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IC 252

B20215
MOHIT VERMA

Assignment 3

Instructor - Satyajit Thakor

Ans 1 (a) $P(X=0) = 0.08$

$P(X=1) = 0.11$

$P(X=2) = 0.27$

$P(X=3) = 0.33$

$P(X=4) = ?$

Since we have a PMF, it's sum for $X=0,1,2,3,4$ must be 1.

hence,

$$\sum_{i=0}^{4} P(X=i) = 1$$

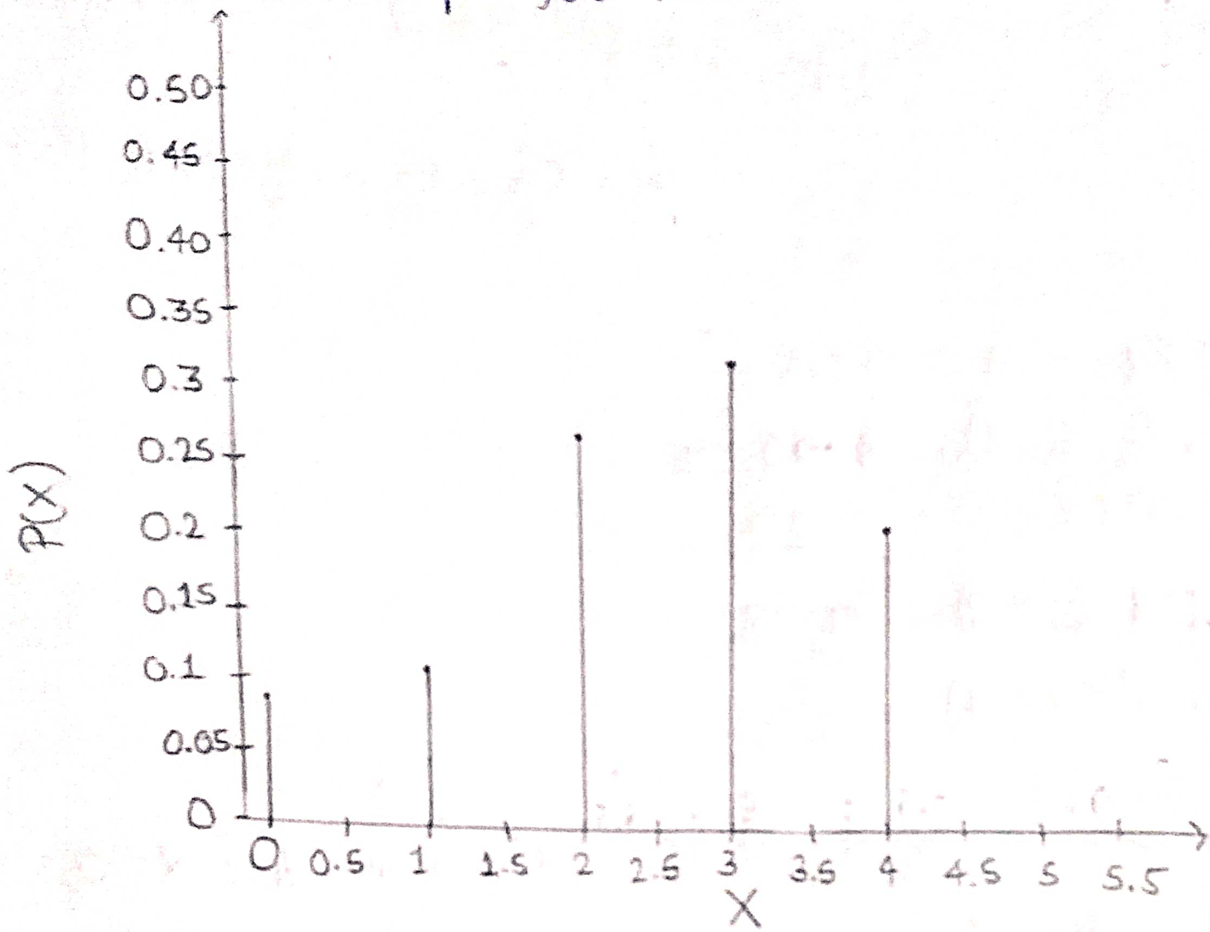
$$P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) = 1$$

$$0.08 + 0.11 + 0.27 + 0.33 + P(X=4) = 1$$

$$P(X=4) = 1 - 0.79 = 0.21$$

(b) The line graph of probability mass function:

