Effect of Multiple Predators on Particle-Based Predator-Prey Interactions

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***Abstract*—Swarm robotics is an emerging field that aims to develop autonomous systems capable of performing complex tasks in uncertain environments in a distributed manner. Understanding the interactions between agents in a swarm and the environment the agent is in is critical for designing effective swarm robotics systems. Many academics turn to nature for novel ideas, and the field of swarm robotics is no exception. Swarming behaviors can be seen in many different corners of the Animal Kingdom, from sheep herding, to schools of fish. In this paper, we investigate the effect of predator-prey interactions on swarm robotics behavior through simulation, which allows for a controlled study of different parameters that would be difficult to replicate in real-world scenarios. By performing a comprehensive mathematical analysis on our simulation results, we are able to generate phase diagrams that describe the emerging behaviors in discrete regions that describe the interactions between multiple predators and prey.**

I. Introduction

Swarm robotics has emerged as a promising field for designing autonomous systems capable of performing complex tasks in uncertain and dynamic environments [1], [2]. A key feature of swarm robotics is the ability of multiple robots to work together in a coordinated manner, resulting in emergent behavior that can be adapted to a wide range of tasks. Understanding the interactions between individuals in a swarm and the ever-changing environment is critical for designing efficient and robust swarm robotics systems. One of the most intriguing aspects of swarm behavior is the predator-prey relationship, which has been widely studied in nature, such as in schools of fish, animal stampedes, herding sheep, or foraging ants as can be seen in Figure 1 [3]–[6]. Many fields look towards nature and the animal kingdom to help solve problems [7]. According to the Biomimicry Institute, "[b]iomimicry is a practice that learns from and mimics the strategies found in nature to solve human design challenges." [8]. By embracing the lessons nature has to offer (biomimicry), we can accelerate our journey toward achieving greater efficiency and innovation in robotics. Turning to nature, helps us learn about efficient hunting, foraging, and evasion strategies that improve the performance of swarm robotic systems.

In this paper, we investigate the impact of various predator-prey interactions with multiple predators on swarm robotics behavior, building upon the work of Chen and Kolokolnikov who explored the impact of single predator-prey interaction strategies [9]. Drawing inspiration from O'Keeffe et al., we analyze our data visually to help us understand the threshold at which behaviors change within swarms of predators and prey [10]. By exploring the effects of multiple predators on swarm behavior, we hope to better understand the mechanics that govern swarm behavior, and produce a model that can accurately describe the behaviors of a swarm with given governing parameters.

Fig. 1. Sheepdogs herding in groups

II. Related Work

Predator-prey interactions are fundamental to the dynamics of many ecological systems, ranging from terrestrial ecosystems to marine environments. Understanding the behavior of predator-prey interactions in groups or swarms, where prey animals exhibit collective behaviors, has garnered increasing attention in recent years. In particular, the work by Chen and Kolokolnikov [11] has provided insights into the dynamics of such interactions using a simple particle-based mathematical model.

Chen and Kolokolnikov initially focused on studying the effects of a single predator on a swarm of prey in their minimal model. They considered a scenario where a single predator interacts with a swarm of prey that move collectively according to alignment, cohesion, and repulsion rules [12]. The prey particles are modeled as a particle (zero physical size) that exhibit flocking behavior, aligning their velocities with their neighbors, maintaining cohesion with nearby particles, and avoiding collisions with each other.

In this paper, we extended this work by introducing multiple predators. The interactions between multiple predators can play a crucial role in shaping the dynamics of the predator-prey system. For example, in a recent study by Ordaz-Rivas et al. [13], a flock of robots with self-cooperation was used to investigate the predator-prey dynamics. In their work, the robots were programmed to exhibit collective behaviors, similar to the flocking behavior of prey particles in Chen and Kolokolnikov's model. The robots cooperatively herded virtual prey particles towards a designated goal area while avoiding obstacles.

The study by Ordaz-Rivas et al. demonstrated that the coordination among multiple predators can significantly impact the efficiency and effectiveness of the predator's behavior [13]. The predators were able to effectively herd the prey by coordinating their movements, and the presence of multiple predators improved the overall performance (capture time and area covered) of the predator-prey system. This highlights the importance of considering interactions among multiple predators in predator-prey systems, as it can lead to emergent behaviors and impact the dynamics of the system.

A previously studied phenomenon is that of predator confusion, which is when a predator is 'confused' as to which individual prey to pursue, as mentioned in [9]. There are also examples in nature and several other papers that study this behavior [14]–[17]. In examples where the predator is confused, the predator oscillates within the flock. In nature a single lion would not be able to catch its prey as it is not fast enough to catch any member of the flock, however with the introduction of many equally fast lions they are suddenly able to catch their prey. In our research, we study the threshold when predator confusion changes into different behaviors.

