USC Viterbi
School of Engineering
Department of Computer Science

CSCI 544
Applied Natural Language Processing

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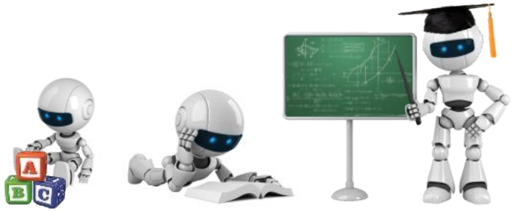


Adapted from slides by Danqi Chen, Michael Collins, and Max Ma

Logistical Notes

- Project Group Formation Deadline: 09/12
  - Most students have joined a group (~220 students).
  - Contact groups with 3 or more members on Excel
  - Do NOT form more than 50 groups
  - Meet weekly, helpful for both HW and project
- Next Lecture
- Paper Selection Deadline: 09/19
  - Only one group!
  - Check and then enter your paper: https://docs.google.com/spreadsheets/d/1_vafG77ijmETCnuVZvKpT35k--5op5wn71GZXgAY7O0
- Project Proposal Deadline: 10/03
  - Think about extending your paper

HMM Assumptions

- Markov Assumption on **S**

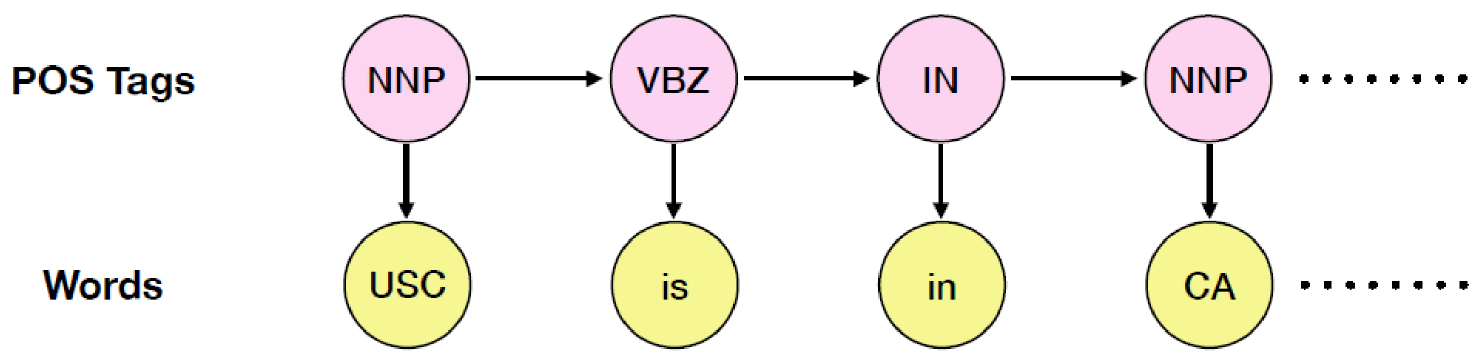
$$P(S_j = s_j \mid S_{j-1} = s_{j-1}, \ldots, S_1 = s_1) = P(S_j = s_j \mid S_{j-1} = s_{j-1})$$

Transition Probabilities

- Conditional independence of **X** and **S**

$$P(X_1 = x_j, \ldots, X_m = x_m \mid S_1 = s_1, \ldots, S_m = s_m) = \prod_{j=1}^{m} P(X_j = x_j \mid S_j = s_j)$$

Emission Probabilities



$$P(S_3 = \text{IN} \mid S_2 = \text{VBZ}, S_1 = \text{NNP}) = P(S_3 = \text{IN} \mid S_2 = \text{VBZ})$$

$$P(\text{USC is in CA} \mid \text{NNP VBZ IN NNP}) = P(\text{USC} \mid \text{NNP}) P(\text{is} \mid \text{VBZ}) P(\text{in} \mid \text{IN}) P(\text{CA} \mid \text{NNP})$$

