Assignment 4

IC252 - IIT Mandi

Submission Deadline: 6 April, 2021

1. Let X denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as

Table 1: $p_{X,Y}(x, y)$

		x		
		1	2	3
y	1	0.05	0.05	0.10
	3	0.05	0.10	0.35
	5	0.00	0.20	0.10

   (a) Evaluate the marginal distribution of X. **[1]**

   (b) Evaluate the marginal distribution of Y. **[1]**

   (c) Find $P(Y = 3|X = 2)$. **[1]**

2. If the joint probability distribution of X and Y is given by

$$p_{X,Y}(x, y) = \frac{x + y}{30},$$

   for $x = 0, 1, 2, 3; y = 0, 1, 2$, find

   (a) $P(X \leq 2, Y = 1)$; **[2]**

   (b) $P(X > 2, Y \leq 1)$; **[2]**

   (c) $P(X > Y)$; **[2]**

   (d) $P(X + Y = 4)$. **[2]**

3. Each student in a certain BTech program was classified according to her year in university (1st, 2nd, 3rd, 4th year) and according to the number of times that she had visited a certain museum (never, once, or more than once). The proportions of students in the various classifications are given in the following table:

	Never	Once	More than once
1st year	0.08	0.10	0.04
2nd year	0.04	0.10	0.04
3rd year	0.04	0.20	0.09
4th year	0.02	0.15	0.10

(a) If a student selected at random from the BTech program is "3rd year", what is the probability that she has never visited the museum? **[2]**

(b) If a student selected at random from the BTech program has visited the museum three times, what is the probability that she is "4th year"? **[2]**

4. A fair coin is tossed four times, and the random variable X is the number of heads in the first three tosses and the random variable Y is the number of heads in the last three tosses.

   (a) What is the joint probability mass function of X and Y? **[2]**

   (b) What are the marginal probability mass functions of X and Y? **[2]**

   (c) Are the random variables X and Y independent? **[1.5]**

5. Two cards are drawn without replacement from a pack of cards, and the random variable X measures the number of hearts drawn and the random variable Y measures the number of clubs drawn.

   (a) What is the joint probability mass function of X and Y ? **[2]**

   (b) What are the marginal probability mass functions of X and Y ? **[2]**

   (c) Are the random variables X and Y independent? **[1.5]**