Degrees of Freedom (DOF)

Definitions:

A degree of freedom (DOF) is the minimum number of independent parameters or variables, required to completely describe the position and configuration of a system.

In mechanics, a dof is the minimum number of independent variables or coordinates needed to completely describe the position and orientation of a mechanical system at any instant. It's the number of ways a system can move independently.

In robotics, DOF is the number of independent movements or axes of motion a robot's joints have, which can be either rotational or linear. The total number of DOFs defines a robot's ability to position and orient itself in space, with a higher DOF allowing for more flexibility, versatility, and complex manipulation.

Each DOF represents a single, independent variable needed to describe the robot's configuration.

Restrained vs Unrestrained Motion in Robotics

Unrestrained motion refers to the ability of a rigid body to move freely, without any physical constraints. In robotics, a completely unrestrained rigid body in 3D space possesses **six degrees of freedom (DOF)**: three for translations (movement along the x, y, and z axes), and three for rotations (pitch, yaw, roll about these axes). This means it can move or rotate in any direction

Restrained motion occurs when certain constraints like physical joints limit the possible movements of the robot or a robot link. The more constraints (restraints) a robot has, the fewer DOFs remain for it to move. For example, if you attach a robot arm segment to the base with a **revolute joint**, that joint only allows rotation about a single axis, so the arm segment loses five DOFs and is only permitted to rotate around that joint's axis.

KINEMATIC PAIR

A **kinematic pair** is the basic unit of a mechanical or robotic mechanism, formed when **two rigid links are connected in such a way that they allow specific relative motion while constraining others**. It defines how two mechanical components interact whether by sliding, turning, rolling, or a combination of these motions.

The **two elements** of a pair are:

- **Driver:** the active link imparting motion.

- **Follower:** the passive link that receives motion.

Type	**Description**	**Example**	**DOF**
Sliding Pair	Linear sliding of one link over another	Piston and cylinder, drawer slides	1 (translation)
Turning (Revolute) Pair	Rotational motion around a fixed axis	Door hinge, robotic elbow	1 (rotation)
Rolling Pair	Surface contact with rolling action	Ball bearings, wheel and ground	1 (rolling)
Screw Pair	Helical motion combining rotation & translation	Bolt and nut, lead screw	1 (helical motion)
Spherical Pair	3D rotation around common center	Ball-and-socket (hip, robot wrist)	3 (rotation)

JOINTS AND LINKS

A robot's **anatomy** is built from two essential structural elements: **joints** and **links**. Together, they form the **kinematic chain**, which determines how the robot moves, positions, and orients its end effector in space.

End effectors are devices attached to the end of a robotic arm that enable it to interact with its environment. Also known as "end-of-arm tooling" (EOAT), they act as the robot's "hands" or "tools" and are designed to perform specific tasks like gripping, welding, painting, or sensing. The type of end effector used depends on the robot's application and the task it needs to perform.

Joints in Robotics

Robot joints are the movable connections between a robot's rigid parts (links) that allow for controlled movement, enabling a robot to perform tasks like rotation, sliding, and pivoting.

Each joint introduces a certain **degree of freedom (DOF)**, representing one independent direction of motion, either rotational or translational.

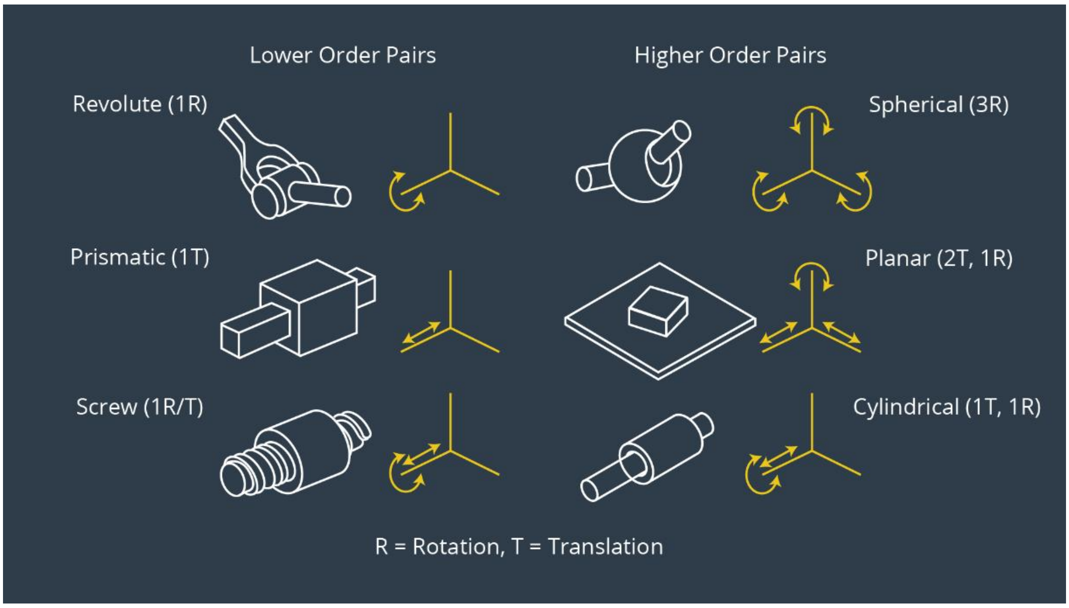
These joints are typically equipped with actuators, sensors, and mechanical components to facilitate controlled movement and precise positioning. Overall, robot joints are essential for the mobility and **functionality of robots** in a wide range of applications, from manufacturing and assembly to exploration and healthcare.