HMM Assumptions

- Joint Distribution of Sequence Pairs in HMMs

$$P(X_1 = x_j, \ldots, X_m = x_m, S_1 = s_1, \ldots, S_m = s_m)$$

$$= \boxed{P(X_1 = x_j, \ldots, X_m = x_m \,|\, S_1 = s_1, \ldots, S_m = s_m)}$$

Output Independence

$$\boxed{\times P(S_1 = s_1, \ldots, S_m = s_m)}$$

Markov Assumption

$$= \prod_{j=1}^{m} P(X_j = x_j \,|\, S_j = s_j)$$

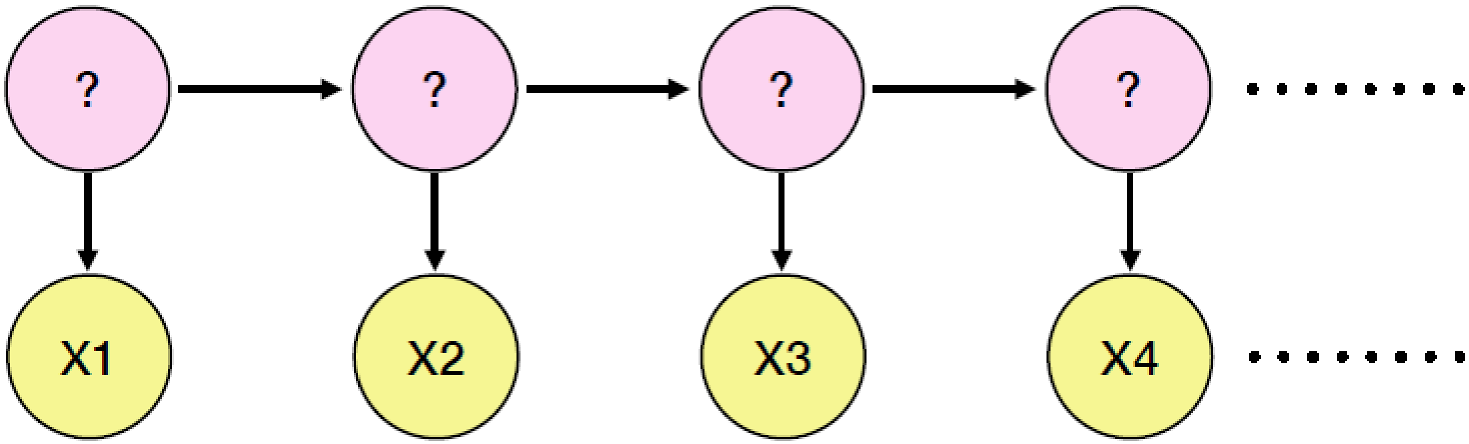
How to model $P(X_j = x_j \,|\, S_j = s_j)$

and $P(S_j = s_j \,|\, S_{j-1} = s_{j-1})$?

$$\times P(S_1 = s_1) \prod_{j=1}^{m} P(S_j = s_j \,|\, S_{j-1} = s_{j-1})$$

Decoding with HMM

- Given an input sequence $x_1, \ldots, x_m$ compute:



