5/06/21

IC 252

MOHIT VERMA

Assignment 11

B20215

Ans1 (a) Since, she has committed Type I error that means the Ho(Null Hypothesis) is true but she has rejected it. Hence the hypothesis must be that "The Training course is effective".

(b) As it is a type II error, we can easily see that the null hypothesis must be "The training course is effective".

Ans2 (a) Our $H_0: p_0 = 0.6$ is true. Hence, Type I error will be denoted by α,

&

α = P(rejection of null Hypothesis when it is true)

let x be a r.v denoting order of raw material arriving late,

α = P(order lates are $x = 0,1,2,3$ when $p = 0.6$)

$$\alpha = P(0 \leq X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= {}^nC_0 (0.6)^{10} + {}^nC_1 (0.6)^9 (0.4) + {}^nC_2 (0.6)^8 (0.4)^2 + {}^nC_3 (0.6)^7 (0.4)^3 \text{ where } n = 10.$$

$\alpha = 0.0548$

(b) $p=0.3$, $p=0.4$, $p=0.5$
the Null hypothesis is false but we are accepting Null hypothesis.
let $p=0.3$
$\beta = l$ orders late must be $x > 3$)

$$\beta = \sum_{i=4}^{10} {}^{10}C_i (0.3)^i (0.7)^{10-i}$$

On Calculating,

$\beta = 0.350$

Similarly for $p=0.4$ & $p=0.5$

$$\beta = \sum_{i=4}^{10} {}^{10}C_i (0.4)^i (0.6)^{10-i}$$

$= 0.6177$

& $\beta = \sum_{i=4}^{10} {}^{10}C_i (0.5)^i (0.5)^{10-i} = 0.8281$

Ans 3

$\mu = 800 \quad \sigma = 40$

let level of significance be 0.03.

So, $H_0 : \mu = 800$

& Alternate hypothesis:
$H_1 : \mu \neq 800$

So, we know that we have sample of 30 bulbs we can approximate it as Normal distribution from central limit theorem.

Hence,

$$Z_\alpha = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{788-800}{40/\sqrt{30}} = -1.63$$

As it is a two tailed test,

$$p = 2 \cdot P(Z \geq |Z_\alpha|) = 2P(Z \geq |-1.64|)$$

$$= 2P(Z \leq -1.64) = 0.1$$

So, $p = 0.1$, hence, it is greater than level of significance, so we will not reject null hypothesis.

Ans4 H_0: $p = 0.7$

(a) Alternative hypothesis is $p > 0.7$.

So, Type I error is rejection of H_0 when it is true, hence

$$\alpha = {}^{12}C_{11}(0.7)^{11}(0.3) + {}^{12}C_{12}(0.7)^{12}$$

$$= \underline{0.085}$$

(b) for β, The H_0 is not rejected when it is false, So, $p = 0.9$

$$\beta = 1 - {}^{12}C_{11} (0.9)^{11} (0.1)^{1} - {}^{12}C_{12} (0.9)^{12}$$

$$= 1 - 0.659$$

$$\beta = 0.341$$

Ans 5

$H_0 : \alpha \geq 1$

$H_1 : \alpha < 1$

Reject H_0 if $X \geq 1$

So, power of test will be

$$1 - \beta = P(X \geq 1 \text{ when } \alpha < 1)$$

So, for $X \geq 1$ we have,

$$= \int_1^\infty \alpha e^{-\alpha x} dx$$

$$= \frac{\alpha}{-\alpha} \left[e^{-\alpha x} \right]_1^\infty$$

$$= -1 (0 - e^{-\alpha})$$

$$= e^{-\alpha}$$

Hence, $1 - \beta = e^{-\alpha}$ where $\alpha \in (0, 1)$

Ans 6

$H_0 : p = 0.2$

$H_1 : p \neq 0.2$

So power of test will be 1 when $p = 0$ which is quite trivial for $p = 0.4$,

$P(Y \geq 7) = 0.7499$

$P(Y \leq 1) \approx 0$

So, power of test will be $= 0.7499$

Similarly for $p = 1$

The power of test will be 1.

Ans 7 $\quad n = 64 \quad \bar{x} = 38 \quad \sigma = 5.8$

$H_0 = 40$

$H_1 : \mu < 40$

So,

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{38 - 40}{5.8/\sqrt{64}} = -2.76$$

$p = P(Z < -2.76) = 0.003$

Since p value is very small, hence we will reject H_0.