03/03/21

IC 252

MOHIT VERMA
B20215

Data Science – II

Assignment – 2

Instructor – Satyajit Thakor

Ans 1 Total no. of ways in which n people can be seated = $n!$

for XY to sit together, other $n-2$ people & X.Y can be seated in $(n-1)!$ ways. Considering XY to be an entity.

So, $P = \frac{2\cdot(n-1)!}{n!}$

2 is multiplied by $(n-2)!$ since XY can permutate in $2!$ ways.

So,

$$P = \frac{2\times(n-1)}{(n)(n-1)} = \frac{2}{n}$$ Ans

Ans 2 For all the aces to be in the same set, the probability for this event can be thought like this -

ways of choosing the set in which four aces should be present = ${}^{4}C_1$

And if there are four aces in a set then rest of 9 cards can be chosen in ${}^{48}C_9$ ways. So, the probability should be,

$$P = \frac{{}^{4}C_1 \times {}^{48}C_9}{{}^{52}C_{13}} = \frac{4 \times {}^{48}C_9}{{}^{52}C_{13}}$$

Ans 3

Total No. of Balls = 24

" " Red balls = 2

if two red balls are choosen, then rest of 8 balls can be chosen in ${}^{22}C_8$ ways.

Total No. of ways of selecting 10 balls

$= {}^{24}C_{10}$

So,

$$P = \frac{{}^{22}C_8}{{}^{24}C_{10}} = 0.163$$

Ans 4

For all n people to have different birthdays we can select n days out of 365 day in ${}^{365}C_n$ ways. which can belong to any one of n people.
Also, there are 365 days on which a person's birthday can lie.

So, probability = $\dfrac{{}^{365}C_n \times n!}{365 \ldots n \text{ times}}$

$$= \frac{365! \times n!}{n!\,(365-n)!} \times \frac{1}{(365)^n}$$

$$P = \frac{365}{365} \times \frac{364}{365} \ldots \times \frac{365-n+1}{365}$$

Ans 5

The chance that the bridge built by Mr. Narayanan will collapse can be found by adding all the possibilities in which bridge can collapse.

1. Let A be event that design is faulty & B be event it is not.

Also, C be event that bridge collapses

So, $P\left(\frac{C}{A}\right) = 0.9$ & $P\left(\frac{C}{B}\right) = 0.2$

From Law of Total Probability,

$\cancel{P(C/B)}$ $P(C) = P(A) \cdot P\left(\frac{C}{A}\right) + P(B) \cdot P\left(\frac{C}{B}\right)$

$= 0.6 \times 0.9 + 0.4 \times 0.2$

$P(C) = \underline{\underline{0.62}}$

<u>Ans 6</u>. For A:

For atleast one 6 to appear when 6 fair die are rolled,

No. of ways in which No. head appears $= \left(\frac{5}{6}\right)^6$

So,

Probability for atleast one head $= 1 - \left(\frac{5}{6}\right)^6$

$= \underline{\underline{0.665}}$

For B: For atleast two 6 to appear,

Probability for No head $= \left(\frac{5}{6}\right)^{12}$

" " One head $= {}^{12}C_1 \times \frac{(5)^{11}}{6^{12}}$

So,

$$P = 1 - \left(\frac{5}{6}\right)^{12} - {}^{12}C_1 \times \frac{5^{11}}{6^{12}}$$

$$\underline{\underline{P = 0.618}}$$

For C: Similarly from previous arguments,

$$P = 1 - \left(\frac{5}{6}\right)^{18} - {}^{18}C_1 \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{17} - {}^{18}C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{16}$$

$$\underline{\underline{P = 0.60}}$$

So, $P(A) > P(B) > P(C)$

So, Event A has highest Probability.

Ans7 (i) Given that X has been selected,
so, there are three red & one blue card left. If second pick is red card than both cards will be red. So,

$$P = \frac{3}{4}$$

(ii) Let A be the event that first red card has been selected & B be the even that second card be red.

$$P(A) = \frac{4}{5}$$

&

$$P\left(\frac{B}{A}\right) = \frac{3}{4}$$

for Both card to be red,

$$P = P(A) \cdot P\left(\frac{B}{A}\right) = \frac{3}{5}$$

Ans8 let A - an Event that ball in bag was green

B - " " " " " " " blue

C - an Event that green ball is taken out

D - an Event that both balls are green.

E - an event that one ball is blue & other is green.

From Baye's Theorem -

$$P(D|C) = \frac{P(C|D) \cdot P(D)}{P(C|D) \cdot P(D) + P(C|G) \cdot P(G)}$$

$$= \frac{\frac{1}{2} \cdot 1}{\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

$$P(D|C) = \frac{2}{3} = 0.66$$

Ans9

A - Event that drawn card is atleast Five.

B - Event that card drawn is ten.

$$P(A) = \frac{6}{10}$$

$$P(A \cap B) = \frac{1}{10}$$

So, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{10}}{\frac{6}{10}} = \frac{1}{6}$

Ans 10 let A_i be the event that card of i^{th} suit is not present in hand.

So, No. of ways of $A_i = {}^4C_1 \cdot {}^{39}C_{13}$

& $P(A_i) = \frac{{}^4C_1 \times {}^{39}C_{13}}{{}^{52}C_{13}}$

But we want to calculate that the hand doesn't have card of one suit. i.e $P(A_1 \cup A_2 \cup A_3 \cup A_4)$.

Hence, From Inclusion-Exclusion principle,

$$P(A_1 \cup A_2 \cup A_3 \cup A_4) = \sum_{i=1}^{4} P(A_i) - \sum_{i=1}^{3} P(A_i \cap A_{i+1}) + \sum_{i=1}^{2} P(A_i \cap A_{i+1} \cap A_{i+2})$$

So,

$$P(A_1 \cup A_2 \cup A_3 \cup A_4) = \frac{{}^4C_1 \times {}^{39}C_{13}}{{}^{52}C_{13}} - \frac{{}^4C_2 \, {}^{26}C_{13}}{{}^{52}C_{13}} + \frac{{}^4C_3 \, {}^{13}C_{13}}{{}^{52}C_{13}}$$