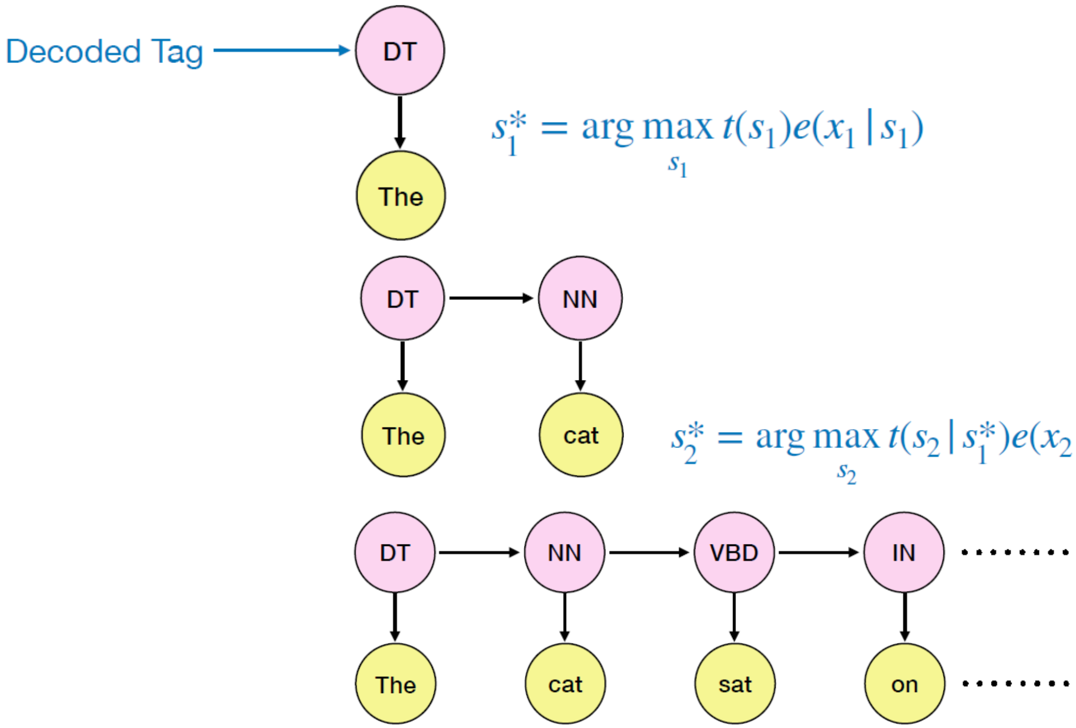
$$S^* = \arg\max_{s_1,\ldots,s_m} p(x_1,\ldots,x_m,s_1,\ldots,s_m) = t(s_1)\prod_{j=2}^{m} t(s_j \mid s_{j-1}) \prod_{j=1}^{m} e(x_j \mid s_j)$$

How can we maximize this over all state sequences?

- Brute-force search: 45^14~10^23 -> +1000 years with THz speed!

Greedy Decoding

- Start from the first word and decode one state at a time



$$s_1^* = \arg\max_{s_1} t(s_1)e(x_1 \mid s_1)$$

$$s_2^* = \arg\max_{s_2} t(s_2 \mid s_1^*)e(x_2 \mid s_2)$$

$$s_j^* = \arg\max_{s_j} t(s_j \mid s_{j-1}^*)e(x_j \mid s_j), \quad \forall j$$

- Local Decisions
- Not guaranteed to produce the overall optimal sequence

Viterbi Decoding

- A dynamic programming algorithm
- The basic data structure will be a table that stores the maximum probability for any state sequence ending in state ***s*** at position ***j***.

$\pi[1, s] = t(s)e(x_1|s)$, and for $j > 1$,

$$\pi[j, s] = \max_{s_1 \ldots s_{j-1}} \left[\underbrace{t(s_1)e(x_1|s_1) \left(\prod_{k=2}^{j-1} t(s_k|s_{k-1})e(x_k|s_k) \right)}_{\text{The value for position } j\text{-}1} \underbrace{t(s|s_{j-1})e(x_j|s)}_{\text{Word } j} \right]$$

- Ex: The man saw the rat: a table with the size 5*#(tag classes)

