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IC 252

Assignment-10

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Ans 1 So, Let X be a r.v for Zinc concentration-

So, $\bar{x} = 2.6$ g/ml $\sigma_x = 0.3$ g/ml

$\bar{x}$ is an estimate of μ_x.

for 99 % confidence, $\alpha = 0.01$

we know,

$$\phi(z_{\frac{\alpha}{2}}) = P(z < z_{\frac{\alpha}{2}}) = 1 - \frac{\alpha}{2}$$

$$\phi(z_{0.005}) = 1 - 0.005 = 0.995$$

from CDF, $z_{0.005} = 2.58$

Hence, 99 % confidence interval of μ is

$$\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

on substituting values,

$$2.6\left(2.5 \times \frac{0.3}{\sqrt{36}}\right) < \mu < 2.6 + \left(2.5 \times \frac{0.3}{\sqrt{36}}\right)$$

$$2.6 - 0.129 < \mu < 2.6 + 0.129$$

$$2.471 < \mu < 2.729$$

Ans 2 We have $n = 30$ & $\bar{x} = 780$

To find a 96 % confidence interval,

$$96 = 100(1-\alpha)$$

$$0.96 = 1-\alpha$$

$$\alpha = \underline{\underline{0.04}}$$

Since, μ, σ_x^2 is given to us & $\bar{x}$ is the mean of sample,

$$\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$\alpha/2 = 0.02$

So, $z_{0.02} = 2.05$, so

$$780 - \frac{2.05 \times 40}{\sqrt{30}} < \mu < 780 + \frac{2.05 \times 40}{\sqrt{30}}$$

So, the required interval will be

$$765.02 < \mu < 794.94$$

Ans 3 we have $n = 10$ & we need to find the maximum likelihood estimator of mean survival time.

PDF for exponential distribution :-

$$f(x) = \lambda e^{-x_1 \lambda}$$

where $L(\lambda) = \lambda e^{-x_1 * \lambda} * \lambda e^{-x_2 * \lambda} * \cdots \lambda e^{-x_n * \lambda}$

$$L(\lambda) = \lambda^n \left(e^{-\lambda \left(\sum_{i=1}^{n} x_i\right)} \right)$$

In order to find the maximum likelihood estimator,

$$\ln(L(\alpha)) = n\ln\alpha + \left(-\alpha\sum_{i=1}^{n} x_i\right)$$

On derivating,

$$\frac{L'(\alpha)}{L(\alpha)} = \frac{n}{\alpha}(1) - \sum_{i=1}^{n} x_i$$

putting $L'(\alpha) = 0$

$$\alpha = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{162}{10} = 16.2$$

So, the given α represents maximumlikeli-
-hood estimator.

Ans 4. let X be a r.v. measuring average score of all students.

So, $\bar{x} = 74.6$ & $\sigma_X = 11.3$

So, 90% confidence interval means

$$90 = 100(1-\alpha)$$
$$\alpha = 0.1$$

& $\alpha/2 = 0.05$

hence, $Z_{\alpha/2} = 1.645$ from Table.

we know; $Z_{\alpha/2}$, $\bar{x}$, $\sigma_{\bar{x}}$. hence we can determine the interval,

$$\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

thence, the required interval is,

$$74.6 - \left(1.64 \frac{(11.3)}{\sqrt{81}}\right) < \mu_x < 74.6 + \left(1.64 \times \frac{11.3}{\sqrt{81}}\right)$$

$$74.6 - 2.06 < \mu_x < 74.6 + 2.06$$

$$\underline{\underline{72.53 < \mu_x < 76.66}}$$