

Assignment 3  
IC252 - IIT Mandi  
Submission Deadline: 22 March, 2021

1. An office has four copying machines, and the random variable  $X$  measures how many of them are in use at a particular moment in time. Suppose that  $P(X = 0) = 0.08$ ,  $P(X = 1) = 0.11$ ,  $P(X = 2) = 0.27$ , and  $P(X = 3) = 0.33$ .
  - (a) What is  $P(X = 4)$ ? [1]
  - (b) Draw a line graph of the probability mass function. [1]
  - (c) Construct and plot the cumulative distribution function. [1]
2. If  $X \sim \text{Pois}(3.2)$ , calculate:
  - (a)  $P(X = 1)$  [1]
  - (b)  $P(X \leq 3)$  [1]
  - (c)  $P(X \geq 6)$  [1]
3. Two cards are drawn at random from a pack of cards with replacement. Let the random variable  $X$  be the number of cards drawn from the heart suit.
  - (a) Construct the probability mass function. [2]
  - (b) Construct the cumulative distribution function. [1]
  - (c) What is the most likely value of the random variable  $X$ ? [1]
4. Two fair dice, one red and one blue, are rolled. A score is calculated to be twice the value of the blue die if the red die has an even value, and to be the value of the red die minus the value of the blue die if the red die has an odd value. Construct and plot the probability mass function and the cumulative distribution function of the score. [3]
5. A communication system consists of  $n$  components, each of which will, independently, function with probability  $p$ . The total system will be able to operate effectively if at least one-half of its components function. For what values of  $p$  is a 5-component system more likely to operate effectively than a 3-component system? [3]
6. Suppose that a random variable  $X$  can take the value 1, 2, or any other positive integer.
  - (a) Is it possible that  $P(X = i) = c/i^2$  for some value of the constant  $c$ ? [1.5]
  - (b) Is it possible that  $P(X = i) = c/i$  for some value of  $c$ ? [1.5]