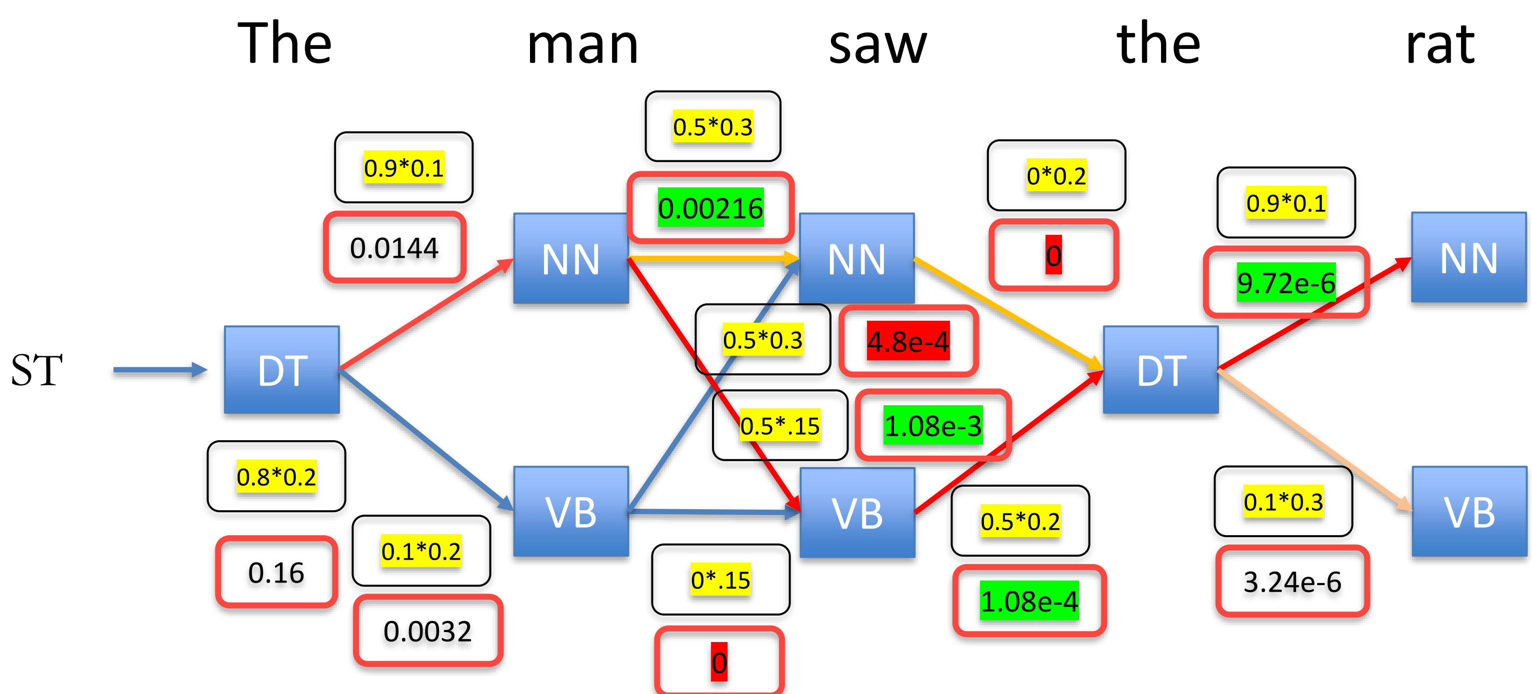
Viterbi Decoding: Example

- Sentence: The man saw the rat -> 5^4 vs 5*4

	DT	NN	VB
S	0.8	0.2	0
DT	0	0.9	0.1
NN	0	0.5	0.5
VB	0.5	0.5	0

	The	man	saw	rat
DT	0.2	0	0	0
NN	0	0.1	0.3	0.1
VB	0	0.2	0.15	0.3

$$p(s_j|s_{j-1})p(x_j|s_j) = t(s_j|s_{j-1})e(x_j|s_j)$$



Viterbi Decoding

- Algorithm

  - Initialization: for $s = 1 \ldots k$

$$\pi[1, s] = t(s)e(x_1|s)$$

  - For $j = 2 \ldots m$, $s = 1 \ldots k$:

$$\pi[j, s] = \max_{s' \in \{1 \ldots k\}} [\pi[j-1, s'] \times t(s|s') \times e(x_j|s)]$$