Graph for PMF



(c) For cumulative distribution function.

$$F_X(x) = P(X \le x)$$

For $x < 0$

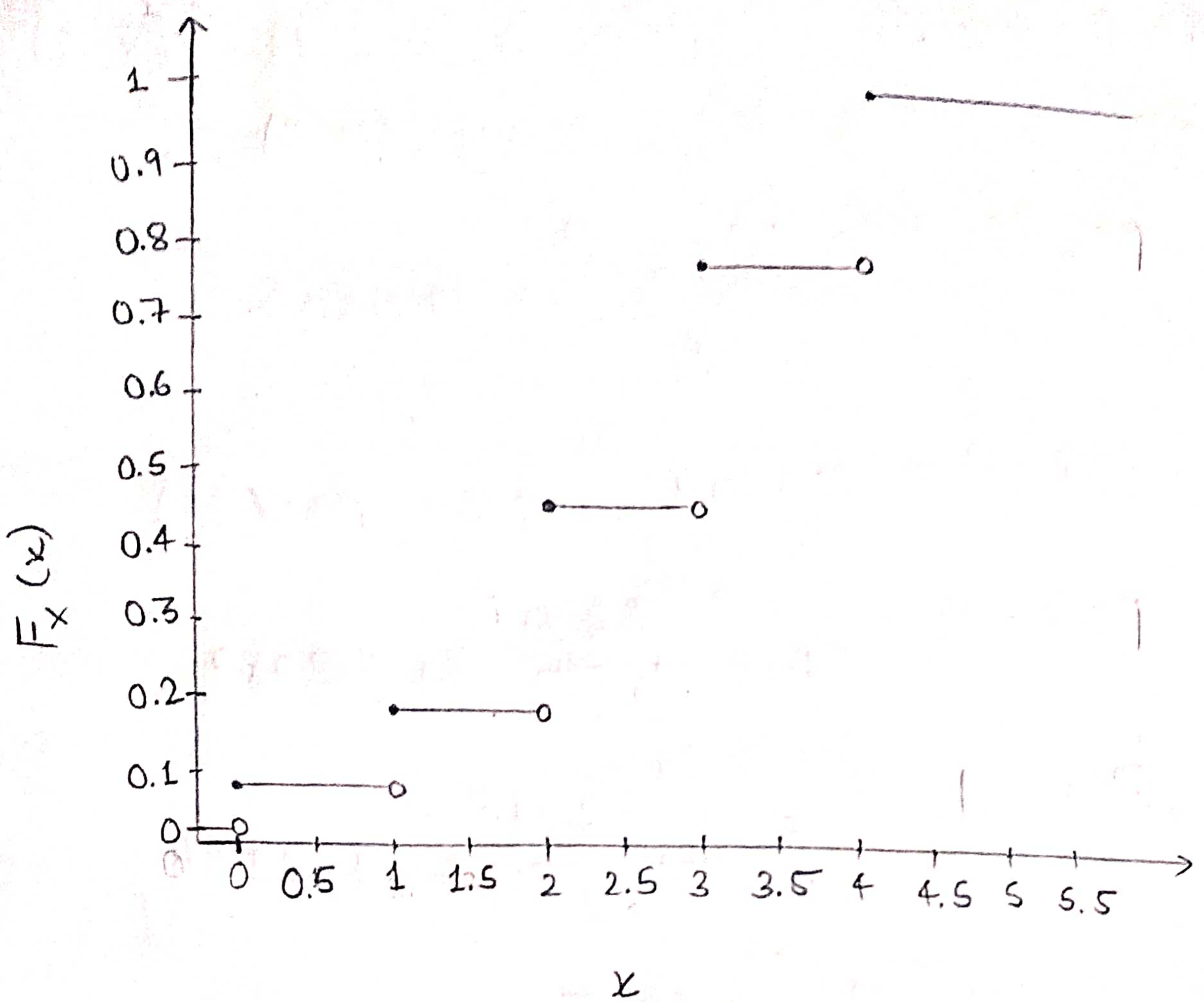
0.08 , for $0 \le x \le 1$

0.19 , for $1 \le x < 2$

0.46 , for $2 \le x < 3$

0.79 , for $3 \le x < 4$

1 for $x \ge 4$



Graph for Cumulative Distribution Function.