Another behavior that may be observed in nature is that of collective hunting. Pitman et. al investigated cooperative hunting behaviors by killer whales [18]. They observed that the whales traveled differently depending on their environment, specifically dependent on the presence of ice. In open water, they travelled in a fairly tight group, while in areas of packed ice, they travel as individuals while 'spying' for seals (prey) on the ice.

One way to analyze data is to observe the changes in behaviors as you test different variables, as was done in the work by O'Keeffe et al. [10]. Their analysis resulted in a phase diagram with discrete regions of observed behaviors.

Drawing inspiration from the study by Ordaz-Rivas et al. [13], our investigation will extend the work of Chen and Kolokolnikov by exploring how the coordination of multiple predators can affect the behavior of prey particles in a swarm robotics system. We will examine how introducing multiple predators into a prey swarm, and introducing repulsion and attraction laws between predators affect coordination strategies and the collective behavior of the prey swarm. Drawing inspiration from the mathematical methodology of O'Keeffe et al. [10], we propose to analyze our data by generating heat maps and phase diagrams. By seamlessly integrating insights from the work of Chen and Kolokolnikov [9] with the findings of Ordaz-Rivas et al. [13], our study broadens the understanding of predator-prey interactions within simulations by introducing multiple predators and interactions between said predators. This introduction creates a better understanding of the intricate dynamics characterizing predator-prey interactions. This endeavor not only seeks to expand the current knowledge base but also aims to make substantial contributions to the refinement of swarm robotics systems tailored for efficient predator-prey tasks.

III. Methodology

A. Single Predator Model

In our investigation we will be adapting the particle-based model used by Chen and Kolokolnikov [9] to account for multiple predators. Chen and Kolokolnikov's initial model [9] consisted of the following equations to determine the velocity of each predator and prey respectively:

$$\frac{dx_i}{dt} = F_{prey-prey} + F_{prey-predator} \tag{1}$$

$$\frac{dz}{dt} = F_{predator-prey} \tag{2}$$

where $x_i \in \mathrm{R}^2$ represents the 2D position of the i^{th} prey, and $z \in \mathrm{R}^2$ represents the 2D position of the predator. With these models, the prey's velocity is the sum of two components, the average force between this prey and every other prey in the swarm $F_{prey-prey}$, and the force between this prey and the predator $F_{prey-predator}$. Chen and Kolokolnikov modeled the forces between x_i and any other prey x_n using a Newtonian-type Short-range repulsion in the form $\frac{x_i - x_n}{|x_i - x_n|^2}$ and linear long range attraction; $a(x_i - x_n)$ with the scalar a allowing for tuning of the repulsion-attraction ratio. By averaging these forced among all the prey in the swarm the resulting $F_{prey-prey}$ can be calculated as:

$$F_{prey-prey} = \frac{1}{N} \sum_{n=1, n \neq i}^{N} \left(\frac{x_i - x_n}{|x_i - x_n|^2} - a(x_i - x_n) \right) \tag{3}$$

Additionally, $F_{prey-predator}$ was similarly modeled using Newtonian-type short range repulsion in the form $b\frac{x_i - z}{|x_i - z|^2}$ where b allows for the scaling of this force. To calculate the velocity of the predator $\frac{dz}{dt}$ Chen and Kolokolnikov decided to use a short-range attraction in the form $\frac{x_n - z}{|x_n - z|^p}$ where p turns each individual interaction into a power law, which decays at large distances. This results in the following model for prey and predators respectively.

$$\frac{dx_i}{dt} = \frac{1}{N} \sum_{n=1, n \neq i}^{N} \left(\frac{x_i - x_n}{|x_i - x_n|^2} - a(x_i - x_n) \right) + b\frac{x_i - z}{|x_i - z|^2} \tag{4}$$

$$\frac{dz}{dt} = \frac{c}{N} \sum_{n=1}^{N} \frac{x_n - z}{|x_n - z|^p} \tag{5}$$

B. Multiple Predator Models

To expand Chen and Kolokolnikov's model to include multiple predators, we adapted $F_{prey-predator}$ to calculate the resulting force as an average of all predators. We maintained the Newtonian-type short range repulsion in the form

$$\frac{1}{M} \sum_{m=1}^{M} b\frac{x_i - z_m}{|x_i - z_m|^2} \tag{6}$$

This resulted in the following formula for calculating each prey and predator velocities:

$$\frac{dx_i}{dt} = \frac{1}{N}\sum_{n=1,n\neq i}^{N}\left(\frac{x_i - x_n}{|x_i - x_n|^2} - a(x_i - x_n)\right) + \frac{b}{M}\sum_{m=1}^{M}\frac{x_i - z_m}{|x_i - z_m|^2} \tag{7}$$

$$\frac{dz_j}{dt} = \frac{c}{N}\sum_{n=1}^{N}\frac{x_n - z_j}{|x_n - z_j|^p} \tag{8}$$