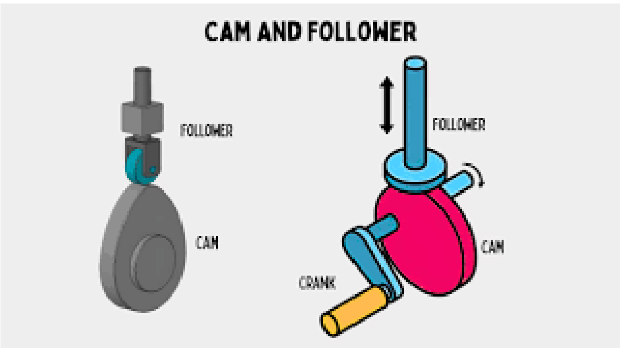
TYPES OF JOINTS IN A ROBOT

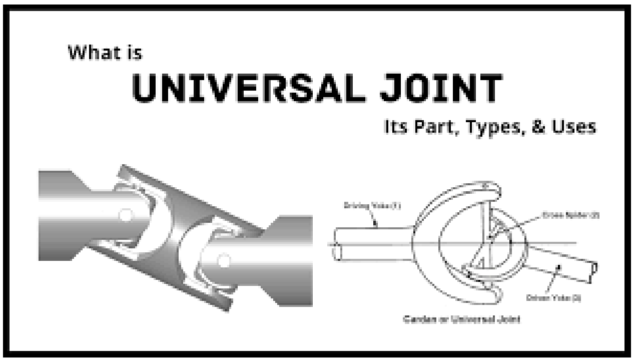
JOINT TYPE	MOTION	KINEMATIC PAIR	APPLICATIONS	DOF	LIMITATION
Revolute (pin) joint	Rotation around one axis	(lower) Turning pair -1	Welding, assembly, rotating tools, Door hinge	1	Limited linear reach
Prismatic (slider) joint	Translational Motion	Lower pair (sliding pair)	Cartesian and cylindrical robots, CNC and 3D printer axes	1	No angular motion possible; increased frictional resistance
Cylindrical joint	Combined rotation and translation along a common axis	*Higher pair*	Cylindrical coordinate robots, telescopic systems	2	Complex sealing and alignment; harder to control precisely than single DOF joints
Spherical joint	Rotations about three perpendicular axes through a common center	*Higher pair (ball-and-socket)*	Robotic wrists, humanoid shoulders, gimbals	3	Structurally complex; difficult to actuate and control multiple angles simultaneously
Screw Joints	Helical motion (rotation + translation with a fixed pitch)	Lower pair (screw pair)	Lead screw actuators, press drives, precise linear motion systems	1	Frictional losses and backlash; limited speed; wear causes positioning errors
Planar	Motion restricted to a plane (2 translations + 1 rotation within plane)	HIgher pair (plane pair)	XY gantry systems, surface-parallel robots	3	Limited to flat motion; cannot move out of plane

Cam-follower	Rolling or sliding contact along a curved surface profile	Lower pair	Automatic machinery, valve trains, automated timing systems	1	Subject to wear and lubrication issues; complex surface design; contact stress causes fatigue
Universal joint	Two sequential revolute joints with perpendicular axes	Lower pair combination	Drive shafts, robotic wrists allowing multi-axis tilt	2	Non-uniform angular velocity; limited angular range; complex kinematic analysis

IMAGES OF ALL JOINTS:







LINKS

A link is a rigid segment that connects two joints, forming the "bones" of the robot's structure. These links are rigid bodies that define the spatial relationship and length between adjacent joints. Together, links and joints allow the robot to move and perform tasks, with the joints enabling relative motion between the links

Types of Links on the basis of number of connections:

- **Binary Link: Connected to two other links by joints. Most links in mechanisms fall in this category.**
- **Ternary Link: Connected to three other links.**
- **Quaternary Link: Connected to four other links.**

GRUBLER'S FORMULA

Grübler's formula (also known as Gruebler's equation) is a fundamental tool in kinematic analysis used to calculate the **mobility (degrees of freedom, DOF)** of planar and spatial mechanisms.