Ans 2

We have, $X \sim Pois(3.2)$

$\lambda = 3.2$

$$P(X=k) = \frac{e^{-\lambda} \lambda^k}{k!}$$

(i) $$P(X=1) = \frac{e^{-3.2} \times (3.2)^1}{1!} = 0.1304$$

Ans 2(ii) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$$P(X=0) = \frac{e^{-3.2} \times (3.2)^0}{0!} = 0.0407$$

$P(X=1) = 0.1304$ from part (i)

$$P(X=2) = \frac{e^{-3.2}(3.2)^2}{2!} = 0.2087$$

$$P(X=3) = \frac{e^{-3.2}(3.2)^3}{3!} = 0.2226$$

Hence, $P(X \leq 3) = \underline{\underline{0.6025}}$

(iii) $P(X \geq 6) = P(X=6) + P(X=7) + \cdots$

So,

$$P(X \geq 6) = \sum_{k=6}^{\infty} \frac{e^{-(3.2)}(3.2)^k}{k!}$$

which also can be found out as

$$P(X \geq 6) = 1 - P(X < 6)$$

$$P(X=4) = \frac{e^{-3.2}(3.2)^4}{4!} = 0.1780$$

$$P(X=5) = \frac{e^{-3.2}(3.2)^5}{5!} = 0.1139$$

So,
$$P(X<6) = 0.6025 + 0.1780 + 0.1139$$
$$= 0.8944$$

& hence, $P(X \geq 6) = 1 - 0.8944$
$$= \underline{\underline{1.054}}$$

<u>Ans3</u> (a) X can take values 0, 1, 2.

So, $P_X(0) = P(X=0) = \frac{39^2}{52^2} = 0.5625$

$P_X(1) = P(X=1) = {}^{2}C_{1}\,\frac{13 \cdot 39}{52^2} = 0.3750$

$P_X(2) = P(X=2) = \frac{13^2}{52^2} = 0,0625$

(b) Cumulative distribution function will be

$F_X(x) = P(X<0) = 0$

$$F_X(x) = \begin{cases} 0.5625, & \text{for } 0 \le x < 1 \\ 0.9375, & \text{for } 1 \le x < 2 \\ 1, & \text{for } 2 \le x \end{cases}$$

(c) The most likely value for random variable X is it's expectation.

$$E(X) = 0 \times (0.562) + 1.(0.3750) + 2.(0.0625)$$
$$= \underline{\underline{0.5}}$$

Ans4

Let random variable be X which is the score after the roll of a dies.

So, if red = 2, 4, 6
& blue = 1, 2, 3, 4, 5, 6

$$P(X=2) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$
$$P(X=4) = 1/2 \times \frac{1}{6} = 1/12$$
$$P(X=6) = 1/2 \times \frac{1}{6} = 1/12$$
$$P(X=8) = 1/2 \times \frac{1}{6} = 1/12$$
$$P(X=10) = 1/2 \times \frac{1}{6} = 1/12$$
$$P(X=12) = \frac{1}{2} \times \frac{1}{6} = 1/12$$

Also, if red(R) = 1, 3, 5
& blue(B) = 1, 2, 3, 4, 5, 6

So,
R = 1, 3, 5 & B = 1, 3, 5
Taking cases respectively,
R - B = 0

So, $P(X=0) = \frac{3}{6} \times \frac{1}{6} = \frac{1}{12}$
Similarly,

Also, $P(X=-1) = \frac{3}{36} = \frac{1}{12}$

$P(X=-2) = \frac{2}{36} = \frac{1}{18}$

$P(X=-3) = \frac{2}{36} = \frac{1}{18}$

$P(X=-4) = \frac{1}{36}$ & $P(X=-5) = \frac{1}{36}$

& when R = 5 & B = 1, similarly.