  - We then have

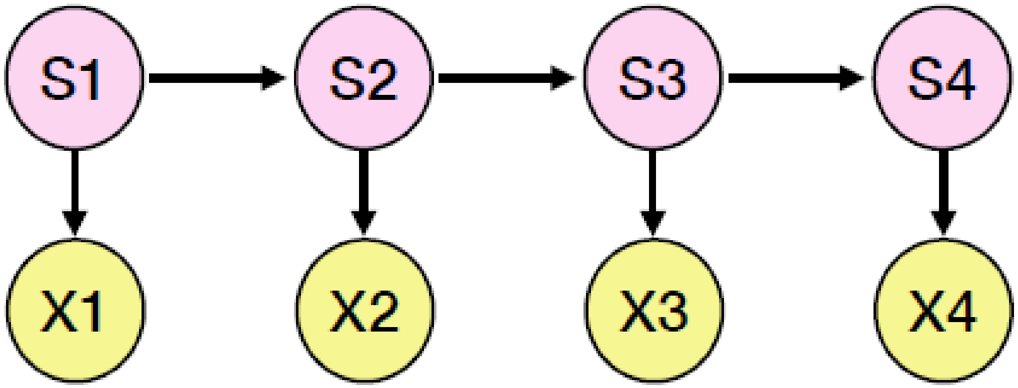
$$\max_{s_1 \ldots s_m} p(x_1 \ldots x_m, s_1 \ldots s_m; \underline{\theta}) = \max_s \pi[m, s]$$

  - The algorithm runs in $O(mk^2)$ time

HMM Limitations

- Transition probabilities are position independent
- Limited dependencies: Markov Assumption

- Is he informed? VS he informed



Generative vs Discriminative Models

- Can we have discriminative models for sequence labeling?

	Generative	**Discriminative**
Classification	Naive Bayes: $P(y)P(x \mid y)$	Logistic Regression: $P(y \mid x)$
Sequence Labeling	HMM: $P(s_1, \ldots, s_n)P(x_1, \ldots, x_n \mid s_1, \ldots, s_n)$	MEMM/CRF: $P(s_1, \ldots, s_n \mid x_1, \ldots, x_n)$

Log-Linear Models

We have some input domain $\mathcal{X}$, and a finite label set $\mathcal{Y}$. Aim is to provide a conditional probability $p(y \mid x)$ for any $x \in \mathcal{X}$ and $y \in \mathcal{Y}$.

A feature is a function $f : \mathcal{X} \times \mathcal{Y} \rightarrow \mathbb{R}$
(Often binary features or indicator functions $f_k : \mathcal{X} \times \mathcal{Y} \rightarrow \{0, 1\}$).

Say we have m features f_k for $k = 1 \ldots m$
$\Rightarrow$ A feature vector $f(x, y) \in \mathbb{R}^m$ for any $x \in \mathcal{X}$ and $y \in \mathcal{Y}$.

We also have a **parameter vector** $v \in \mathbb{R}^m$

We define

$$p(y \mid x; v) = \frac{e^{v \cdot f(x,y)}}{\sum_{y' \in \mathcal{Y}} e^{v \cdot f(x,y')}}$$

$$\log p(y \mid x; v) = \underbrace{v \cdot f(x, y)}_{\text{Linear term}} - \underbrace{\log \sum_{y' \in \mathcal{Y}} e^{v \cdot f(x,y')}}_{\text{Normalization term}}$$

POS Features for Log-Linear Models

$S = S_i, S_2, \ldots, S_n$	NNP	$\boxed{\text{VBZ}}$	IN	NNP
$X = X_i, X_2, \ldots, X_n$	USC	is	in	California

- We can use the current word, POS, and the surrounding words and their POS

Number of Features

$$f_1 = \begin{cases} 1, & \text{if } x_i = \text{is}, \ s_i = \text{VBZ} \\ 0, & \text{otherwise} \end{cases}$$

Vocabulary size $\longrightarrow VK$

POS tag size $\uparrow$

$$f_2 = \begin{cases} 1, & \text{if } x_{i-1} = \text{USC}, \ x_i = \text{is}, \ s_{i-1} = \text{NNP}, \text{ and } s_i = \text{VBZ} \\ 0, & \text{otherwise} \end{cases}$$