As many predators do not act alone while hunting flocking prey we decided to further introduce interactions between predators. As a result we updated (8) to include a $F_{predator-predator}$ term:

$$\frac{dz}{dt} = F_{predator-prey} + F_{predator-predator} \tag{9}$$

When developing the $F_{predator-predator}$ term, we introduced two components. First we integrated another Newtonian-type short-range repulsion between z_j and z_m (current predator and another predator) in the form $d\frac{z_j - z_m}{|z_j - z_m|^2}$. Second we introduced a linear long range attraction in the form $e(z_j - z_m)$. These components were introduced so predators do not overlap with one another and reduce their effective coverage of the swarm, while still maintaining an attraction to one another allowing them to achieve coordinated motions. We also introduced d and e parameters to scale these repulsive and attractive forces respectively. As a result our model for multiple predators with interactions is as follows:

$$\frac{dx_i}{dt} = \frac{1}{N}\sum_{n=1,n\neq i}^{N}\left(\frac{x_i - x_n}{|x_i - x_n|^2} - a(x_i - x_n)\right) + \frac{b}{M}\sum_{m=1}^{M}\frac{x_i - z_m}{|x_i - z_m|^2} \tag{10}$$

$$\frac{dz_j}{dt} = \frac{c}{N}\sum_{n=1}^{N}\frac{x_n - z_j}{|x_n - z_j|^p} + \frac{1}{M}\sum_{m=1}^{M}\left(d\frac{z_j - z_m}{|z_j - z_m|^2} + e(z_j - z_m)\right) \tag{11}$$

C. Model Integration and Behavior

To collect data on the behavior of swarms with multiple predators we implemented the provided equations in Python using Numpy and Matplotlib. Figure 2 shows an example of a simulation with three predators at a given time step. See Appendix A for the source code.

While verifying the implementation of these formulas we observed several interesting behaviors as shown in Figure 3. Notably as in Chen and Kolokolnikov's work we found that depending on the model's parameters, the behavior would fall into 3 categories including, prey escaping, predator confusion, and chaotic behaviors. When predators are not able to move fast enough, the prey escape their short range attraction and successfully evade capture. Alternatively if the predators can move too fast then their motion becomes chaotic. This chaotic motion can be partially attributed to the $F_{predator-prey}$ power law. The power law causes long-range decay of the forces, but also causes short-range explosion of forces. In between these two states, the predators exhibit certain amounts of confusion, however we saw 3 distinct forms of confusion. First, the predators could reach a stable state where all forces acting upon them cancel out and they cease to move or chase any prey. Second, the predators could fall into unstable oscillations where they move in smaller circles around various areas of the flock. Lastly, when $d > e$, we saw significantly more circling behavior. We define circling behavior as when the velocities of each predator end up tangent to the edge of the flock, resulting in a circling motion around the centerpoint of the flock (see Figure 8).

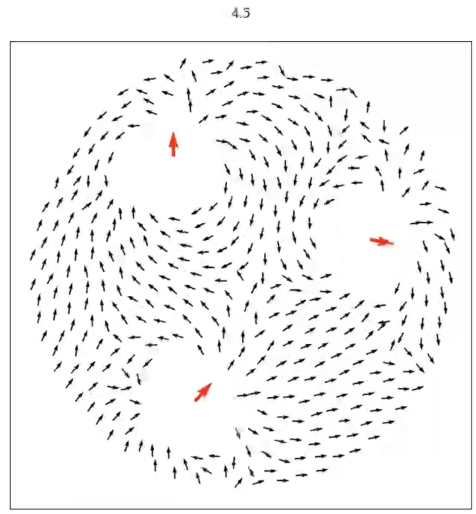

Fig. 2. Sample Visualization

D. Data Collection Methods

In our experiments we wanted to find two metrics of the predators'. Firstly we wanted to measure the predators effectiveness by implementing a kill radius r. This served as a zone where, if any prey fell within the specified radius of any predator, then the predator swarm was labeled as successfully catching prey. Secondly, after simulation, we manually classified each model's behavior, as can be seen in Figure 3.

To gather this data we randomly initialized N prey and M predators in an evenly distributed circle about the origin of the field with the same random seed. This ensured the starting position of predators and prey remained consistent between simulations and any behaviors were reproducible. Next we ran each model for a specified number of time steps T, recorded whether any predators caught a prey, and exported the visualization of the model to a video file. This allowed us to observe behaviors after running the model once and return to any models for further analysis.

