Assignment 7

IC252 - IIT Mandi

Submission Deadline: 29 April, 2021

1. Suppose that you are waiting for a friend to call you and that the time you wait in minutes has an exponential distribution with parameter $\lambda = 0.1$.

   (a) What is the expectation of your waiting time? **[1]**

   (b) What is the probability that you will wait longer than 10 minutes? **[1]**

   (c) What is the probability that you will wait less than 5 minutes? **[1]**

2. A new battery supposedly with a charge of 1.5 volts actually has a voltage with a uniform distribution between 1.43 and 1.60 volts.

   (a) What is the expectation of the voltage? **[1]**

   (b) What is the standard deviation of the voltage? **[1]**

   (c) What is the CDF of the voltage? **[1]**

   (d) What is the probability that a battery has a voltage less than 1.48 volts? **[1]**

   (e) If a box contains 50 batteries, what are the expectation and variance of the number of batteries in the box with a voltage less than 1.5 volts? **[2]**

3. Suppose that $Z \sim N(0, 1)$, i.e., Z has the standard normal distribution. Find:

   (a) $P(Z \leq -0.77)$ **[1]**

   (b) $P(Z \geq 0.32)$ **[1]**

   (c) $P(-0.82 \leq Z \leq 1.80)$ **[1]**

   (e) $P(|Z| \geq 0.91)$ **[1]**

   (f) The value of x for which $P(Z \leq x) = 0.23$ **[1]**

   (g) The value of x for which $P(Z \geq x) = 0.51$ **[1]**

   (h) The value of x for which $P(|Z| \geq x) = 0.42$ **[1]**

   For this problem, use the table of CDF for finding numerical solutions:

   `https://www.mathsisfun.com/data/standard-normal-distribution-table.html`

   OR

   `https://www.math.arizona.edu/~rsims/ma464/standardnormaltable.pdf`

4. For the air conditioner maintenance problem discussed in Lecture 21,

   (a) Suppose that a location has only one air conditioner that needs servicing. What is the conditional PMF of the service time required? **[2]**

(b) Suppose that a location requires a service time of two hours. What is the conditional PMF of the number of air conditioner units serviced? **[2]**

(c) find the correlation between X and Y. **[2.5]**

5. Consider the mining problem discussed in Lectures 22-23.

   (a) Show that $P(0.8 \leq X \leq 1, 25 \leq Y \leq 30) = 0.092$. **[2.5]**

   (b) Show that the iron content has an expected value of 27.36 and a standard deviation of 4.27. **[2.5]**

6. Suppose that two continuous r.v.s X and Y have the joint PDF

$$f_{X,Y}(x, y) = c(e^{x+y} + e^{2x-y})$$

   for $1 \leq x \leq 2$ and $0 \leq y \leq 3$, and $f_{X,Y}(x, y) = 0$ elsewhere.

   (a) What is the value of c? **[2.5]**

   (b) What is $P(1.5 \leq X \leq 2, 1 \leq Y \leq 2)$? **[2.5]**

   (c) Construct the marginal PDFs $f_X(x)$ and $f_Y(y)$. **[2.5]**

   (d) Are the r.v.s X and Y independent? **[2.5]**

   (e) If $Y = 0$, what is the conditional PDF of X? **[2.5]**