Formula for Planar mechanisms:

$F= 3(n-1)-2l-h$

Where,

F: DOF

n= total number of links

l= number of lower pairs (joints having 1 dof)

h=number of higher pairs(joints having 2 dof)

Formula for Spatial Mechanisms:

$M=6(N-1-J) + I = 1 \sum J fi$

Where,

M *= mobility or degrees of freedom (DOF) of the mechanism,*

N *= number of links (including ground),*

J *= number of joints,*

f_i *= degrees of freedom of the* i^{th} *joint (each spatial rigid body has 6 DOFs).*

MECHANISMS

In robotics, a **mechanism** is the physical, mechanical structure that enables a robot to perform motion and interact with its environment. Mechanisms transform input forces and movement into a desired set of output forces and movements to achieve a specific task. They are the functional skeleton of a robot, comprised of interconnected links and joints.

A mechanism is an assembly of links and joints where:

- **Links** are rigid bodies.
- **Joints** allow constrained relative motion.
  Together, they form a **kinematic chain** that controls the robot's degrees of freedom (DOF) and motion paths to accomplish tasks such as pick-and-place, welding, or tool manipulation.

COMMON MECHANISMS AND THEIR DOF

In robotics, **mechanisms** can be classified into several types based on their **kinematic structure, joint arrangement, and motion capabilities**. The main types of robotic mechanisms are:

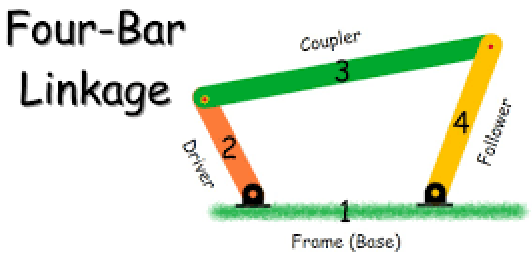
1. Four-Bar Linkage

- **Structure:** Four rigid links connected in a loop by four revolute joints.
- **Function:** Converts rotary motion into desired output; widely used in levers, pumps, suspension.
- **DOF Calculation:**
  - Links (N) = 4
  - Joints (J) = 4 (all revolute joints)

- Using Grübler's formula for planar mechanisms:

$$M = 3(N - 1) - 2J = 3(4 - 1) - 2 \times 4 = 9 - 8 = 1$$

- **Result:** 1 DOF; one input controls the motion.

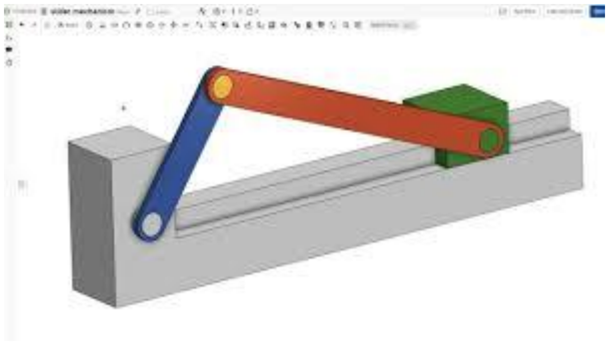


2. Slider-Crank Mechanism

- **Structure:** Similar to four-bar, but one link moves in linear sliding motion (prismatic joint).
- **Function:** Converts rotary motion into linear motion or vice versa; key in engines.
- **DOF Calculation:**
  - Links (N) = 4
  - Joints (J) = 4 (3 revolute + 1 prismatic)
  - Applying Grübler's formula:

$$M = 3(4 - 1) - 2 \times 4 = 1$$

  - Same as four-bar, 1 DOF.