While running our experiments we found that running models with larger numbers of prey took exceedingly long to run and export. As a result we used Numba to pre-compile the models into optimized machine code to expedite the collection

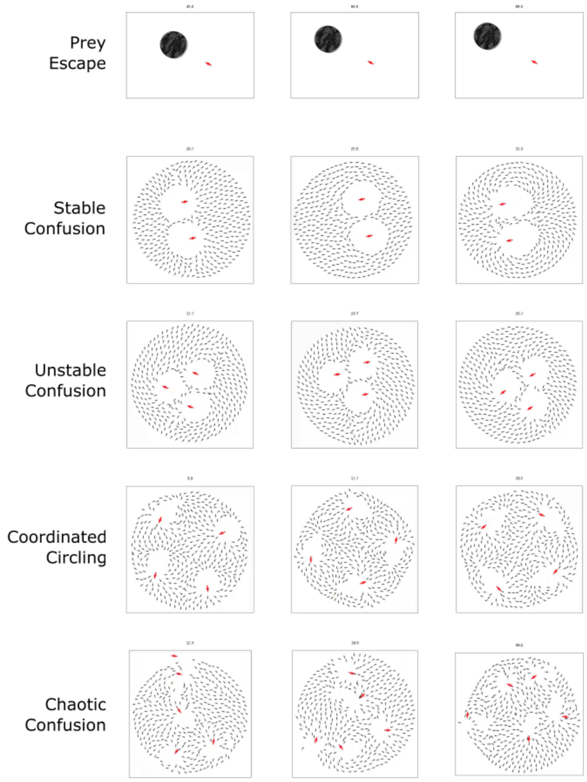

Fig. 3. Observed behaviors characterization

of data. Additionally we developed a pipeline that allowed simulation frames to be directly sent in argb form to hardware encoders on a computer's CPU/GPU using ffmpeg, reducing the export time of videos to 10% of Matplotlib's default implementation. This allowed us to investigate more variables and effects in the given time.

After exporting these videos, we watched every simulation and manually labeled the observed behaviors (by utilizing the examples outlined in Figure 3 and recorded the results in CSV files for later parsing and visualization. Once the data was amassed we further used Matplotlib to generate heatmaps of catching success and behaviors as discussed in later sections. Our data sets can be found in the Github linked in III-C.

IV. Experiments

We simulated several thousand experiments grouping into three main categories. First, we replicated the work of Chen and Kolokolnikov in order to evaluate the performance of our model and to make sure it worked correctly. Second, we introduced multiple predators into the environment without any interaction between one another. Lastly, we introduced multiple predators with interaction between them, same as the interaction between prey. This allowed us to simulate prey that wanted to work together and maintain some distance between one another.

A. Replication of Previous Work

We first wanted to replicate the work of Chen and Kolokolnikov, as it would be a good starting point to make sure that our simulation was going to be an accurate comparison. For this experiment, we used equations (4) and (5). We set the parameters to $N = 400$, $a = 1$, $b = 0.2$, $p = 3$, and $c \in [0.15, 0.4, 0.8, 1.5, 2.5]$.

Figure 4 shows the results that we obtained from our simulation using the same parameters from Chen and Kolokolnikov [9]. For simplification, we reduced the number of intermediate steps, but we were able to closely replicate the previous work. In the first row, $c = 0.15$, the swarm escapes completely. In the second row, $c = 0.4$, the predator catches up with the swarm but gets 'confused' as the swarm forms a stable ring around it. In the third row, $c = 0.8$, the predator catches up with the swarm; the swarm forms an unstable ring around it where regular oscillations are observed. In the fourth row, $c = 1.5$, the predator did not catch up with prey but complex periodic patterns can be seen. In the fifth row, $c = 2.5$, the predator 'catches' the prey and chaotic behavior is observed.

B. Multiple Predators without Predator-Predator Interaction

The first extension to the previous work that we wanted to investigate was the effect of introducing multiple predators to the simulation. We first introduced more predators without predator-predator interactions. The main behavior we wanted to investigate was whether a greater number of 'slower' predators can catch a prey. In other words, we explored, for a same c value (short-range attraction gain), whether introducing more predators led to catching prey.

For this experiment, we used the equations (7) and (8). We set the constant parameters to $N = 400$, $a = 1$, $b = 0.2$, $p = 3$, $r = 0.025$. The variable parameters were combinations of $c \in [0.15, 0.4, 0.8, 1.5, 2.5]$ and $M \in [1, 2, 3, 4, 5]$.

In Figure 5, we observe the catching behavior for multiple predators without predator-predator interactions and for different c gains. For one predator, there was only a prey catch at $c = 2.5$. For 2-4 predators, there were catches for $c = 0.8, 1.5, 2.5$. For 5 predators, there were catches for all c values except for $c = 0.15$. This shows that a greater number of predators does in fact lead to more catches for the same short-range attraction gain and with no interactions between the predators. This can be explained by analyzing the flocking behavior of multiple predators with no interaction in Figure 6. We observe that higher number of predators show more unstable confusion and chaotic confusion for different values of c. Both of these behaviors, chaotic confusion in particular, lead to more prey catches.

C. Multiple Predators with Predator-Predator Interaction

The next extension that we investigated was the effect of multiple predators with predator-predator interactions. The main aspect we wanted to investigate was emergent behaviors from forces within the predator swarm.

