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SC 252

Assignment - 9

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Ans 1

Sample mean $= \frac{\sum_{i=1}^{n} x_i}{n}$

where x_i = data values.

So,

mean $= \frac{103.63}{50} = \underline{\underline{2.07}}$

& Sample variance $= \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$

$= \underline{\underline{0.005}}$

Ans 2

$E(X) = \mu$

Variance $= \sigma^2$

The mean for $X_1 + X_2 + X_3 + X_4 + \dots X_n$ will be,

mean $= E(X_1 + X_2 + \dots X_n) = (E(X_1) + E(X_2) + \dots E(X_n))$

$= n\mu$

Also,

Variance $= Var(X_1 + X_2 + X_3 + \dots X_n) = Var(X_1) + \dots + Var(X_n)$

$= n\sigma^2$

Ans 3 $E(X_1) = \mu$, $Var(X_1) = 10$, $E(X_2) = \mu$ & $Var(X_2) = 15$

(1) $\hat{\mu}_1 = \frac{X_1}{2} + \frac{X_2}{2}$

(2) $\hat{\mu}_2 = \frac{X_1}{6} + \frac{X_2}{3} + 9$

(a) Let us assume that X_1 & X_2 are Independent.

So, $E(\hat{\mu}_1) = E\left(\frac{X_1}{2}\right) + E\left(\frac{X_2}{2}\right) = \frac{E(X_1)}{2} + \frac{E(X_2)}{2}$

$= \frac{\mu}{2} + \frac{\mu}{2} = \mu$

Hence, mean is same for X_1 & X_2 so point estimate is unbiased.

$E(\hat{\mu}_2) = E\left(\frac{X_1}{6}\right) + E\left(\frac{X_2}{3}\right) + E(9) = \frac{E(X_1)}{6} + \frac{E(X_2)}{3} + 9$

$= \frac{\mu}{6} + \frac{\mu}{3} + 9 = \frac{\mu}{2} + 9 \neq E(X_i)$

So, point estimate is biased with bias of, $\left(\frac{\mu}{2} + 9 - \mu\right) = 9 - \frac{\mu}{2}$

similarly,

$var(\hat{\mu}_1) = Var\left(\frac{X_1}{2} + \frac{X_2}{2}\right)$

$= \frac{Var(X_1)}{4} + \frac{Var(X_2)}{4}$

$$\frac{10}{4} + \frac{15}{4} = \frac{25}{4} = 6.25$$

& $Var(\hat{\mu}_2) = \frac{Var(X_1)}{36} + \frac{Var(X_2)}{9} + 0$

$$= \frac{10}{36} + \frac{15}{9} = 1.944$$

Hence, the second point estimate has smaller variance.

<u>Ans4</u> We have $X \sim B(n,p)$, the expected value of X is $E(X) = np$

$$\hat{\Theta} = \hat{p} = \frac{X}{n}$$

is an unbiased point estimate with success probability p, we need to show $E(\hat{\Theta}) = E\left(\frac{X}{n}\right) = p$

$$E\left(\frac{X}{n}\right) = \frac{1}{n} E(X)$$

$$= \frac{np}{n}$$

$$= p$$ Hence proved.

Ans5 $Var(\theta) = \sigma_x^2$

Hence, the point estimator of θ is:

$$\hat{\theta} = \hat{\sigma}_x^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$$

(c) We know that,

$$\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$ is an unbiased

estimator. Therefore,

$$bias = \frac{E\left[\sum_{i=1}^{n} (x_i - \bar{x})^2\right]}{n} - \sigma_x^2$$

$$= \frac{E\left[\sum_{i=1}^{n} (x_i - \bar{x})^2\right]}{n-1} \times \frac{n-1}{n} - \sigma_x^2$$

$$= \frac{n\sigma_x^2 - \sigma_x^2 - n\sigma_x^2}{n} = -\frac{\sigma_x^2}{n}$$

Hence, proved.