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IC 252 B20215

Assignment - 6

MOHIT VERMA

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Ans 1

$$f_X(x) = \begin{cases} c(4x - 2x^2), & 0 \le x \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

(a) The value of c for this to be a valid PDF,

$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1$$

So,

$$= c \times \int_0^2 (4x - 2x^2)\, dx$$

$$= c \left[\frac{4x^2}{2} - \frac{2x^3}{3} \right]_0^2$$

$$= c \left[8 - \frac{8 \times 2}{3} \right]$$

$$= \frac{8c}{3} = 1$$

So, $c = \frac{3}{8}$

(b) $P(X>1) = \int_1^2 f_X(x)\,dx$

$$= \int_1^2 \frac{3}{8}(4x-2x^2)\,dx$$

$$= \int_1^2 \frac{3}{8}\left[2x^2 - \frac{2x^3}{3}\right]$$

$$= \frac{3}{8}\left[\left(8-\frac{16}{3}\right)-\left(2-\frac{2}{3}\right)\right]$$

$$= \frac{3}{8}\left[\frac{8}{3}-\frac{4}{3}\right]$$

$$= \underline{\underline{\frac{1}{2}}}$$

Ans 2

$$f_X(x) = \begin{cases} 40.976 - 16x - 30e^{-x}, & 1.95 \le x \le 2.20; \\ 0 & , \text{elsewhere} \end{cases}$$

(a) for f_X to be a valid PDF,

$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$ & $f_X(x) \ge 0$ for all x,

Now, $= \int_{-\infty}^{\infty} 40.976 - 16x - 30e^{-x}\,dx$

$$= \int_{1.95}^{2.2} \left[40.976x - \frac{16x^2}{2} + 30e^{-x}\right]$$

$= 54.75 - 53.75 = \underline{\underline{1}}$

(b) for the milk container to be underweight,

$$P(X<2) = \int_{1.95}^{2.0} f_X(x)\,dx$$

$$= \int_{1.95}^{2.0} (40.976 - 16x - 30e^{-x})\,dx$$

$$= \left[40.976x - 8x^2 + 30e^{-x}\right]_{1.95}^{2.0}$$

$$= 54.0120 - 53.7514$$

$$= \underline{\underline{0.2605}}$$

<u>Ans 3</u> (a) It should be continuous as we can assume that the height of a person can belong to a certain interval which is continuous.

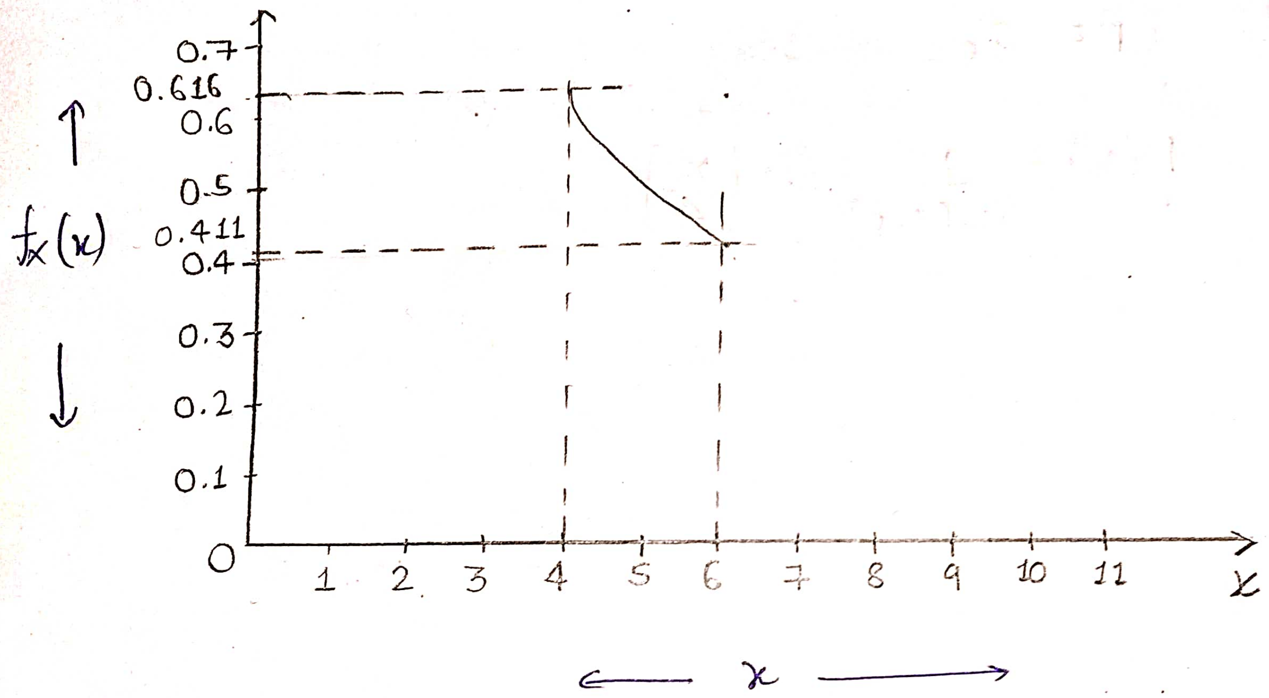
(b) It should be discrete random variable since in each course he can get a distinct grade.