We ran 256 experiments for this investigation and used equations (7) and (11). The constant parameters were set to

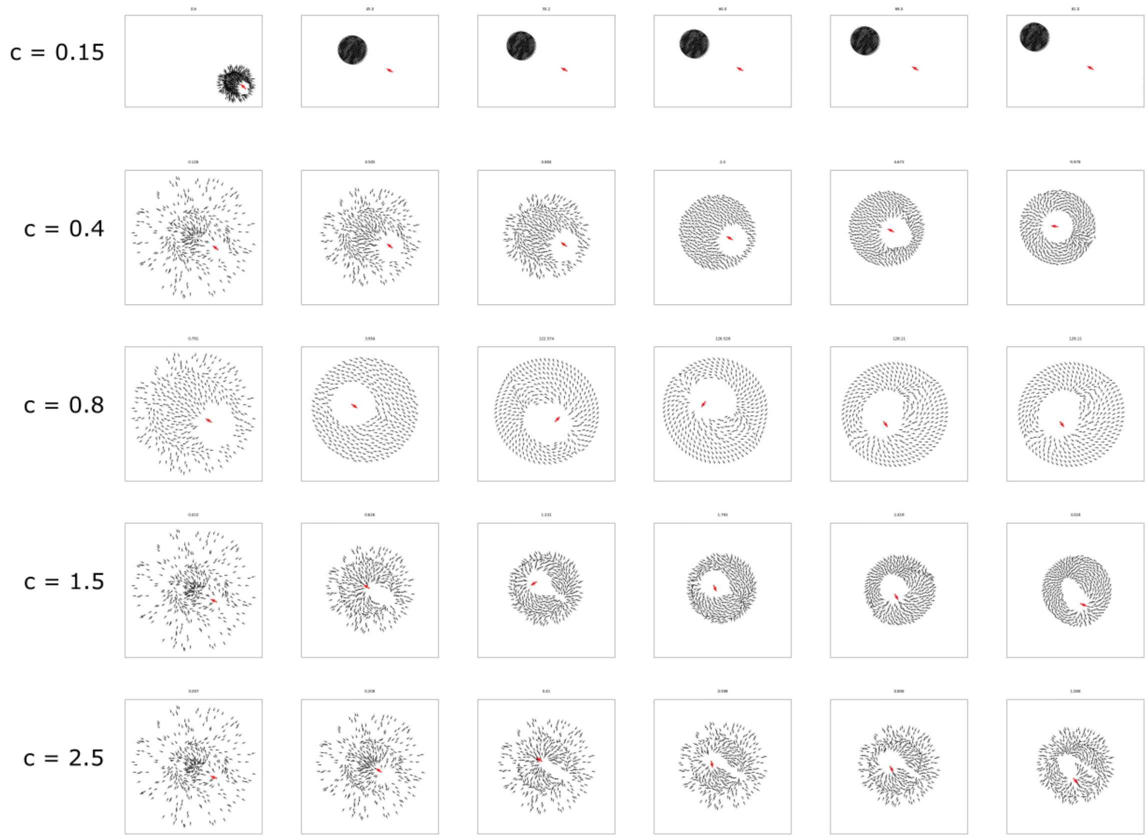

Fig. 4. Our replication of previous work by Chen and Kolokolnikov.

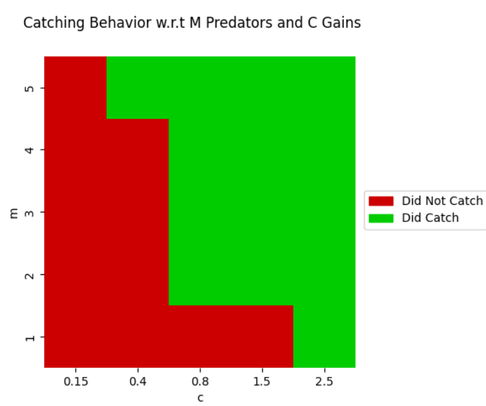

Fig. 5. Catching behavior for multiple predators without predator-predator interactions and with different c gains.

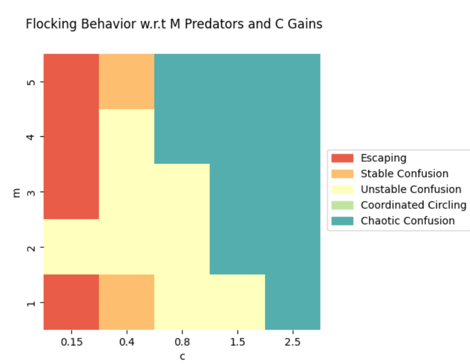

Fig. 6. Flocking behavior for multiple predators without predator-predator interactions and with different c gains.

$N = 400$, $a = 1$, $b = 0.2$, $p = 3$, $r = 0.005$. In this experiment (Equation 11), the main 'tuning' gains are c, d, and e. Hence, the parameters under study are combinations of $M \in [1, 2, 3, 4, 5]$, $c \in [0.1, 0.2, ..., 0.8]$, $d \in [1.1, 1.0, ..., 0.4]$, and $e \in 1.5 - d$.

