Mensuration

IMPORTANT FORMULAE:

Rectangle :

- Area = lb
- Perimeter = 2(l+b)

Square :

- Area = a×a
- Perimeter = 4a

Parallelogram:

- Area = l × h
- Perimeter = 2(l+b)

Triangle :

Area =b×h/2 or √s(s-a)(s-b)(s-c)................where s=a+b+c/2

Right angle Triangle :

- Area =1/2(bh)
- Perimeter = b+h+d

Isosceles right angle triangle :

- Area = ½. a2
- Perimeter = 2a+d.........where d=a√2

Equilateral Triangle :

- Area = $\sqrt{3}. a^2/4$ or $\frac{1}{2}(ah)$....where h = $\sqrt{3}/2$
- Perimeter = 3a

Trapezium :

- Area = $1/2h(a+b)$
- Perimeter = Sum of all sides

Rhombus :

- Area = $d1 \times d2/2$
- Perimeter = 4l

Mensuration

Quadrilateral:
Area =1/2 × Diagonal × (Sum of offsets)

Circle :
- Area = πr^2 or $\pi d^2/4$
- Circumference = $2\pi r$ or πd
- Area of sector of a circle = $(\theta \pi r^2)/360$

Sphere:
- Volume: $V = 4/3\ \pi r^3$
- Surface Area: $S = 4\pi r^2$

Hemisphere :

- Volume = $2/3 \pi r^3$
- Curved surface area(CSA) = $2 \pi r^2$
- Total surface area = TSA = $3 \pi r^2$

Right Circular Cylinder :

- Volume of Cylinder = $\pi r^2 h$
- Lateral Surface Area (LSA or CSA) = $2\pi r h$
- Total Surface Area = TSA = $2 \pi r (r + h)$
- Volume of hollow cylinder = $\pi r h(R^2 - r^2)$

Right Circular cone :

- Volume = $1/3 \pi r^2h$
- Curved surface area: CSA= $\pi r l$
- Total surface area = TSA = $\pi r(r + l)$

Some other Formula :

- Area of Pathway running across the middle of a rectangle = $w(l+b-w)$
- Perimeter of Pathway around a rectangle field = $2(l+b+4w)$
- Area of Pathway around a rectangle field $=2w(l+b+2w)$
- Perimeter of Pathway inside a rectangle field $=2(l+b-4w)$
- Area of Pathway inside a rectangle field $=2w(l+b-2w)$
- Area of four walls = $2h(l+b)$

- Circumradius of equilateral Triangle $R = a / \sqrt{3}$
- Inradius of equilateral Trangle $r = a / 2\sqrt{3}$

- Circumradius of Square $R = a / \sqrt{2}$
- Inradius of Square $r = a / 2\sqrt{2}$

CHAPTER 10

Mensuration

It is one of the easiest chapters, which contributes almost 6-8% problems in Quantitative Aptitude Section of CAT. Besides there are several other aptitude tests which include plethora of questions from this topic itself.

Therefore it is advised that those students who are not so good in other sections such as algebra or sort of logical questions they must emphasise on this chapter. Even the questions asked from this chapter are not as much complex as they are in Geometry.

10.1 Mensuration

Definition : Mensuration is a science of measurement of the lengths of lines, areas of surfaces and volumes of solids.

Planes : Planes are two dimensional i.e. these two dimensions are namely length and breadth. These occupy surface.

Solids : Solids are three dimensional, namely length, breadth and height. These occupy space.

Conversion of Important Units

1 km = 10 hectometre = 100 decametre
= 1000 metre = 10,000 decimetre
= 1,00,000 centimetre = 10,00,000 millimetre

1 hectare = 10,000 square metre

1 are = 100 square metre

1 square hectometre = 100 square decametre

1 square decametre = 100 square metre

1 square metre = 100 square decimetre

1 square decimetre = 100 square centimetre

1 square centimetre = 100 square millimetre

$\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$, $\sqrt{6} = 2.45$

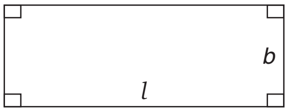
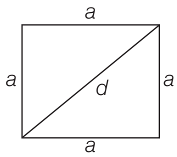
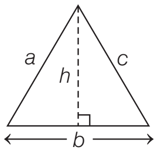
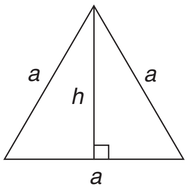
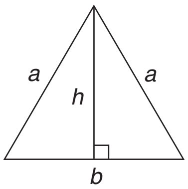
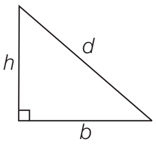
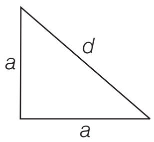
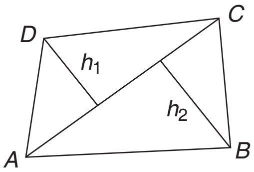
Weight = Volume × density