(c) It should be a continuous random variable since it's thickness can be within a continuous interval.

Ans4

$$f_X(x) = \begin{cases} \dfrac{1}{x\log_e(1.5)}, & 4 \le x \le 6 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Plot of the PDF is shown—



(b) To check area under PDF is 1,

$$= \int_{-\infty}^{\infty} f_X(x)\,dx$$

$$= \int_4^6 \frac{1}{x\ln(1.5)}\,dx$$

$$= \frac{1}{\ln(1.5)}\left[\ln x\right]_4^6$$

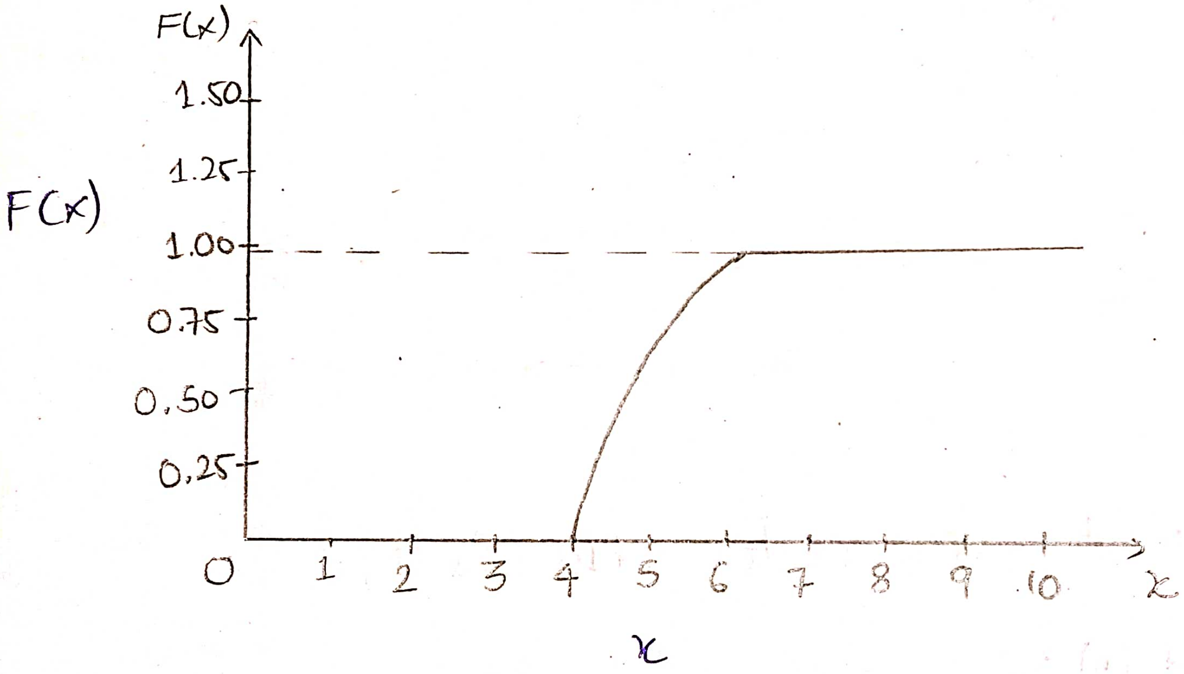
$$= \frac{1}{\ln(1.5)} \times \ln\left(\frac{6}{4}\right) = \underline{\underline{1}}$$

(c) $P(4.5 \le X \le 5.5) = \int_{4.5}^{5.5} f_X(x)\,dx$

$= \frac{1}{\ln(1.5)} \ln\left(\frac{5.5}{4.5}\right) = \underline{\underline{0.494914}}$

(d) Now, CDF is,

$P_X(x) = \frac{1}{\ln(1.5)} \ln\left(\frac{x}{4}\right)$



Plot for CDF.