Figure 7 illustrates how different combinations of d and e for multiple predators with different c values does not necessarily lead to more prey catches. All of the plots look very similar. However, the resulting behaviors from the different combinations of d and e clearly change, as shown in Figure 8. The main observation for the resulting behaviors is how, for combinations of higher d values and lower e values, the dominating behavior is coordinated circling. For lower d values and higher e values, the dominating behavior is unstable confusion, though we do see more stable confusion as well but no coordinated circling. d and e values that are closer to each

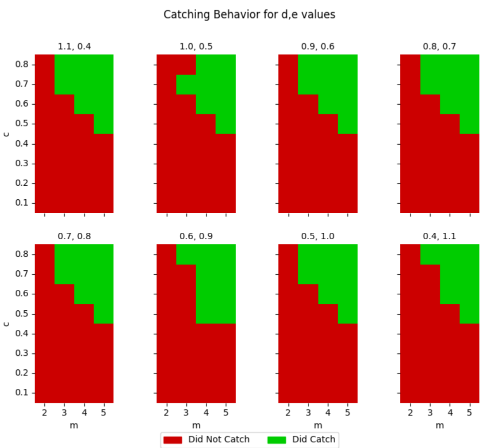

Fig. 7. Catching behavior for multiple predators with predator-predator interactions and different c gains.

other result in a more balanced mix of all behaviors.

Although for different ratios of d and e values there is a clear difference in flocking behaviors, the catching behavior across these experiments was very similar.

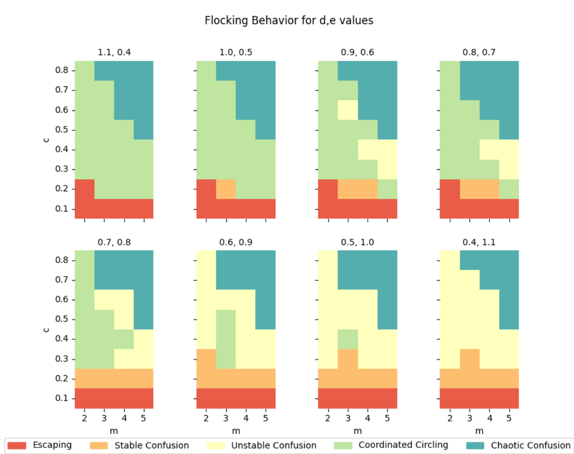

Fig. 8. Flocking behavior for multiple predators with predator-predator interactions and different c gains.

D. Behavioral Classification by Velocity

In order to gain a comprehensive understanding of the predator-prey dynamics within the experiment, a systematic approach was taken to examine the behavior of randomly selected prey. The absolute velocity, represented by the norm of the velocity vector, and the angular position of a random prey were tracked throughout the experiment. The resulting graphical representations, as illustrated in Figure 9, serve as invaluable visualizations that encapsulate the diverse range of behaviors observed in the previous section.

One noteworthy observation is the correlation between specific behaviors and distinct patterns evident in the graphs. For instance, during instances of escape behavior, a discernible similarity emerges in the shape of the velocity graph, mirroring the characteristics depicted in Figure 9. This consistent pattern not only reaffirms the accuracy of the behavioral categorization but also highlights the potential for leveraging these graphical representations as diagnostic tools for identifying and understanding specific predator-prey interactions. The ability to categorize behaviors through the analysis of velocity and angular position graphs not only facilitates a qualitative understanding but also lays the groundwork for more profound characterization and extensive mathematical analysis.

V. Conclusions

Our work showed that the introduction of additional predators within an environment can change the behaviors of the swarm drastically. Even changing the forces between agents can have a drastic impact on the behaviors of the swarm. The study of predator-prey interactions within simulation for swarm robotics provides valuable insights into adaptability and emergent behaviors of predator-prey swarms. Balancing adaptability and robustness is crucial for swarm systems, allowing them to effectively navigate changing environments. While challenges exist in translating simulations to physical implementations, this research field has the potential to advance our understanding of collective behavior and contribute to the development of advanced robotic systems.

A. Our Contribution

As outlined in section IV, we were able to determine that the introduction of multiple predators meant that it was easier for predators to get a kill, as well as altered their behaviors when changing the gains of Equation 9. We believe that we created a more accurate particle-physics based mathematical model that can be used to simulate swarm interactions of predators and prey, and better models that are used for robotic hearding of animals [19]. Utilizing mathematical analysis outlined by O'Keeffe [10] and by graphing the velocity and orientation, we were able to generate phase diagrams that help describe the boundaries between emerging behaviors, and generate graphs that show how the velocities and behaviors are closely related.