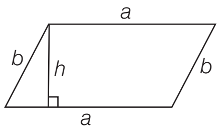
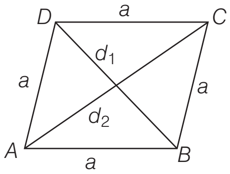
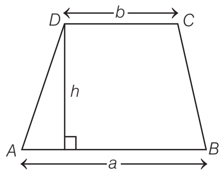
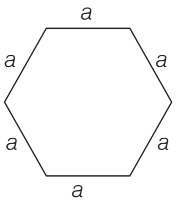
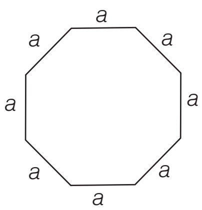
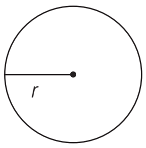
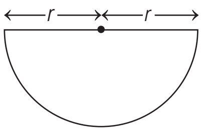
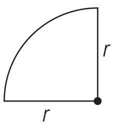
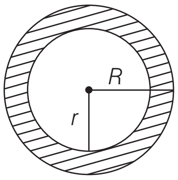
Chapter Checklist

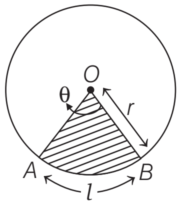
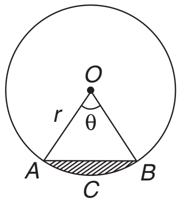
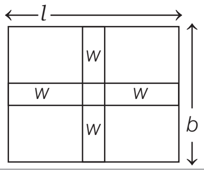
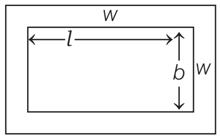
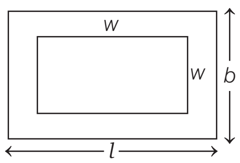
- Mensuration
- 2-D Figures (Planes)
- Rectangles and Squares
- Triangles
- Parallelogram, Rhombus and Trapezium
- Circles
- 3-D Figures (Solids)
- Cuboid and Cube
- Cylinder and Cone
- Sphere, Prism and Pyramid
- **CAT Test**

10.2 2-D Figures

Table 2-D Figures (Plane figures)

Name	Figure	Nomenclature	Area	Perimeter
Rectangle		$l \to$ length $b \to$ breadth	$l \times b = lb$	$2l + 2b = 2(l + b)$
Square		$a \to$ side $d \to$ diagonal $d = a\sqrt{2}$	(i) $a \times a = a^2$ (ii) $\frac{d^2}{2}$	$a + a + a + a = 4a$
Triangle (Scalene)		a, b and c are three sides of triangle and s the semiperimeter, where $s = \left(\frac{a + b + c}{2}\right)$ b is the base and h is the altitude of triangle	(i) $\frac{1}{2} \times b \times h$ (ii) $\sqrt{s(s-a)(s-b)(s-c)}$ (Hero's formula)	$a + b + c = 2s$
Equilateral triangle		$a \to$ side $h \to$ height or altitude $h = \frac{\sqrt{3}}{2} a$	(i) $\frac{1}{2} \times a \times h$ (ii) $\frac{\sqrt{3}}{4} a^2$	$3a$
Isosceles triangle		$a \to$ equal sides $b \to$ base $h \to$ height or altitude $h = \frac{\sqrt{4a^2 - b^2}}{2}$	(i) $\frac{1}{2} \times b \times h$ (ii) $\frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$	$2a + b$
Right angled triangle		$b \to$ base $h \to$ altitude/height $d \to$ diagonal $d = \sqrt{b^2 + h^2}$	$\frac{1}{2} \times b \times h$	$b + h + d$
Isosceles right angled triangle		$a \to$ equal sides $d \to$ diagonal $d = a\sqrt{2}$	$\frac{1}{2} a^2$	$2a + d$
Quadrilateral		AC is the diagonal and h_1, h_2 are the altitudes on AC, from the vertices D and B, respectively.	$\frac{1}{2} \times AC \times (h_1 + h_2)$	$AB + BC + CD + AD$

