Question relating to use of FORM in context of Compton scattering

A. Context

Figure (1) is a Feynman diagram corresponding to one of the processes contributing to Compton scattering. The amplitude associated with Fig. (1) is (Griffiths 2008, Example 7.4 and problem 7.30)

$$\mathcal{M}_1 = \frac{g_e^2}{(p_1 - p_3)^2 - m^2c^2}\ [\bar{u}_4\, \not{\epsilon}_2\ (\not{p}_1 - \not{p}_3 + mc)\ u_1\, \not{\epsilon}_3^*\,] \tag{1}$$

where $\bar{u}_4$ is the (adjoint) spinor associated with the outgoing electron, u_1 is the spinor representing the incoming electron, $\not{\epsilon}_2 \equiv \epsilon_{\mu(2)}\gamma^\mu$ where $\epsilon_{\mu(2)}$ are the components of the polarization 4-vector for the incoming photon (etc). Spin and momentum labels have been dropped, i.e. more completely, for example, $\bar{u}_4 = \bar{u}^{(s_4)}(p_4)$ with two possible spin states ($s_4 = 1, 2$). Implicitly, the mc factor is multiplied by the identity matrix.

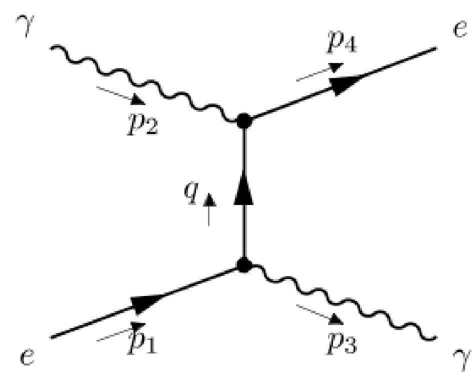

Figure 1: One of the two lowest-order processes contributing to Compton scattering, p_1 designating the four-momentum of the incoming electron, etc.

The matrix element for Compton scattering involves (among others) the contribution

$$\mathcal{M}_1\mathcal{M}_1^* = \left(\frac{g_e^2}{(p_1 - p_3)^2 - m^2c^2}\right)^2\ [\bar{u}_4\Gamma_1 u_1]\ [\bar{u}_4\Gamma_1 u_1]^* \tag{2}$$

where

$$\Gamma_1 = \not{\epsilon}_2\ [\not{p}_1 - \not{p}_3 + mc]\ \not{\epsilon}_3^*\ . \tag{3}$$

After some manipulation one obtains

$$|\mathcal{M}_1|^2 = A^2\, \bar{u}_4\, \not{\epsilon}_2\ [\not{p}_1 - \not{p}_3 + mc]\ \not{\epsilon}_3^*\, u_1\ \bar{u}_1\, \not{\epsilon}_3\ [\not{p}_1 - \not{p}_3 + mc]\ \not{\epsilon}_2^*\, u_4\ , \tag{4}$$

where

$$A = \frac{g_e^2}{(p_1 - p_3)^2 - m^2c^2}\ . \tag{5}$$

Then averaging over the initial spin states and summing over final spin states, one obtains

$$\langle|\mathcal{M}_1|^2\rangle = \frac{A^2}{4} Q_{\mu\lambda} Q_{\nu\kappa} \, \mathrm{Tr}\left[\gamma^\mu \left(\not{p}_1 - \not{p}_3 + mc\right) \gamma^\nu \left(\not{p}_1 + mc\right) \gamma^\kappa \left(\not{p}_1 - \not{p}_3 + mc\right) \gamma^\lambda \left(\not{p}_4 + mc\right)\right] \tag{6}$$

where

$$Q_{\mu\lambda} = \sum_{s=1,2} \epsilon_\mu^{(s)} \, \epsilon_\lambda^{(s)*} \, .$$

Regarding $Q_{\mu\lambda}$, recall that the photon polarization vector $\epsilon^\mu = (\epsilon^0, \epsilon)$ with $\epsilon^0 = 0$ and that the completeness relation (for the photon polarization three vectors) is

$$\sum_{s=1,2} \epsilon_i^{(s)} \, \epsilon_j^{(s)*} = \delta_{ij} - \hat{p}_i \hat{p}_j \tag{7}$$

where $\hat{p}_i$ is the unit vector $\hat{p}_i = p_i/|p_i|$ (no summation). Hence

$$Q_{\mu\lambda} = \begin{cases} 0 \text{ , if } \mu = 0 \text{ or } \lambda = 0 \\ \delta_{\mu\lambda} - \hat{p}_\mu \hat{p}_\lambda \text{ , otherwise .} \end{cases} \tag{8}$$

Eqns. (6–8) concur with the solution manual written by D. Griffiths (available at www.academia.edu and elsewhere). I have attempted to evaluate Eq. (6) using FORM, and the program and it's output are shown below.

B. Evaluation of Eq. (6) using FORM

The following FORM program is intended to evaluate Eq. (6), albeit excluding the factor A^2. The output is shown in red.

```
FORM 4.2 (Mar 15 2019) 64-bits                   Run: Sun Oct 13 12:18:42 2019
    ***  compton_scattering_singleterm.frm  JDW 13 Oct. 2019 ***
    Dimension 4;
    Vectors p1, p2, p3, p4;
    Symbols mc;
    Indices alpha,beta,gamma,delta,rho,kappa,lambda,mu,nu;
    AutoDeclare Vector p;
    Off Statistics;

    Local traceM1M1   = div_(1,4)*d_(mu,lambda)*d_(nu,kappa)
                     *g_(1,mu)*(p1(alpha)*g_(1,alpha)-p3(alpha)*g_(1,alpha) + mc)
                     *g_(1,kappa)*(p1(beta)*g_(1,beta)+mc)*g_(1,nu)
                     *(p1(gamma)*g_(1,gamma)-p3(gamma)*g_(1,gamma) + mc)
                     *g_(1,lambda)*(p4(delta)*g_(1,delta)+mc);
    Trace4,1;
```

```
.sort;

id p3.p3=0;
id,multi p1.p1=mc^2;
id,multi p1.p3=p2.p4;
id,multi p3.p4=p1.p2;

AntiBracket mc;
Print;
.sort;

traceM1M1 =
    + p1.p2*p2.p4 * ( 8 )
    + p1.p2 * ( 8*mc^2 )
    + p1.p4 * (  - 8*mc^2 )
    + p2.p4 * (  - 16*mc^2 )
    + 16*mc^4;
.end
```

My concern is that this result disagrees with that cited by others, e.g. Millar 2014, who for the same quantity gives:

$$\left[16(mc)^4 + 16(mc)^2(p_1 \cdot p_2) - 8(mc)^2(p_1 \cdot p_4) - 8(mc)^2(p_2 \cdot p_4) + 8(p_1 \cdot p_2)(p_4 \cdot p_2)\right] \ . \tag{9}$$

References

Griffiths, D. 2008. *Introduction to Elementary Particles*. Second edn. Wiley-VCH.