B. Future Work

Though our work was short, we took some time to discuss our thoughts on future work that could be implemented into this model, or features that could used to ease the analysis process. First, using some form of machine learning to identify the behaviors as seen in Figure 3 in a more efficient manner. This would allow for an automated process that would necessarily require humans to go through and characterize the behaviors, saving time, energy, and headaches. Work by Mattson et. al shows the viability of human-in-the-loop methods versus strictly automated processes [20]. Second, we think it would be more realistic to introduce 2^{nd} order equations of motion (acceleration) into the model. This would make the movements of the predators and prey more realistic. Third, we would like to analyze the *catch rate* of the different parameters. We want to use the kill radius, defined in Section III-C, to eliminate any prey in the zone, then define a metric as to how long

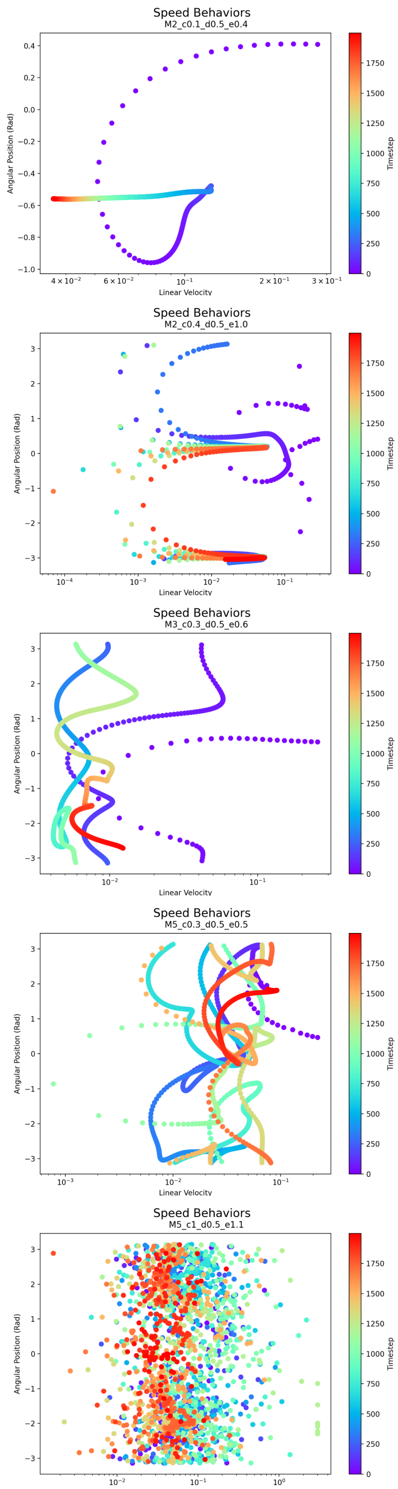

Fig. 9. Velocity behavior showing the classifications (from top to bottom): Escaping, Stable Confusion, Unstable Confusion, Coordinated Circling, and Chaotic Confusion

it takes to reduce the number of prey by a certain amount. This will allow us to qualitatively analyze the performance of the gains. Fourth, we want to perform differential analysis of the motion equations to determine steady state behaviors and stability of the system. Lastly, we think it would be interesting to investigate heterogeneous predators and prey, i.e. predators and prey with different maximum speeds/accelerations.

C. *Concluding Remarks*

Overall, our implementation of a particle-based mathematical model of predator-prey interactions within swarms has been instrumental in examining the intricate dynamics of these interactions. By manipulating the attractive and repulsive gains in our simulation, we have gained valuable insights into the nuanced behaviors exhibited by both predators and prey. Moreover, the introduction of additional predators has shed light on how the presence of multiple predators impacts the overall dynamics of the system. Our findings provided a deeper understanding of the underlying mechanisms at play in predator-prey interactions within swarm robotics.

We recognize that our current implementation is just the beginning of a broader journey in studying predator-prey interactions within swarm robotics. There are still many unanswered questions and unexplored aspects that offer exciting avenues for future investigation. In conclusion, our particle-based mathematical model of predator-prey interactions within swarms has provided a platform for in-depth analysis and understanding of the complex behaviors exhibited in these systems. The exploration of attractive and repulsive gains, as well as the introduction of multiple predators, has allowed us to unravel the dynamics of the interactions and gain valuable insights.

APPENDIX

A. *GitHub Repository*

Source code is available on GitHub.

B. *Dataset*

The video dataset can be found in this Google Drive.

REFERENCES

[1] M. Dorigo, G. Theraulaz, and V. Trianni, “Swarm robotics: Past, present, and future [point of view],” *Proceedings of the IEEE*, vol. 109, no. 7, pp. 1152–1165, 2021.