Name	Figure	Nomenclature	Area	Perimeter
Parallelogram		a and b are sides adjacent to each other. $h \to$ distance between the parallel sides	$a \times h$	$2(a+b)$
Rhombus		$a \to$ each equal side of rhombus d_1 and d_2 are the diagonals $d_1 \to BD$ $d_2 \to AC$	$\frac{1}{2} \times d_1 \times d_2$	$4a$
Trapezium		a and b are parallel sides to each other and h is the perpendicular distance between parallel sides	$\left(\frac{a+b}{2}\right) \times h$	$AB + BC + CD + AD$
Regular hexagon		$a \to$ each of the equal side	$\frac{3\sqrt{3}}{2} a^2$	$6a$
Regular octagon		$a \to$ each of equal side	$2a^2 (1 + \sqrt{2})$	$8a$
Circle		$r \to$ radius of the circle $\pi = \frac{22}{7} = 3.1416$ (approx)	πr^2	$2\pi r$ (called as circumference)
Semicircle		$r \to$ radius of the circle	$\frac{1}{2} \pi r^2$	$\pi r + 2r$
Quadrant		$r \to$ radius	$\frac{1}{4} \pi r^2$	$\frac{1}{2} \pi r + 2r$
Ring or circular path (shaded region)		$R \to$ outer radius $r \to$ inner radius	$\pi (R^2 - r^2)$	(outer) $\to 2\pi R$ (inner) $\to 2\pi r$

Name	Figure	Nomenclature	Area	Perimeter
Sector of a circle		$O \to$ centre of the circle $r \to$ radius $l \to$ length of the arc $\theta \to$ angle of the sector $l = 2\pi r\left(\dfrac{\theta}{360°}\right)$	(i) $\pi r^2\left(\dfrac{\theta}{360°}\right)$ (ii) $\dfrac{1}{2} r \times l$	$l + 2r$
Segment of a circle		$\theta \to$ angle of the sector $r \to$ radius $AB \to$ chord $ACB \to$ arc of the circle	Area of segment ACB (minor segment) $= r^2\left(\dfrac{\pi\theta}{360°} - \dfrac{\sin\theta}{2}\right)$	$2r\left[\dfrac{\pi\theta}{360°} + \sin\left(\dfrac{\theta}{2}\right)\right]$
Pathways running across the middle of a rectangle		$l \to$ length $b \to$ breadth $w \to$ width of the path (road)	$(l + b - w)\,w$	$2(l + b) - 4w$ $= 2[l + b - 2w]$
Outer pathways		$l \to$ length $b \to$ breadth $w \to$ widthness of the path	$(l + b + 2w)\,2w$	(inner) $\to 2\,(l + b)$ (outer) $\to 2\,(l + b + 4w)$
Inner path		$l \to$ length $b \to$ breadth $w \to$ widthness of the path	$(l + b - 2w)\,2w$	(outer) $\to 2\,(l + b)$ (inner) $\to 2\,(l + b - 4w)$

10.3 Rectangles and Squares

1. Area of a rectangle = length × breadth = $l \times b$
2. Area of a square = $(\text{side})^2 = \dfrac{1}{2}(\text{diagonal})^2$

$$= \frac{1}{2} d^2 = a^2$$

3. Diagonal of a rectangle

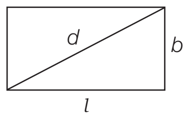
$$= \sqrt{(\text{length})^2 + (\text{breadth})^2} = \sqrt{l^2 + b^2}$$

4. Diagonal of a square = $\sqrt{\text{side}^2 + \text{side}^2} = \text{side}\sqrt{2} = a\sqrt{2}$
5. Perimeter of a rectangle $= 2$ (length + breadth) $= 2(l + b)$
6. Perimeter of a square $= 4 \times \text{side} = 4a$
7. Area of four walls of a room $= 2\,(l + b) \times h$

Exp. 1) The length and breadth of a rectangular room are 15 m and 8 m respectively :
(a) Find the perimeter of room.
(b) Find the area of the floor of room.
(c) Find the maximum possible length of the rod that can be put on the floor.

Solution
(a) Perimeter = 2(15 + 8) = 46 m
(b) Area of floor = 15 × 8 = 120 m^2
(c) Length of diagonal = $\sqrt{(15)^2 + (8)^2} = 17\,\text{m}$

NOTE The maximum possible length of any rod that can be placed on a rectangular floor is equal to the diagonal of the floor of the room.



Here $d > l$

Exp. 2) The side of a square shaped garden is 20 m. Find the :
(a) area of the garden
(b) perimeter (or boundary) of the garden
(c) maximum possible distance between any two corners of the garden.

Solution (a) Area = $(\text{side})^2 = (20)^2 = 400\ m^2$ (square metre)
(b) Perimeter = 4 × side = 4 × 20 = 80 m
(c) Diagonal = side $\sqrt{2} = 20 \times \sqrt{2} = 20 \times 1.414 = 28.28$ m