

Assignment 6  
IC252 - IIT Mandi  
Submission Deadline: 22 April, 2021

1. Suppose that  $X$  is a continuous r.v. whose PDF is given by

$$f_X(x) = \begin{cases} c(4x - 2x^2), & 0 \leq x \leq 2; \\ 0, & \text{elsewhere.} \end{cases}$$

(a) What is the value of the constant  $c$  (for  $f_X$  to be a valid PDF)? [2]

(b) Find  $P(X > 1)$ . [1]

2. Milk containers have label printed “2 liters”. But, the PDF of the amount of milk deposited in a milk container by a dairy factory is

$$f_X(x) = \begin{cases} 40.976 - 16x - 30e^{-x}, & 1.95 \leq x \leq 2.20; \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Is  $f_X$  a valid PDF? [2]

(b) What is the probability that a container produced by the dairy factory is underweight? [2]

3. Consider a random variable measuring the following quantities. In each case state with reasons whether you think it more appropriate to define the random variable as discrete or as continuous.

(a) A person’s height [1]

(b) A student’s course grade [1]

(c) The thickness of a metal plate [1]

4. A random variable  $X$  takes values between 4 and 6 with a probability density function

$$f_X(x) = \begin{cases} \frac{1}{x \log_e(1.5)}, & 4 \leq x \leq 6; \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Make a plot of the PDF (you may use some programming tools to plot functions). [1]

(b) Check that the total area under the probability density function is equal to 1. [2]

(c) What is  $P(4.5 \leq X \leq 5.5)$ ? [2]

(d) Find the CDF and plot it (you may use some programming tools to plot functions). [2]

(e) What is the expected value of this random variable? [1.5]

(f) What is the median of this random variable? [1.5]

(g) What is the variance of this random variable? [2]

(h) What is the standard deviation of this random variable? [1]

5. (a) For  $X \sim N(\mu, \sigma^2)$ , verify that, its PDF is symmetric around the mean, i.e., [1]

$$f_X(\mu - x) = f_X(\mu + x).$$

(b) For  $X \sim N(0, 1)$ , verify that [1.5]

$$\Phi_X(-x) = 1 - \Phi_X(x).$$

6. **Optional (advanced):** For  $X \sim N(\mu, \sigma^2)$ , i.e.,  $X$  with PDF

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

verify that

(a) the mean is  $\mu$  and [2.5]

(b) the variance is  $\sigma^2$ . [2.5]