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IC 252

MOHIT VERMA
B20215

Assignment 8

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Ans1 let X be a r.v which denotes the time taken to serve a customer.

$\mu_x = 75$ seconds. $\sigma_x = 7.3$ seconds

From Chebyshev inequality,

$$P(|X-\mu| \geq a) \leq \frac{\sigma^2}{a^2}$$

&

$$P(|X-\mu| \leq a) \geq 1 - \frac{\sigma^2}{a^2}$$

$$1 - \frac{\sigma^2}{a^2} = 0.89$$

$$\frac{\sigma^2}{a^2} = 0.11$$

$$\frac{\sigma}{a} = 0.33 \simeq \frac{1}{3}$$

$$a = 3\sigma$$

So,

$|X-\mu| \leq a$

$-a \leq X-\mu \leq a$

replacing $a = 3\sigma$,

$-3\sigma + \mu \leq X \leq 3\sigma + \mu$

So,

$-3(7.3) + 75 \leq X \leq 3(7.3) + 75$

$53.1 \leq X \leq 96.9$.

So, the time interval would be

$[53.1, 96.9]$.

Ans 2 let X be a r.v denoting the length of iron bars being rolled out.

$\mu_X = 110.8$ cm

$\sigma_X = 0.5$ cm

So, $\mu - a \leq X \leq \mu + a$

So, $\mu - a = 109.55$

$110.8 - 109.55 = a$

$a = 1.25$ cm

So,

$$P(|X-\mu| \le a) \ge 1 - \frac{\sigma^2}{a^2}$$

So, lower bound $= 1 - \frac{\sigma^2}{a^2}$

$$= 1 - \frac{(0.5)^2}{(1.25)^2}$$

$$= 1 - (0.4)^2 = \underline{\underline{0.84}}$$

lower bound $= 0.84$

Ans 3

$\mu_X = 8 \qquad \sigma_X = 3$

(a) $P(-4 \le X \le 20)$

from chebyshev's inequality,

$$P(|X-\mu| \le a) \ge 1 - \frac{\sigma^2}{a^2}$$

$\mu - a = -4$

$8 - a = -4$

$a = +12$

So,

$P(-4 \leq X \leq 20) \geq 1 - \frac{9}{(12)^2}$

$P(-4 \leq X \leq 20) \geq 0.9375$

(b) $P(|X-8| \geq 6)$

we have $a = 6$

so,

$P(|X-\mu| \geq a) = \frac{\sigma^2}{a^2}$

$= \frac{9}{6^2} = 0.25$

Ans 4

$\int_0^\infty y e^{-\frac{y^2}{2}} dy = 1$

L.H.S $= \int_0^\infty y e^{-\frac{y^2}{2}} dy$

let $y^2 = t$

$2y\, dy = dt$

On substitution,

$$= \int_0^\infty y \cdot e^{-t/2} \frac{dt}{2y}$$

$$= \frac{1}{2}\int_0^\infty e^{-t/2} dt$$

$$= \frac{(-2)}{2}\left[e^{-t/2}\right]_0^\infty$$

$$= -\left[e^{-t/2}\right]_0^\infty = [0-1]\times(-1)$$

$$= \underline{\underline{1}} = R.H.S$$

<u>Ans 5</u> let X be a r.v. denoting the no: of cattle showing adverse reaction.

$n = 500{,}000$, $p = 0.0005$

So, $E(X) = p = 0.0005$ & $\sigma_x^2 = p(1-p) = 0.0005 \times 0.9995$

considering X to be a binomial random variable.

Now, The data can be approximated as a Normal distribution:

$$Y \sim N(n\mu, n\sigma^2) = N(n\mu_x, n\sigma_x^2)$$

$= N(500000 \times 5\times10^{-4},\ 5\times10^{5}\times10^{-4}\times 0.9995)$

$= N(250, 249.9)$

So, $Y = \mu + \sigma Z = 250 + \sqrt{249.9}\, Z$

$= 250 + 15.82$

So, $P(X \leq 300) \approx P(Y \leq 300) = P(250 + 15.82) \leq 300)$

$= P(Z \leq 3.164) = 0.99916$

Ans6 (a) $P(X \geq 8)$ where $X \sim B(10, 0.7)$

$P(X \geq 8) = {}^{10}C_8 (0.7)^8 (0.3)^2 + {}^{10}C_9 (0.7)^9 (0.3)$

$+ {}^{10}C_{10} (0.7)^{10}$

$= 0.3823$

and from approximation-

$\mu_X = p = 0.7$,

& $\sigma_X^2 = 0.7 \times 0.3 = 0.21$

So, r.v. X can be approximated by-

$X \sim B(n,p) \approx N(n\mu_X, n\sigma_X^2) = N(7, 2.1)$

So, the required probability will be,

$$P(X \geq 8) \approx P(Y \geq 7.5) = P((7 + \sqrt{2.1}\, Z) \geq 7.5)$$

where Z is standard Normal distribution.

$$P(7 + \sqrt{2.1}\, Z) \geq 7.5 = 1 - (P(7 + 1.449\, Z < 7.5))$$

$$= 1 - P(Z < 0.345) \approx 1 - 0.63 = \underline{0.365}$$

(b) $X \sim B(15, 0.3)$

$$P(2 \leq X \leq 7) = {}^{15}C_2 (0.3)^2 (0.7)^{13} + {}^{15}C_3 (0.3)^3 \times (0.7)^{12}$$

$$+ {}^{15}C_4 (0.3)^4 (0.7)^{11} + {}^{15}C_5 (0.3)^5 (0.7)^{10}$$

$$+ {}^{15}C_6 (0.3)^6 (0.7)^9 + {}^{15}C_7 (0.3)^7 (0.7)^8$$

$$= \underline{0.9147}$$

Doing similarly as in part (a) $Y \sim N(4.5, 3.15)$ & we can find required probability

$$P(2 \leq X \leq 7) \simeq P(1.5 \leq Y \leq 7.5) = P(1.5 \leq 4.5 + \sqrt{3.15}\, Z \leq 7.5)$$

$$= P(-3 \leq 1.77482 \leq 3) = P(Z \geq 1.6903) - P(Z \geq 1.6903)$$

$\simeq 0.909$

Ans 7

X be a r.v. denoting a question attempted being right or wrong.

$P(X=0) = \frac{3}{4} = 0.75$ [$X=0$ denoting answer being wrong]

$P(X=1) = \frac{1}{4} = 0.25$ [$X=1$ denoting answer being correct]

$n = 60$

$\mu_X = E(X) = 0.25$

$\sigma_X^2 = \frac{3}{4} \times \frac{1}{4} = 0.1875$

So, $X \sim B(0.25, 0.1875) \simeq N(15, 11.25)$

So, $Y = 15 + \sqrt{11.25}\, Z$

where Z is standard Normal distribution.

So, $P(X \geq 30) \simeq P[Y \geq 29.5]$

$= 1 - P[15 + \sqrt{11.25}\, Z \leq 29.5]$

$= 1 - P\left[Z \leq \frac{29.5 - 15}{\sqrt{11.25}}\right]$

$\simeq 0$