$P(X=4) = \frac{1}{36}$ $\quad$ $P(X=2) = \frac{2}{36}$

$P(X=1) = \frac{2}{36}$

$P(X=3) = \frac{1}{36}$

Now, finding total probability for all cases,

$P(X=-5) = \frac{1}{36}$ $\quad$ $P(X=-1) = \frac{3}{36}$ $\quad$ $P(X=3) = 1/36$

$P(X=-4) = \frac{1}{36}$ $\quad$ $P(X=0) = \frac{3}{36}$ $\quad$ $P(X=4) = \frac{4}{36}$

$P(X=-3) = \frac{2}{36}$ $\quad$ $P(X=1) = \frac{2}{36}$ $\quad$ $P(X=6) = \frac{3}{36}$

$P(X=-2) = \frac{2}{36}$ $\quad$ $P(X=2) = \frac{5}{36}$ $\quad$ $P(X=8) = \frac{3}{36}$

$F_X(x) = 0.333 \qquad 0 \leq x < 1$

$F_X(x) = 0.389 \qquad 1 \leq x < 2$

$F_X(x) = 0.527 \qquad 2 \leq x < 3$

$F_X(x) = 0.555 \qquad 3 \leq x < 4$

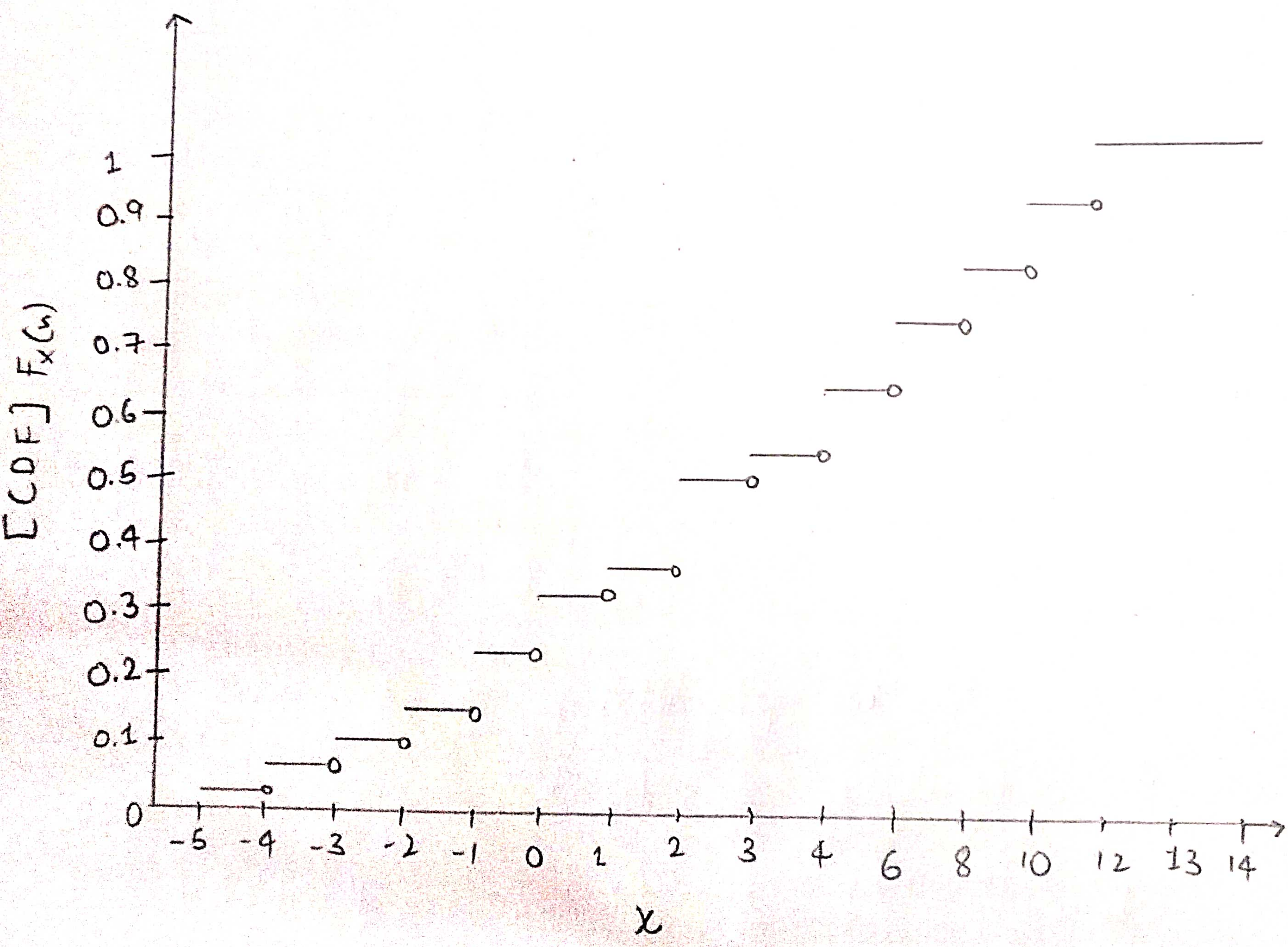
$F_X(x) = 0.666 \qquad 4 \leq x < 6$

$F_X(x) = 0.75 \qquad 6 \leq x < 8$

$F_X(x) = 0.833 \qquad 8 \leq x < 10$

$F_X(x) = 0.916 \qquad 10 \leq x < 12$

$F_X(x) = 1 \qquad x \geq 12$



Ans 5 No. of components is n.

& each of them function independently for probability = p.

Probability that 5- component system will operate effectively

$$P_5(p) = {}^5C_3\, p^3(1-p)^2 + {}^5C_4\, p^4(1-p) + {}^5C_5\, p^5$$

& for 3 component system.

$$P_3(p) = {}^3C_2\, p^2(1-p) + {}^3C_3\, p^3$$

Hence, for 5 component system to operate more effectively than 3 component system,

$${}^5C_3\, p^3(1-p)^2 + {}^5C_4\, p^4(1-p) + {}^5C_5\, p^5 > {}^3C_2\, p^2(1-p) + {}^3C_3\, p^3$$

$$10p^3 + 10p^5 - 20p^4 + 5p^4 - 5p^5 + p^5 > 3p^2 - 3p^3 + p^3$$

$$6p^5 + 12p^3 - 15p^4 - 3p^2 \geq 0$$

