Assignment 9

IC252 - IIT Mandi

Submission Deadline: 28 May, 2021

1. The data set for milk container weights (in liters) for 50 randomly chosen containers is as follows:

   $$1.958, 1.951, 2.107, 2.092, 1.955, 2.162, 2.168, 2.134, 1.971,$$
   $$2.072, 2.049, 2.017, 2.117, 1.977, 2.034, 2.062, 2.110, 1.974,$$
   $$1.992, 2.018, 2.135, 2.107, 2.084, 2.169, 2.085, 2.018, 1.977,$$
   $$2.116, 1.988, 2.066, 2.126, 2.167, 1.969, 2.198, 2.078, 2.119,$$
   $$2.088, 2.172, 2.133, 2.112, 2.066, 2.128, 2.142, 2.042, 2.050,$$
   $$2.102, 2.000, 2.188, 1.960, 2.128.$$

   Find the sample mean and sample variance of weight of the container. **[2]**

2. Related to Slide 5, Lecture 26: Show that, if $X_1, \ldots, X_n$ is a sequence of independent identically distributed random variables with a mean μ and a variance σ^2, then $X_1 + \ldots + X_n$ has the mean $n\mu$ and a variance $n\sigma^2$. **[2]**

3. Suppose that $E(X_1) = \mu$, $\text{Var}(X_1) = 10$, $E(X_2) = \mu$, and $\text{Var}(X_2) = 15$, and consider the point estimates

   $$(1)\ \hat{\mu} = \frac{X_1}{2} + \frac{X_2}{2}$$
   $$(2)\ \hat{\mu} = \frac{X_1}{6} + \frac{X_2}{3} + 9$$

   (a) Calculate the bias of each point estimate. Is any one of them unbiased? **[2.5]**

   (b) Calculate the variance of each point estimate. Which one has the smallest variance? **[2.5]**

4. Suppose that $X \sim B(n, p)$ (or $\text{Bin}(n, p)$). Then, show that

   $$\hat{\Theta} = \hat{p} = \frac{X}{n}$$

   is an unbiased point estimate of the success probability p. **[2]**

5. If $X_1, \ldots, X_n$ is a sample of observations from a probability distribution with a variance σ_X^2, then show that the sample variance $\hat{\Theta} = \hat{\sigma}_X^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}$ has the bias of

   $$-\frac{\sigma_X^2}{n}$$

   for point estimate of the population variance $\theta = \sigma_X^2$. **[2]**