Assignment 8

IC252 - IIT Mandi

Submission Deadline: 22 May, 2021

1. The time taken to serve a customer at a fast-food restaurant has a mean of 75.0 seconds and a standard deviation of 7.3 seconds. Use Chebyshev inequality to calculate the time interval that has 89% probability of containing a particular service time. **[2]**

2. A machine produces iron bars whose lengths have a mean of 110.8 cm and a standard deviation of 0.5 cm. Use Chebyshev's inequality to obtain a lower bound on the probability that an iron bar chosen at random has a length between 109.55 cm and 112.05 cm. **[2]**

3. A random variable X has a mean $\mu_X = 8$, a variance $\sigma_X^2 = 9$, and an unknown probability distribution. Find

   (a) $P(-4 \leq X \leq 20)$, **[1]**

   (b) $P(|X - 8| \geq 6)$. **[1]**

4. Homework in Slide 6, Lecture 24: Show that **[1.5]**

$$\int_0^\infty y e^{-\frac{y^2}{2}} dy = 1.$$

5. Cattle are vaccinated to help prevent the spread of diseases among them. Suppose that a particular vaccine has a probability of 0.0005 of causing a serious adverse reaction when administered to an animal. Suppose that the vaccine is to be administered to 500,000 head of cattle. Find the approximate value of the probability that at most 300 animal suffer serious adverse reaction? **[2.5]**

6. Calculate the following probabilities both exactly and by using a normal approximation:

   (a) $P(X \geq 8)$ where $X \sim B(10, 0.7)$ **[2.5]**

   (b) $P(2 \leq X \leq 7)$ where $X \sim B(15, 0.3)$ **[2.5]**

7. A multiple-choice test consists of a series of questions, each with four possible answers. If there are 60 questions, estimate the probability that a student who guesses blindly at each question will get at least 30 questions right. **[2.5]**