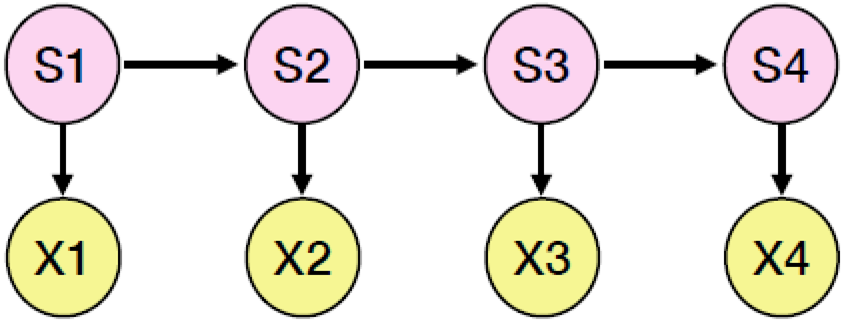
V^2K^2

$$f_3 = \begin{cases} 1, & \text{if } x_{i-1} = \text{USC}, \ x_i = \text{is}, \ x_{i+1} = \text{in}, \ s_{i-1} = \text{NNP}, \text{ and } s_i = \text{VBZ} \\ 0, & \text{otherwise} \end{cases}$$

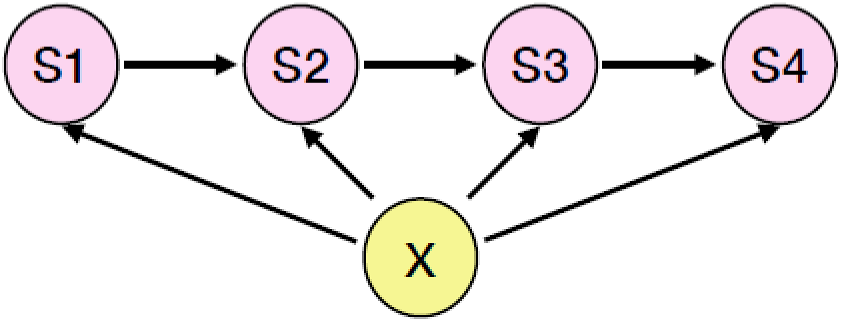
V^3K^2

Maximum-Entropy Markov Models (MEMMs)

- We assume that the hidden states are generated from the observations
- States are hidden and by observing data, we want to assign a probability for any possible state
- We still use Markov assumption on states
- Context is considered to determine POS



HMM



MEMM

Maximum-Entropy Markov Models (MEMMs)

- Markov assumption on ***S***

$$p(s_1, \ldots, s_m | x_1, \ldots, x_m) = \prod_{j=1}^{m} p(s_j | s_1, \ldots, s_{j-1}, x_1, \ldots, x_m)$$ chain rule

$$= \prod_{j=1}^{m} p(s_j | s_{j-1}, x_1, \ldots, x_m)$$ Markov assumption

- We model each term using a log-linear model

$$p(s_j | s_{j-1}, x_1, \ldots, x_m) = \frac{\exp(v \cdot f(x_1, \ldots, x_m, i, s_{j-1}, s_j))}{\sum_{s'_j \in \mathbb{S}} \exp(v \cdot f(x_1, \ldots, x_m, i, s_{j-1}, s'_j))}$$

MEMM: Training

- We use a training dataset and solve for optimal ***v***

$$p(s_1, \ldots, s_m | x_1, \ldots, x_m) = \prod_{j=1}^{m} \frac{\exp(v \cdot f(x_1, \ldots, x_m, i, s_{j-1}, s_j))}{\sum_{s_j' \in \mathbb{S}} \exp(v \cdot f(x_1, \ldots, x_m, i, s_{j-1}, s_j'))}$$