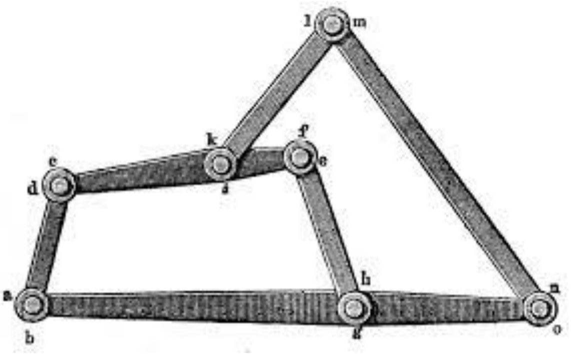


3. Six-Bar Linkage

- **Structure:** Six links connected by revolute joints; more complex motion than four-bar.
- **DOF Calculation:**
    - Links (N) = 6
    - Joints (J) = 7
    - DOF:

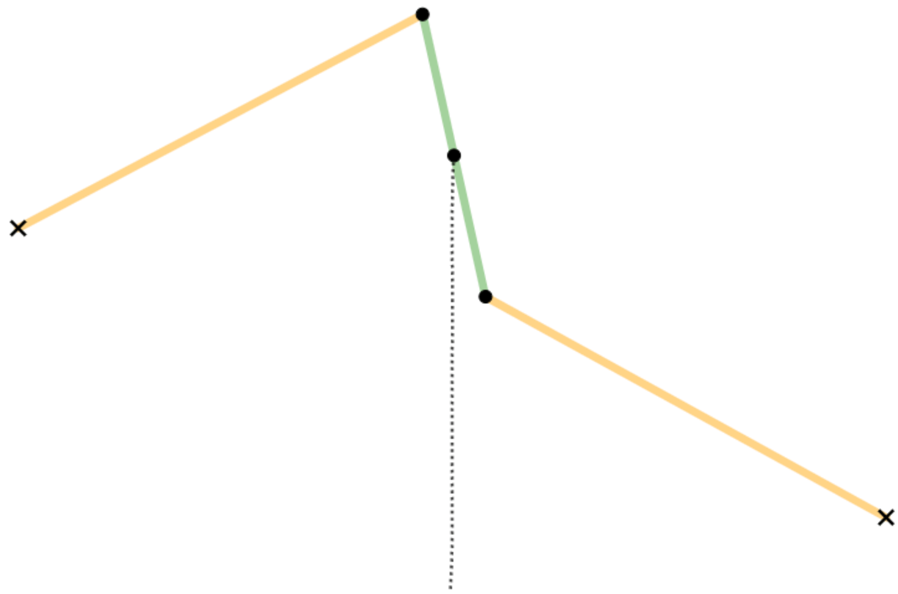
$$M = 3(6 - 1) - 2 \times 7 = 15 - 14 = 1$$

- Generally designed for 1 DOF.



4. Watt Mechanism

- It is a specialized **four-bar linkage** designed to approximate straight-line motion.
- DOF is same as 4-bar linkage & **1 DOF**.



5. **Whitworth Quick Return Mechanism**

The Whitworth Quick Return Mechanism is designed to convert rotary motion into oscillating reciprocating motion where the **return stroke is faster than the forward stroke**. This reduces non-cutting time in shaping and slotting machines, improving productivity.

- DOF : 1

6. Elliptical Trammel

An elliptical trammel is a mechanical linkage that converts rotary motion into a precise **elliptical motion**. It consists of two perpendicular sliding bars fixed in a cross shape and a rod attached sliding along both.

DOF: 1

7. Geneva Mechanism

Also called the Geneva drive, this mechanism converts continuous rotary motion into **intermittent rotary motion**. It consists of a drive wheel with a pin that engages slots on the driven wheel to rotate it in steps.

- DOF:

BONUS TASK:

STEWART PLATFORM

A Stewart platform is a parallel manipulator that uses six independently actuated legs to connect a base plate to a mobile top plate, allowing for precise six-degrees-of-freedom movement (three translational and three rotational). It is used for applications requiring high precision, such as flight simulators, medical systems, and motion control, where adjusting the length of the legs controls the position and orientation of the top plate.

Components and operation

- **Base plate:** A fixed platform that is mounted to the ground or a stable surface.
- **Top plate:** A rigid body, or mobile plate, that is positioned and oriented by the legs.
- **Actuated legs:** Six legs, typically hydraulic or electric linear actuators, connect the base and top plates, often using universal joints at both ends.
- **Control:** The lengths of the legs are adjusted by the actuators to control the position and orientation of the top plate in six degrees of freedom.

Key features

- **Parallel mechanism:** The legs are connected in parallel, which provides high rigidity and precision.
- **Six degrees of freedom (6-DOF):** The platform can move in X, Y, and Z directions and rotate around X, Y, and Z axes (roll, pitch, yaw).
- **High payload capacity:** Stewart platforms can handle heavy loads while maintaining high precision, making them ideal for applications like flight simulators.

Applications

- Flight and driving simulators
- Motion control systems
- Vibration and oscillation testing
- Medical equipment (e.g., surgical robotics)
- Satellite and antenna orientation

- Machining and precision positioning