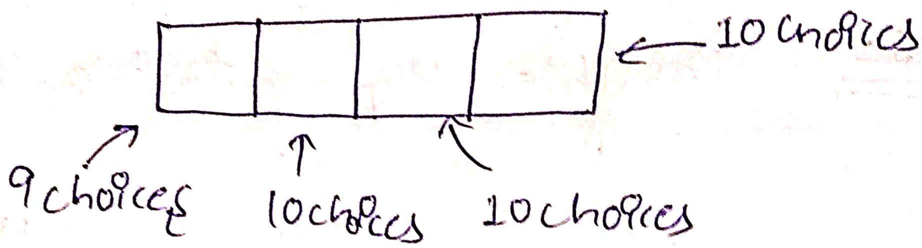
Since, there should be one representative from each domain,
Hence Total no. of different committes formed will be = ${}^3C_1 \times {}^2C_1 \times {}^4C_1 = 3 \times 2 \times 4 = \underline{24}$

Ans2 5 different colored marble can be arranged be found out by chosing one marble out of five for first position, then one marble out of four for second position and so on. Hence,

No. of ways = $5 \times 4 \times 3 \times 2 \times 1 = \underline{120}$

or $\underline{5!}$

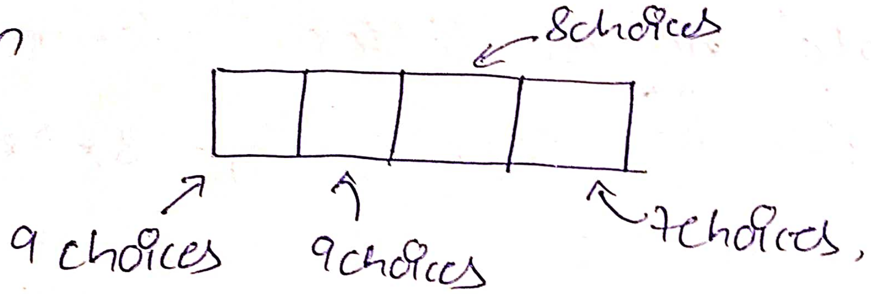
Ans 3 (a) if repitition is allowed then for a four digit No. first place can't be zero, hence,



So,

Total No. of ways = $\underline{\underline{9 \times 10^3}}$

(b) if repititions are not allowed, then again

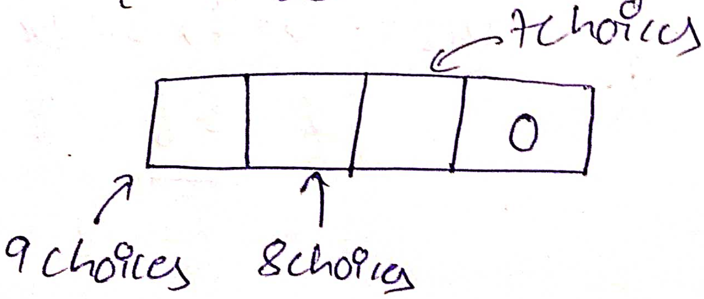


So,

Total ways = ${}^9C_1 \times {}^9C_1 \times {}^8C_1 \times {}^7C_1$

$= \underline{\underline{4536}}$

(c) for this case



So, Total ways = ${}^{9}C_1 \times {}^{8}C_1 \times {}^{7}C_1 = \underline{\underline{504}}$

Ans4

The No. of elements contained by sample space can be counted by 52 choose 5 or ${}^{52}C_5$. i.e

$${}^{52}C_5 = \frac{52!}{47!\,5!} = \underline{\underline{2598960}} \text{ ways}$$

Ans5 (a) In this case if we take marbles one by one with Replacement then we have four possible events

i.e RR, RG, GR, GG

So, Sample Space (S) = {RR, RG, GR, GG}

Hence 4 points.

(b) If we take them one by one without replacement the sample space will remain same i.e S = {RR, RG, GR, GG}

So, 4 points.

(c) If we take two marbles together, sample case will reduce to 3 i.e,

S = {RR, GG, RG}

So, 3 points.

Ans6 (a) If the book of particular subject must all stand together we can consider them as single entity & books can be permutated inside the entity.

So, Total ways = $3! \times 4! \times 6! \times 2!$

where $3!$ is no. of ways to permutate books of subjects (entity)

& $4!, 6!$ & $2!$ are no. of ways to permute books of maths, physics & Chemistry.

So, Total = 207360 ways

(b) for only mathematics books to stand together, we'll consider maths book as a group in which books can be permutated in it. while books of other subject can be permutated freely.

So,

Total ways = $9! \times 4!$ = 8709120 ways

Ans 7

$$^{m+n}C_k = \sum_{i=0}^{k} {}^{m}C_i \times \sum_{j=0}^{k} {}^{n}C_{k-j}$$

$$RHS = \sum_{i=0}^{k} \frac{m!}{(m-i)!\, i!}\left(\sum_{j=0}^{k} \frac{n!}{(k-j)!\,(n-k+j)!}\right)$$

We know that $(1+x)^n = {}^nC_0 x^0 + {}^nC_1 x^1 + \dots {}^nC_n x^n$.

So, coeff. of x^k is ${}^nC_k x^k$.

Since we have (i)

We can consider it to be the product of two binomial expansion.

$$(1+x)^m (1+x)^n = \left(\sum_{k=0}^{m} {}^mC_k x^k\right)\left(\sum_{k=0}^{n} {}^nC_k x^k\right) \quad (ii)$$

But in (i) we have product upto some k^{th} Term.

& $(1+x)^m (1+x)^n = (1+x)^{m+n}$

A coefficient of x in $(1+x)^{m+n}$ can be written as ${}^{m+n}C_k x^k$.

from (ii)

$$ {}^{m+n}C_k x^k = \left(\sum_{j=0}^{k} {}^mC_j x^j\right)\left(\sum_{j=0}^{k} {}^nC_{k-j} x^{k-j}\right)$$

$$ = \sum_{j=0}^{k} ({}^mC_j)({}^nC_{k-j}) . x^k$$

put $x=1$

$$ {}^{m+n}C_k = \sum_{j=0}^{k} ({}^mC_j)({}^nC_{k-j})$$

Hence proved.