- Potential solution: gradient-based techniques

$$\max_v L(v) = \sum_{i=1}^{N} \log P(X_i, S_i; v)$$

$$= \sum_{i=1}^{N} v \cdot f(X_i, S_i) - \sum_{i=1}^{N} \log \sum_{s' \in \mathbb{S}} e^{v \cdot f(X_i, S')}$$

$$\frac{\partial L(v)}{\partial v_k} = \underbrace{\sum_{i=1}^{N} f_k(X_i, S_i)}_{\text{Empirical counts}} - \underbrace{\sum_{i=1}^{N} \sum_{S' \in \mathbb{S}} f_k(X_i, S') p(S' | X_i; v)}_{\text{Expected counts}}$$

MEMM: Decoding

- Viterbi Algorithm

Goal: for a given input sequence $x_1, \ldots, x_m$, find

$$\arg\max_{s_1,\ldots,s_m} p(s_1 \ldots s_m | x_1 \ldots x_m)$$

$\pi[j, s]$ will be a table entry that stores the maximum probability for any state sequence ending in state s at position j. More formally:

$$\pi[j, s] = \max_{s_1 \ldots s_{j-1}} \left(p(s | s_{j-1}, x_1 \ldots x_m) \prod_{k=1}^{j-1} p(s_k | s_{k-1}, x_1 \ldots x_m) \right)$$

MEMM: Decoding

- Viterbi Algorithm

  - Initialization: for $s \in \mathcal{S}$

    $$\pi[1, s] = p(s|s_0, x_1 \ldots x_m)$$

    where s_0 is a special "initial" state.

  - For $j = 2 \ldots m$, $s = 1 \ldots k$:

    $$\pi[j, s] = \max_{s' \in \mathcal{S}} \left[\pi[j-1, s'] \times p(s|s', x_1 \ldots x_m)\right]$$

  - We then have

    $$\max_{s_1 \ldots s_m} p(s_1 \ldots s_m | x_1 \ldots x_m) = \max_s \pi[m, s]$$

MEMM vs HMM

- Performance: 96.9% vs 96.4% on WSJ
- Transition probability modeling

$$p(s_j \mid s_{j-1}, x_1, \ldots, x_m) = \frac{\exp(v \cdot f(x_1, \ldots, x_m, i, s_{j-1}, s_j))}{\sum_{s' \in \mathbb{S}} \exp(v \cdot f(x_1, \ldots, x_m, i, s_{j-1}, s'_j))} \quad \text{vs} \quad p(s_j \mid s_{j-1}) p(x_j \mid s_j)$$

- MEMM feature vector encode contextual information
- Training procedure for parameter estimation in MEMMs is more expensive than in HMMs

Conditional Random Fields (CRFs)

- Motivation

- HMM Assumption:

$$p(s_1, \ldots, s_m \mid x_1, \ldots, x_m) = \prod_{j=1}^{m} p(s_j \mid s_1, \ldots, s_{j-1}, x_1, \ldots, x_m)$$ chain rule

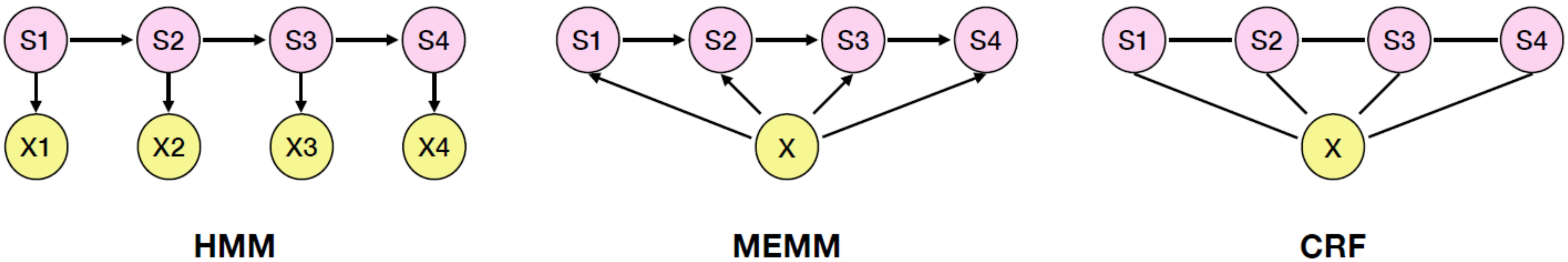
$$= \prod_{j=1}^{m} p(s_j \mid s_{j-1}, x_1, \ldots, x_m)$$ Markov assumption

- Can we build a log-linear model to directly model the conditional distribution?