[2] Y. Zhang, D. wei Gong, and J. hua Zhang, “Robot path planning in uncertain environment using multi-objective particle swarm optimization,” *Neurocomputing*, vol. 103, pp. 172–185, 2013. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0925231212007722

[3] J. K. Parrish and L. Edelstein-Keshet, “Complexity, pattern, and evolutionary trade-offs in animal aggregation,” *Science*, vol. 284, no. 5411, pp. 99–101, 1999. [Online]. Available: https://www.science.org/doi/abs/10.1126/science.284.5411.99

[4] B. L. Penzhorn, “A long-term study of social organisation and behaviour of cape mountain zebras equus zebra zebra,” *Zeitschrift für Tierpsychologie*, vol. 64, no. 2, pp. 97–146, 1984. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1439-0310.1984.tb00355.x

[5] J. F. A. Traniello, “Foraging strategies of ants,” *Annual Review of Entomology*, vol. 34, no. 1, pp. 191–210, 1989. [Online]. Available: https://doi.org/10.1146/annurev.en.34.010189.001203

[6] A. Waters, F. Blanchette, and A. D. Kim, “Modeling huddling penguins,” *PLOS ONE*, vol. 7, no. 11, pp. 1–8, 11 2012. [Online]. Available: https://doi.org/10.1371/journal.pone.0050277

[7] E. Lurie-Luke, "Product and technology innovation: What can biomimicry inspire?" *Biotechnology Advances*, vol. 32, no. 8, pp. 1494–1505, 2014. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0734975014001517
[8] "Biomimicry institute," https://biomimicry.org/what-is-biomimicry/, accessed: 2023-11-18.
[9] Y. Chen and T. Kolokolnikov, "A minimal model of predator–swarm interactions," *Journal of The Royal Society Interface*, vol. 11, no. 94, p. 20131208, 2014. [Online]. Available: https://royalsocietypublishing.org/doi/abs/10.1098/rsif.2013.1208
[10] K. P. O'Keeffe, H. Hong, and S. H. Strogatz, "Oscillators that sync and swarm," *Nature Communications*, vol. 8, no. 1, p. 1504, Nov. 2017. [Online]. Available: https://doi.org/10.1038/s41467-017-01190-3
[11] D. Chakraborty, S. Bhunia, and R. De, "Survival chances of a prey swarm: how the cooperative interaction range affects the outcome," *Sci Rep*, vol. 10, no. 1, p. 8362, May 2020. [Online]. Available: https://doi.org/10.1038/s41598-020-64084-3
[12] C. W. Reynolds, *Flocks, Herds, and Schools: A Distributed Behavioral Model*. New York, NY, USA: Association for Computing Machinery, 1998, p. 273–282. [Online]. Available: https://doi.org/10.1145/280811.281008
[13] E. Ordaz-Rivas, A. Rodriguez-Liñan, and L. Torres-Treviño, "Flock of robots with self-cooperation for prey-predator task," vol. 101, no. 2, p. 39. [Online]. Available: https://doi.org/10.1007/s10846-020-01283-0
[14] R. S. Olson, A. Hintze, F. C. Dyer, D. B. Knoester, and C. Adami, "Predator confusion is sufficient to evolve swarming behaviour," vol. 10, no. 85, p. 20130305. [Online]. Available: https://royalsocietypublishing.org/doi/10.1098/rsif.2013.0305
[15] M. Milinski, "Can an experienced predator overcome the confusion of swarming prey more easily?" *Animal Behaviour*, vol. 27, pp. 1122–1126, 1979. [Online]. Available: https://www.sciencedirect.com/science/article/pii/0003347279900605
[16] J. M. Jeschke and R. Tollrian, "Effects of predator confusion on functional responses," *Oikos*, vol. 111, no. 3, pp. 547–555, 2005. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1600-0706.2005.14118.x
[17] J. M. Jeschke and R. Tollrian, "Prey swarming: which predators become confused and why?" *Animal Behaviour*, vol. 74, no. 3, pp. 387–393, 2007. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S000334720700187X
[18] R. L. Pitman and J. W. Durban, "Cooperative hunting behavior, prey selectivity and prey handling by pack ice killer whales (orcinus orca), type b, in antarctic peninsula waters," vol. 28, no. 1, pp. 16–36. [Online]. Available: https://onlinelibrary.wiley.com/doi/10.1111/j.1748-7692.2010.00453.x
[19] A. J. King, S. J. Portugal, D. Strömbom, R. P. Mann, J. A. Carrillo, D. Kalise, G. de Croon, H. Barnett, P. Scerri, R. Groß, D. R. Chadwick, and M. Papadopoulou, "Biologically inspired herding of animal groups by robots," *Methods in Ecology and Evolution*, vol. 14, no. 2, pp. 478–486, 2023. [Online]. Available: https://besjournals.onlinelibrary.wiley.com/doi/abs/10.1111/2041-210X.14049
[20] C. Mattson, J. C. Clark, and D. S. Brown, "Exploring behavior discovery methods for heterogeneous swarms of limited-capability robots," 2023. [Online]. Available: https://arxiv.org/abs/2310.16941