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IC-252	B20215
DS II	MOHIT VERMA

Assignment 4

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Ans1 (a) Marginal of X can be calculated as

i) $P(X=1) = P(X=1, Y=1) + P(X=1, Y=3) + P(X=1, Y=5)$

$= 0.05 + 0.05 + 0.00 = \underline{0.10}$

ii) $P(X=2) = P(X=2, Y=1) + P(X=2, Y=3) + P(X=2, Y=5)$

$= 0.05 + 0.10 + 0.20 = \underline{0.35}$

iii) $P(Y=3) = P(X=3, Y=1) + P(X=3, Y=3) + P(X=3, Y=5)$

$= 0.10 + 0.35 + 0.10 = \underline{0.55}$

(b) Similarly as in part (a).

i) $P(Y=1) = P(Y=1, X=1) + P(Y=1, X=2) + P(Y=1, X=3)$

$= 0.05 + 0.05 + 0.10 = \underline{0.20}$

ii) $P(Y=3) = P(Y=3, X=1) + P(Y=3, X=2) + P(Y=5, X=3)$

$= 0.05 + 0.10 + 0.35 = \underline{0.50}$

iii) $P(Y=5) = P(Y=5, X=1) + P(Y=5, X=2) + P(Y=5, X=3)$

$= 0.00 + 0.20 + 0.10 = \underline{0.30}$

(c) $P(Y=3 \mid X=2) = \dfrac{P(Y=3, X=2)}{P(X=2)} = \dfrac{0.10}{0.35} = \dfrac{2}{7}$

<u>Ans2</u>

$$P_{X,Y}(x,y) = \frac{x+y}{30}$$

(a) $P(X \le 2, Y=1) = P(X=0, Y=1) + P(X=1, Y=1) + P(X=2, Y=1)$

Also, $P_{X,Y}(x,y) = \dfrac{x+y}{30}$

$$P(X \le 2, Y=1) = \frac{0+1}{30} + \frac{1+1}{30} + \frac{2+1}{30} = \frac{6}{30} = \frac{1}{5}$$

(b) $P(X>2, Y \le 1) = P(X=3, Y=0) + P(X=3, Y=1)$

$$= \frac{3+0}{30} + \frac{3+1}{30} = \frac{7}{30}$$

(c) $P(X>Y) = P(X=1, Y=0) + P(X=2, Y=0) + P(X=3, Y=0) + P(X=2, Y=1) + P(X=3, Y=1) + P(X=3, Y=2)$

$$= \frac{1}{30} + \frac{2}{30} + \frac{3}{30} + \frac{3}{30} + \frac{4}{30} + \frac{5}{30}$$

$$= \frac{18}{30} = \frac{3}{5}$$

d) $P(X+Y=4) = P(X=3, Y=1) + P(X=2, Y=2)$

$= \frac{8}{30} = \frac{4}{15}$

Ans3 (a) Let X be a random variable denoting the year of the student in B.Tech Programme

so, $X = 1, 2, 3, 4$

& Y be a random variable such that $Y = 0, 1, 2$ where $Y=0$ denotes never visiting a museum, $Y=1$ denotes visiting once & $Y=2$ denotes visiting more than once.

So, for the given question probability will be,

$$P(Y=0 \mid X=3) = \frac{P(Y=0, X=3)}{P(X=3)}$$

Now $P(X=3) = P(X=3, Y=0) + P(X=3, Y=1) + P(X=3, Y=2)$

$= 0.04 + 0.20 + 0.09$

$= 0.33$

from Table $P(Y=0, X=3) = 0.04$

So, $P(Y=0 \mid X=3) = \frac{0.04}{0.33} = \frac{4}{33}$

(b) we have to find $P(X=4 | Y=2)$

i.e $= \dfrac{P(X=4, Y=2)}{P(Y=2)}$

So, $P(Y=2) = 0.04 + 0.04 + 0.09 + 0.10$

$= 0.27$

$P(X=4, Y=2) = 0.10$

So, $P(X=4 | Y=2) = \dfrac{0.1}{0.27} = \underline{\underline{\dfrac{10}{27}}}$

Ans4 (a) The joint PMF will be -

The possible outcomes are,

1. HHHH
2. HHHT
3. HHTH
4. HTHH
5. THHH
6. TTHH
7. THTH
8. THHT
9. HTHT
10. HHTT
11. HTTT
12. THTT
13. TTHT
14. TTTH
15. HTTH
16. TTTT

X \ Y	0	1	2	3	P(X)
0	1/16	1/16	0	0	2/16
1	1/16	3/16	2/16	0	6/16
2	0	2/16	3/16	1/16	6/16
3	0	0	1/16	1/16	2/16
P(Y)	2/16	6/16	6/16	2/16	Sum 1

Joint PMF for X & Y.

(b) Marginal probability mass Function of X:

$P(X=0) = 2/16$

$P(X=1) = 6/16$

$P(X=2) = 6/16$

$P(X=3) = 2/16$

Similarly, marginal PMF of Y:

$P(Y=0) = 2/16$

$P(Y=1) = 6/16$

$P(Y=2) = 6/16$

$P(Y=3) = 2/16$

(c) for X & Y to be independent,

$P(X=0, Y=0) = P(X=0) \cdot P(Y=0)$

$P(X=1) = \frac{6}{16}$ $\quad P(Y=1) = \frac{6}{16}$

& $P(X=1, Y=1) = \frac{3}{16}$

& $P(X=1) \cdot P(Y=1) = \frac{36}{256} = \frac{18}{128} = \frac{9}{64}$

So, $P(X=1, Y=1) \neq P(X=1) \cdot P(Y=1)$

Hence X & Y are not independent r.v's.

Ans5 The different outcomes-

$P(X=0, Y=0)$ i.e no heart & no clubs,

$= \frac{\cancel{26}}{2\cancel{52}} \times \frac{25}{51} = \frac{25}{102}$

& $P(X=0, Y=1) = 2 \cdot \frac{13}{52} \times \frac{26}{51} = \frac{13}{51} = \frac{26}{102}$

$P(X=0, Y=2) = \frac{13 \cdot 12}{52 \cdot 51} = \frac{3}{51} = \frac{1}{17} = \frac{6}{102}$

$P(X=1, Y=0) = \frac{26}{102}$ (distribution of X & Y is similar)

$P(X=2, Y=0) = \frac{6}{102}$

So, PMF is shown in table.

(a)

Y / X	0	1	2	P(X)
0	25/102	26/102	6/102	57/102
1	26/102	13/102	0	39/102
2	6/102	0	0	6/102
P(Y)	57/102	39/102	6/102	(sum) 1

Joint PMF for X, Y.

(b) marginal PMF for X

$P(X=0) = \frac{57}{102}$

$P(X=1) = \frac{39}{102}$

$P(X=2) = \frac{6}{102}$

For Y

$P(Y=0) = \frac{57}{102}$ $P(Y=2) = \frac{6}{102}$

$P(Y=1) = \frac{39}{102}$

(c) For r.v. X & Y to be independent

$$P(X=x) \cdot P(Y=y) = P(X=x, Y=y)$$

let $x=1, y=1$

so, $P(X=1) = \frac{39}{102}$ $\quad P(Y=1) = \frac{39}{102}$

& $P(X=1, Y=1) = \frac{13}{102}$

but $P(X=1) \cdot P(Y=1) = \left(\frac{39}{102}\right)^2$

i.e $P(X=1, Y=1) \neq P(X=1) \cdot P(Y=1)$

so, X & Y are not independent r.v's.