Conditional Random Fields (CRFs)

- Formulation

- Can we have more complex models?



$$p(s_1, \ldots, s_m \,|\, x_1, \ldots, x_m) = \frac{\prod_{j-1}^{m} \exp(v \cdot f(x_1, \ldots, x_m, i, s_{j-1}, s_j))}{\sum_{s'_1, \ldots, s'_m \in \mathbb{S}} \prod_{j=1}^{m} \exp(v \cdot f(x_1, \ldots, x_m, i, s'_{j-1}, s'_j))}$$

CRFs: Training and Decoding

- Training is similar to MEMM
- Decoding: Viterbi Algorithm

- We replace the maximum operation with summation operation

$$p(s_1, \ldots, s_m \mid x_1, \ldots, x_m) = \frac{\prod_{j-1}^{m} \exp(v \cdot f(x_1, \ldots, x_m, i, s_{j-1}, s_j))}{\boxed{\sum_{s'_1, \ldots, s'_m \in \mathbb{S}} \prod_{j=1}^{m} \exp(v \cdot f(x_1, \ldots, x_m, i, s'_{j-1}, s'_j))}}$$

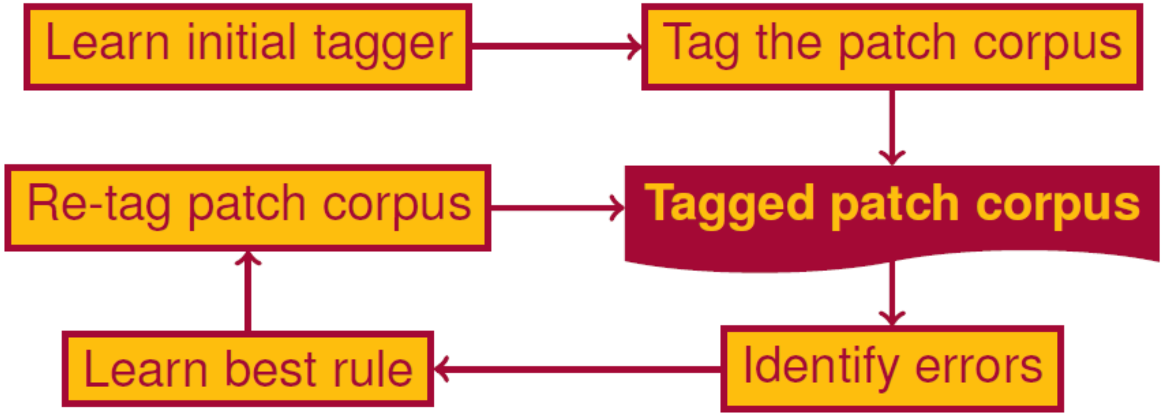
$$\pi[j, s] = \boxed{\sum_{s_1, \ldots, s_{j-1}}} \left[\prod_{k=1}^{j-1} \exp(v \cdot f(x_1, \ldots, x_m, k, s_{k-1}, s_k)) \right] \exp(v \cdot f(x_1, \ldots, x_m, k, s_{j-1}, s)))$$

Performance Comparison

	POS Tagging	NER
HMM	96.4%	75.3
MEMM	96.9%	85.9
CRF	97.3%	88.7

Brill Tagger

- Brill (1992): A Simple Rule-Based Part of Speech Tagger
- Explainable Model



- Procedure for finding best rule
- Find most common error (e.g., noun tagged as verb)
- Find best rule to correct that error

- Rules must be applied in the order they are learned