$$2p^5 - 5p^4 + 4p^3 - p^2 > 0$$

$$p^2(2p^3 - 5p^2 + 4p - 1) > 0$$

On factorizing,

$$p^2(p-1)(p-1)(2p-1) > 0$$

$$p^2(p-1)^2(2p-1) > 0$$

Since $p^2(p-1)^2 > 0$ for all p,

$$p > \frac{1}{2}$$

So, values of p should be greater than 0.5.

Ans 6

X can take 1, 2, or any other positive integer.

(a) $P(X=i) = \frac{c}{i^2}$ is a valid PMF only if it is a valid PMF.

So, $P_X(x) > 0$ if $x = x_i$ for some i. otherwise 0.

&

$$\sum_{i=1}^{\infty} P_X(x_i) = 1$$

$x_1 = 1, x_2 = 2, \ldots$ in our case.

So, $\frac{c}{i^2} \geq 0$

Hence $c \geq 0$. but $c \neq 0$, since $\sum_{i=1}^{\infty} P(X=i) = 0$.

P $P(X=i) = \frac{c}{i^2}$

$$\sum_{i=1}^{\infty} P(X=i) = 1$$

Hence,

$$\frac{c}{1^2} + \frac{c}{2^2} + \frac{c}{3^2} + \cdots\cdots = 1$$

$$c\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\cdots\right) = 1$$

we know that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\cdots = \frac{\pi^2}{6}$$

So,

$$\frac{\pi^2}{6} \times c = 1$$

$c = \frac{6}{\pi^2} = 0.6079$ So, Yes, it is possible

(b) Again checking the validity for PMF.

$\frac{c}{i} > 0$

or $c > 0$.

Also, $\sum_{i=1}^{\infty} P(X=i) = 1$

$\sum_{i=1}^{\infty} P(X=i) = \frac{c}{1} + \frac{c}{2} + \frac{c}{3} + \frac{c}{4} + \cdots$

$\sum_{i=1}^{\infty} P(X=i) = c\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots\right)$

But, this series $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

is diverging to ∞,

Hence there exists no value for c that can make the PMF valid.

No, it is not possible.