(e) $E(X) = \int_4^6 x \cdot \frac{1}{x \ln(1.5)}\,dx = \left[\frac{x}{\ln(1.5)}\right]_4^6 = \frac{2}{\ln(1.5)} = \underline{4.932}$

(f) Median of the random variable,

$$\int_4^a f_X(x)\,dx = \int_a^6 f_X(x)\,dx$$

$$\int_4^a \frac{1}{x\,\cancel{\ln(1.5)}}\,dx = \int_a^6 \frac{1}{x\,\cancel{\ln(1.5)}}\,dx$$

$$[\ln(x)]_4^a = [\ln x]_a^6$$

$$\ln(a) - \ln(4) = \ln(6) - \ln(a)$$

$$2\ln(a) = \ln(6) + \ln(4)$$

$$\ln(a) = 1.58$$

$$a = e^{1.58} = \underline{4.89}$$

(g) Variance of random variable is,

$$Var(X) = E(X^2) - (E(X))^2$$

for $E(X^2) = \int_4^6 x^2 \cdot \frac{1}{x\ln(1.5)}\,dx$

$$= \int_4^6 \frac{x}{\ln(1.5)}\,dx$$

$$= \left[\frac{x^2}{2\ln(1.5)}\right]_4^6$$

$$= \frac{1}{\ln(1.5)}\left(\frac{36}{2} - \frac{16}{2}\right)$$

$$= \frac{10}{\ln(1.5)} = 24.66$$

So,

$$Var(X) = 24.66 - (4.93)^2$$

$$= \underline{\underline{0.338}}$$

(b) The standard deviation is,

$$\sigma = \sqrt{Var(X)}$$

$$\sigma = \sqrt{0.338} = \underline{\underline{0.581}}$$

<u>Ans 5</u>

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

$$X \sim N(\mu, \sigma^2)$$

(a) for mean,

$$\text{mean} = \int_{-\infty}^{\infty} x \cdot f_X(x)\, dx = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} dx$$

let, $d = \frac{x-\mu}{\sigma\sqrt{2}}$

$$\text{mean} = \frac{\cancel{\sigma\sqrt{2}}}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma\sqrt{2}+\mu) e^{-t^2} dt$$

$$= \frac{1}{\sqrt{\pi}} \left(\int_{-\infty}^{\infty} \sigma\sqrt{2}\, t e^{-t^2} dt \right) + \frac{\mu}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2} dt$$

$$= \frac{1}{\sqrt{\pi}} \left(\sigma\sqrt{2} \left[\frac{1}{2} [e]_{-\infty}^{-t^2} + \mu\sqrt{\pi} \right) dt = \frac{\mu\sqrt{\pi}}{\sqrt{\pi}} = \underline{\underline{\mu}}$$

Hence, Proved.

(b) for variance is σ^2,

$$\text{Var}[x] = E(x^2) - E(x)$$

On evaluating, $E[x^2]$,

$$E(x^2) = \frac{1}{\sigma\sqrt{2\pi}} \left[\int_{-\infty}^{\infty} \left(x^2 e^{-(x-\mu)^2/2\sigma^2} \right) dx \right]$$

let, $b = \frac{x-\mu}{\sqrt{2}\sigma}$

So,

$$E[x^2] = \frac{\sqrt{2}\sigma}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} (\sqrt{2}\sigma t + \mu)^2 e^{-t^2} dt - \mu^2$$

$$= \frac{1}{\sqrt{\pi}\sigma} \Big(2\sigma^2 \int_{-\infty}^{\infty} t^2 e^{-t} dt + 2\sqrt{2}\sigma\mu \int_{-\infty}^{\infty} e^{-t^2} dt$$

$$+ \mu^2 \int_{-\infty}^{\infty} e^{-t^2} dt \Big)$$

$$= \frac{1}{\sqrt{\pi}\sigma} \Big(2\sigma^2 \int_{-\infty}^{\infty} t^2 e^{-t^2} dt + 2\sqrt{2}\sigma\mu . 0 \Big) + \mu^2$$

$$= \frac{2\sigma^2}{\sqrt{\pi}\sigma} \int_{-\infty}^{\infty} t^2 e^{-t^2} dt + \mu^2$$

$$= \frac{2\sigma^2}{\sqrt{\pi}\sigma} \Big[\frac{-t}{2} e^{-t^2} \Big]_{-\infty}^{\infty} + \frac{1}{2} \int_{-\infty}^{\infty} t^2 e^{-t^2} dt + \mu^2$$

$$= \frac{2\sigma^2}{\sqrt{\pi}\sigma} \times \frac{1}{2} \int_{-\infty}^{\infty} t^2 e^{-t^2} dt + \mu^2 = \frac{\cancel{2}\sigma^2}{\sqrt{\cancel{\pi}\cancel{\sigma}}} \times \frac{1}{\cancel{2}} \sqrt{\cancel{\pi}} + \mu^2$$

$= \sigma^2 + \mu^2$

So, $\quad var[x] = E(x^2) - (E[x])^2$

$= \sigma^2 + \mu^2 - \mu^2$

$= \underline{\underline{\sigma^2}}$